

# 1 Heuristic Analysis

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## 1.1 Overall comparison

The final performance is as follows, in terms of average of winning: 1. **Custom 3**: 71.4% 2. **Custom 1**: 67.1% 3. **Reference**: 65.7% 4. **Custom 2**: 62.9%

The simplest heuristic (ratio of remaining legal moves) outperformed the improved and second ones. The third heuristic, a bit more sophisticated as we will see below, outperformed all the others. The final recommendation is to choose the third heuristic over the others, including the reference one, since: (i) such heuristic just performed worse than the reference against Random and AB Improved games, as shown in Table I; (ii) when using it, it never loses the majority of games against, i.e., its worst result was a draw (5x5); and finally (iv) such heuristic is easy to follow, fast to implement, computationally light and intuitive, as we will see below.

When the agent uses the first heuristic for scoring, it also performed better than the benchmark. However, it has the drawback of recording a majority of loses in one game (AB Improved). The second heuristic is the worst performer of all custom heuristics and, on top of that, is not any better than the reference. Thus, the choice of heuristics would be in the following order: Custom 3, Custom 1, Reference. The Custom 3 was left out since, once again, we already have a benchmark that performs better.

Table I: Heuristics comparison in number of wins out of 10 games

Match No.	Opponent	Reference	Custom 1	Custom 2	Custom 3
1	Random	10	8	9	9
2	MM Open	6	6	8	9
3	MM Center	9	8	8	10
4	MM Improved	6	8	6	7
5	AB Open	4	6	3	5
6	AB Center	5	7	6	5
7	AB Improved	6	4	4	5

### 1.1.1 Heuristic 1

The first heuristic follows the intuition that the more legal moves left at any given moment, the better, but with a twist. It actually compares the active player's legal moves at the given time with its opponent's. Therefore, if the opponent has more legal moves left than the active player, then the active player's score is worsened.

The implementation is calculated with natural logarithms of the remaining legal moves with biased by one unit (because there can be zero remaining moves and its logarithm is undefined).

$$score = \log(M_a + 1) - \log(M_o + 1),$$

where  $M_i$  is the remaining legal moves  $M$  of the player  $i$ , where  $a$  stands for active player and  $o$  for opponent.

### 1.1.2 Heuristic 2

The second heuristic is the (Euclidean) distance between players. It follows that, presumably, the further a player is from its opponent, the better. Surely, it also depends on the remaining legal moves, but this is left to the next heuristic.

$$score = \sqrt{(x_a - x_o)^2 + (y_a - y_o)^2},$$

where  $(x_i, y_i)$  are the position's coordinates from the player  $i$ , where  $a$  stands for active player and  $o$  for opponent.

### 1.1.3 Heuristic 3

The third heuristic is an extension of the other two. It is the difference of each player's to the center weighted by its remaining legal moves at the given time. It follows that the more central positioned a player is, the better options it has, but to ensure it has more option.

$$score = \log(D_a M_a + 1) - \log(D_o M_o + 1),$$

where  $M_i$  is the remaining legal moves  $M$  of the player  $i$ , and  $D_i$  stands for the player's  $i$  distance from the center of the board, where  $a$  stands for active player and  $o$  for opponent