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THE ROLE OF TWITTER IN THE US FINANCIAL MARKET

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Dissertação apresentada à Escola de Economia de São Paulo como requisito à obtenção do título de Mestre em Economia

Área de concentração:
Economia, Econometria

Orientador:
Prof. Dr. Bruno Cara Giovannetti

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1 Introduction

The field of behavioral economics aims to better understand the behaviors of economic agents in many typical day-to-day situations - it is interesting how this field relates to psychology. In fact, pioneer researchers from psychology, such as Kahneman and Tversky, demonstrated interesting findings in the 1970's. They have shown in Tversky and Kahneman (1971) that people assume that a small sample is highly representative of the population and, therefore, make erroneous inference. In their future works, in Kahneman and Tversky (1973) and Tversky and Kahneman (1974), they discovered what is called *representativeness heuristic* which people under uncertainty tend to find familiar patterns to make, hence, biased, judgments based on that.

Surely, this findings could have interesting applications for investors, specially those who use news as their main source of information. Overconfident investor could overweight their private information about certain stocks with respect to public data. As demonstrated in Daniel, Hirshleifer and Subrahmanyam (1998), their belief update when the outcome is analyzed is asymmetrical, since outcomes that comply with their previous belief will increase their confidence, but those that do not will only decrease the confidence modestly.

Shiller (2000) argues that although the media vehicles - newspapers, magazines, TV, the Internet - are detached observers of the market, significant market events in the financial market, specially, will only occur if there is an synchrony among large groups of people and the media is the vehicle to spread it. He also points out that the media urges to attract audiences (as it is its inherent objective) and they may reinforce ideas that are not supported by evidence, create debates on a variety of topics that would otherwise not be as attractive, comment on market events with the so called specialists or celebrities - i.e. media might act as speculative propagator.

Besides the speculative property of the media, Tetlock (2007) found that the overall pessimism of a financial-related column of the *Wall Street Journal* has some predictive power on daily stock returns as well as on trading volume. Moreover, Engelberg and Parsons (2011) found the relationship between local news coverage and trading volumes on local brokerage accounts. It is also supported in García (2013) that people are more sensitive during recession and, therefore, news have more predictive power on such periods.

It is clear that the behaviorists literature found compelling evidence that economic agents are not fully rational. With the advent of the Internet, information is more accessible than ever before and such biases can be, at least, further studied. It is interesting to notice that we can easily identify trending topics and how fast they spread and we can now easily

quantify it. Asur and Huberman (2010) shown that box-offices revenues predictions based on Twitter chatter outperform traditional market models. Choi and Varian (2012) studied how Google Trends, the trending keywords searched on Google's search engine, can help to *predict the present*, or *nowcasting*. They have shown that the search trends can help predict automobile sales, unemployment claims, travel destination planing and consumer confidence. Bollen, Mao and Pepe (2011) could construct a sentiment index based on Twitter posts and how the audience respond to economical, political and cultural events.

This work revisits, in some sense, the findings of the traditional literature on investors behavior and its relationship to the news in the Modern Era of social media. The main goal of this work is to identify if the overall sentiment of Twitter users has some relationship with the financial market, such as returns, trading volume and volatility, or if the users comments are solely backward-looking and reflects the overall market events.

To construct a sentiment index from Twitter corpora, the General Inquirer software, a computer-assisted approach to analyze text content proposed by Stone et al. (1962), took care of semantic categorization of each word of each tweet, from March 2007 up to October 2017. Then, Principal Component Analysis (PCA) was carried out in order to extract the latent state of the content. The first principal component is interpreted as measure of overall engagement whilst the second is related to optimism, together they are responsible to 72.4% of the total variance. To relate them with the financial market of the United States, we collected data from the S&P500 index - which is composed by the 500 largest companies listed on the main stock exchanges and it is often referred as the gauge of the US Economy - and the VIX index, a volatility index that serve as a proxy for the volatility of the S&P500 index.

We estimate the effects of sentiment and financial market variables on each other using a Vector Autoregressive (VAR) framework. Such model enables all the variables to be seen as endogenous and estimate the effects on each other separately. The variables used were Engagement Factor (scores from the first principal component), Optimism Factor (score from the second principal component), excess of returns of the S&P500 with respect to the 3-month Treasury Bill yield, on daily frequency, the trading volume of the S&P500 volume and the VIX index. We found that the impacts of the market changes are higher on the sentiment index than the opposite. This strongly supports that the social media reflects market events as opposed to be the drivers of them.

It is also true that the S&P500 index is composed by big companies that may be more resilient on the speculative nature of social media in the sense that social media may not bring new information that could fundamentally change the track of those companies stock prices. In that way, we could not find enough evidence that the sentiment on Twitter could drive or predict prices of such companies. On future works, however, it would be interesting to study the relationship between sentiment and smaller companies.

This work is organized as follows: chapter 2 reviews the behavioral economics literature as well as related works on media and financial market; chapter 3 describes the data collection method and descriptive statistics of them; chapter 4 provides the data preparation and the model used; chapter 5 contains an analysis of the results obtained; chapter 6 proposes a final reflection and grounds for extensions of this work. A brief theory review on PCA and Time series can be found in Appendix A and Appendix B, respectively. All the analysis and modelling were done in open source tools such as Python and R programming languages and source codes can be found in <https://github.com/fernandoramacciotti/fgv-mpe-dissertacao>.

2 Literature Review

Investors have always wanted to understand better how the financial markets move and, if possible, predict the next move so that they can build their optimal portfolio. Since the investor plays a essential role when it comes to investing, the area of behavioral finance has been trying to correlate investors sentiment and behavior to the financial market. A great summary of behavioral economics and its application can be found in DellaVigna (2007).

2.1 Psychology

Psychological findings play an important role in behavioral economics. The so called *Law of small numbers* was first shown by Tversky and Kahneman (1971) - people tend to assume that a small random sample drawn from a population is highly representative. Therefore, when making prediction under uncertainty this erroneous inference can lead to systematic errors. They also showed that people tend to overweight recent and salient piece of information with respect to prior beliefs. Also, under uncertainty, people tend to make judgments by looking at familiar patterns being confident about its high representativeness in the population - an anomaly called *representativeness heuristic* in Kahneman and Tversky (1973), Tversky and Kahneman (1974).

A formal approach, using Bayesian inference, to model the inference by the law of small numbers was presented in Rabin (2002). It is assumed that each information that a person receives, call it a *signal*, is drawn from a urn of $N < \infty$ elements without replacement. Therefore, the smaller N , the more believer in the law of small number the person is. An example of how this works is the classical coin flip experiment. For instance, a person might prior believes that if a given coin is fair, then each pair of flips the number of heads must be equal to the number of tails. So, if he observes two heads in a row he might believe that the coin is biased. The same framework would apply for anything else, say security analysts forecasts, stock prices changes and so on.

Investor psychology is modeled on psychological grounds in Barberis, Shleifer and Vishny (1998). They assume that the earnings of an asset follows a random walk, but the investor does not know it and. Rather, he believes they switch between two possible states: mean-reversion and trend. On top of it, the transition rules, or probabilities, are fixed in his mind: for any given period, firm's earnings are inertial, i.e. they tend to remain in their current state than switching. So, the investor look at firm's earning at each period and updates his belief about the state in a Bayesian fashion. *Conservatism* and *representativeness heuristics*, which are psychological findings, both play roles while

updating the investor beliefs. The first suggests that people are slow to change their opinions and it takes some time, and evidence, to an investor change their belief of which state the firm's earnings are in. The latter helps to explain that people tend to see familiar patterns where they might not exist. In this case, if the earnings follow a truly random walk, a sequence of positive earnings might happen only due to chance, but for the investor it might resemble a positive trend pattern.

Another theory involving psychological evidence is suggested in Daniel, Hirshleifer and Subrahmanyam (1998), where investor overconfidence is explored. An overconfident investor will likely overweight the precision of his private information and underestimate the publicly available data. Such behavior will cause the stock price to overreact and the price will move to the full information move over the subsequent dates, as publicly information arises. The investor also asymmetrically updated their confidence observing the outcomes - if the publicly signal confirms their trading signal, then he becomes more confident, but if it is not confirmed, then confidence falls modestly.

2.2 Sentiment index

The overall attitude of investor towards the financial market is what we will call the investor sentiment. Some papers showed that the discounts of closed-end funds are correlated with sentiment index as shown in Lee, Shleifer and Thaler (1991) and Chopra et al. (1993) - actually they argue that closed-end fund discount rate are a measure of sentiment index. Neal and Wheatley (1998) take measures of sentiment index and test their ability to predict companies' financial returns. Among other findings, they found a positive relation between closed-end fund discounts and small companies expected returns, but no relation with large firms. This is consistent, since most at the time of the study, most small firm stocks were held by individual investors and institutions held the large firm stocks (see Lee, Shleifer and Thaler (1991)). Otoo et al. (1999) studied the positive correlation between consumer sentiment and stock prices and raised a question about which one is the cause of the other. With individual data from a Michigan survey, they concluded that people tend to use the stock prices movement as an leading indicator of the future instead of consumer sentiment primarily influences stock prices.

2.3 Media influence

It is natural to assume that media is used by investor so that can track their investments performance (such as stock prices) and be informed about new events that might affect their portfolio. Tetlock (2007) measured how a popular *Wall Street Journal* column correlates with the financial stock market. The words from the column were extracted and categorized into 77 categories predetermined by a psychological dictionary.

Since a word can belong to several categories, a principal components analysis (PCA) was employed to keep the most important component of the the variance-covariance matrix of the word categories. The resultant factor was called the *Pessimism Media Factor*. A vector autoregressive (VAR) model was used to estimate the links between such media factor and the stock market. High levels of pessimism predicts downward pressure on prices. Moreover, high or low levels of pessimism factor are followed by high trading volume. Finally, to close the cycle, low returns lead to high levels of pessimism.

Similarly, García (2013) analyzed the effects of two columns from the *New York Time* on the financial market. It is important to notice that these columns are viewed as description about recent market performance and speculation about its near future and not new piece of information. As Tetlock (2007), he also find that trade volume is predicted by extreme content, either positive or negative, providing additional evidence that traders are not fully rational. If they were so, and symmetrically informed, a publicly signal such as a newspaper column should not affect trading volume. The novel finding of the paper is that people tend to be more sensitive during recession and, therefore, the predictability power of the news content is much higher during economic downturns.

What if the same information event is covered by different media content sources? Engelberg and Parsons (2011) studied the effect of local media coverage on local brokerage accounts, from 1991 to 1996, a pre-Internet period. The main finding is that if the earnings announcement for a given S&P 500 firm is locally covered or not strongly influence the firm's local trading, controlling for all fixed effects (e.g. firm-city closeness, timezones that might affect publishing date). The trading volume increases from 8% up to 50%. The evidence, however, supports prediction power from media coverage on trading volume only for trading volume on the same day that the coverage occurs. Moreover, they found that the local media coverage predicts trading only for *day 1*, i.e. the first possible day that the coverage can occur. For coverage initiated on future possible dates, there is no evidence of predictability.

Shiller (2000) describes how the media can relate to the financial market. The author argues that the media can actively drive public attention and they can support speculative price movements while they put their efforts to make news interesting to their audience.

2.4 Digital Era

The Internet today can serve as the main source of information, specially when it comes to news. Microblogging on *Twitter*, for instance, has become more popular and widely used as a channel to spread news. Bollen, Mao and Pepe (2011) studied how textual analysis on posts from from *Twitter* users, or *tweets*, measures the public mood and how

it responds to social, political, economical and cultural events on the short term. Bollen, Mao and Zeng (2011) also carried a textual analysis from twitters and found that some *Calm* and *Happiness* moods can predict the value of the *Down Jones Industrial Average* (*DJIA*). In fact, they are Granger causative of the value of the DJIA for a period of 3 to 4 days.

2.5 Other related work

Monetary policy and market sentiment has been explored by Chague et al. (2015). The paper has also conducted a textual analysis, but now from the Brazilian Central Bank's Monetary Policy Committee press releases. Similarly to Tetlock (2007), they used PCA to build a measure of the communication sentiment, the *Optimism Factor* of the policy makers. They found that when policy makers are more optimistic, the long-term future interest rates drops. However, when Optimism is rather low, the market expectations responds with higher volatility on future interest rate.

Also related to monetary policy, Kurov (2010) found that the effects of monetary policy changes on the United States depends on market sentiment. Essentially, the study found that monetary policy changes in bull markets have little effect on stock returns and investor sentiment, but large effects in bear markets.

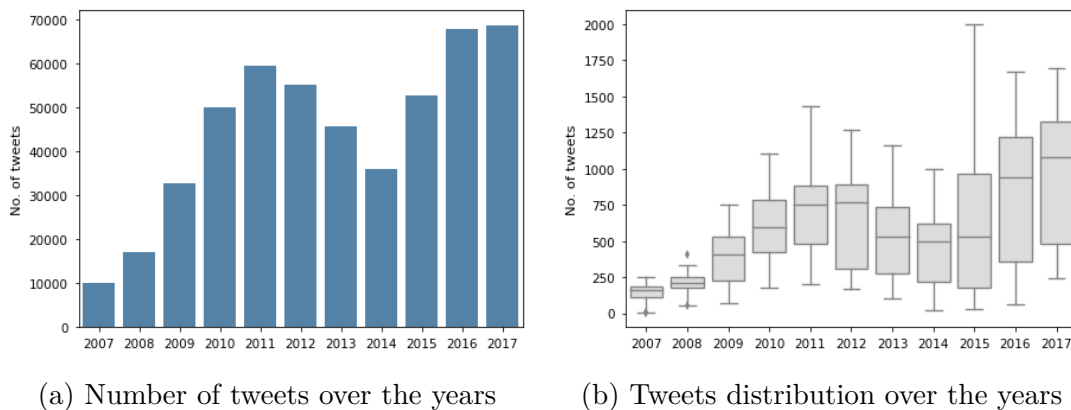
3 Data

The dataset studied is composed by social media posts on Twitter¹ (or tweets), the S&P500 index and the VIX index. We can, therefore, split our datasets into *non-structured* and *structured* data. The former is the textual data from tweets that needs to be processed somehow in order to fit any statistical and mathematical model. The latter is viewed and the historical time series of the financial data. We here just describe the raw data and further data preparation in order to ensure interpretable meanings and be corrected specified for our models is presented in ??

3.1 Non-structured data

Twitter was founded in 2006 and has increased popularity ever since, specially when it comes to news. Tweets can now be seen as a newsfeed and is often used as a source of trending topics. Therefore, the motivation to relate its data to the financial is straightforward.

Figure 3.1 – Distribution of tweets over the years



We collected a variety of tweets that might be in the light of financial market practitioners. Our dataset has 495,259 tweets, from March 21, 2007 to October 31, 2017. Our sample was collected by filtering posts with at least one of the following keywords related to the US market: *NASDAQ*, *NYSE*, *Dow Jones Industrial Average* and *Financial Market*. On top of it, we also collected tweets from the official accounts of the main financial news sources of the US, such as *Wall Street Journal*, *Financial Times Finance News*, *CNBC*, *NY Times Business*. Note that both collections methods are not mutually exclusive, so we had to ensure uniqueness by dropping duplicates. However, one or more

¹ <<https://twitter.com>>

users can post exactly the same tweet or even *retweet* it (i.e. reproduce someone's tweet referencing the source). Thus, our tweets are unique per user and we do want to have duplicated tweets from different accounts as this may be a sign of a trending topic.

Figure 3.1 depicts the evolution of the number of tweets per year. We can clearly see that at the beginning, years of 2007 and 2008, the numbers of tweets are much smaller than in the following years. For some reason, the number of tweets in our sample decreased after 2011, but began increasing again in 2015 and kept like this ever since. Table 3.1 contains the descriptive statistics of each year. We can also clearly see that the news providers dominate our sample, NY Times and CNBC being the most active accounts, as depicted in Figure 3.2.

Table 3.1 – Descriptive monthly statistics of the Twitter dataset

Year	N	Monthly Avg	St. Dev.	Min	Max
2007	10,123	1,012.3	237.5	412	1,254
2008	17,196	1,433.0	215.5	845	1,645
2009	32,663	2,721.9	634.3	1,647	3,479
2010	50,105	4,175.4	868.9	3,092	5,566
2011	59,423	4,951.9	862.1	3,634	7,080
2012	55,160	4,596.7	831.2	2,576	5,841
2013	45,745	3,812.1	1,025.4	2,438	6,322
2014	36,009	3,000.8	1,284.3	708	4,526
2015	52,566	4,380.5	2,498.0	664	7,650
2016	67,655	5,637.9	2,331.3	988	7,568
2017	68,614	6,861.4	1,213.2	4,192	8,573

Regarding seasonality, it is possible to notice that users are less active on the weekends. It seems reasonable, since we are taking samples related to the financial market and it is closed during weekends. When we look for monthly seasonality, however, there is no apparent distinction on users activity, as depicted in Figure 3.3.

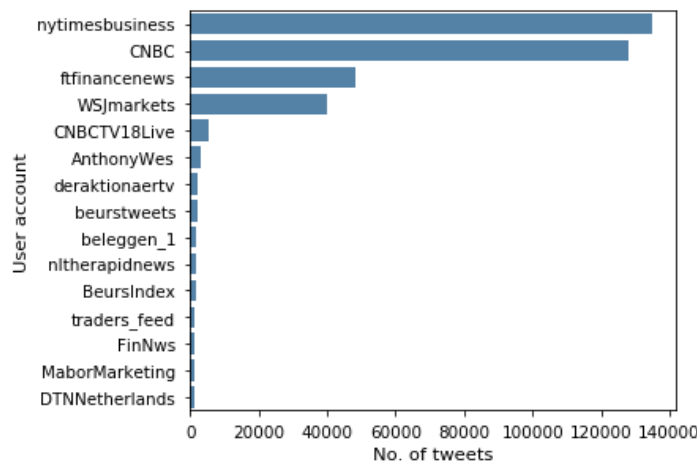
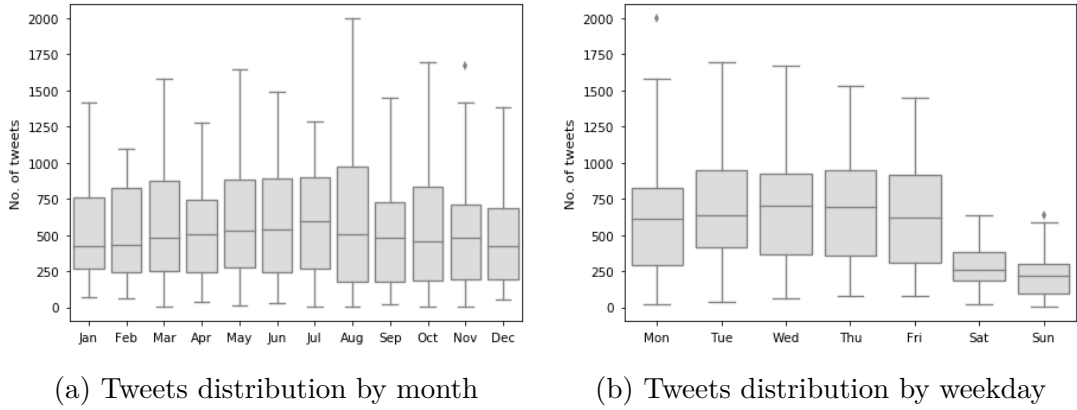


Figure 3.2 – No. of tweets per user

Figure 3.3 – Intra-year seasonality patterns

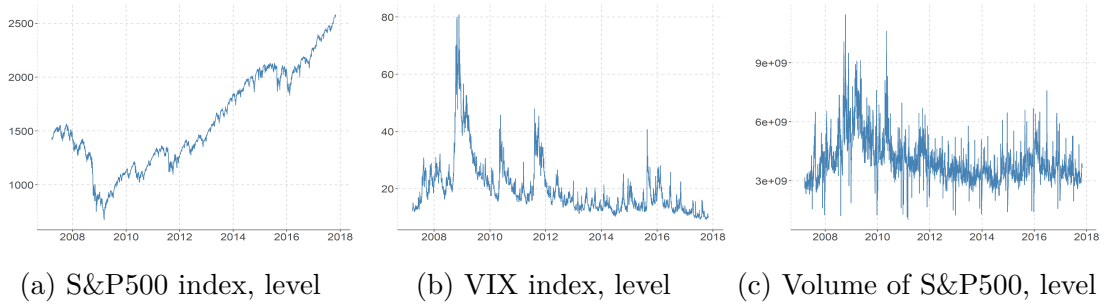


3.2 Structured data

The financial data collected is composed by daily adjusted closing price and volume of the S&P500 index² and daily values of the VIX index³, taken from Yahoo Finance⁴, for the same period as the tweets collected, i.e. from March 21, 2007 to October 31, 2017 - a total of 2,651 samples (or business working days).

The S&P500 is traditionally used as indicator of the US economy, since it is a weighted composite of the 500 largest company by market capitalization. The VIX index is a measure of the implied volatility of the S&P500. Since it measures volatility, it is associated with uncertainty of the markets, i.e. periods with great volatility are, in general, when people are more risk averse. A summary can be found on Table 3.2.

Figure 3.4 – Level value of S&P500, VIX and S&P500 trading volume



The S&P500 index increased by 79.5% over the period considered. It is interesting to notice that during the Financial Crisis of 2008-09, the S&P500 level dropped considerably. In fact, as depicted in Figure 3.4, it decreased by 53% between the beginning of 2008 and the bottom-most level in 2009 (March 9, 2009). The VIX level in such period also have

² <<https://us.spindices.com/indices/equity/sp-500>>

³ <<http://www.cboe.com/vix>>

⁴ <<https://finance.yahoo.com>>

increased by 114% (i.e. from 23.2 up to 49.7), although it has peaked up to a level of 80 slightly afterwards. Trading volume of S&P500 is also affected by volatile periods.

Table 3.2 – Descriptive statistics of financial data over the considered period

Financial Data	N	Period Avg	St. Dev.	Min	Max
S&P500 Index	2,651	1,589.5	451.3	676.5	2,581.1
VIX Index	2,651	20.1	9.6	9.2	80.9
S&P500 Volume	2,651	4.0	1.2	1.0	11.5

Note: S&P500 volume figures in billion

4 Methodology and model

In this chapter, we describe our research methodology and the model used to analyze the data. However, data cleansing and preparation is needed to ensure interpretable and consistent results.

4.1 Data Preparation

Our goal is to have a sentiment index given by our Twitter dataset, specified in section 3.1 and, as already pointed out, we need to process such textual data. We used a dictionary approach to classify, measure and create a sentiment index of our tweets, similarly to what was done by Chague et al. (2015) and Tetlock (2007). Regarding our structured data, presented on section 3.2, we manipulated our historical series in order to ensure stationarity and also give them interpretable meaning. The following subsections discuss the procedures taken.

4.1.1 General Inquirer

The General Inquirer (GI), first introduced by Stone et al. (1962), is a computer-assisted system for content analysis which uses dictionaries lookups for word categorization and can also handle disambiguation. Today, the distributed software has two built-in psychological dictionaries, Harvard-IV and Lasswell, that maps thousands of words into 182 semantic groups, such as *Negative*, *Positive*, *Pleasure*, *Passive*, *Power* and so on. It is worth noticing the the categories are not mutually exclusive, i.e. one word can belong to more than one category. Also, one word alone can have multiple meanings that is given by the context and each one of possible meanings can belong to different categories - that is why disambiguation plays an essential role.

The software analyzes each word of our corpora and map them into one of the categories of the dictionary and the output is given in both the absolute and relative frequency of words in each category. We have grouped the tweets on a daily basis, generating one file per day with all the posts of the corresponding day and then passed all the files to the GI software. Finally, we can have a quantifiable meaning for our textual data.

4.1.2 Latent sentiment

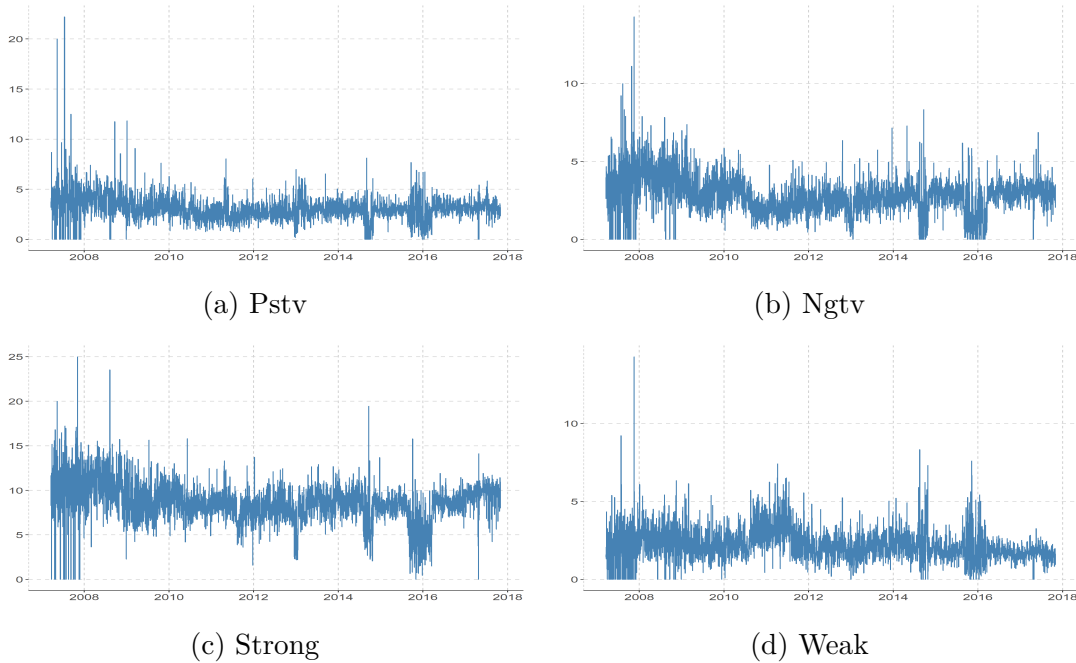
Similarly to the approaches in Chague et al. (2015) and Tetlock (2007), we carry a Principal Component Analysis (PCA) in order to extract the latent sentiment of our sample of tweets already categorized by the GI. The PCA technique aims dimensionality

reduction by finding orthogonal vectors that captures the variance of the original data and are a linear combination of the original variables. A formal definition is found in Appendix A.

For simplicity sake, we studied just the four main categories of the Harvard-IV dictionary: (i) *Pstv*, related to positive words; (ii) *Ngvtv*, related to negative words; (iii) *Strong*, related to word that show strength, power, control, authority, etc.; and (iv) *Weak*, related to words that show weakness, submission, dependence, vulnerability, etc. In Figure 4.3 we can see how each category evolves over time and in Table 4.1 we can see there is no pair of variables with more than 40% of correlation, which is good since we want to avoid redundant variables (i.e. highly correlated).

Remember that Figure 3.1 showed a growth tendency in the number of tweets in our dataset. It is straightforward to realize that the absolute frequency of each GI category will also follow such trend. Therefore, we used the relative frequency to avoid feed our PCA algorithm with the trend of our number of tweets. Also, before carry a PCA, we also scaled (to zero mean and unit variance) our GI output so that none of the categories' variance prevails over the other (see Appendix A).

Figure 4.1 – GI categories relative frequency over time



After running a PCA, we could reduce our dimension from 4 to 2, capturing 72.4% of the original variance. Table 4.2 summarizes the PCA results, giving the loading coefficients, associated eigenvalue λ and the variance explained. The first principal component, namely *PC1* has all its coefficients greater than zero and there is not a coefficient that strong dominates the others - we could interpret this component as the *Engagement* of Twitter users. Now the second component assigns positive weights to *Pstv* and *Strong* categories,

Table 4.1 – Correlation structure between selected GI categories

	Pstv	Ngvtv	Strong	Weak
Pstv	1.00			
Ngvtv	0.26	1.00		
Strong	0.40	0.46	1.00	
Weak	0.03	0.34	0.14	1.00

but negative to *Ngvtv* and *Weak* - since they are opposite pairs of categories and there is positive weight to positive categories, we interpret it as a measure of *Optimism*. From Figure 4.2 it is clear the cluster as a result of the second principal axis.

Table 4.2 – PCA results

GI category	PC1	PC2	PC3	PC4
Pstv	.46	.54	.67	.20
Ngvtv	.58	-.22	-.41	.66
Strong	.58	.25	-.40	-.67
Weak	.34	-.77	.47	-.27
λ	1.86	1.03	.62	.48
Cum. explained Variance	46.6%	72.4%	87.9%	100%

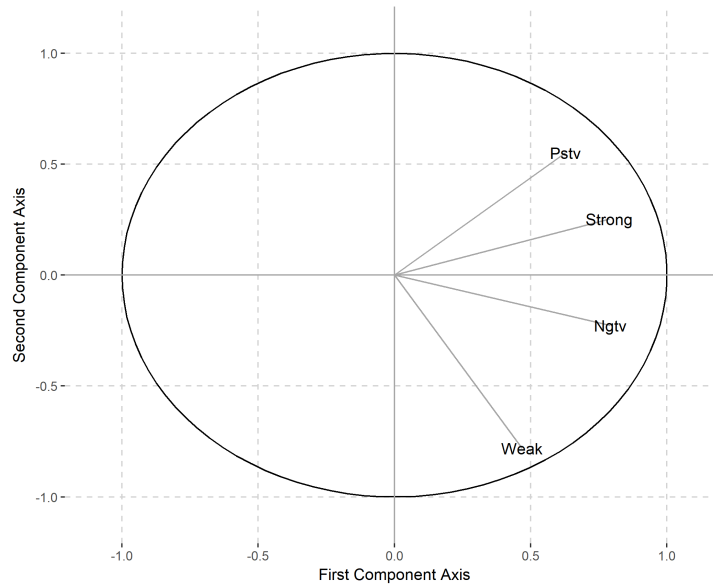


Figure 4.2 – Visualization of the first two principal axis

We need now to project our original data into this new coordinate system formed by the first two components:

$$\mathbf{t}_i = \mathbf{X}\mathbf{w}_i, \quad (4.1)$$

where \mathbf{X} is our $(N \times p)$ original scaled data, \mathbf{w}_i the $(p \times 1)$ loading vector associated with the i -th principal component and \mathbf{t}_i is defined as the (centered) *score*, or the projected data, into the i -th principal axis, of size $(N \times 1)$ (see Appendix A). In our case, $p = 4$ categories and $N = 3,878$ is the length of our tweets dataset, resampled on a daily frequency.

Note, again, that since we will use only 2 principal components to project our data, we reduced our original dataset \mathbf{X} of size $(N \times 4)$ to our score matrix \mathbf{T} of size $(N \times 2)$, as $\mathbf{T} = [\mathbf{t}_1 \ \mathbf{t}_2]$. Since \mathbf{t}_1 contains the original data projected on the principal axis related to *Engagement*, we called those scores *Engagement Factor (EF)*, and the \mathbf{t}_2 scores form the *Optimism Factor (OF)*. Their evolution over the considered period can be seen in Figure 4.3 and its statistics in Table 4.3.

Figure 4.3 – Original data projected on the first two principal axis

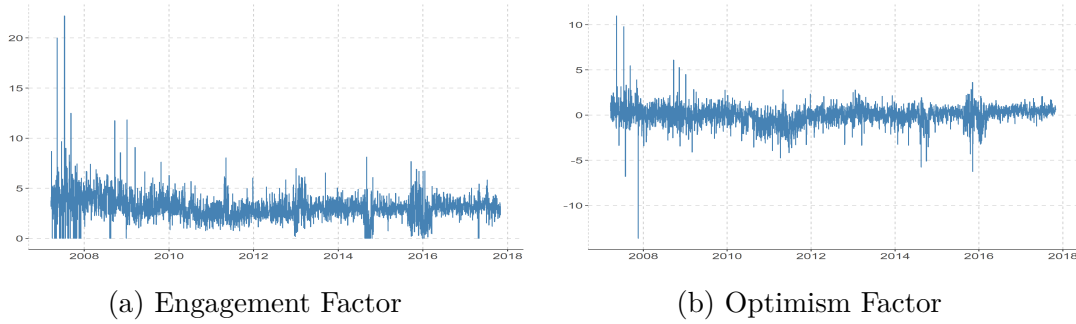


Table 4.3 – Descriptive statistics factors

Factor	N	Period Avg	St. Dev.	Min	Max
<i>EF</i>	3,878	0.0	1.37	-5.50	7.09
<i>OF</i>	3,878	0.0	1.01	-13.64	11.0

It is interesting to note how the loadings we get differ from the one in Chague et al. (2015) and Tetlock (2007), where the principal axis obtained already relate to optimism or pessimism. This was expected, since the corpora analyzed in those studies were Central Bank communication reports and columns from a newspaper, respectively in Chague et al. (2015) and Tetlock (2007). With that, it is reasonable to assume that a specific columns on a given day will have a latent sentiment towards the view of the author as well as the Central Banks reports will need to address monetary policy perspectives that carry an inherent sentiment. When we analyze thousands of tweets per day, we capture the overall sentiment since there are lots of people with different opinions expressing different sentiments about the current scenario. Thus, an overall engagement of the people is the element with the highest variance.

4.1.3 Time series

We now have structured and non-structured data as time series format. However, before proceed to the modelling phase, we need to ensure stationarity so that our model is consistent.

A time series is weakly stationary if it has time invariant components up to the second moment. Although there are more ways of stationarity, from now on we will omit the term *weak(ly)* from *weakly stationary time series* and when we say that a series is stationary or not, we are referring to weak stationarity. Such kind of stationarity is usually enough to most models, specially the one we will use. Formal statistical tests provide a useful method to check whether a time series has a unit root or not, i.e. if it is an *Integrated Process*. In fact, an integrated process of order d , i.e. $I(d)$ has d unit roots and needs to be differenced d times to become stationary. Additionally to such unit root tests, analyzing the Autocorrelation and Partial Autocorrelation Functions (ACF and PACF, respectively) is very insightful (see Appendix B for a time series review).

In general, for financial data the level value does not provide meaningful interpretation and we usually are interested in the change of such variables. For indexes and stock prices, the percentage change is a measure of return, defined as:

$$R_t = \frac{P_t}{P_{t-1}} - 1, \quad (4.2)$$

where R_t is the return over the period $[t-1, t]$ and P_t is the level value (e.g. price of stocks) at time t . It is also usual to express the *log* returns:

$$r_t = \log(P_t) - \log(P_{t-1}) = p_t - p_{t-1}, \quad (4.3)$$

where $\log X$ is the natural logarithm of X . The lowercase notation, such as r_t or p_t , is defined as the logarithm of the level value, e.g. $p_t = \log(P_t)$ and $r_t = \log(R_t)$.

Also, it is usual to express return of a financial asset as the excess return with respect to a *risk-free* investment. In the case of the US market, it is common to use the yield of the 3-month US Treasury Bill as a proxy for the risk free rate in the United States. Therefore, we define the *excess of return* r_t^{xs} of a financial asset as:

$$r_t^{xs} = r_t - r_t^f, \quad (4.4)$$

where r_t^{xs} denotes the excess ("xs") of returns of a financial asset r_t with respect to the return of a risk-free investment r_t^f , the yield of 3-month US Treasury Bills in our case, provided by the US Treasury¹.

In Table 4.4 we can see the test statistics for the Augmented Dickey-Fuller, with null hypothesis that the series has unit root and the alternative is that it is stationary. It

¹ <<https://www.quandl.com/data/USTREASURY/YIELD-Treasury-Yield-Curve-Rates>>

was filled by "n/a" where a series transformation is either not applicable or does not make economic sense. For instance, the *EF* and *OF* factors have negative and zero values, where the *log* is not defined. Also, for the excess of log return series, it does make sense to apply logarithm again and its first difference would be an uncommon measure of the acceleration of the S&P500 level. The test was done for the series in level, *log* transformed and first difference of the *log*. Note that the first difference of the *log* of the level of S&P500 is the same as the *log return* of the S&P500 and the level of the excess of return of the S&P500 is useful only on the level and *log*, since it is already a differenced series. Although the level of VIX and S&P500 are stationary, we prefer to use its first differenced of *log*, since it provides a measure of percentage change. Note that S&P500 is not stationary at the level and *log*, as expected, since in Figure 3.4 we see a clearly upward trend over the years, but when differenced it becomes stationary as well as the excess of return series. Despite the peaks of VIX and S&P500 Volumes, they can be considered stationary even at the level. Both created factors, *EF* and *OF* are also stationary.

Table 4.4 – Augmented Dickey-Fuller test statistics

Series	Level	log	Δ	$\Delta \log$
<i>EF</i>	-6.16***	n/a	-23.58***	n/a
<i>OF</i>	-8.23***	n/a	-25.83***	n/a
S&P500	-2.12	-2.39	-14.27***	-14.27***
VIX	-4.21***	-15.46***	-15.46***	-16.13***
S&P500 Volume	-6.26***	-7.16***	-19.57***	-18.90***
Excess of log return S&P500	-14.30***	n/a	n/a	n/a

Note: *p<0.1; **p<0.05; ***p<0.01

log is the the log transformed series and Δ means the first difference

The series we propose to work on are, therefore, the factors *EF* and *OF*, obviously, in addition to the first difference of the VIX and S&P500 Volume and the excess of return of the S&P500. The reason to choose the first difference of the *log* for the VIX and Volume, in spite of their stationarity behavior on the level, is because it can be thought as a rate of change and, thus, easier to interpret. The choice to work with level of *EF* and *OF* is because they are stationary at level and their intuition are hard enough to grasp at level that taking a transformed series would make it even harder.

4.2 Model

In our model, we simultaneously estimate the relationships among returns, sentiment and volume. A model that allow this kind of structure is a VAR (Vector Autoregressive) model, see Box et al. (2015), Durbin and Koopman (2012), Enders (2014) and Lütkepohl (2005).

A VAR of order p , i.e. $\text{VAR}(p)$, is modeled as (see Appendix B):

$$\Phi(L)\mathbf{X}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t, \quad (4.5)$$

where $\mathbf{X}_t = (x_{1t}, \dots, x_{nt})'$ is an $(n \times 1)$ vector with n time series variables, L is the lag operator such that $L^k x_t = x_{t-k}$, $\Phi(L) = \mathbf{I}_n - \mathbf{A}_1 L - \dots - \mathbf{A}_p L^p$ is the lag polynomial where each \mathbf{A}_i is an $(n \times n)$ coefficient matrix, \mathbf{c} is an $(n \times 1)$ constant vector and $\boldsymbol{\varepsilon}_t$ is an $(n \times 1)$ vector of unobservable zero-mean white noise process.

In our model we have:

$$\mathbf{X}_t = \begin{bmatrix} EF_t \\ OF_t \\ Ret_t \\ \Delta Vlm_t \\ \Delta Vix_t \end{bmatrix}, \quad (4.6)$$

where EF_t is the Engagement Factor (i.e. the scores of the first principal component), OF_t is the Optimism Factor (i.e. the scores of the second principal component), Ret_t is the excess of return of the S&P500 calculated as described in Equation 4.3, ΔVlm_t is defined as the first difference of the log of the S&P500 trading volume and ΔVix_t is defined as the first difference of the log of the VIX index. As tested in subsection 4.1.3, all of these time series are stationary and, thus, the roots of the lag polynomial $\Phi(L)$ from Equation 4.5 lie outside the unit circle and, hence, the our VAR model is stable (see section B.7 from Appendix B).

The only parameter that is left to define is the order p of our VAR model. In order to estimate it, we estimated multiple multiples, up to the lag $p = 10$ and check which model order is the most informative, measured by information criteria (IC) statistics. From Table 4.5 we would choose the order $p = 10$, $p = 5$ and $p = 5$ according to Akaike Information Criteria (AIC), Hannan-Quinn (HQ) and Schwarz Criterion (SC), respectively. We choose $p = 5$, since it is more intuitive to analyze effects lasting the equivalent of a business week (5 business day in a week). At this point, it is worth noticing that since we are merging all time series, we chose to drop weekends and holidays that were presented in the Twitter dataset but not in the financial data, remaining with a total of 2,646 samples.

Table 4.5 – VAR order selection according to IC statistics for each $\text{VAR}(p)$

IC \ p	1	2	3	4	5	6	7	8	9	10
AIC(p)	-18.824	-18.963	-19.047	-19.140	-19.198	-19.197	-19.212	-19.212	-19.220	-19.229
HQ(p)	-18.799	-18.918	-18.983	-19.055	-19.093	-19.073	-19.067	-19.046	-19.035	-19.024
SC(p)	-18.757	-18.840	-18.869	-18.906	-18.909	-18.852	-18.811	-18.755	-18.708	-18.662

Each equation of our VAR model can be thought as a standard regression and can be estimated by OLS (ordinary least squares). Our model equations are represented as

follows:

$$EF_t = c_1 + \alpha_1 L_{1-5}(EF_t) + \beta_1 L_{1-5}(OF_t) + \gamma_1 L_{1-5}(Ret_t) + \eta_1 L_{1-5}(\Delta Vlm_t) + \theta_1 L_{1-5}(\Delta Vix_t) + \varepsilon_{1,t} \quad (4.7)$$

$$OF_t = c_2 + \alpha_2 L_{1-5}(EF_t) + \beta_2 L_{1-5}(OF_t) + \gamma_2 L_{1-5}(Ret_t) + \eta_2 L_{1-5}(\Delta Vlm_t) + \theta_2 L_{1-5}(\Delta Vix_t) + \varepsilon_{2,t} \quad (4.8)$$

$$Ret_t = c_3 + \alpha_3 L_{1-5}(EF_t) + \beta_3 L_{1-5}(OF_t) + \gamma_3 L_{1-5}(Ret_t) + \eta_3 L_{1-5}(\Delta Vlm_t) + \theta_3 L_{1-5}(\Delta Vix_t) + \varepsilon_{3,t} \quad (4.9)$$

$$\Delta Vlm_t = c_4 + \alpha_4 L_{1-5}(EF_t) + \beta_4 L_{1-5}(OF_t) + \gamma_4 L_{1-5}(Ret_t) + \eta_4 L_{1-5}(\Delta Vlm_t) + \theta_4 L_{1-5}(\Delta Vix_t) + \varepsilon_{4,t} \quad (4.10)$$

$$\Delta Vix_t = c_5 + \alpha_5 L_{1-5}(EF_t) + \beta_5 L_{1-5}(OF_t) + \gamma_5 L_{1-5}(Ret_t) + \eta_5 L_{1-5}(\Delta Vlm_t) + \theta_5 L_{1-5}(\Delta Vix_t) + \varepsilon_{5,t} \quad (4.11)$$

where the operator $L_{1-5}(\cdot) = (L + \dots + L^5)(\cdot)$ is defined here as the lag operator from 1 to 5, c_i and ε_i the constants of the vectors \mathbf{c}_t and $\mathbf{\varepsilon}_t$ from Equation 4.5, α_i , β_i , γ_i , η_i and θ_i are (1×5) vectors composed by the coefficients of each lag of the series of \mathbf{X}_t and part of each matrix \mathbf{A}_i from Equation 4.5.

5 Results

Since we are primarily interested in the social media impact on the financial market, we begin by assessing the coefficients related to the media factors, i.e. α_i , β_i from Equation 4.9, Equation 4.10 and Equation 4.11. Then we inspect the responsiveness of the social media with respect to market events, i.e. Equation 4.7 and Equation 4.8. The complete estimates from all equations of our model can be found in Table A.1.

It is interesting to note from our estimates from Table 5.1 the overall engagement tested significant from its immediate effect on the next days' excess of return of the S&P500 index. At 5% significance level, the impact of a one standard deviation change in the engagement factor is a negatively impacts 0.1 basis point in next day's S&500 excess of returns and 0.3 basis point, at 10% level, the VIX index, where 1 basis point is equivalent to 0.01%. All the other previous days of the engagement and optimism factors have statistical zero impact on returns and VIX. However, previous engagement and optimism factor affect the next day's respective factor, i.e. a positive change on today's engagement increases future engagements up to a week.

Table 5.1 – Social Media factors estimates

	<i>Dependent variable:</i>		
	<i>Ret</i>	ΔVix	ΔVlm
EF_{t-1}	−.001** (.0003)	.003* (.002)	−.004 (.004)
EF_{t-2}	.0002 (.0003)	.001 (.002)	−.006 (.004)
EF_{t-3}	−.0002 (.0003)	.002 (.002)	.001 (.004)
EF_{t-4}	−.0002 (.0003)	−.002 (.002)	.001 (.004)
EF_{t-5}	.0001 (.0003)	−.002 (.002)	.006* (.004)
OF_{t-1}	.0003 (.0003)	.001 (.002)	.0003 (.005)
OF_{t-2}	−.0002 (.0004)	.0001 (.002)	.008* (.005)
OF_{t-3}	.0004 (.0004)	−.0005 (.002)	.002 (.005)
OF_{t-4}	.00004 (.0004)	.001 (.002)	−.010** (.005)
OF_{t-5}	.0002 (.0003)	.002 (.002)	.005 (.005)
Observations	2,646	2,646	2,646
R ²	.033	.028	.206
Adjusted R ²	.023	.019	.198
Residual Std. Error (df = 2620)	.013	.073	.170
F Statistic (df = 25; 2620)	3.544***	3.023***	27.172***

Note:

*p<0.1; **p<0.05; ***p<0.01
Standard deviations in parenthesis

When we analyze the trading volume, we can only reject the null hypothesis for the previous week's engagement and both lags 2 and 4 for the optimism. To summarize, a one-standard deviation change in the engagement factor increases 0.6 basis points the next week's S&P500 trading volume. A one-standard deviation change in optimism, on the other hand, increases trading volume by 0.8 basis point with a two-day delay, but

subsequently decreases 1 basis point (the 4th lag impact). This is consistent to what was studied in Tetlock (2007), where a positive change in the pessimism factor (or a decrease in optimism) first increases volume due to a group of liquidity traders, but the equilibrium is restored in the following days (as in our case).

The estimates in Table 5.1 can serve as evidence of a primarily conclusion that the social media do not contain new information about fundamentals, and, therefore, do not influence the price. Therefore, it is possible that the social media sentiment is backward-looking, serving just as informative source of past events regarding returns.

Table 5.2 – Market effects on social media sentiment

	<i>Dependent variable:</i>	
	<i>EF</i>	<i>OF</i>
Ret_{t-1}	.073*** (.020)	.001 (.018)
Ret_{t-2}	.017 (.024)	.088*** (.021)
Ret_{t-3}	-.328 (2.052)	-3.081* (1.764)
Ret_{t-4}	-.280** (.113)	.041 (.097)
Ret_{t-5}	.341 (.357)	-.059 (.307)
Vlm_{t-1}	.142*** (.020)	.037** (.017)
Vlm_{t-2}	.004 (.024)	.102*** (.021)
Vlm_{t-3}	1.039 (2.054)	-2.153 (1.766)
Vlm_{t-4}	-.158 (.112)	.040 (.096)
Vlm_{t-5}	.347 (.357)	-.181 (.307)
VIX_{t-1}	.212*** (.019)	-.005 (.017)
VIX_{t-2}	.012 (.023)	.106*** (.020)
VIX_{t-3}	-1.127 (2.039)	.205 (1.753)
VIX_{t-4}	-.012 (.101)	.078 (.087)
VIX_{t-5}	-.691* (.353)	.313 (.304)
Observations	2,646	2,646
R ²	.574	.212
Adjusted R ²	.570	.204
Residual Std. Error (df = 2620)	.881	.758
F Statistic (df = 25; 2620)	141.344***	28.159***

Note:

*p<0.1; **p<0.05; ***p<0.01
Standard deviations in parenthesis

From Table 5.2 we can reject the null hypothesis for previous days' effects of market events (returns, volume and volatility) on both sentiment factors in some way. The overall engagement is positively responsive for a 1% change in market variables of the very previous day - VIX index increase, i.e. volatility and therefore uncertainty increase accounts for 0.2 standard deviation in engagement factor. It is interesting to note, however, the sign reversal impact of some variables - it seems that social media overall engagement have short memory for market events. For example, the increase in VIX of the previous week negatively impacts the engagement factor, while the very previous day increases it. This is also true for returns.

Regarding optimism, on the other hand, it always is positive correlated to the

change direction of the events, that is, an increase in returns, volume or volatility also increases the optimism of the social media. The optimism, however, seems to have a slower response of volatility and returns compared to engagement. The very previous day does not immediately change the current optimism, just the day after. The exception the volume that affect both the current and the day after.

The residuals of our regressions are serially uncorrelated and resemble a white noise process. Such robustness confirms that we could there is no meaningful information in the error term of Equation 4.5. Diagnostic plots of each regression can be found in Appendix A.

6 Final remarks and future work

The main goal of this work was to assess the role of Twitter in the US financial market, specifically related to one of the main index of US economy, the S&P500. Our approach carefully extracted a curated list of tweets that could serve as source of information for all types of investor as well as tweets of random users related to the economy and the financial market.

This study fundamental difference from the literature is that since it collects an amount of data substantially larger than previous work with newspaper columns, the first and principal latent variable resulted from PCA is, of course, overall engagement, while for datasets from a single source, such as newspaper columns in García (2013) and Tetlock (2007), is the pessimism/optimism (our second factor).

It is interesting to notice that the information posted on Twitter seems more backward-looking. Such finding tells us that Twitter users use it as a mean to describe past events and does not necessarily contain new information. Our study found that the today's sentiment can affect tomorrow's returns, volume and volatility only in a small scale compared to what those market variables affects the sentiment.

It is worth noticing that the market variables we chose to measure the impact of the Twitter is the S&P500 index, i.e. the weighted equity value of the biggest companies listed in the biggest exchanges in the US. It is hard to change the inertia of the index.

Future extensions of this study could assess the impact of social media on smaller companies. Asset factor modeling introduced and constantly refined by Fama and French (2015) can capture smaller stocks effects relative to big ones, as done in Tetlock (2007). The literature has also effectively investigated local effects, as in Engelberg and Parsons (2011), and it could be interesting to group tweets from a specific region and check their local influence.

It is interesting to notice that the advent of new machine and deep learning techniques can predict many things with high accuracy levels. However, there is a trade-off between accurate and interpretable results. Future works needs to blend the benefit from high accuracy levels of sophisticated algorithms and explanatory modelling.

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Appendix

APPENDIX A – Principal Component Analysis Review

Principal Component Analysis (PCA) is a statistical procedure that orthogonally transform the data to a new coordinate system with coordinates sorted in descended order of variance of the original data - called principal components. First known papers to describe the technique are dated back to the beginning of the 20th century (see Pearson (1901)). Further developed by Hotelling (1933), the motivation was to found a found a fundamental set of independent variables from a correlated set of data. In essence, PCA is very useful for dimensionality reduction of the data set and achieves it by finding characteristic latent vectors and roots (see Dunteman (1989)). PCA is often related to Singular Value Decomposition (SVD) and is usually used for computational methods of PCA components calculation. For simplicity's sake, we are approaching the derivation for the population and not for samples, i.e. there is no bias correction of $\frac{n}{n+1}$, although conclusion are the same. For further and in-depth considerations of PCA and its applications, see Jolliffe (1986) and Hastie, Tibshirani and Friedman (2001). Formal mathematical motivation is shown below and use properties and theorems from Linear Algebra. For a review in Linear Algebra, please refer to Hoffman and Kunze (1971).

A.1 Principal Component Analysis

In practice, the first Principal Component (*PC*) is trying to find the direction given by the unit vector \mathbf{w}_1 such that the data projection on it, i.e. $\mathbf{t}_1 = \mathbf{X}\mathbf{w}_1$, has the highest variance. The second *PC* is trying to find the direction given by the unit vector \mathbf{w}_2 with the second most highest variance for the projected data on it, being additionally constraint to be orthogonal do \mathbf{w}_1 . The third *PC* follows the same rationale and is orthogonal do both \mathbf{w}_1 and \mathbf{w}_2 . This continues up to the p -th, and last, *PC*.

Usually, \mathbf{w}_i are called *loadings* and $\mathbf{t}_i = \mathbf{X}\mathbf{w}_i$ the *scores* associated with the i -th *PC*, for $i \in \{1, \dots, p\}$.

Formally, the first *PC* maximizes $Var(\mathbf{t}_1) = Var(\mathbf{X}\mathbf{w}_1) = \mathbf{v}_1' \mathbf{\Sigma} \mathbf{v}_1$, being $\mathbf{\Sigma} = \mathbf{X}'\mathbf{X}$ the covariance matrix of \mathbf{X} . Now, remember that \mathbf{w}_1 is a unit vector, so the maximization is subject to $\mathbf{w}_1' \mathbf{w}_1 = 1$. Using Lagrange Multipliers for constraint optimization, we want to maximize:

$$L(.) = \mathbf{v}_1' \mathbf{\Sigma} \mathbf{v}_1 - \lambda(\mathbf{w}_1' \mathbf{w}_1 - 1), \quad (\text{A.1})$$

where λ is the Lagrange Multiplier. Hence, we find the optimal \mathbf{w}_1 as follows:

$$\frac{\partial L(\cdot)}{\partial \mathbf{w}_1} = (\mathbf{\Sigma} - \lambda \mathbf{I}_p) \mathbf{w}_1 = \mathbf{0} \quad (\text{A.2})$$

It is clear now that the Lagrange Multiplier is an eigenvalue of $\mathbf{\Sigma}$ associated with the eigenvector \mathbf{w}_1 . Since the first *PC* is the one that captures the most variance of the data, it is associated with largest λ_1 eigenvalue of $\mathbf{\Sigma}$.

The procedure to find the remaining *PCs* is the same, with additional constraints that the \mathbf{w}_k must be orthogonal to each one of the $k - 1$ vectors of loadings. The eigenvalues are found in such fashion that $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$. That illustrates why is useful to standardize our variables (zero mean and unit variance). If we have a variable with a wide range of possible values (i.e. high intrinsic variance) and all the other with relative small variance, then such variable prevails when calculating the loadings, since its variance is high relative to the others.

It also follows that, with λ_i being the variance of the $i - th$ principal component (*PC*), the sum of all λ_i is equal to the total variances σ_i^2 of the original variables:

$$\sum_{i=1}^p \lambda_i = \sum_{i=1}^p \sigma_i^2 \quad (\text{A.3})$$

Now consider the the covariance matrix $\mathbf{\Sigma} = \mathbf{X}'\mathbf{X}$ of size $(p \times p)$. Since $\mathbf{\Sigma}$ is symmetric, from the Spectral Theorem, we can rewrite it as:

$$\mathbf{\Sigma} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}' , \quad (\text{A.4})$$

where \mathbf{V} is a $(p \times p)$ matrix with orthonormal columns, the eigenvectors, and $\mathbf{\Sigma}$ is a $(p \times p)$ diagonal matrix with eigenvalues in descended order. It follows that such decomposition can also be seen as:

$$\mathbf{\Sigma} = \sum_{i=1}^p \lambda_i \mathbf{v}_i \mathbf{v}_i' \quad (\text{A.5})$$

This is very insightful once the covariance matrix is decomposed by contributions $\lambda_i \mathbf{v}_i \mathbf{v}_i'$ of each *PC*. Remember that when doing a PCA, we are interesting in project our centered data set \mathbf{X} of size $(n \times p)$ with n observations of p variables on a new coordinates system in such fashion that, hopefully, the first $m \ll p$ principal components explain and capture most of the variance of \mathbf{X} and the principal components are orthogonal to each other with unit norm. As i increases, the contribution of each *PC* decreases and therefore one can choose the first m *PCs* that explain a certain desired threshold of explained variance of the data set and work with a smaller dimension.

A.2 Singular Value Decomposition

Now, consider again a generic matrix \mathbf{X} ($n \times p$) with n observations of p variables' measurements, properly demeaned. Such matrix can be decomposed as follows:

$$\mathbf{X} = \mathbf{U}\mathbf{L}\mathbf{A}', \quad (\text{A.6})$$

where \mathbf{L} is a ($r \times r$) diagonal matrix; \mathbf{U} and \mathbf{A} each of which with orthonormal columns¹ and dimensions ($n \times r$) and ($p \times r$), respectively; and $r = \text{rank}(\mathbf{X}) \leq p$. The diagonal values of \mathbf{U} are often called singular values of \mathbf{X} and \mathbf{U} and \mathbf{A} are often called left and right singular vectors of \mathbf{X} for the corresponding singular values, respectively.

It is worth noticing that some left singular vectors collapse to zero if \mathbf{X} does not have full rank (i.e. $r < p$), since the last $(p - r)$ eigenvalues of $\mathbf{X}'\mathbf{X}$ are zero (and therefore the last $(p - r)$ terms of Equation A.5) - in general there are r singular values and the same number of left singular vectors.

A.3 PCA meets SVD

SVD is often employed to calculate the *PCs*, being both efficient and numerically robust. Now take the data set \mathbf{X} and rewrite the its covariance matrix $\mathbf{X}'\mathbf{X}$ in terms of the resultant matrices from SVD in Equation A.6, i.e.:

$$\begin{aligned} \mathbf{X}'\mathbf{X} &= (\mathbf{U}\mathbf{L}\mathbf{A}')'\mathbf{U}\mathbf{L}\mathbf{A}' \\ &= \mathbf{A}\mathbf{L}'\mathbf{U}'\mathbf{U}\mathbf{L}\mathbf{A}' \\ &= \mathbf{A}(\mathbf{L}'\mathbf{L})\mathbf{A}' \end{aligned} \quad (\text{A.7})$$

The result is actually very insightful. One can note that the ($r \times r$) matrices $\mathbf{X}'\mathbf{X}$ and $\mathbf{L}'\mathbf{L}$ are similar². Similar matrices share the same eigenvalues. This proves Equation A.3, as the singular values σ_i of \mathbf{X} are in the diagonal matrix \mathbf{L} and the eigenvalues λ_i of $\mathbf{X}'\mathbf{X}$:

$$\lambda_i = \sigma_i^2, \quad (\text{A.8})$$

for $i \in \{1, \dots, r\}$.

To summarize, find \mathbf{U} , \mathbf{L} and \mathbf{A} that satisfy Equation A.6. Then, \mathbf{L} and \mathbf{A} give us the square roots of the eigenvalues and the eigenvectors of $\mathbf{X}'\mathbf{X}$, respectively, and, thus, the standard deviations and loading vectors of the *PCs*. Moreover, with \mathbf{U} we get the *PC* scores, since $\mathbf{X}\mathbf{A} = \mathbf{U}\mathbf{L}\mathbf{A}'\mathbf{A} = \mathbf{U}\mathbf{L}$ and $\mathbf{X}\mathbf{A}$ is a matrix whose columns contain the scores of the corresponding *PC*. A more detailed proof can be found in Jolliffe (1986).

¹ A generic matrix \mathbf{A} of size ($n \times m$) with orthonormal columns follows the property $\mathbf{A}\mathbf{A}' = \mathbf{I}_n$, i.e. $\mathbf{A}' = \mathbf{A}^{-1}$

² Two generic ($n \times n$) matrices \mathbf{A} and \mathbf{B} are similar if there exists an invertible ($n \times n$) matrix \mathbf{P} such that $\mathbf{A} = \mathbf{P}^{-1}\mathbf{B}\mathbf{P}$

APPENDIX B – Time Series Modelling

Review

A time series is a sequence of observations in time. Let $x_t, x_{t+1}, x_{t+2}, \dots, x_{t+N-1}$ be a set of N successive observations taken at equidistant time intervals. The time of each observation is denoted by the subscript. More generally, we can write the same sequence as $x_{t_0+h}, x_{t_0+2h}, x_{t_0+3h}, \dots, x_{t_0+Nh}$, where t_0 is the origin of our series and h is the time interval of each measurement. In general, we denote x_t a observation made at time t .

This appendix is intended to provide a quick review for stationary time series as well as ARIMA models. For further reference, extensions and applications of time series, please use the same references used for this work Box et al. (2015), Durbin and Koopman (2012), Enders (2014) and Lütkepohl (2005).

B.1 Stationarity

Consider the observations $x_{t_1+h}, x_{t_2+h}, \dots, x_{t_N+h}$ of a stochastic process $\{X_t\}$ and let $F(x_{t_1+h}, x_{t_2+h}, \dots, x_{t_N+h})$ be the cumulative distribution of the joint distribution of such process. Then, $\{X_t\}$ is said to be *strictly stationary* if for any time shift s , backward or forward, the joint distribution remains unaffected, as follows:

$$F(x_{t_1+h}, x_{t_2+h}, \dots, x_{t_N+h}) = F(x_{t_1-s+h}, x_{t_2-s+h}, \dots, x_{t_N-s+h}) \quad (\text{B.1})$$

This means that all moments of $\{X_t\}$ are time invariant.

For practical purposes, such as time series modelling, it is enough to have up to second order time-invariant finite moments - the process is then *weakly stationary*. Thus, we have time-invariant expected value, variance and covariance structure between time lags. Formally we have (ENDERS, 2014):

$$E[x_t] = E[x_{t-s}] = \mu \quad (\text{B.2})$$

$$E[(x_t - \mu)^2] = E[(x_{t-s} - \mu)^2] = \sigma_x^2 \quad (\text{B.3})$$

$$E[(x_t - \mu)(x_{t-k} - \mu)] = E[(x_{t-s} - \mu)(x_{t-s-k} - \mu)] = \gamma_s, \quad (\text{B.4})$$

for all k, s , where μ and σ_x^2 are the expected value and variance of x_t and γ_s is the autocovariance between x_t and x_{t-s} . Note that when $s = 0$, γ_0 is equal to the variance of x_t . We also define the *autocorrelation coefficient* ρ_s between x_t and x_{t-s} as:

$$\rho_s \equiv \frac{\gamma_s}{\gamma_0} \quad (\text{B.5})$$

From now on, we will use the terms weakly stationary and stationary interchangeably.

B.2 Moving-average process

A moving-average process of order q , i.e. $MA(q)$, is defined as:

$$x_t = \sum_{i=1}^q \theta_i \varepsilon_{t-i}, \quad (\text{B.6})$$

where $\{\varepsilon_t\}$ is a white-noise process, i.e., zero-mean and constant variance and θ_i 's are the parameters of the model. One can, therefore, prove that a white-noise process is stationary, for all θ_i and with infinite q . Moreover, even for a infinite-order process, $MA(\infty)$, one can prove that the process is stationary if its respective coefficients are absolute summable¹, but it is out of the scope of this work to prove it.

B.3 Autoregressive process

A autoregressive process of order p , i.e. $AR(p)$, is defined as:

$$x_t = c + \sum_{i=1}^p \phi_i x_{t-i} + \varepsilon_t, \quad (\text{B.7})$$

where ε_i is a white-noise, c is a constant and ϕ_i 's are the parameters of the model.

Let L be the lag operator such that $L^k x_t = x_{t-k}$, we can rewrite Equation B.7 as:

$$\phi(L)x_t = c + \varepsilon_t, \quad (\text{B.8})$$

where $\phi(L) = (1 - \sum_{i=1}^p \phi_i L^i)$ is defined as the lag polynomial. One can prove that a $AR(p)$ process is stationary if the roots of the lag polynomial lie outside the unit circle.

B.4 Integrated process and unit roots

A process is said to be integrated of order d , i.e. $I(d)$, if it becomes stationary after being differenced d times. Moreover, an $AR(p)$ with d unit roots is an integrated process of order d .

For instance, consider an $AR(p)$ process with d unit roots. We can write such process as:

$$\begin{aligned} \phi(L)x_t &= \phi'(L)(1-L)^d x_t \\ &= \phi'(L)\Delta^d x_t = c + \varepsilon, \end{aligned} \quad (\text{B.9})$$

¹ A sequence is absolute summable if $\sum_{i=0}^{\infty} |a_i| < \infty$

where $\Delta^d x_t = (1 - L)^d x_t$ and $\phi'(L)$ is the reduced lag polynomial without the d unit root terms. Thus, one can prove that the process $\Delta^d x_t$ is stationary.

There are formal statistical tests for unit roots, and therefore, stationarity. One of the most used is the Augmented Dickey-Fuller test, where null hypothesis is that there is a unit root with the alternative that the series is stationary (or trend-stationary, depending on how the statistic is built). It is out of the scope of this work to show how the statistic is calculated.

B.5 ARIMA

A process can be composed by a $AR(p)$ and $MA(q)$ parts, after being differenced d times. Thus, it can be modelled as an $ARIMA(p, d, q)$. Stationarity of such process is determined by the roots of the lag polynomial of the $AR(p)$ part, since the series has been already differenced d times to be stationary and $MA(q)$ process are always stationary. Its mathematical representation is:

$$\phi(L)\Delta^d x_t = c + \theta(L)\epsilon_t, \quad (B.10)$$

where $\phi'(L)$ is lag polynomial for the $AR(p)$ part, Δ^d is the Integrated part, $\theta(L)$ is the lag polynomial for the $MA(q)$ part and c is the constant term.

B.6 Autocorrelation and Partial Autocorrelation Functions

Consider again Equation B.5 the gives us the correlation coefficient for each time shift s . The autocorrelation function (ACF), or sometimes called correlogram, can be seen as a plot of ρ_s against s .

To illustrate it, suppose and $AR(1)$ process. Although not explicit, x_t is also correlated to x_{t-2} , through ρ_1 , i.e. $\rho_2 = (\rho_1)^2$. Therefore, given the stationary conditions for a $AR(1)$ process, it should converge geometrically to zero, as $|\rho_1| < 1$. The partial autocorrelation function (PACF), on the other hand, is the same plot, but controlled by the effect of the other correlation coefficients. That is to say, for a $AR(1)$ process, the PACF has only one significant coefficient at lag 1 and all the other are statistically zero. For a $MA(1)$ process the functions are inverted. Table B.1 summarizes the plots for each process, given stationary series.

Table B.1 – Properties of ACF and PACF

ACF		PACF
MA(q)	Spikes up to lag q	Decays towards zero (may oscillate)
AR(p)	Decays towards zero (may oscillate)	Spikes up to lag p
ARMA(p,q)	Decay after lag q (may oscillate)	Decay after lag p

Note: Adapted from Enders (2014)

B.7 Vector Autoregressive process

A Vector Autoregressive process (VAR), is a generalization of a AR process. A VAR of order p , i.e. VAR(p), is described as follows:

$$\Phi(L)\mathbf{X}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t, \quad (\text{B.11})$$

where $\mathbf{X}_t = (x_{1t}, \dots, x_{nt})'$ is an $(n \times 1)$ vector with n time series variables, $\Phi(L) = \mathbf{I}_n - \mathbf{A}_1L - \dots - \mathbf{A}_pL^p$ is the lag polynomial where each \mathbf{A}_i is an $(n \times n)$ coefficient matrix, \mathbf{c} is an $(n \times 1)$ constant vector and $\boldsymbol{\varepsilon}_t$ is an $(n \times 1)$ vector of unobservable zero-mean white noise process.

VAR models are very useful since it allows multiple time series to be modelled together and allows to study the correlation between them. All n time series variable is assumed to be endogenous, since there are n equations of each variable depending on up to p lags of itself and all the other variables. The parameters of each equation can be estimated with a standard OLS approach. It is widely used to study macroeconomic effects and understand the underlying covariance structure.

A VAR(p) model is stable if the roots of the lag polynomial $\Phi(L)$ lie outside of the unit circle - as it is for regular AR(p) models - and it assumed that \mathbf{X}_t is stationary.

Annex

ANNEX A – VAR results

Table A.1 – Complete VAR estimates

Dependent variable:					
	<i>EF</i> (1)	<i>OF</i> (2)	<i>Ret</i> (3)	ΔVm (4)	ΔVtr (5)
<i>EF</i> _{<i>t</i>-1}	.338*** (.019)	.022 (.017)	-.001** (.0003)	.003* (.002)	-.004 (.004)
<i>EF</i> _{<i>t</i>-2}	.102*** (.020)	-.009 (.018)	.0002 (.0003)	.001 (.002)	-.006 (.004)
<i>EF</i> _{<i>t</i>-3}	.073*** (.020)	.001 (.018)	-.0002 (.0003)	.002 (.002)	.001 (.004)
<i>EF</i> _{<i>t</i>-4}	.142*** (.020)	.037*** (.017)	-.0002 (.0003)	-.002 (.002)	.001 (.004)
<i>EF</i> _{<i>t</i>-5}	.212*** (.019)	-.005 (.017)	.0001 (.0003)	-.002 (.002)	.006* (.004)
<i>OF</i> _{<i>t</i>-1}	.093*** (.024)	.217*** (.020)	.0003 (.0003)	.001 (.002)	.0003 (.005)
<i>OF</i> _{<i>t</i>-2}	.042* (.024)	.113*** (.021)	-.0002 (.0004)	.0001 (.002)	.008* (.005)
<i>OF</i> _{<i>t</i>-3}	.017 (.024)	.088*** (.021)	.0004 (.0004)	-.0005 (.002)	.002 (.005)
<i>OF</i> _{<i>t</i>-4}	.004 (.024)	.102*** (.021)	.00004 (.0004)	.001 (.002)	-.010** (.005)
<i>OF</i> _{<i>t</i>-5}	.012 (.023)	.106*** (.020)	.0002 (.0003)	.002 (.002)	.005 (.005)
<i>Ret</i> _{<i>t</i>-1}	-.2736 (2.036)	3.757** (1.751)	-.173*** (.029)	.290* (.169)	-.348 (.394)
<i>Ret</i> _{<i>t</i>-2}	-.5.512*** (2.055)	-.425 (1.766)	-.053* (.030)	.043 (.171)	.011 (.397)
<i>Ret</i> _{<i>t</i>-3}	-.328 (2.052)	-3.081* (1.764)	.064** (.030)	.079 (.171)	.100 (.397)
<i>Ret</i> _{<i>t</i>-4}	1.039 (2.054)	-2.153 (1.766)	-.035 (.030)	-.142 (.171)	-.670* (.397)
<i>Ret</i> _{<i>t</i>-5}	-1.127 (2.039)	.205 (1.753)	-.058** (.029)	.110 (.169)	.701* (.394)
ΔVm_{t-1}	.006 (.102)	-.015 (.088)	.0005 (.001)	-.011 (.008)	-.484*** (.020)
ΔVm_{t-2}	-.093 (.112)	-.037 (.096)	.001 (.002)	-.010 (.009)	-.301*** (.022)
ΔVm_{t-3}	-.280** (.113)	.041 (.097)	-.0002 (.002)	-.015 (.009)	-.239*** (.022)
ΔVm_{t-4}	-.158 (.112)	.040 (.096)	-.001 (.002)	-.009 (.009)	-.172*** (.022)
ΔVm_{t-5}	-.012 (.101)	.078 (.087)	.001 (.001)	-.013 (.008)	-.083*** (.020)
ΔVtr_{t-1}	.472 (.356)	-.770** (.306)	-.008 (.005)	-.075** (.030)	.176** (.069)
ΔVtr_{t-2}	-.942*** (.357)	-.090 (.307)	-.004 (.005)	-.054* (.030)	.134* (.069)
ΔVtr_{t-3}	.341 (.357)	-.059 (.307)	.009* (.005)	-.033 (.030)	.159** (.069)
ΔVtr_{t-4}	.347 (.357)	-.181 (.307)	.004 (.005)	-.074** (.030)	.048 (.069)
ΔVtr_{t-5}	-.691* (.353)	.313 (.304)	.002 (.005)	-.019 (.029)	.093 (.068)
<i>const</i>	.015 (.017)	-.020 (.015)	.0003 (.0002)	-.0.0000 (.001)	.002 (.003)
Observations	2,646	2,646	2,646	2,646	2,646
R ²	.574	.212	.033	.028	.206
Adjusted R ²	.570	.204	.023	.019	.198
Residual Std. Error (df = 2620)	.881	.758	.013	.073	.170
F Statistic (df = 25; 2620)	141.344***	28.159***	3.544***	3.023***	27.172***

Note:
*p<0.1; **p<0.05; ***p<0.01
Standard deviations in parenthesis

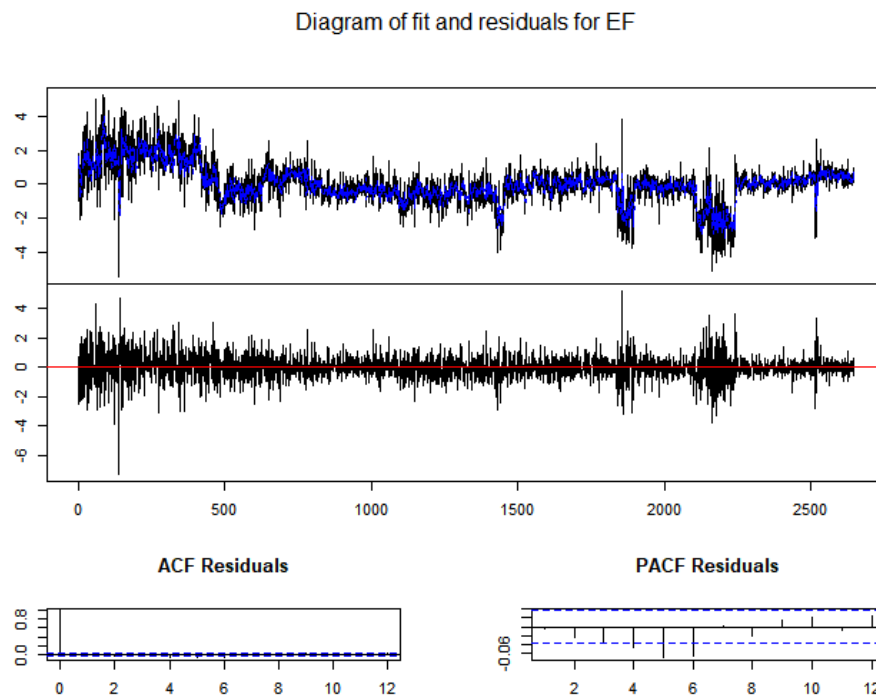


Figure A.1 – Residual diagnostic plots from Engagement Factor estimation

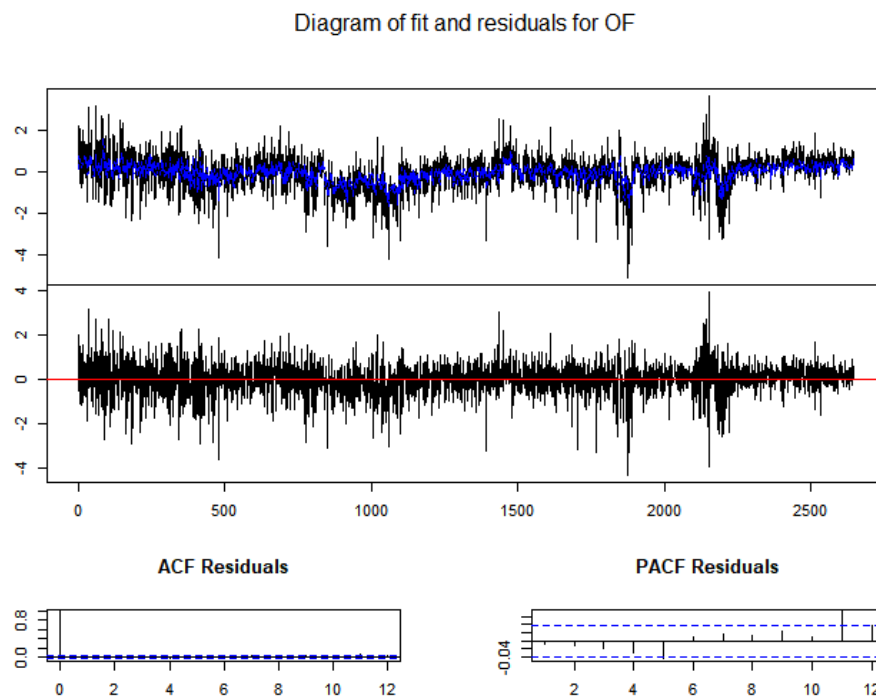


Figure A.2 – Residual diagnostic plots from Optimism Factor estimation

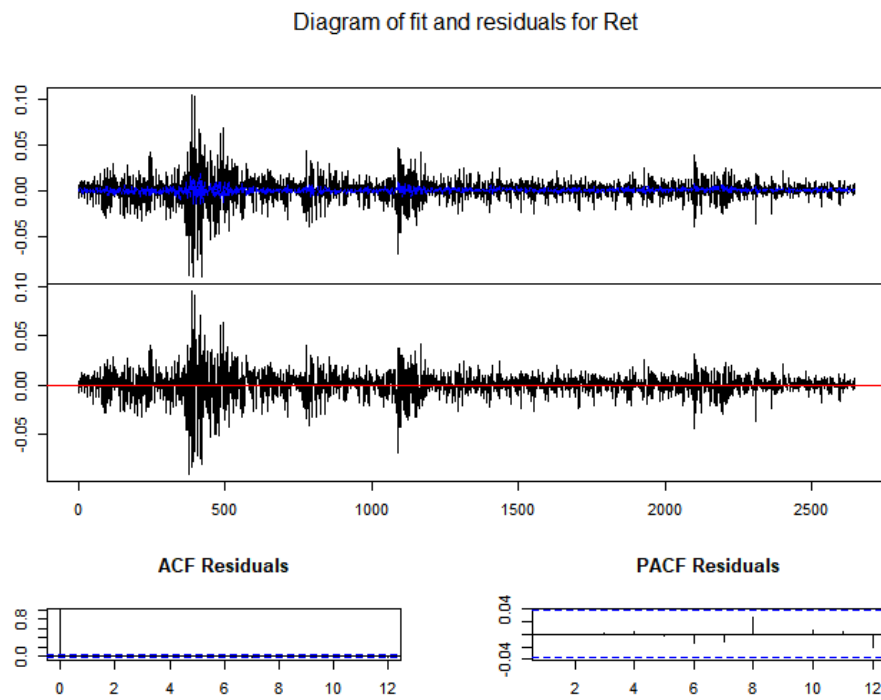


Figure A.3 – Residual diagnostic plots from Returns estimation

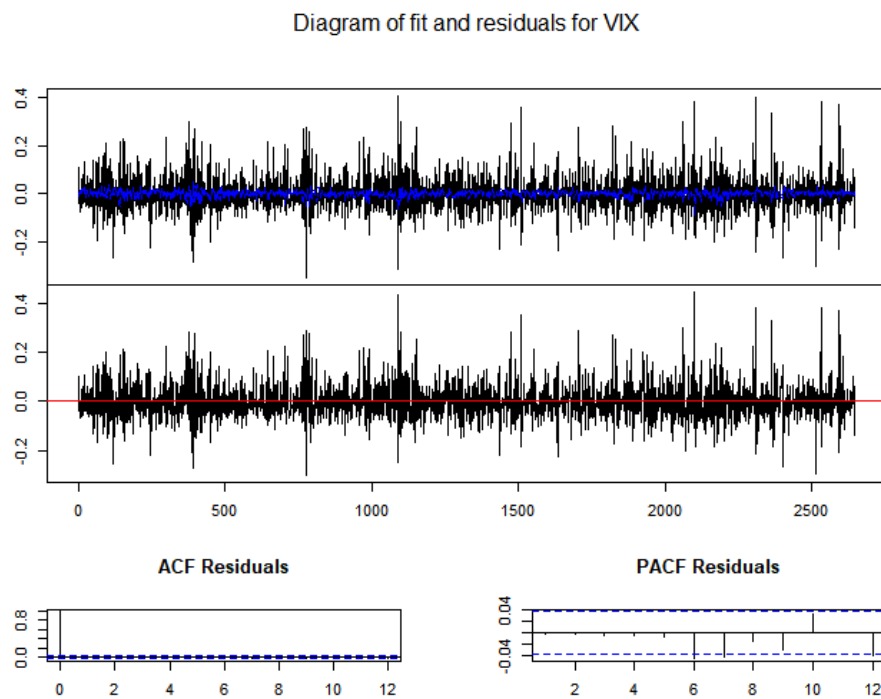


Figure A.4 – Residual diagnostic plots from VIX estimation

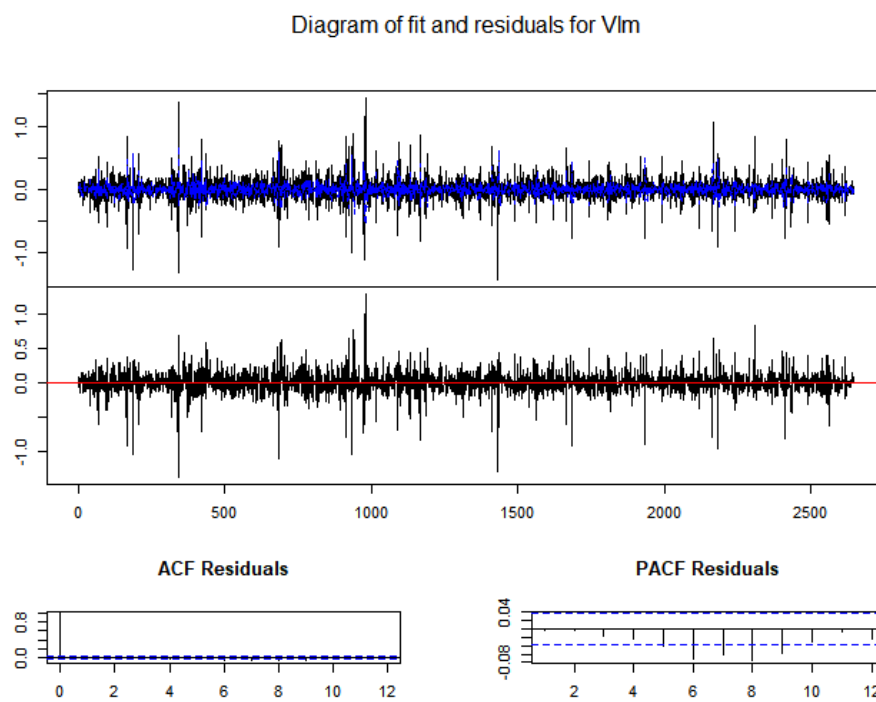


Figure A.5 – Residual diagnostic plots from Volume estimation