A Heston implementation

A Thesis

Presented to

The Division of Applied Mathematics

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M.Sc. of Mathematics

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_____

Advisor F. Name

Acknowledgements

I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class (for LaTeX) and the R bookdown package, in general.

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Abstract

The preface pretty much says it all.

Second paragraph of abstract starts here.

Dedication

You can have a dedication here if you wish.

Chapter 1

Introduction

Welcome to the *R Markdown* thesis template. This template is based on (and in many places copied directly from) the Reed College LaTeX template, but hopefully it will provide a nicer interface for those that have never used TeX or LaTeX before. Using *R Markdown* will also allow you to easily keep track of your analyses in **R** chunks of code, with the resulting plots and output included as well. The hope is this *R Markdown* template gets you in the habit of doing reproducible research, which benefits you long-term as a researcher, but also will greatly help anyone that is trying to reproduce or build onto your results down the road.

Hopefully, you won't have much of a learning period to go through and you will reap the benefits of a nicely formatted thesis. The use of LaTeX in combination with *Markdown* is more consistent than the output of a word processor, much less prone to corruption or crashing, and the resulting file is smaller than a Word file. While you may have never had problems using Word in the past, your thesis is likely going to be about twice as large and complex as anything you've written before, taxing Word's capabilities. After working with *Markdown* and **R** together for a few weeks, we are confident this will be your reporting style of choice going forward.

Why use it?

R Markdown creates a simple and straightforward way to interface with the beauty of LaTeX. Packages have been written in **R** to work directly with LaTeX to produce nicely formatting tables and paragraphs. In addition to creating a user friendly interface to LaTeX, *R Markdown* also allows you to read in your data, to analyze it and to visualize it using **R** functions, and also to provide the documentation and commentary on the results of your project. Further, it allows for **R** results to be passed inline to the commentary of your results. You'll see more on this later.

Who should use it?

Anyone who needs to use data analysis, math, tables, a lot of figures, complex cross-references, or who just cares about the final appearance of their document should use *R Markdown.* Of particular use should be anyone in the sciences, but the user-friendly nature of *Markdown* and its ability to keep track of and easily include figures, automatically generate a table of contents, index, references, table of figures, etc. should make it of great benefit to nearly anyone writing a thesis project.

Chapter 2

Literature Review

This chapter presents the concepts of stochastic calculus, from the historic conception of how it first arose through the basic principles and applications in finance. More precisely, we address the classical Black-Scholes model and its limitations and the Heston model. This model is also well known, it introduces the concept of stochastic volatility which brings us closer to reality.

2.1 Stochastic Calculus

Stochastic calculus arises from stochastic processes and allows the creation of a theory of integration where both the integrand and integrator terms are stochastic processes. Stochastic calculus, also known as, Itô calculus due to the name of it's creator, the Japanese mathematician Kiyosi Itô in the 1940s and 1950s is used for modelling financial options and in another wide variety of fields [1]. In this chapter we present the historical contexts in which the tools and models used arise, but our focus is introducing the concepts and notations that will be further used in our work.

2.1.1 Brownian Motion

The Brownian motion is the name given to the irregular motion observed in the motion of pollen particles suspended in fluid resulting from particle collision with atoms or molecules. It is named after Robert Brown, the first to have observed the movement in 1828. He noted two characteristic in the pollen movement [1]:

- the path of a givern particle is very irregular, having a tangent at no point

- the motion of two distinct particles appear to be independent

The first quantitative works in brownian motion come from an interest in stock price fluctuation by Bachelier in 1900. Albert Einstein also leaned over the subject and in 1905 derived the transition density for Brownian motion from molecular-kinectic theory of heat [1,2].

In 1923, the Wiener process was coined in honor of Norbert Wiener mathematical proof of existence of the brownian motion and stating its properties as follows [3]:

- $W_0 = 0$

- The change in W, given by $\Delta W = W_{t+1} - W_t$, is normally distributed with mean Wero and standard deviation $\sqrt{\Delta t}$, meaning that $\Delta W = \epsilon\sqrt{\Delta t}$, where ϵ is $N(0, 1)$.

- If the increment Δt_1 does not overlap with the time increment Δt_2, then ΔW_1 and ΔW_2 are independent.

- The process is continuous, meaning that there are no jumps in the process.

- The process is a Markov process. This means that the conditional expectation of W_{t+1} given its entire history is equal to the conditional expectation of W_{t+1} given today's information. This can be written as: $E[W_{t+1}|W_1, ..., W_t] = E[W_{t+1}|W_t]$.

- Consider the time interval $[0, t]$ with n equally spaced intervals given by $t_i = \frac{it}{n}$. Then the paths of the Brownian motion have unbounded variation, this means that they are not differentiable and go towards infinity as n increases. The quadratic variation is given by $\sum_{i=1}^{n}\left(Z_{t_i} - Z_{t_{i-1}}\right)^2 \to t$, meaning that when n increases it stays constant at t.

2.1.2   Itô's Lemma

Let X_t be a real-valued stochastic process that satisfies [4–6]:

$$X_t = X_0 + \int_0^t \mu_t dt + \int_0^t \sigma_t dW_t \tag{2.1}$$

for some μ_t, σ_t and $t \in [0, T]$. This equation is often rewritten in its differential stochastic form:

$$dX_t = \mu_t dt + \sigma_t dW_t \tag{2.2}$$

for $0 \leq t \leq T$.

Theorem

Assume that X_t has a stochastic differential given by:

$$dX_t = \mu_t dt + \sigma_t dW_t \tag{2.3}$$

for μ_t, σ_t and $t \in [0, T]$. Assume $u : \mathbb{R} \times [0, T] \to \mathbb{R}$ is continuous and that $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial x}$, $\frac{\partial^2 u}{\partial x^2}$ exist and are continuous.

$$Y_t := u(X_t, t)$$

Then Y has the following stochastic differential:

$$dY_t = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dX_t + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma_t^2 dt$$

$$= \left(\frac{\partial u}{\partial t} + \mu_t\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}\sigma_t^2\right)dt + \sigma_t\frac{\partial u}{\partial x}dW_t$$

(2.4)

where the argument of u, $\frac{\partial u}{\partial x}$ and $\frac{\partial^2 u}{\partial x^2}$ above is (X_t, t) .

Equation (2.4) is the stochastic equivalent to the chain rule, also known as Itô's formula or Itô's chain rule. The proof to this theorem is based on the Taylor expansion of the function $f(X_t, t)$ [4,5]. For practical uses you should write out a second-order Taylor expansion for the function to be analyzed and apply the 2.1 multiplication table [1].

Table 2.1: Box calculus

	dt	dW_t
dt	0	0
dW_t	0	dt

the **??** multiplication table.
??

```
par(mar = c(4, 4, 0.1, 0.1))
plot(pressure, pch = 19, type = "b")
```
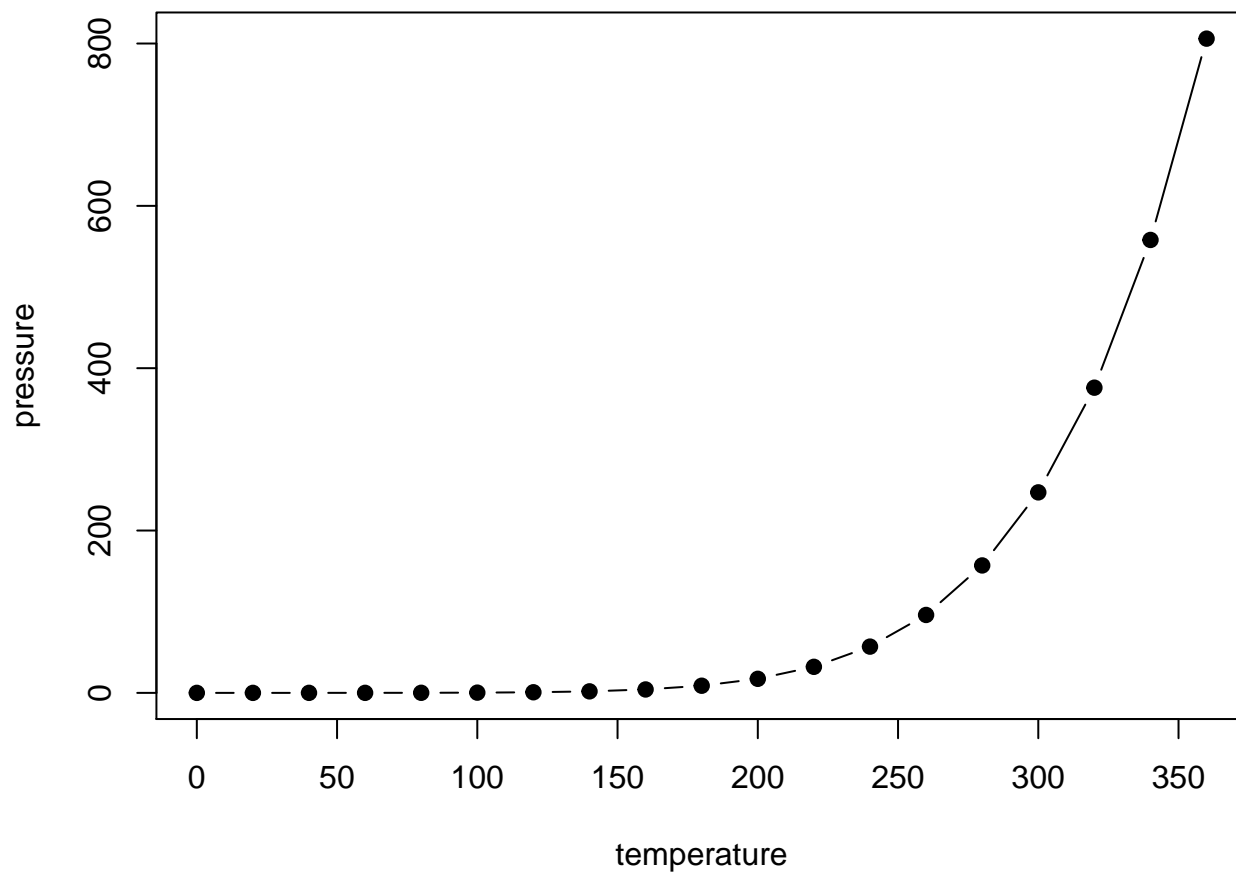
2.2   Black-Scholes Model

- Model

2.2.1   Derivative Contracts

European Call and Put

2.2.2   Limitations

2.3   Heston Model

Chapter 3

Mathematics and Science

3.1 Math

T_EX is the best way to typeset mathematics. Donald Knuth designed T_EX when he got frustrated at how long it was taking the typesetters to finish his book, which contained a lot of mathematics. One nice feature of *R Markdown* is its ability to read LaTeX code directly.

If you are doing a thesis that will involve lots of math, you will want to read the following section which has been commented out. If you're not going to use math, skip over or delete this next commented section.

3.2 Chemistry 101: Symbols

Chemical formulas will look best if they are not italicized. Get around math mode's automatic italicizing in LaTeX by using the argument `$\mathrm{formula here}$`, with your formula inside the curly brackets. (Notice the use of the backticks here which enclose text that acts as code.)

So, $\mathrm{Fe_2^{2+}Cr_2O_4}$ is written `$\mathrm{Fe_2^{2+}Cr_2O_4}$`.
Exponent or Superscript: O^-
Subscript: CH_4

To stack numbers or letters as in Fe_2^{2+}, the subscript is defined first, and then the superscript is defined.
Bullet: $CuCl \bullet 7H_2O$
Delta: Δ
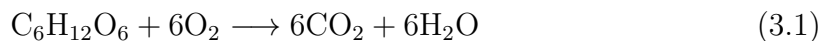Reaction Arrows: $\longrightarrow$ or $\xrightarrow{solution}$
Resonance Arrows: $\leftrightarrow$
Reversible Reaction Arrows: $\rightleftharpoons$

3.2.1 Typesetting reactions

You may wish to put your reaction in an equation environment, which means that LaTeX will place the reaction where it fits and will number the equations for you.

$$C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O \tag{3.1}$$

We can reference this combustion of glucose reaction via Equation (3.1).

3.2.2   Other examples of reactions

$NH_4Cl_{(s)} \rightleftharpoons NH_{3(g)} + HCl_{(g)}$

$MeCH_2Br + Mg \xrightarrow[below]{above} MeCH_2 \bullet Mg \bullet Br$
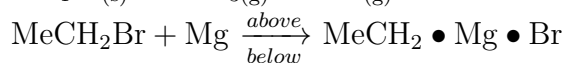
3.3   Physics

Many of the symbols you will need can be found on the math page `http://web.reed.edu/cis/help/latex/math.html` and the Comprehensive LaTeX Symbol Guide (`http://mirror.utexas.edu/ctan/info/symbols/comprehensive/symbols-letter.pdf`).

3.4   Biology

You will probably find the resources at `http://www.lecb.ncifcrf.gov/~toms/latex.html` helpful, particularly the links to bsts for various journals. You may also be interested in TeXShade for nucleotide typesetting (`http://homepages.uni-tuebingen.de/beitz/txe.html`). Be sure to read the proceeding chapter on graphics and tables.

Chapter 4

Tables, Graphics, References, and Labels

4.1 Tables

In addition to the tables that can be automatically generated from a data frame in **R** that you saw in [R Markdown Basics] using the `kable` function, you can also create tables using *pandoc*. (More information is available at `http://pandoc.org/README.html#tables`.) This might be useful if you don't have values specifically stored in **R**, but you'd like to display them in table form. Below is an example. Pay careful attention to the alignment in the table and hyphens to create the rows and columns.

Table 4.1: Correlation of Inheritance Factors for Parents and Child

Factors	Correlation between Parents & Child	Inherited
Education	-0.49	Yes
Socio-Economic Status	0.28	Slight
Income	0.08	No
Family Size	0.18	Slight
Occupational Prestige	0.21	Slight

We can also create a link to the table by doing the following: Table 4.1. If you go back to [Loading and exploring data] and look at the `kable` table, we can create a reference to this max delays table too: Table **??**. The addition of the (`\#tab:inher`) option to the end of the table caption allows us to then make a reference to Table `\@ref(tab:label)`. Note that this reference could appear anywhere throughout the document after the table has appeared.

4.2   Figures

If your thesis has a lot of figures, *R Markdown* might behave better for you than that other word processor. One perk is that it will automatically number the figures accordingly in each chapter. You'll also be able to create a label for each figure, add a caption, and then reference the figure in a way similar to what we saw with tables earlier. If you label your figures, you can move the figures around and *R Markdown* will automatically adjust the numbering for you. No need for you to remember! So that you don't have to get too far into LaTeX to do this, a couple **R** functions have been created for you to assist. You'll see their use below.

    In the **R** chunk below, we will load in a picture stored as `reed.jpg` in our main directory. We then give it the caption of "Reed logo", the label of "reedlogo", and specify that this is a figure. Make note of the different **R** chunk options that are given in the R Markdown file (not shown in the knitted document).

```r
include_graphics(path = "figure/reed.jpg")
```


Figure 4.1: Reed logo

    Here is a reference to the Reed logo: Figure 4.1. Note the use of the `fig:` code here. By naming the **R** chunk that contains the figure, we can then reference that figure later as done in the first sentence here. We can also specify the caption for the figure via the R chunk option `fig.cap`.

Below we will investigate how to save the output of an **R** plot and label it in a way similar to that done above. Recall the `flights` dataset from Chapter **??**. (Note that we've shown a different way to reference a section or chapter here.) We will next explore a bar graph with the mean flight departure delays by airline from Portland for 2014. Note also the use of the `scale` parameter which is discussed on the next page.

```
flights %>% group_by(carrier) %>%
  summarize(mean_dep_delay = mean(dep_delay)) %>%
  ggplot(aes(x = carrier, y = mean_dep_delay)) +
  geom_bar(position = "identity", stat = "identity", fill = "red")
```
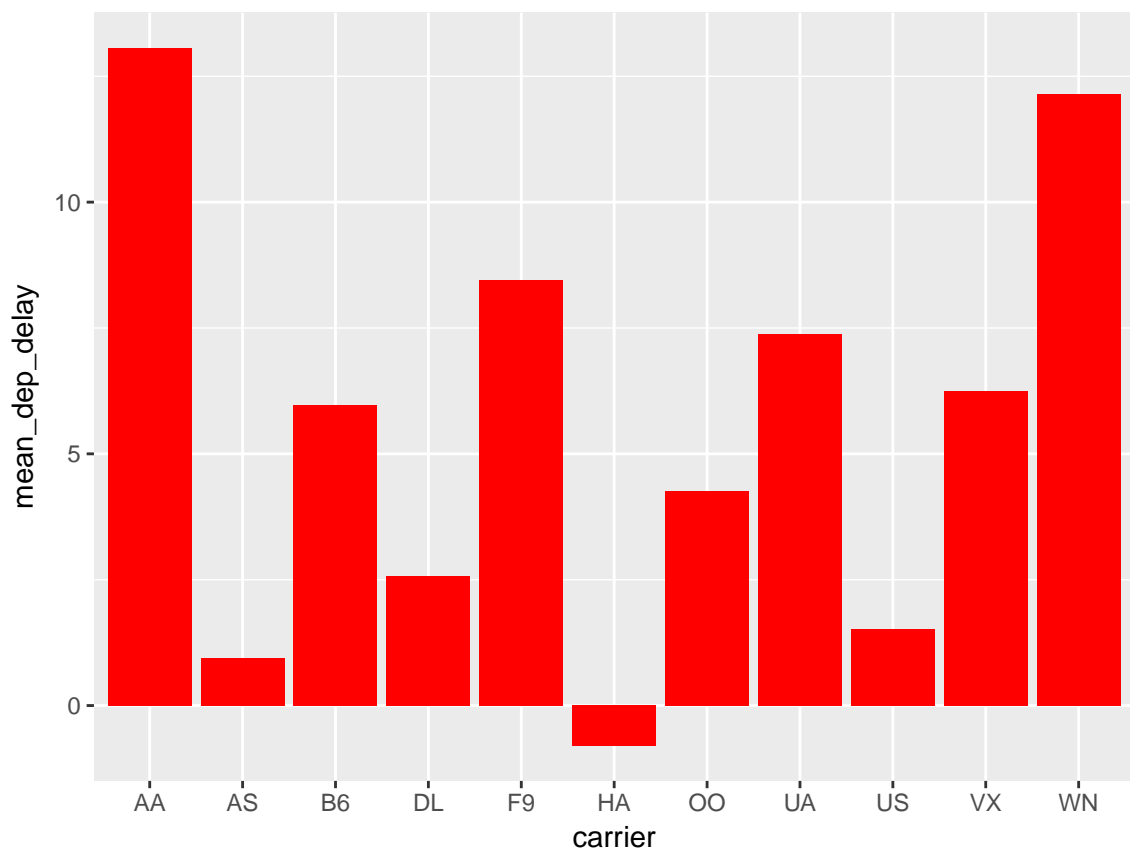

Figure 4.2: Mean Delays by Airline

Here is a reference to this image: Figure 4.2.

A table linking these carrier codes to airline names is available at `https://github.com/ismayc/pnwflights14/blob/master/data/airlines.csv`.

Next, we will explore the use of the `out.extra` chunk option, which can be used to shrink or expand an image loaded from a file by specifying `"scale= "`. Here we use the mathematical graph stored in the "subdivision.pdf" file.

Figure 4.3: Subdiv. graph

Here is a reference to this image: Figure 4.3. Note that `echo=FALSE` is specified so that the **R** code is hidden in the document.

More Figure Stuff

Lastly, we will explore how to rotate and enlarge figures using the `out.extra` chunk option. (Currently this only works in the PDF version of the book.)

Figure 4.4: A Larger Figure, Flipped Upside Down

As another example, here is a reference: Figure 4.4.

4.3   Footnotes and Endnotes

You might want to footnote something.[1] The footnote will be in a smaller font and placed appropriately. Endnotes work in much the same way. More information can be found about both on the CUS site or feel free to reach out to `data@reed.edu`.

4.4   Bibliographies

Of course you will need to cite things, and you will probably accumulate an armful of sources. There are a variety of tools available for creating a bibliography database (stored with the .bib extension). In addition to BibTeX suggested below, you may want to consider using the free and easy-to-use tool called Zotero. The Reed librarians have created Zotero documentation at `http://libguides.reed.edu/`

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[1]footnote text

`citation/zotero`. In addition, a tutorial is available from Middlebury College at `http://sites.middlebury.edu/zoteromiddlebury/`.

R Markdown uses *pandoc* (`http://pandoc.org/`) to build its bibliographies. One nice caveat of this is that you won't have to do a second compile to load in references as standard LaTeX requires. To cite references in your thesis (after creating your bibliography database), place the reference name inside square brackets and precede it by the "at" symbol. For example, here's a reference to a book about worrying: [**???**]. This `Molina1994` entry appears in a file called `thesis.bib` in the `bib` folder. This bibliography database file was created by a program called BibTeX. You can call this file something else if you like (look at the YAML header in the main .Rmd file) and, by default, is to placed in the `bib` folder.

For more information about BibTeX and bibliographies, see our CUS site (`http://web.reed.edu/cis/help/latex/index.html`)[2]. There are three pages on this topic: *bibtex* (which talks about using BibTeX, at `http://web.reed.edu/cis/help/latex/bibtex.html`), *bibtexstyles* (about how to find and use the bibliography style that best suits your needs, at `http://web.reed.edu/cis/help/latex/bibtexstyles.html`) and *bibman* (which covers how to make and maintain a bibliography by hand, without BibTeX, at `http://web.reed.edu/cis/help/latex/bibman.html`). The last page will not be useful unless you have only a few sources.

If you look at the YAML header at the top of the main .Rmd file you can see that we can specify the style of the bibliography by referencing the appropriate csl file. You can download a variety of different style files at `https://www.zotero.org/styles`. Make sure to download the file into the csl folder.

Tips for Bibliographies

- Like with thesis formatting, the sooner you start compiling your bibliography for something as large as thesis, the better. Typing in source after source is mind-numbing enough; do you really want to do it for hours on end in late April? Think of it as procrastination.
- The cite key (a citation's label) needs to be unique from the other entries.
- When you have more than one author or editor, you need to separate each author's name by the word "and" e.g. `Author = {Noble, Sam and Youngberg, Jessica}`,.
- Bibliographies made using BibTeX (whether manually or using a manager) accept LaTeX markup, so you can italicize and add symbols as necessary.
- To force capitalization in an article title or where all lowercase is generally used, bracket the capital letter in curly braces.
- You can add a Reed Thesis citation[3] option. The best way to do this is to use the phdthesis type of citation, and use the optional "type" field to enter "Reed thesis" or "Undergraduate thesis."

---

[2]**???**

[3]**???**

4.5   Anything else?

If you'd like to see examples of other things in this template, please contact the Data @ Reed team (email `data@reed.edu`) with your suggestions. We love to see people using *R Markdown* for their theses, and are happy to help.

Conclusion

If we don't want Conclusion to have a chapter number next to it, we can add the `{-}` attribute.

More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

Appendix A

The First Appendix

This first appendix includes all of the R chunks of code that were hidden throughout the document (using the `include = FALSE` chunk tag) to help with readibility and/or setup.

In the main Rmd file

```r
# This chunk ensures that the thesisdown package is
# installed and loaded. This thesisdown package includes
# the template files for the thesis.
if(!require(devtools))
  install.packages("devtools", repos = "http://cran.rstudio.com")
if(!require(thesisdown))
  devtools::install_github("ismayc/thesisdown")
library(thesisdown)
```

In Chapter 4:

```r
# This chunk ensures that the thesisdown package is
# installed and loaded. This thesisdown package includes
# the template files for the thesis and also two functions
# used for labeling and referencing
if(!require(devtools))
  install.packages("devtools", repos = "http://cran.rstudio.com")
if(!require(dplyr))
    install.packages("dplyr", repos = "http://cran.rstudio.com")
if(!require(ggplot2))
    install.packages("ggplot2", repos = "http://cran.rstudio.com")
if(!require(ggplot2))
    install.packages("bookdown", repos = "http://cran.rstudio.com")
if(!require(thesisdown)){
  library(devtools)
  devtools::install_github("ismayc/thesisdown")
  }
```

```r
library(thesisdown)
flights <- read.csv("data/flights.csv")
```

Appendix B

The Second Appendix, for Fun

References

[1] U.F. Wiersema, Brownian motion calculus, John Wiley & Sons, 2008.

[2] I. Karatzas, S. Shreve, Brownian motion and stochastic calculus, Springer Science & Business Media, 2012.

[3] A.D. Helgadóttir, L. Ionescu, Option pricing within the heston model, (2016).

[4] Z. Tong, Option pricing with long memory stochastic volatility models, PhD thesis, Université d'Ottawa/University of Ottawa, 2012.

[5] L.C. Evans, An introduction to stochastic differential equations, American Mathematical Soc., 2012.

[6] J.M. Steele, Stochastic calculus and financial applications, Springer Science & Business Media, 2012.