Numerical methods for stochastic volatility models:

Heston model

A Thesis
Presented to
The Division of Applied Mathematics
Getulio Vargas Foundation

In Partial Fulfillment
of the Requirements for the Degree
M.Sc. of Mathematics

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julho 23, 2017

Approved for the Division
(Mathematics)

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Acknowledgements

I want to thank a few people.

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Abstract

The preface pretty much says it all.

Second paragraph of abstract starts here.

Dedication

You can have a dedication here if you wish.

Chapter 1

altadvisor: ‘Your Other Advisor’

Chapter 2

Literature Review

Chapter 3

The Heston Model Implementation

3.1 Characteristic Function

The Heston model characteristic function is firstly presented in the 1993 Steven Heston's paper [1] and is described below [2]:

$$f(S_t, V_t, t) = e^{A(T-t)+B(T-t)S_t+C(T-t)V_t+i\phi S_t} \tag{3.1}$$

If we let $\tau = T - t$, then the explicit form of the Heston characteristic function is:

$$\begin{aligned}
f(i\phi) &= e^{A(\tau)+B(\tau)S_t+C(\tau)V_t+i\phi S_t} \\
A(\tau) &= ri\phi\tau + \frac{k\theta}{\sigma^2}\left[-(\rho\sigma i\phi - k - M)\tau - 2\ln\left(\frac{1-Ne^{M\tau}}{1-N}\right)\right] \\
B(\tau) &= 0 \\
C(\tau) &= \frac{(e^{M\tau}-1)(\rho\sigma i\phi - k - M)}{\sigma^2(1-Ne^{M\tau})}
\end{aligned}$$

Where:

$$\begin{aligned}
M &= \sqrt{(\rho\sigma i\phi - k)^2 + \sigma^2(i\phi + \phi^2)} \\
N &= \frac{\rho\sigma i\phi - k - M}{\rho\sigma i\phi - k + M}
\end{aligned}$$

This function is the driving force behind the following formula, that calculates the fair valur of a European call option at time t, given a strike price K, that expires at time T [2]:

$$\begin{aligned}
C =& \frac{1}{2}S(t) + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty \Re\left[\frac{K^{-i\phi}f(i\phi+1)}{i\phi}\right]d\phi \\
&- Ke^{-r(T-t)}\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left[\frac{K^{-i\phi}f(i\phi)}{i\phi}\right]d\phi\right)
\end{aligned} \tag{3.2}$$

3.2 Euler Scheme - Full Truncation

We present here the Euler Scheme - Full Truncation algorithm [3] along with some insights on how it was implemented in the R programming language [4]. The Euler discretization brings approximation paths to stock prices and variance processes. If we set $t_0 = 0 < t_1 < \cdots < t_M = T$ as partitions of a time interval of M equal segments of lenght δt, we have the following discretization for the stock price:

$$S_{t+1} = S_t + rS_t + \sqrt{V_t} S_t Z_s \tag{3.3}$$

And for the variance process:

$$V_{t+1} = V_t + k(\theta - V_t) + \sigma \sqrt{V_t} Z_v \tag{3.4}$$

Z_s being a standard normal random variable, i.e. $N\ (0, 1)$, we set Z_t and Z_v as two independent standard normal random variables and Z_s and Z_v having correlation ρ. This means we can write $Z_s = \rho Z_v + \sqrt{1 - \rho^2} Z_t$

3.3 Karl Jaeckel

3.4 Exact Algorithm

3.5 Teste

```
Call <- function(S,X,tau,r,q,sigma){

  # S = spot
  # X = strike
  # tau = time to maturity
  # r = riskfree rate
  # q = dividend yield
  # sigma = standard deviation

  d1 = (log(S/X) + (r - q + sigma^2/2) * tau) / (sqrt(sigma^2 * tau))
  d2 = d1 - sqrt(sigma^2 * tau)
  c0 = S * exp(-q * tau) * pnorm(d1,0,1) -
       X * exp(-r * tau) * pnorm(d2,0,1)

  return(c0)
}
```

```r
setwd("../rnmethods")
source("../rnmethods/R/euler_heston.R")
```

Chapter 4

This chunk ensures that the thesisdown package is

Chapter 5

Conclusion

Chapter 6

The First Appendix

References

[1] S.L. Heston, A closed-form solution for options with stochastic volatility with applications to bond and currency options, Review of Financial Studies. 6 (1993) 327–343.

[2] R. Dunn, P. Hauser, T. Seibold, H. Gong, Estimating option prices with heston's stochastic volatility model, (2014).

[3] M. Broadie, Ö. Kaya, Exact simulation of stochastic volatility and other affine jump diffusion processes, Operations Research. 54 (2006) 217–231.

[4] R Core Team, R: A language and environment for statistical computing, R Foundation for Statistical Computing, Vienna, Austria, 2017. `https://www.R-project.org/`.