A Heston implementation

A Thesis
Presented to
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M.Sc. of Mathematics

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I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class (for LaTeX) and the R bookdown package, in general.

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Abstract

The preface pretty much says it all.

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Dedication

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Chapter 1

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Chapter 2

Literature Review

This chapter presents the concepts of stochastic calculus, from the historic conception of how it first arose through the basic principles and applications in finance. More precisely, we address the classical Black-Scholes model and its limitations and the Heston model. This model is also well known, it introduces the concept of stochastic volatility which brings us closer to reality.

2.1 Stochastic Calculus

Stochastic calculus arises from stochastic processes and allows the creation of a theory of integration where both the integrand and integrator terms are stochastic processes. Stochastic calculus, also known as, Itô calculus due to the name of it's creator, the Japanese mathematician Kiyosi Itô in the 1940s and 1950s is used for modelling financial options and in another wide variety of fields (1). In this chapter we present the historical contexts in which the tools and models used arise, but our focus is introducing the concepts and notations that will be further used in our work.

2.1.1 Brownian Motion

The Brownian motion is the name given to the irregular motion observed in the motion of pollen particles suspended in fluid resulting from particle collision with atoms or molecules. It is named after Robert Brown, the first to have observed the movement in 1828. He noted two characteristic in the pollen movement (1):

- the path of a givern particle is very irregular, having a tangent at no point
- the motion of two distinct particles appear to be independent

The first quantitative works in brownian motion come from an interest in stock price fluctuation by Bachelier in 1900. Albert Einstein also leaned over the subject and in 1905 derived the transition density for Brownian motion from molecular-kinectic theory of heat (1,2).

In 1923, the Wiener process was coined in honor of Norbert Wiener mathematical proof of existence of the brownian motion.

- $W_0 = 0$
- W has independent increments: $W_{t+u} - W_t$ is independent of $\sigma(W_s : s \le t)$ for $u \ \ge 0$
- W has Gaussian increments: $W_{t+u} - W_t$ is normally distributed with mean 0 and variance u, $W_{t+u} - W_t \sim \mathcal{N}(0, u)$
- W has continuous paths: With probability 1, W_t is continuous in t.

2.1.2 Itô's Lemma

2.2 Black-Scholes Model

- Model

2.2.1 Derivative Contracts

European Call and Put

2.2.2 Limitations

2.3 Heston Model

Chapter 3

Mathematics and Science

Chapter 4

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Conclusion

Chapter 5

The First Appendix

References

1 WIERSEMA, U.F. **Brownian motion calculus**. 2008.

2 KARATZAS, I.; SHREVE, S. **Brownian motion and stochastic calculus**. 2012.