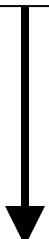
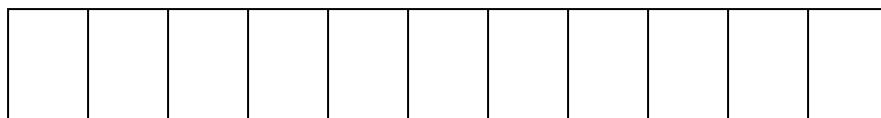


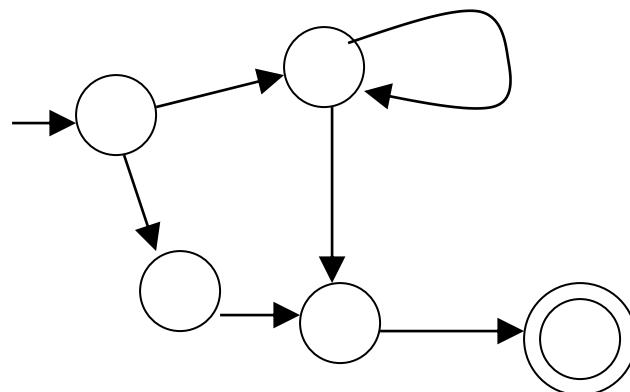
Pushdown Automata PDAs

Pushdown Automaton -- PDA

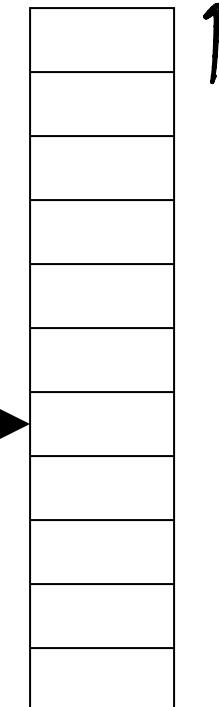
Input String



States



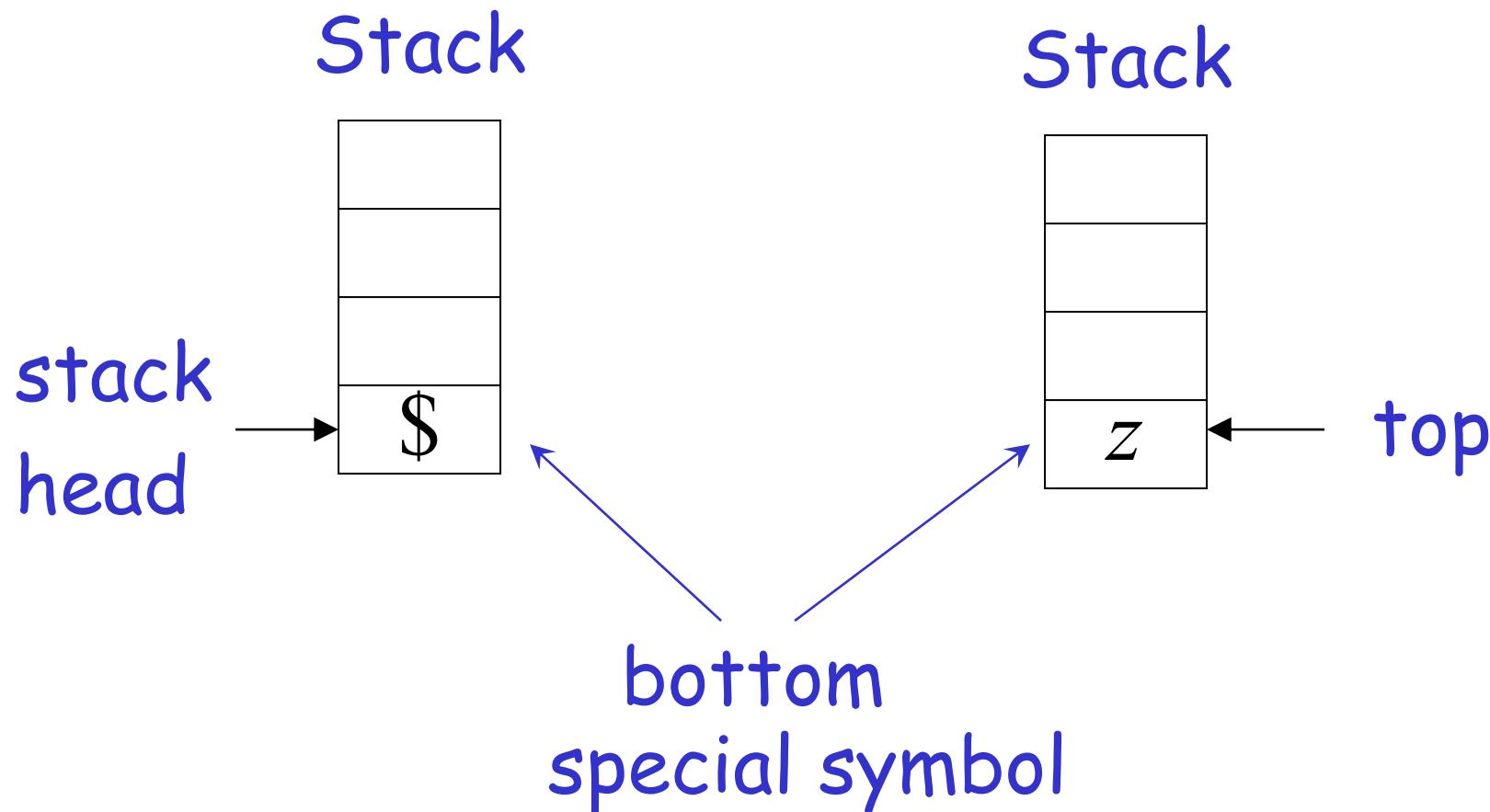
Stack ထားလုပ်များ



ကြော်ခိုက် စတင်ရှိနိုင်
stack တို့ကော်
ဆောင်

Initial Stack Symbol

ຖិន្នន័យ (ចំណាំការពាយ)
Initial define symbol

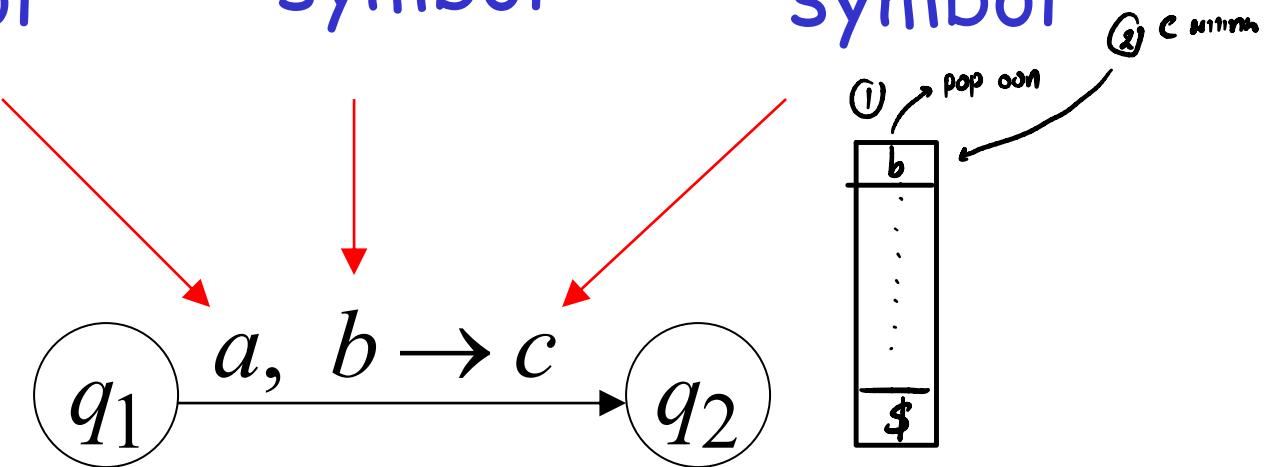


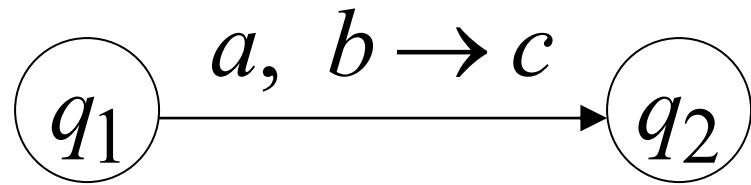
The States

Input
symbol

Pop
on
symbol

Push
on
symbol



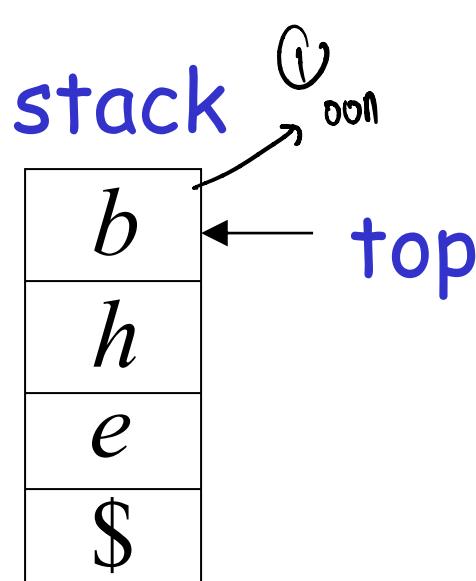


check state, input, Top stack
(ກິດສະຖານົມໃນພົນກຳກົງລົງ)

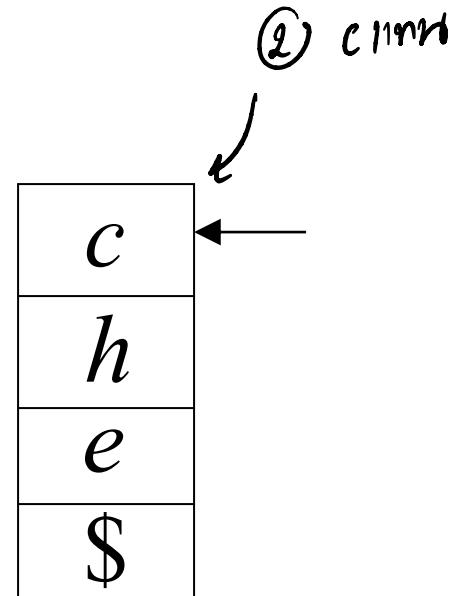
input

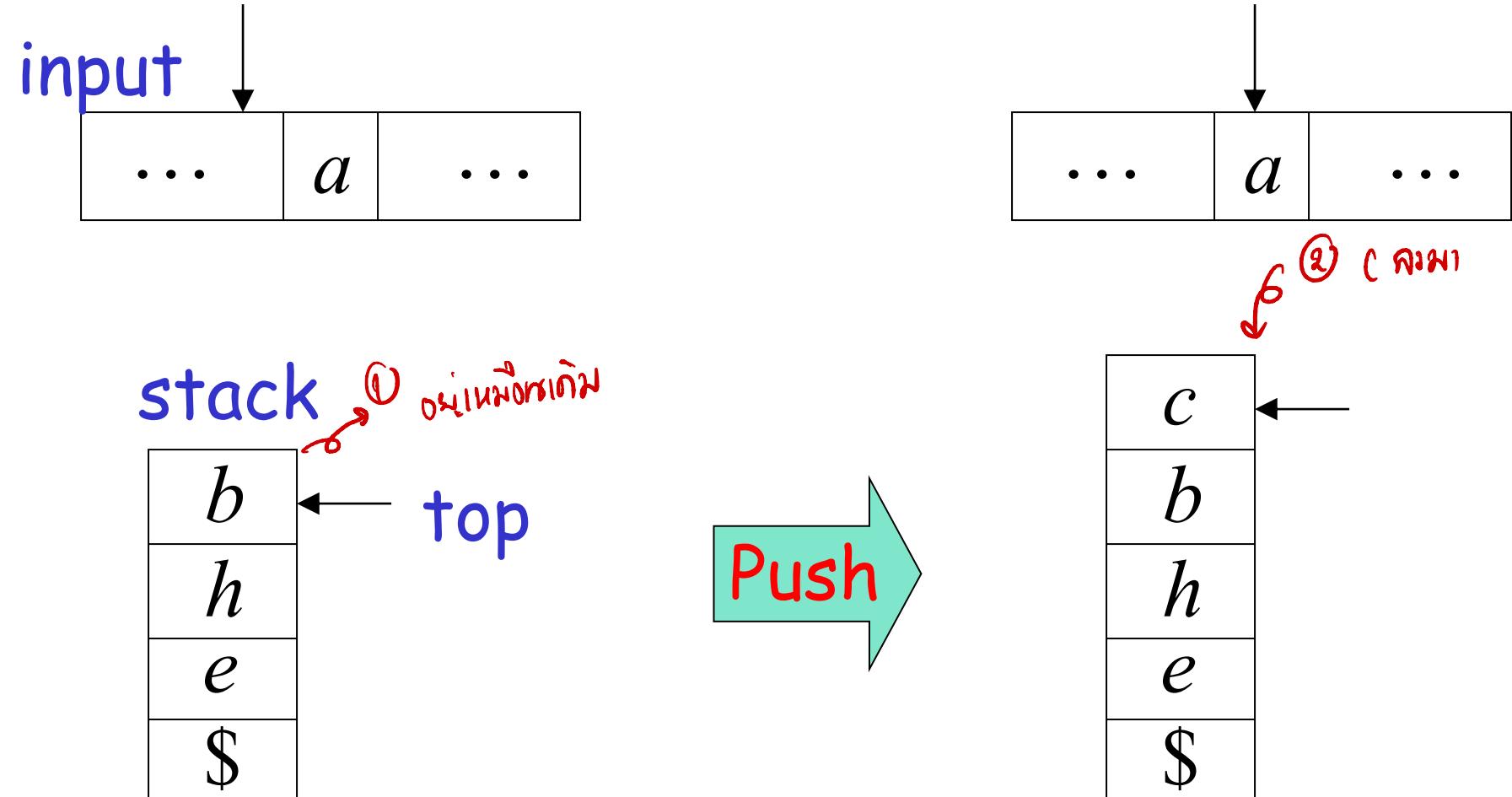
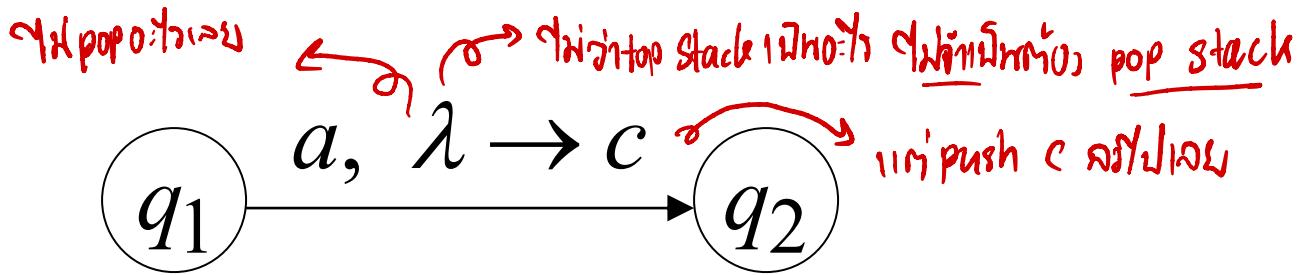
...	a	...
-----	-----	-----

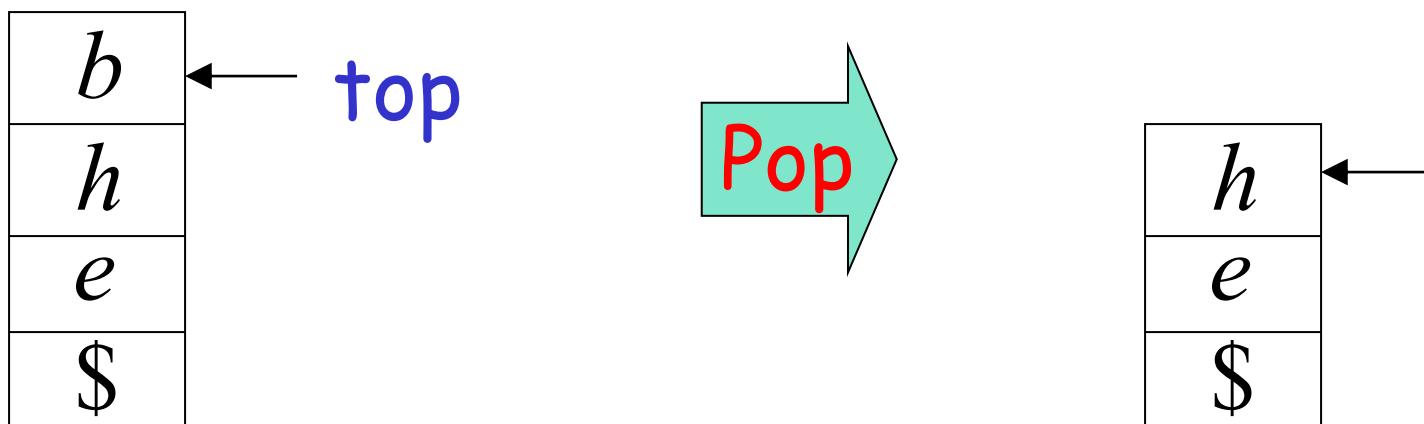
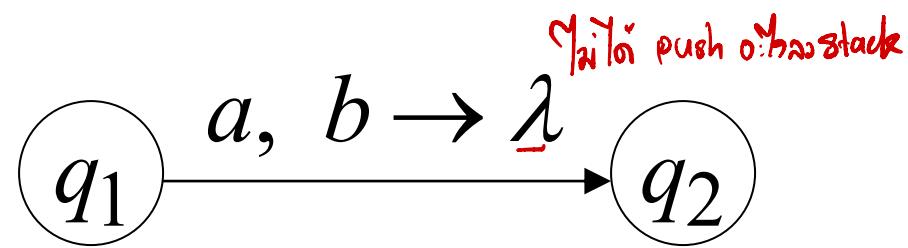
...	a	...
-----	-----	-----

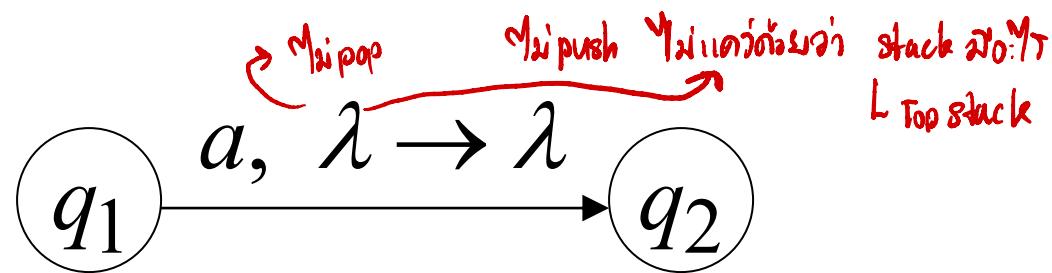


Replace

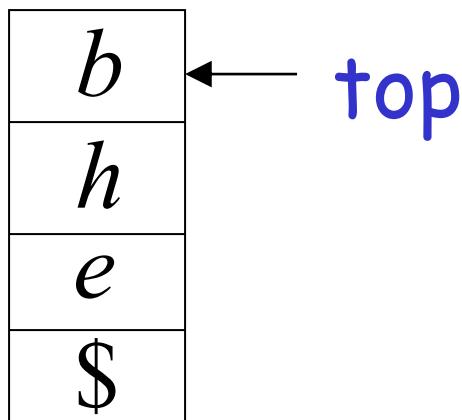




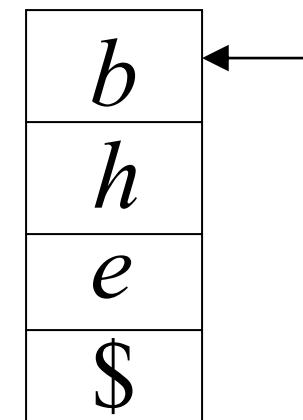




stack



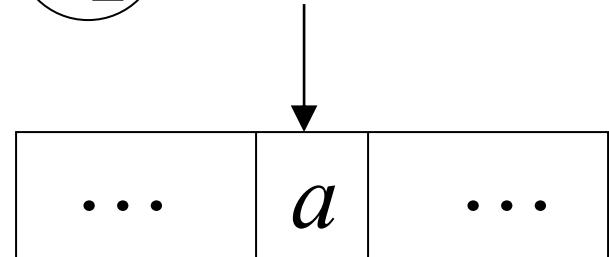
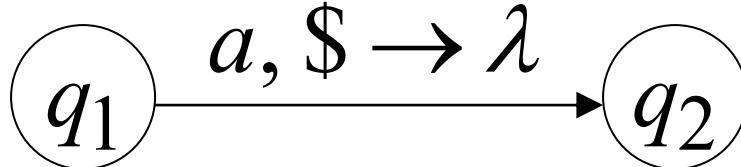
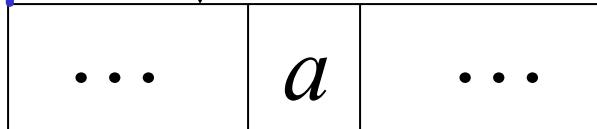
No Change



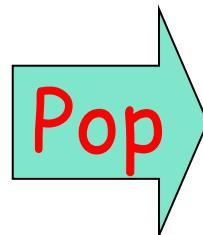
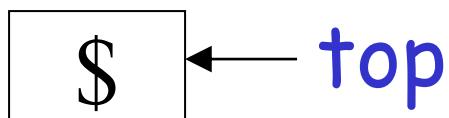
A Possible Transition

លោកអ្នក

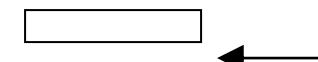
input



stack

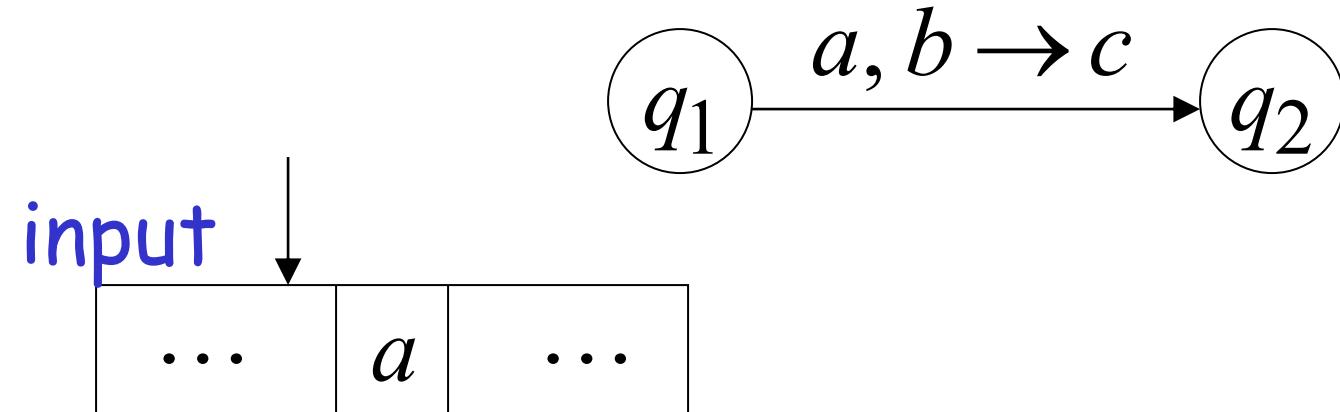


empty

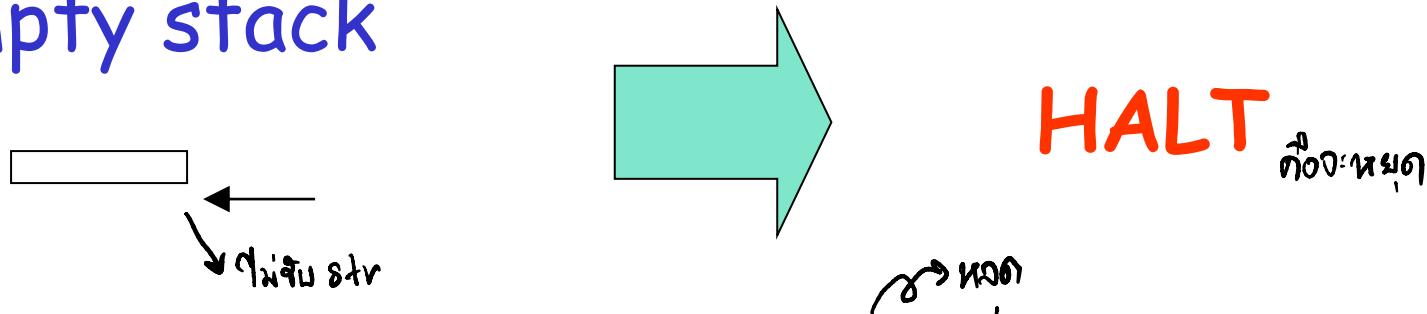


ការណើនីវាទុកសង្គចំបង្ហាញ

A Bad Transition

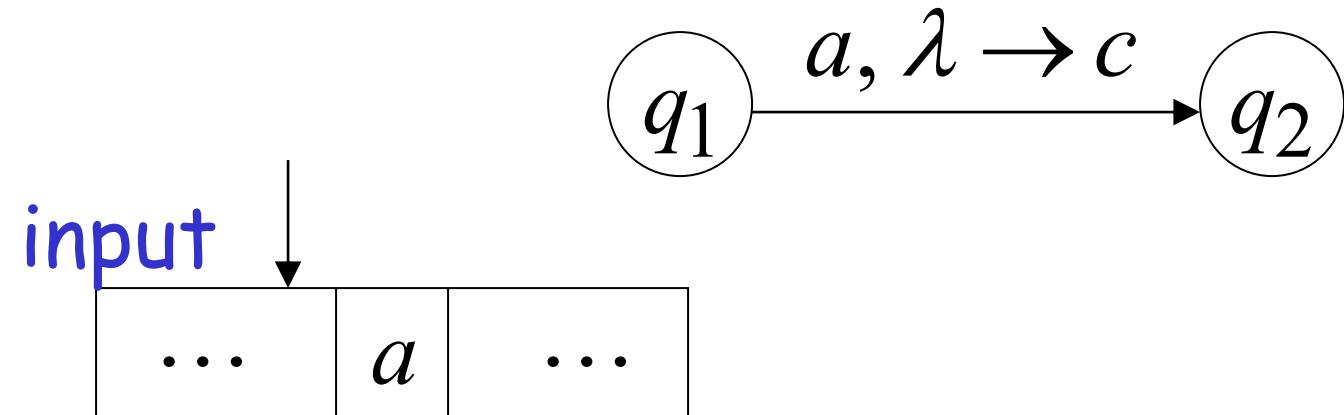


Empty stack

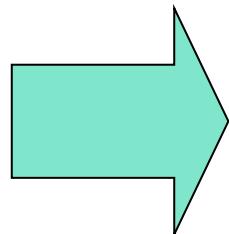
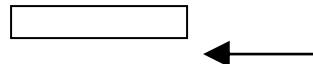


The automaton **Halts** in state q_1
and **Rejects** the input string
stv

A Bad Transition



Empty stack

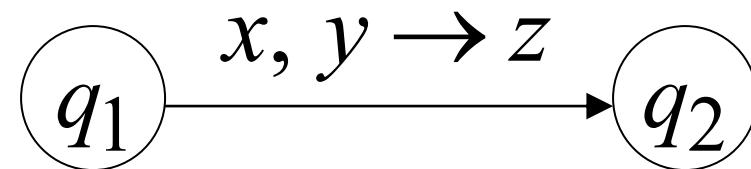


HALT

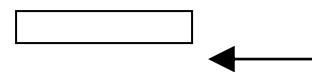
The automaton **Halts** in state q_1
and **Rejects** the input string

2/25 *օրոշումներ*
No transition is allowed to be followed

When the stack is empty



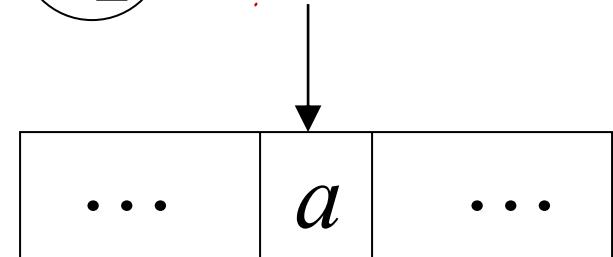
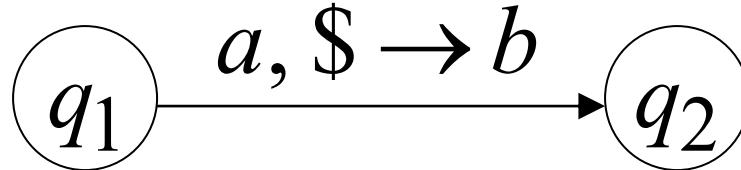
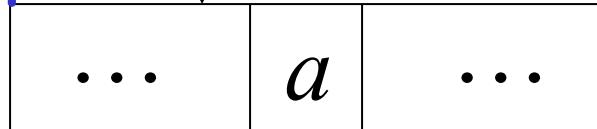
Empty stack



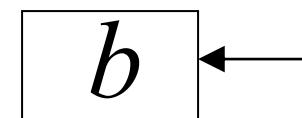
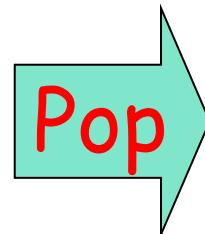
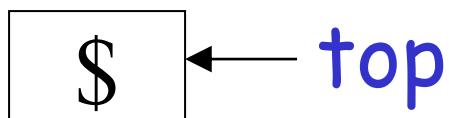
A Good Transition

ជានេះ
ត្រូវ Halt
នៅពេលការស្វែងរក

input

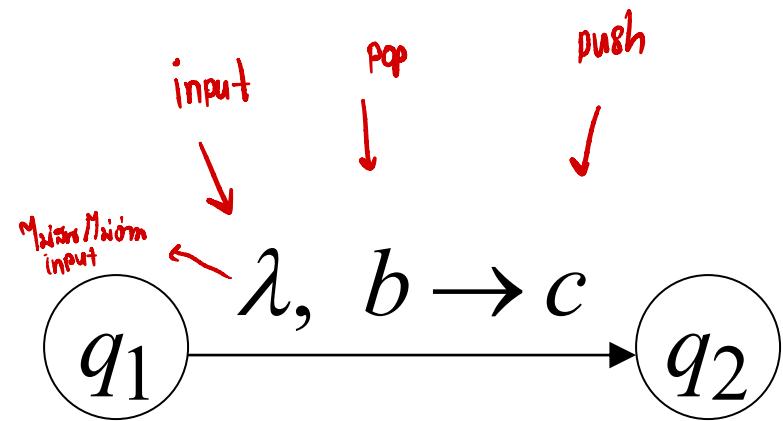
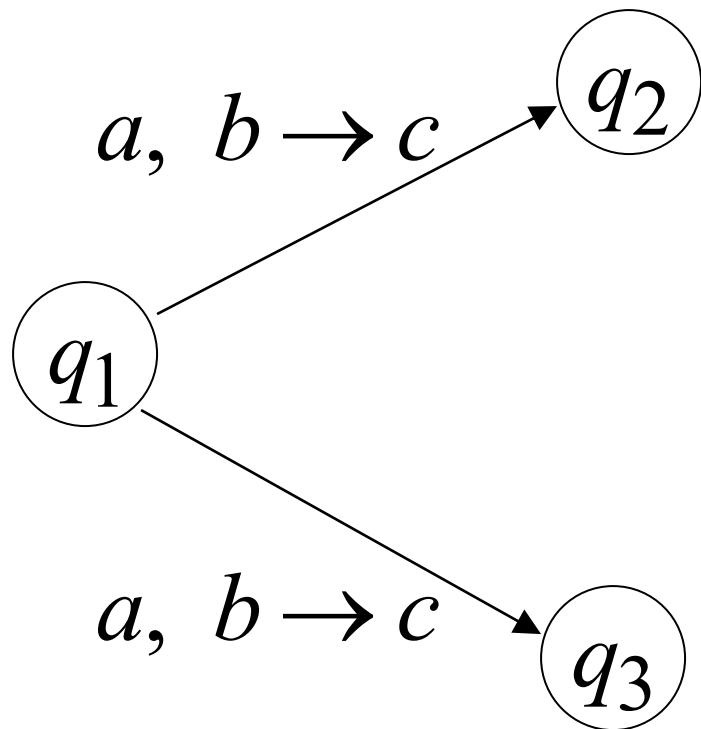
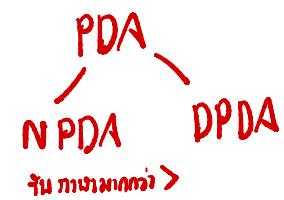


stack



Non-Determinism

မြန်မာစာမျက်နှာ

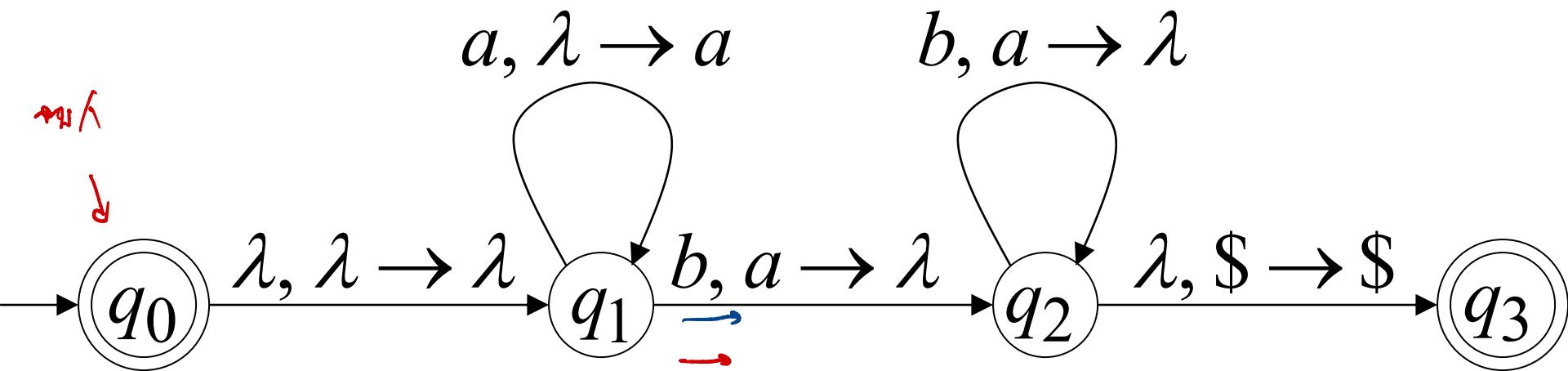
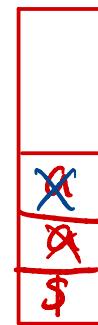


λ – transition
 $\lambda, \lambda \longrightarrow \lambda$

These are allowed transitions in a
Non-deterministic PDA (NPDA)

NPDA: Non-Deterministic PDA

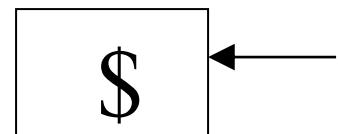
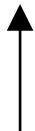
Example:



Execution Example: Time 0

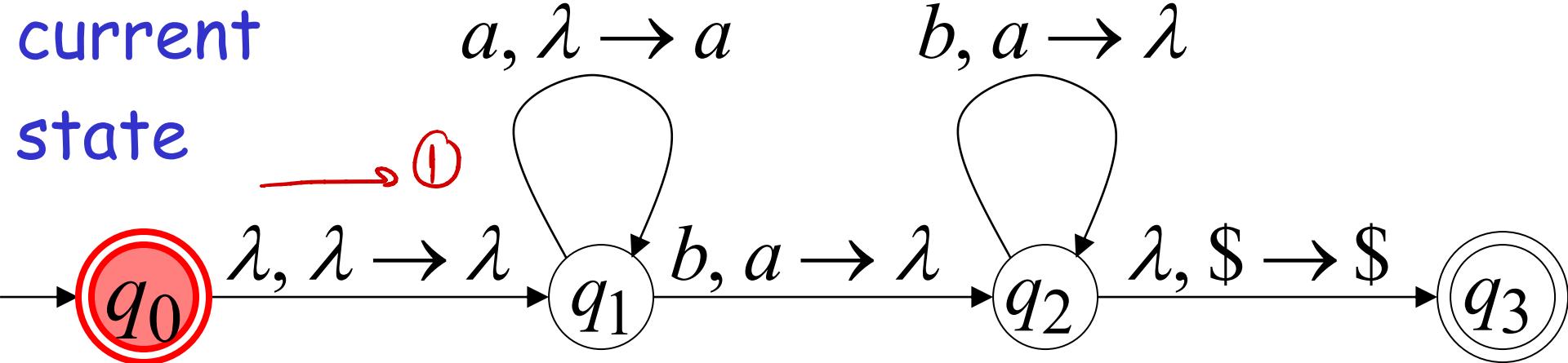
Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



Stack

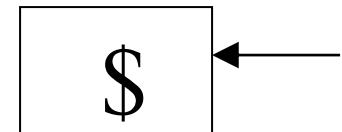
current
state



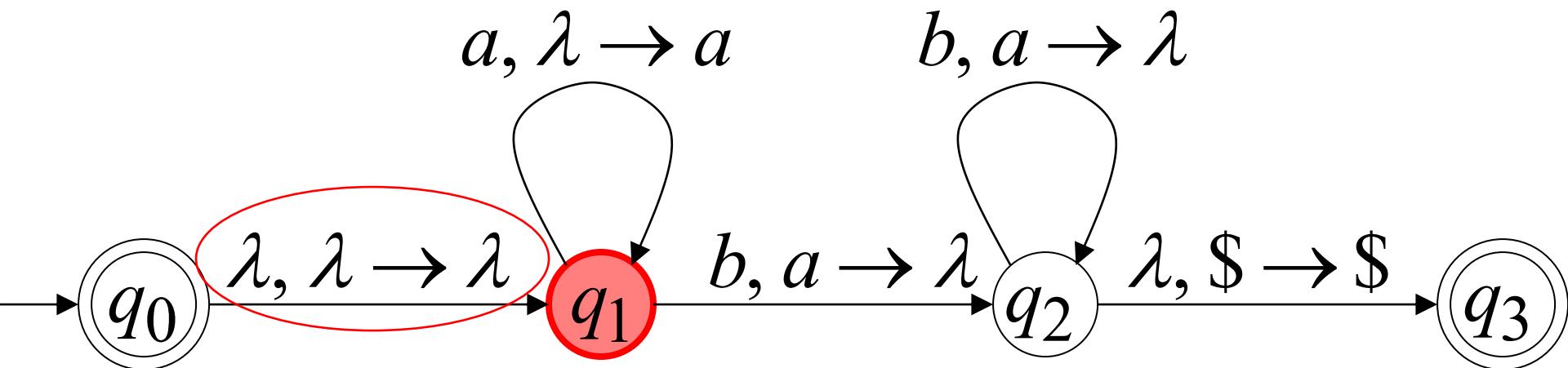
Time 1

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



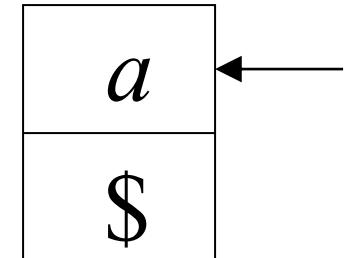
Stack



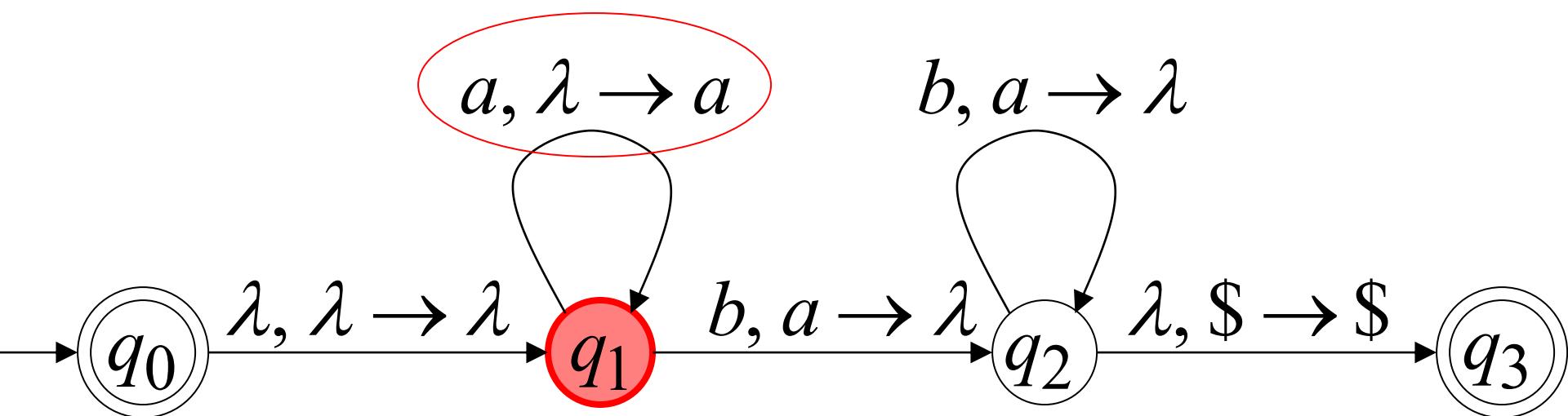
Time 2

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



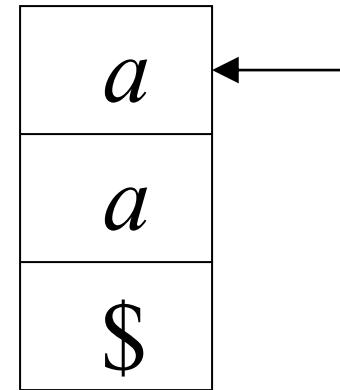
Stack



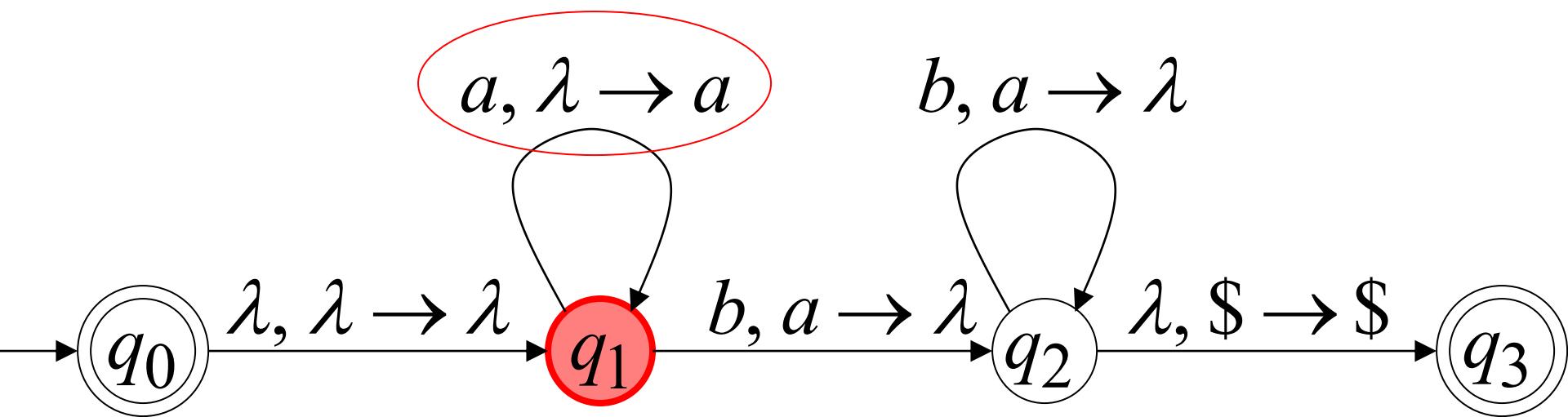
Time 3

Input

a	a	a	b	b	b
---	---	---	---	---	---



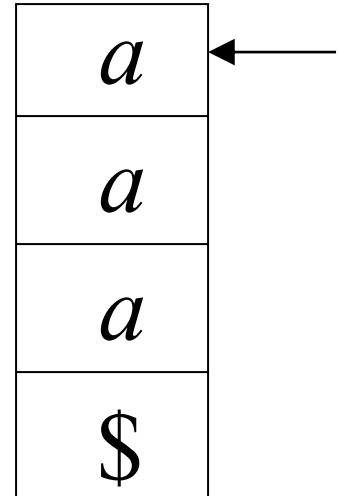
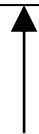
Stack



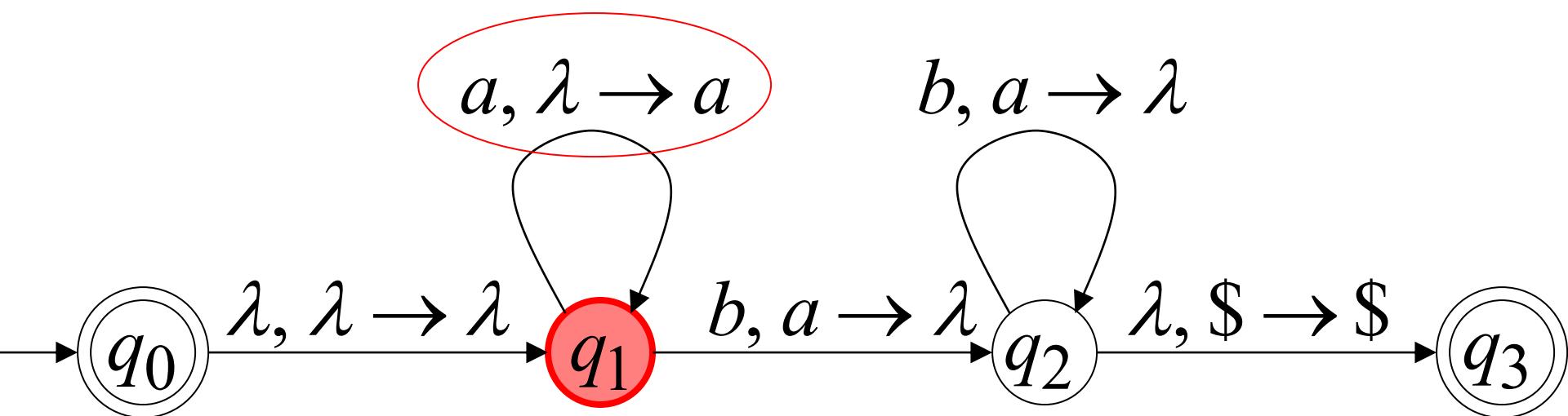
Time 4

Input

a	a	a	b	b	b
---	---	---	---	---	---



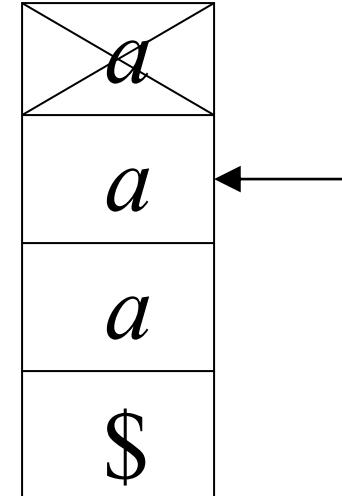
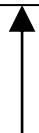
Stack



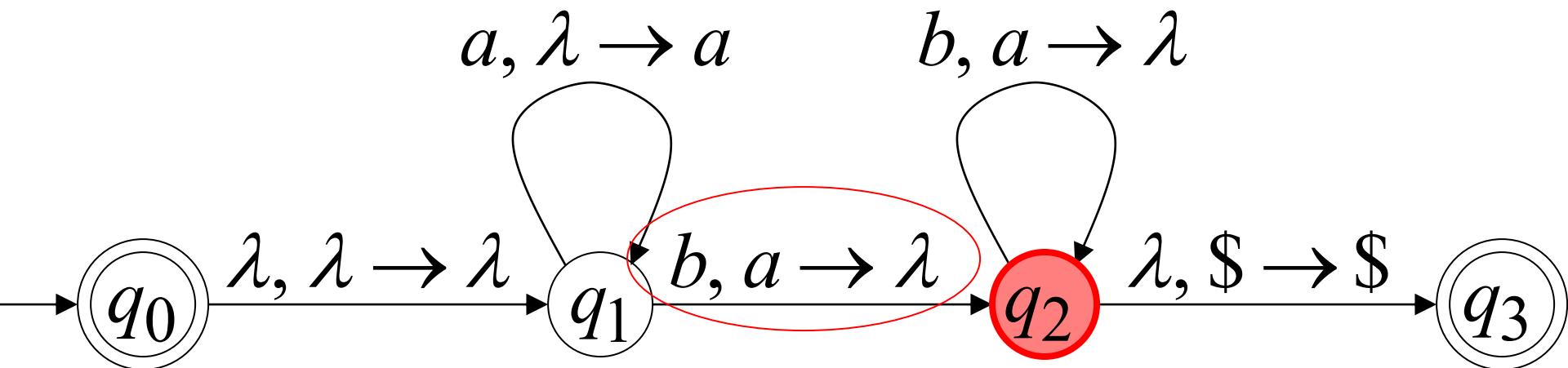
Time 5

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



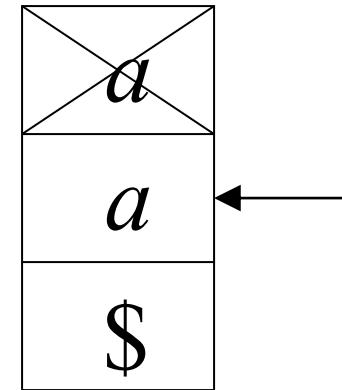
Stack



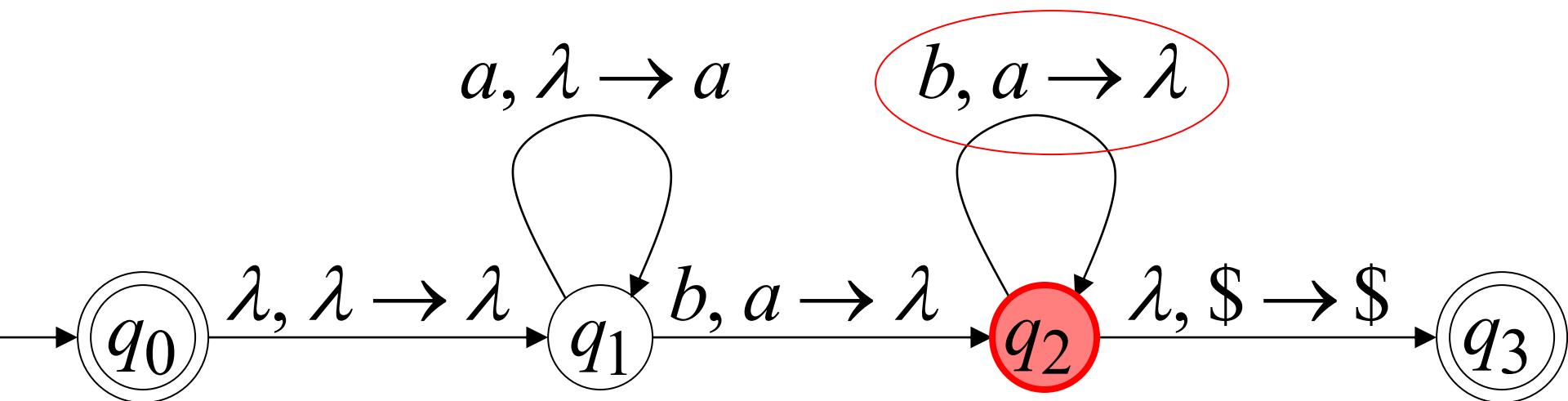
Time 6

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



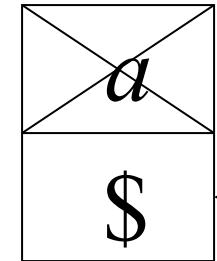
Stack



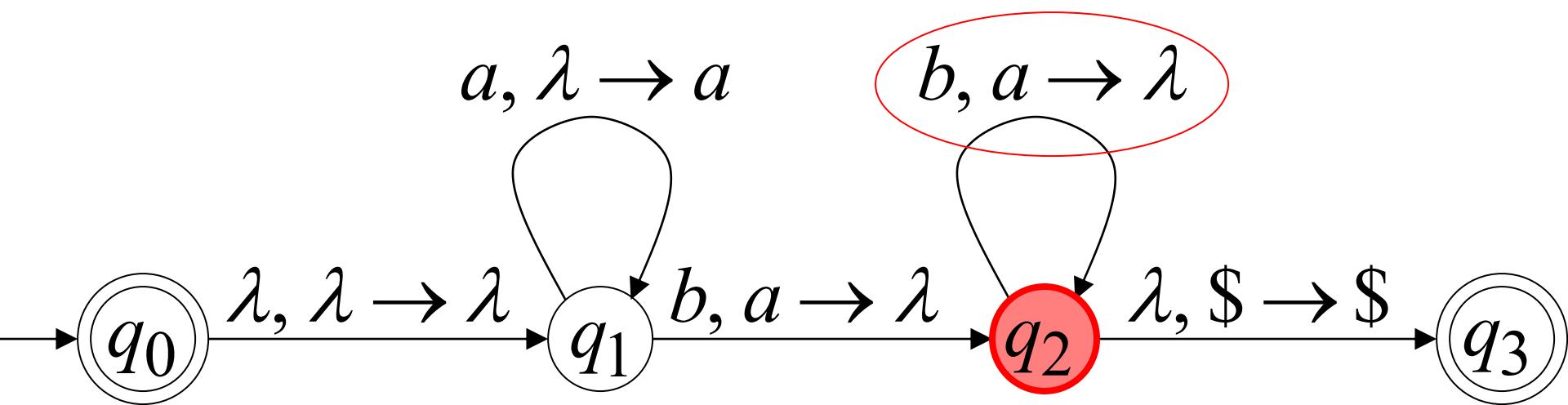
Time 7

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



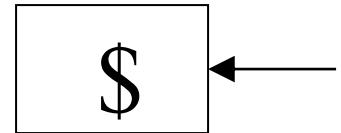
Stack



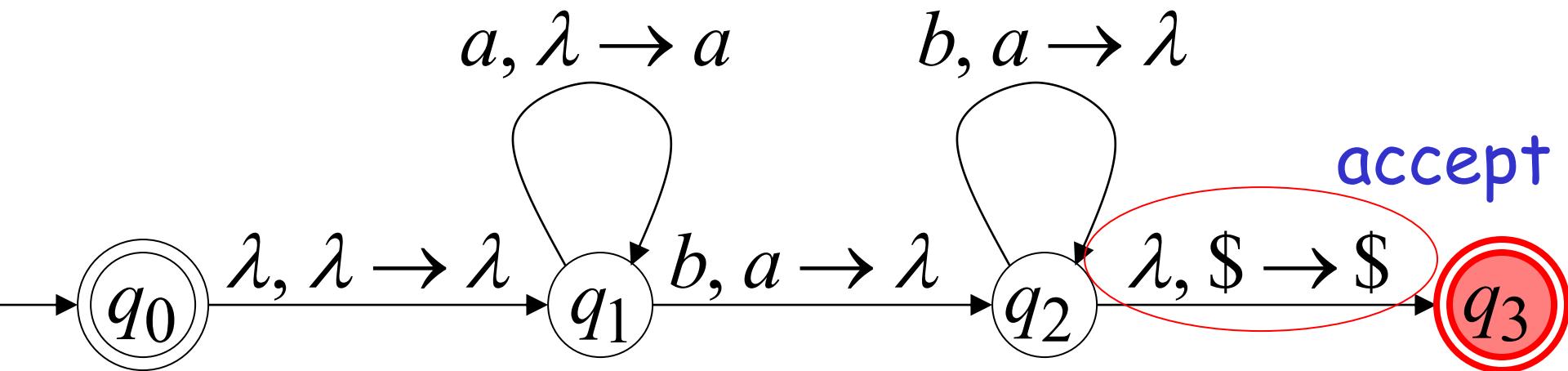
Time 8

Input

a	a	a	b	b	b
-----	-----	-----	-----	-----	-----



Stack



A string is accepted if there is
a computation such that:

Input

All the input is consumed

AND

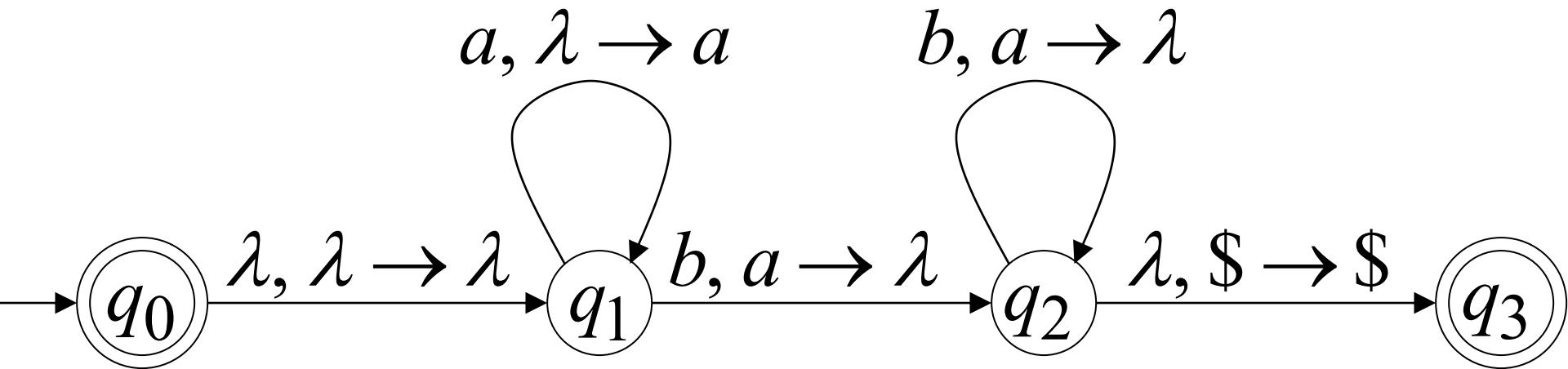
in final state

The last state is a final state

⇒ "accept"

At the end of the computation,
we do not care about the stack contents

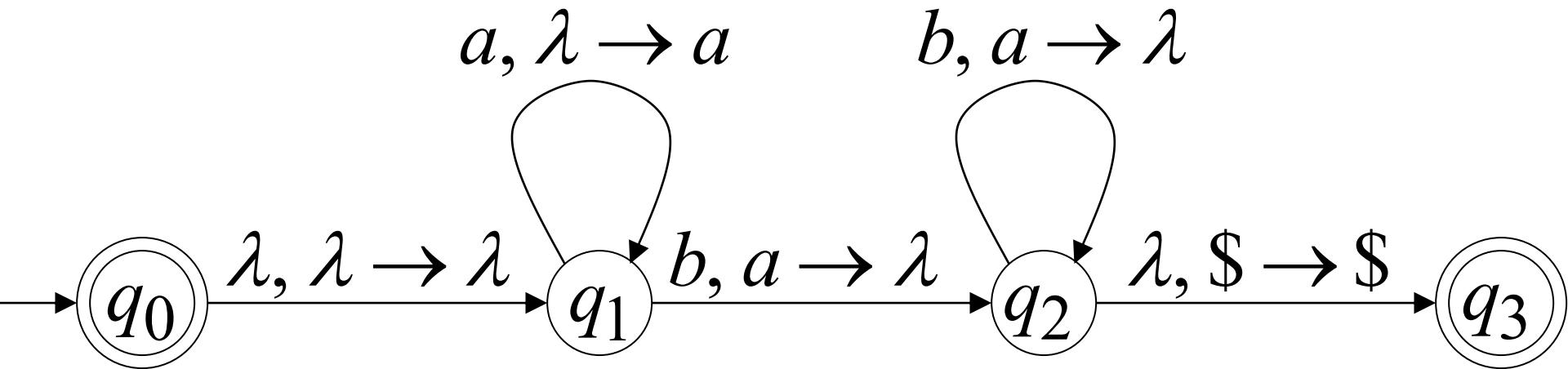
The input string $aaabbbb$
is accepted by the NPDA:



In general,

$$L = \{a^n b^n : n \geq 0\}$$

is the language accepted by the NPDA:



Another NPDA example

ការក្រោងការងារដីបន្លំ

NPDA M

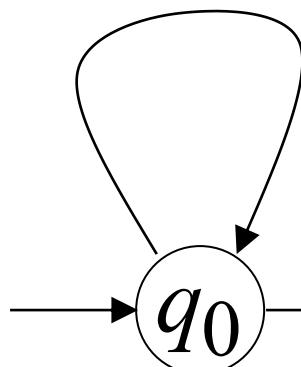
$$L(M) = \{ww^R\}$$

$w = \{a, b\}^*$

ទទួលខ្លួនឯក push នា រឿងនេះ

$$a, \lambda \rightarrow a$$

$$b, \lambda \rightarrow b$$



លេចក្បងការងារ Str

$\lambda, \lambda \rightarrow \lambda$
មែនកំពើការងារ នៅថ្ងៃពេលយ៉ាវកំណត់
នៅថ្ងៃមែនសម្រាប់ State

$$\left. \begin{array}{l} a, a \rightarrow \lambda \\ b, b \rightarrow \lambda \end{array} \right\} \text{ឱ្យក្នុងការងារ 1 នូវ 1}$$

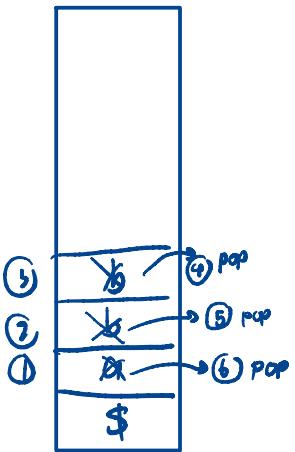
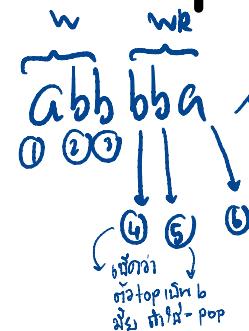
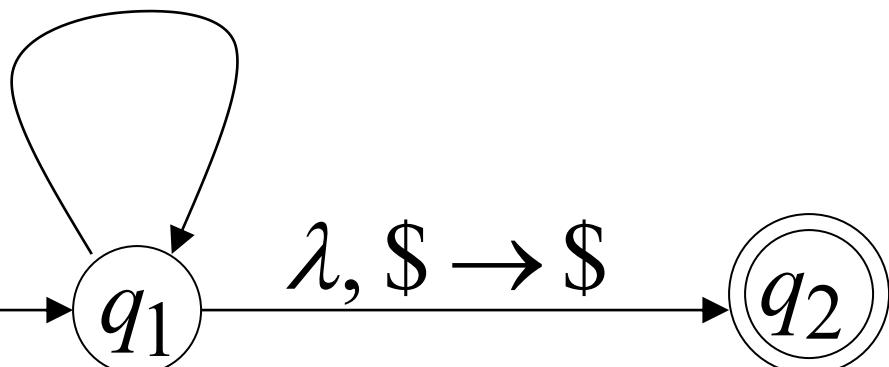
ឱ្យក្នុងការងារ 1 នូវ 1
ឱ្យក្នុងការងារ 1 នូវ 1

$$\left. \begin{array}{l} a, \text{pop} \\ b, \text{pop} \end{array} \right\} \text{ឱ្យក្នុងការងារ 1 នូវ 1}$$

ឱ្យក្នុងការងារ 1 នូវ 1
ឱ្យក្នុងការងារ 1 នូវ 1

$$\left. \begin{array}{l} \text{push} \\ \text{push} \end{array} \right\} \text{ឱ្យក្នុងការងារ 1 នូវ 1}$$

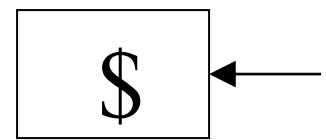
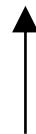
ឱ្យក្នុងការងារ 1 នូវ 1
ឱ្យក្នុងការងារ 1 នូវ 1



Execution Example: Time 0

Input

a	b	b	a
-----	-----	-----	-----

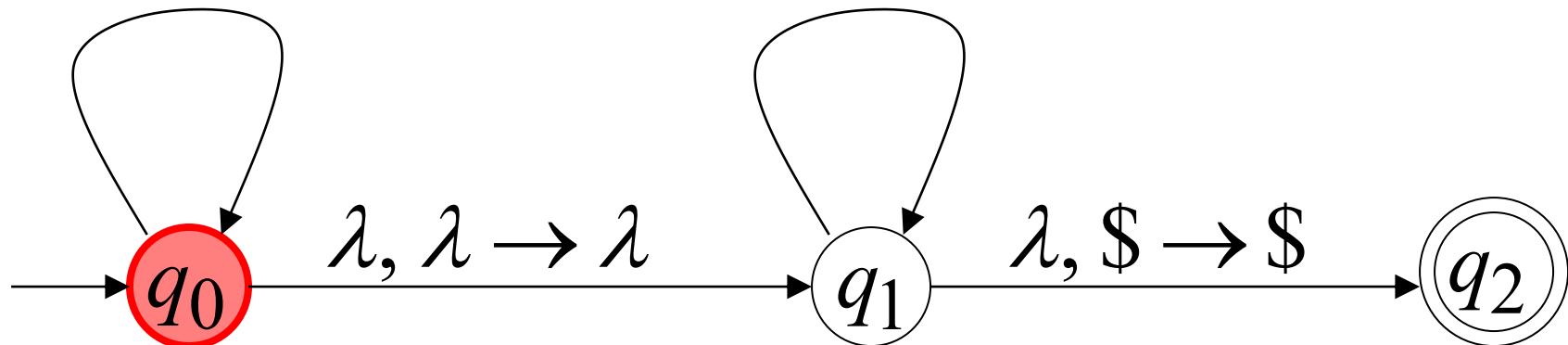


$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



Time 1

Input

a	b	b	a
-----	-----	-----	-----

①

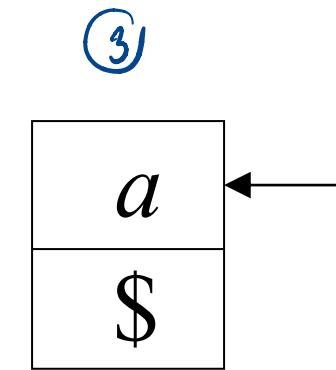
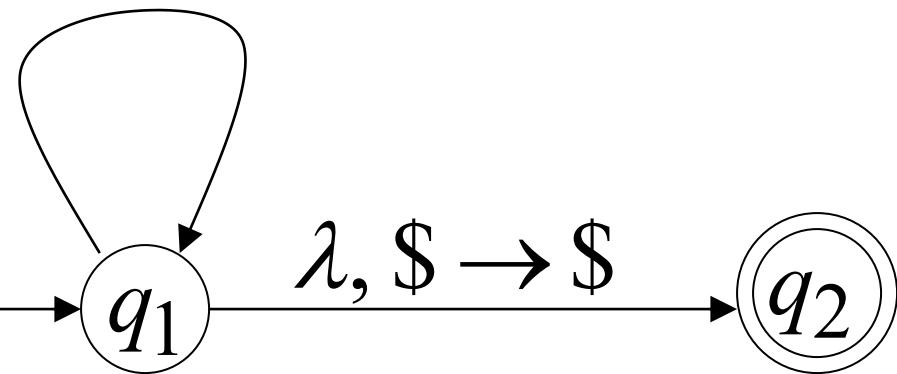
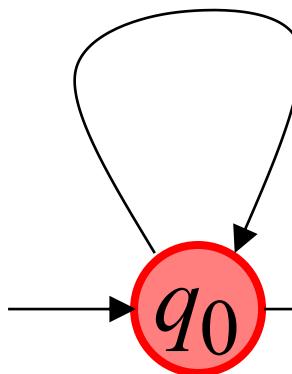
②

$a, \lambda \rightarrow a$

$b, \lambda \rightarrow b$

$a, a \rightarrow \lambda$

$b, b \rightarrow \lambda$

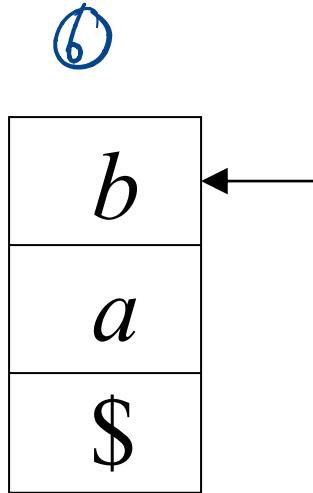


Time 2

Input

a	b	b	a
-----	-----	-----	-----

④



Stack

$$a, \lambda \rightarrow a$$

$$b, \lambda \rightarrow b$$

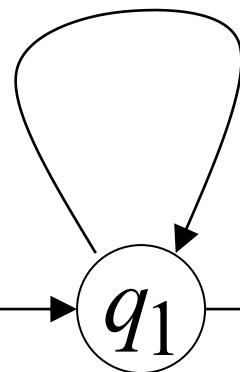
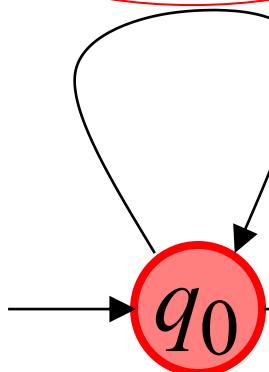
⑤

$$a, a \rightarrow \lambda$$

$$b, b \rightarrow \lambda$$

$$\lambda, \lambda \rightarrow \lambda$$

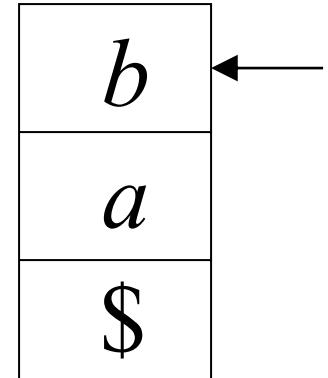
$$\lambda, \$ \rightarrow \$$$



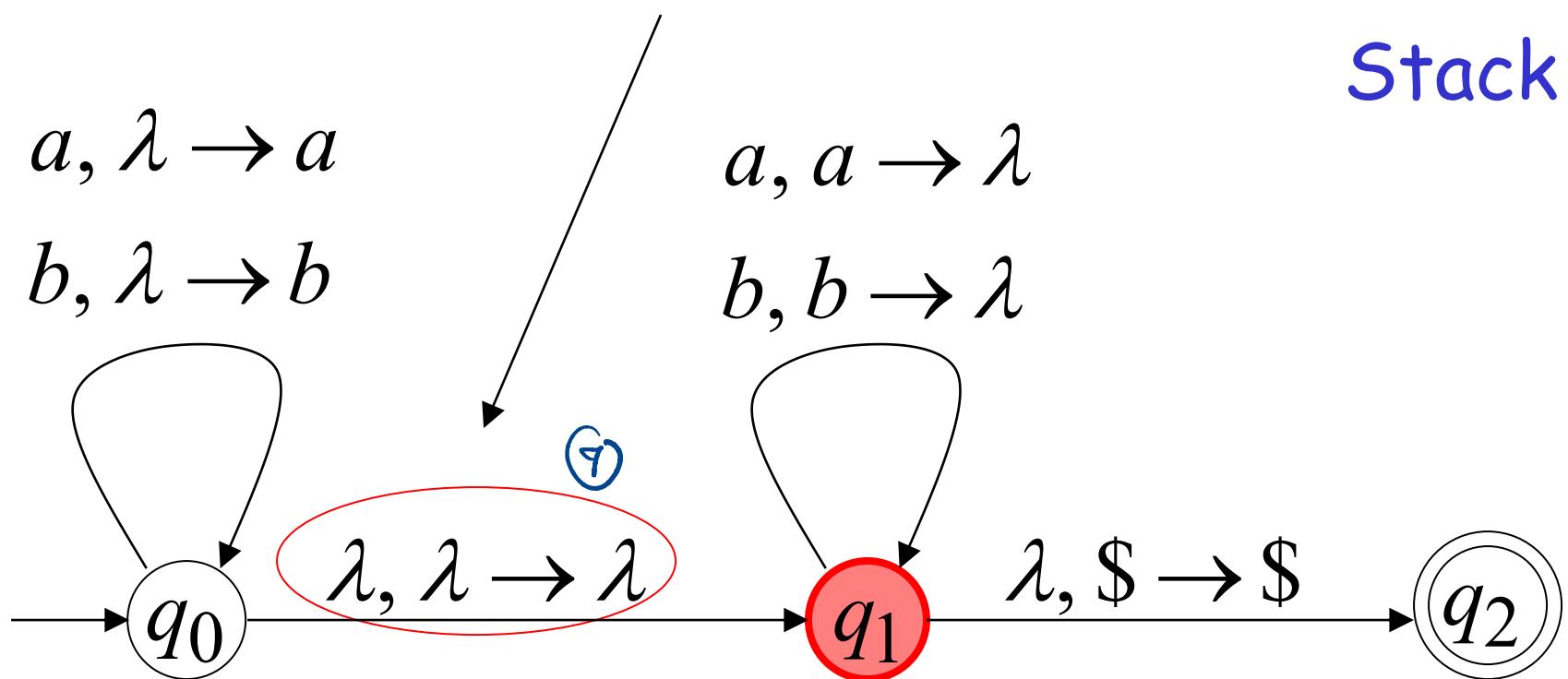
Time 3

Input

a	b	b	a
---	---	---	---



Guess the middle
of string



Time 4

Input

a	b	b	a
-----	-----	-----	-----

⑨

⑩

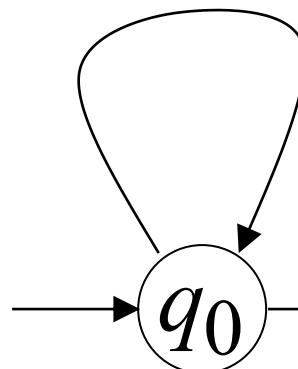
b
a
\$



Stack

$$a, \lambda \rightarrow a$$

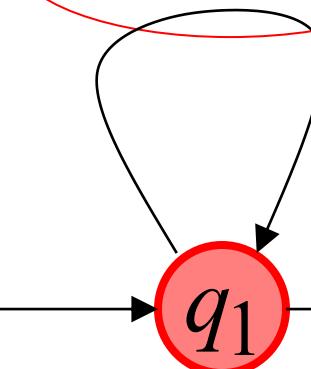
$$b, \lambda \rightarrow b$$



$$\lambda, \lambda \rightarrow \lambda$$

$a, a \rightarrow \lambda$ ⑩

$b, b \rightarrow \lambda$



$$\lambda, \$ \rightarrow \$$$



Time 5

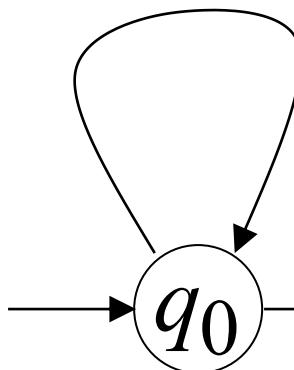
Input

a	b	b	a
-----	-----	-----	-----



$$a, \lambda \rightarrow a$$

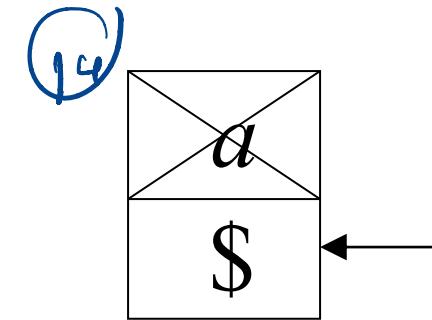
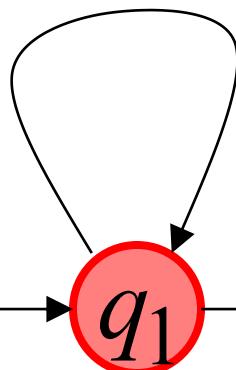
$$b, \lambda \rightarrow b$$



13

$a, a \rightarrow \lambda$

$$b, b \rightarrow \lambda$$



Stack

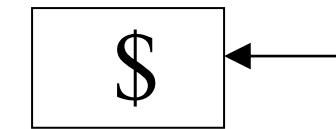
$$\lambda, \$ \rightarrow \$$$



Time 6

Input

a	b	b	a
-----	-----	-----	-----



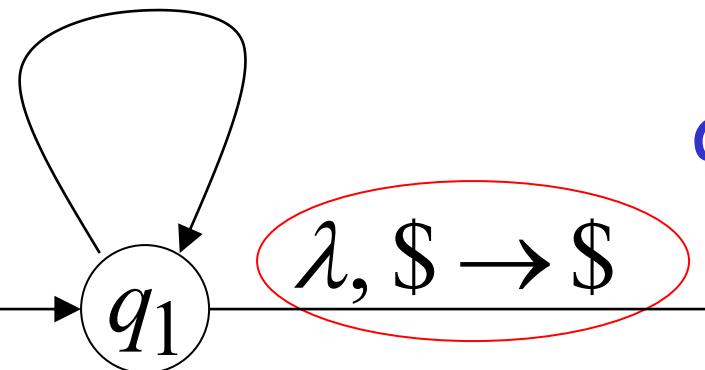
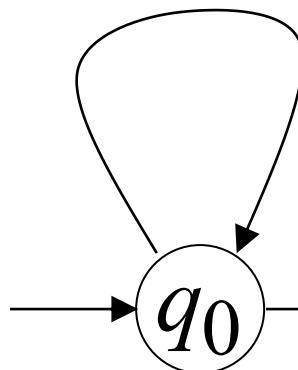
Stack

$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



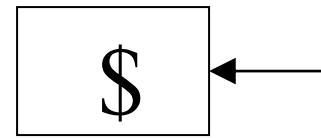
accept

Rejection Example:

Time 0

Input

a	b	b	b
-----	-----	-----	-----

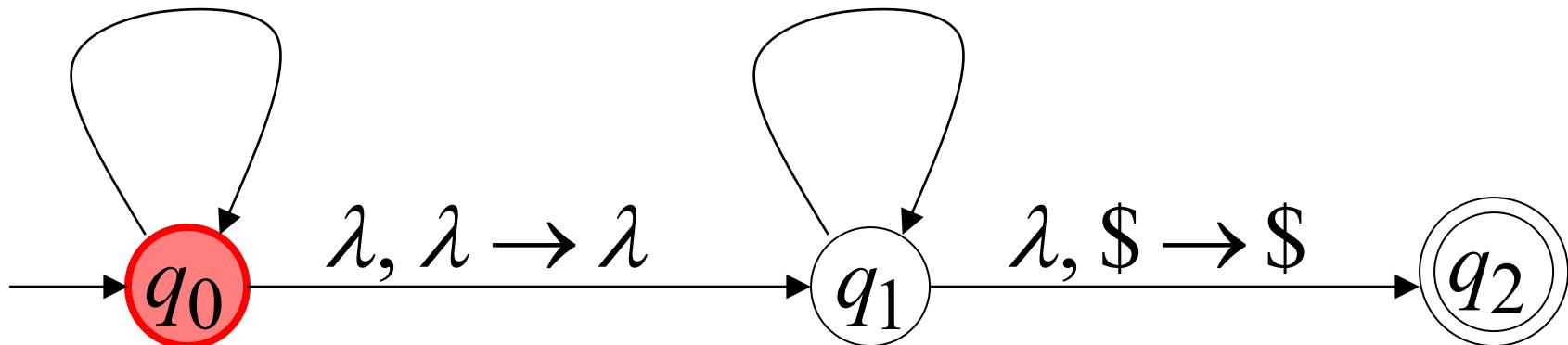


$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

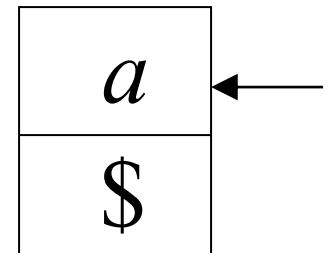
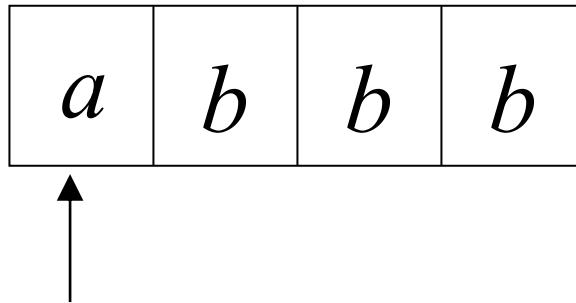
$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



Time 1

Input



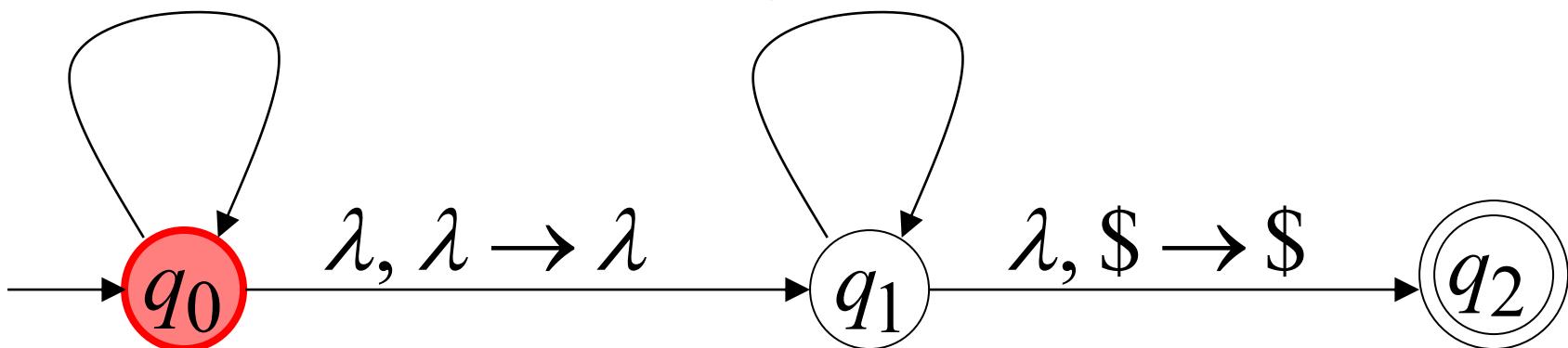
Stack

$a, \lambda \rightarrow a$

$a, a \rightarrow \lambda$

$b, \lambda \rightarrow b$

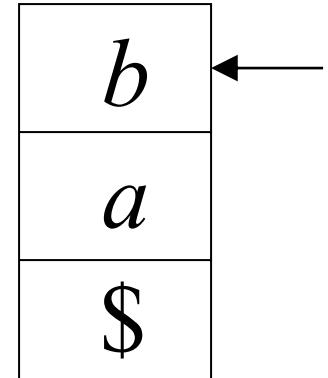
$b, b \rightarrow \lambda$



Time 2

Input

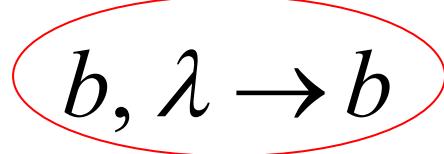
a	b	b	b
-----	-----	-----	-----



Stack

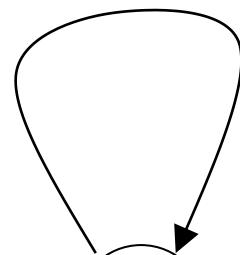
$$a, \lambda \rightarrow a$$

$$b, \lambda \rightarrow b$$



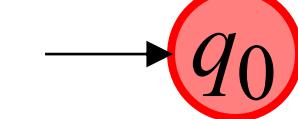
$$a, a \rightarrow \lambda$$

$$b, b \rightarrow \lambda$$



$$\lambda, \lambda \rightarrow \lambda$$

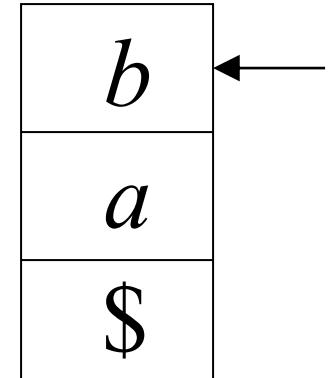
$$\lambda, \$ \rightarrow \$$$



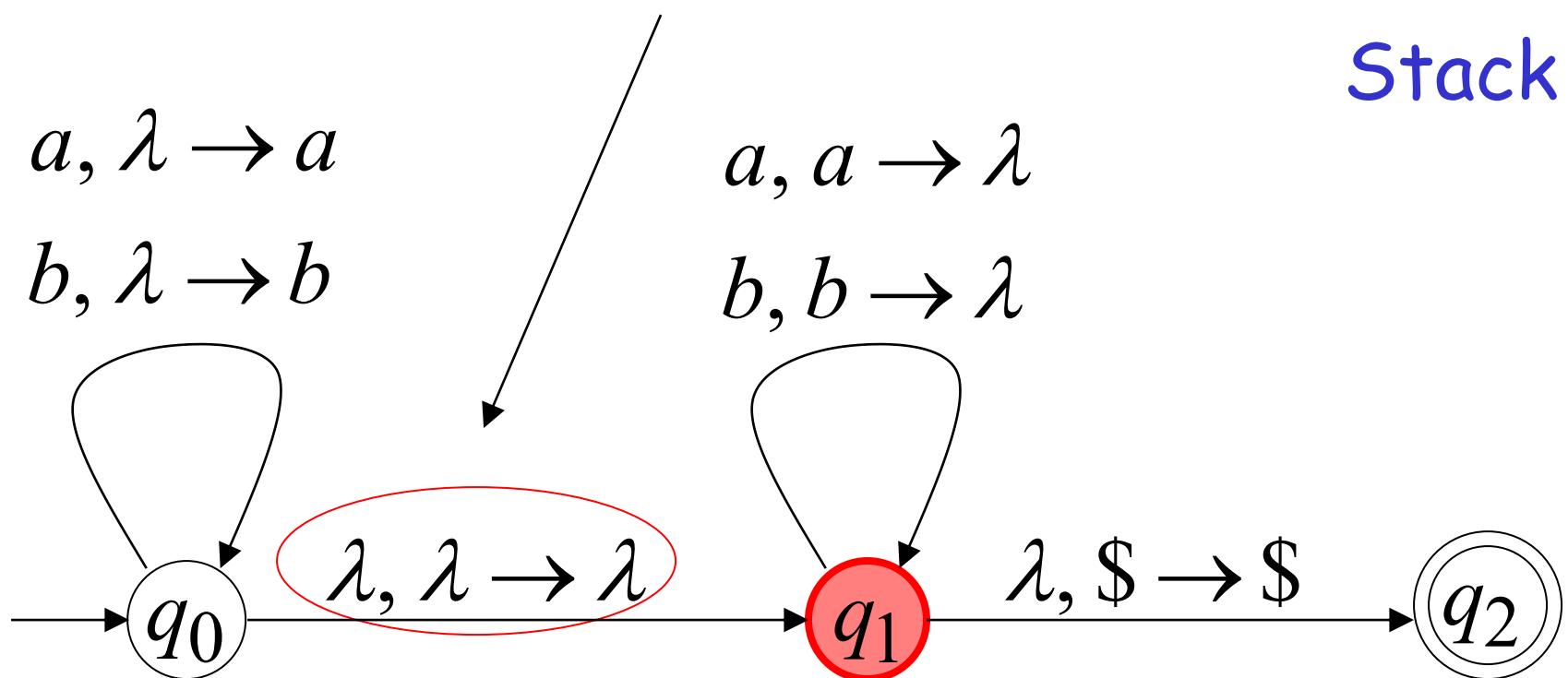
Time 3

Input

a	b	b	b
---	---	---	---



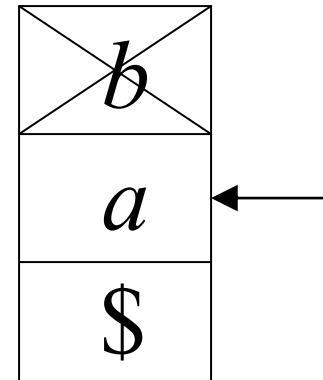
Guess the middle
of string



Time 4

Input

a	b	b	b
-----	-----	-----	-----



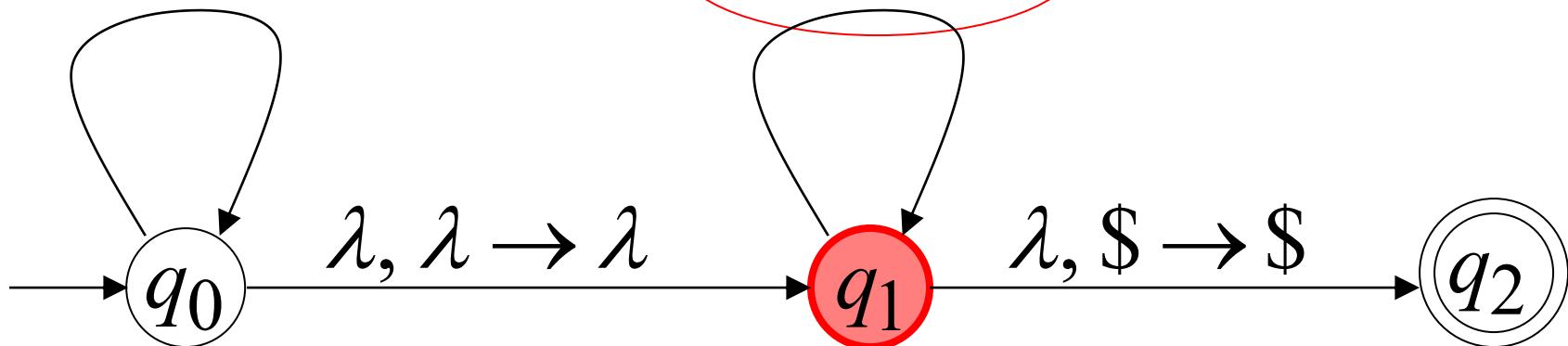
Stack

$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



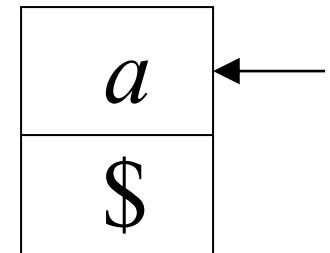
Time 5

Input

a	b	b	b
---	---	---	---

There is no possible transition.

Input is not consumed

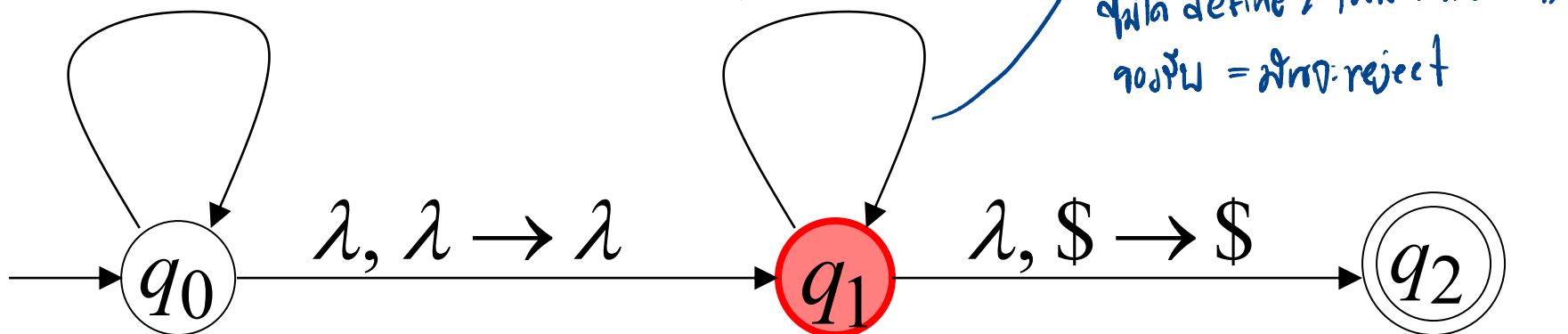


$$a, \lambda \rightarrow a$$

$$b, \lambda \rightarrow b$$

$$a, a \rightarrow \lambda$$

$$b, b \rightarrow \lambda$$

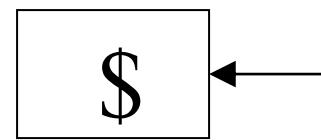


Another computation on same string:

Input

a	b	b	b
---	---	---	---

Time O $\log n$

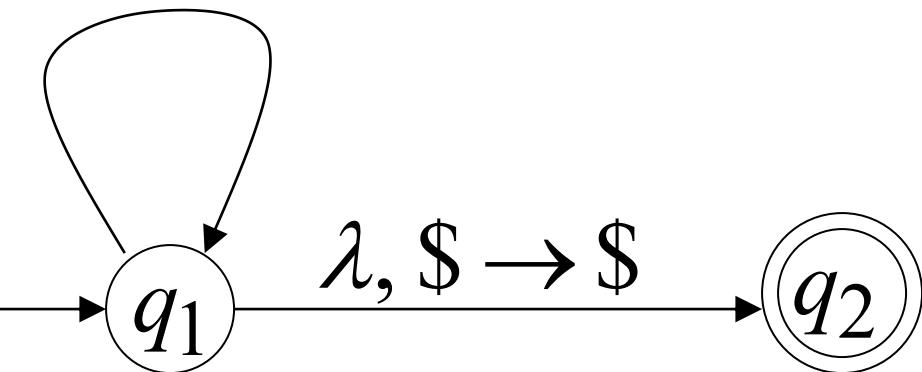
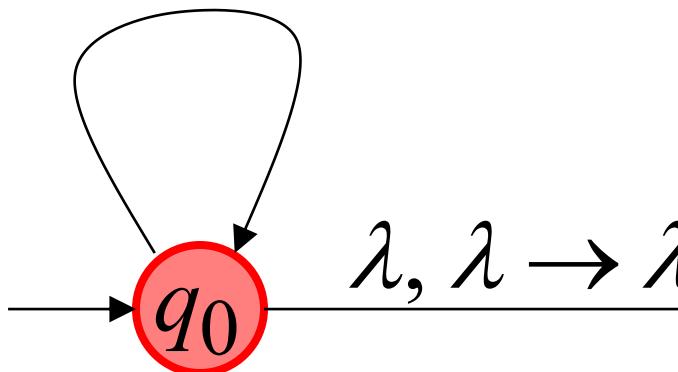


$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

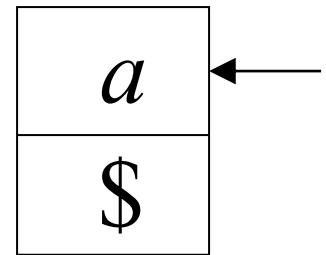
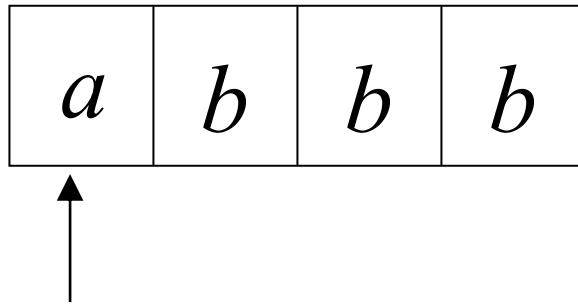
$$b, b \rightarrow \lambda$$



Stack

Time 1

Input



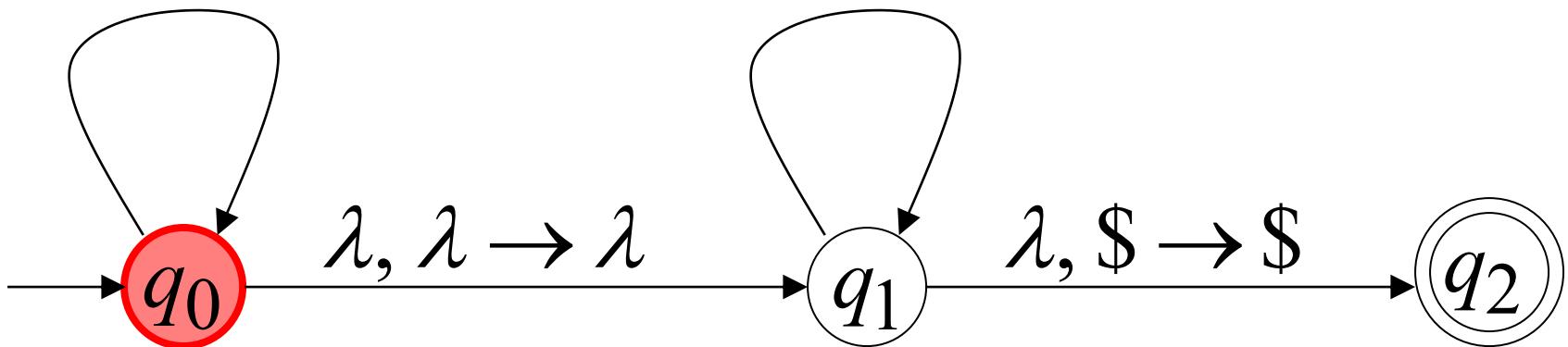
Stack

$a, \lambda \rightarrow a$

$a, a \rightarrow \lambda$

$b, \lambda \rightarrow b$

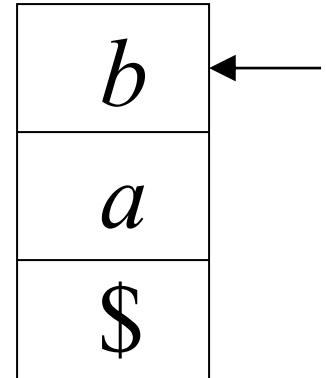
$b, b \rightarrow \lambda$



Time 2

Input

a	b	b	b
-----	-----	-----	-----



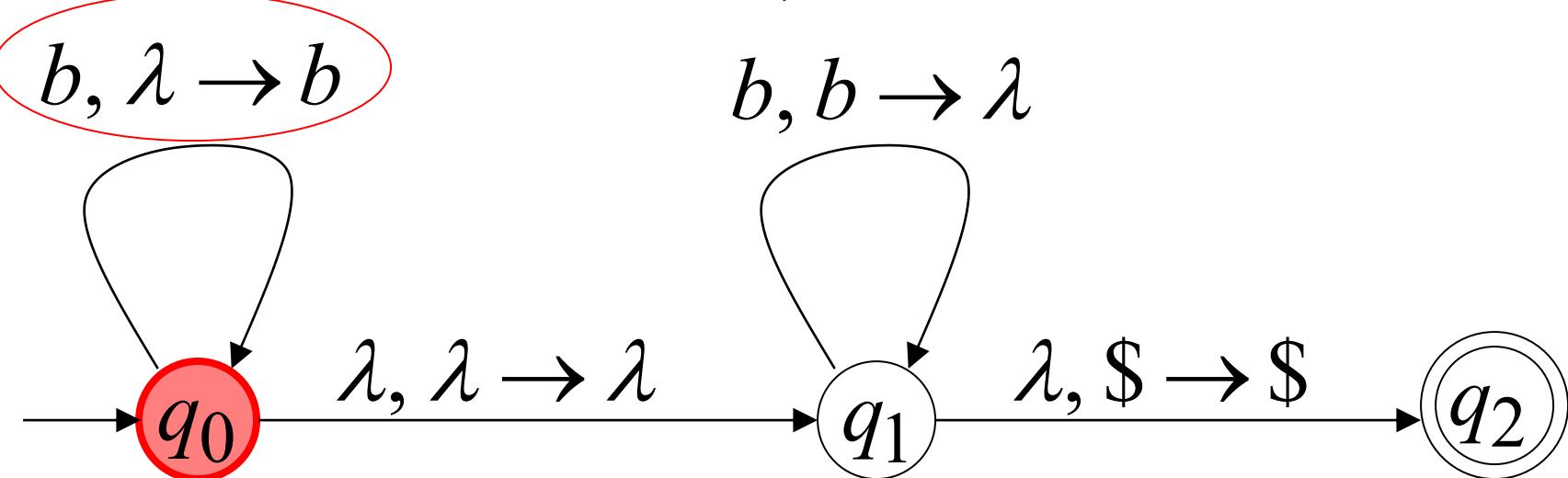
Stack

$$a, \lambda \rightarrow a$$

$$b, \lambda \rightarrow b$$

$$a, a \rightarrow \lambda$$

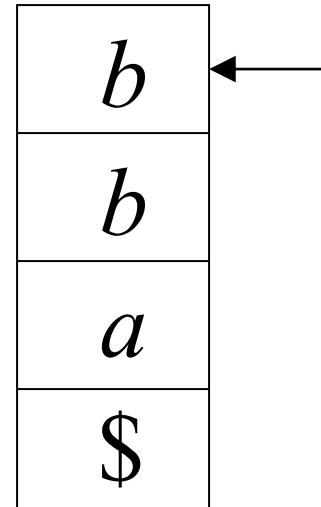
$$b, b \rightarrow \lambda$$



Input

a	b	b	b
---	---	---	---

Time 3



Stack

$$a, \lambda \rightarrow a$$

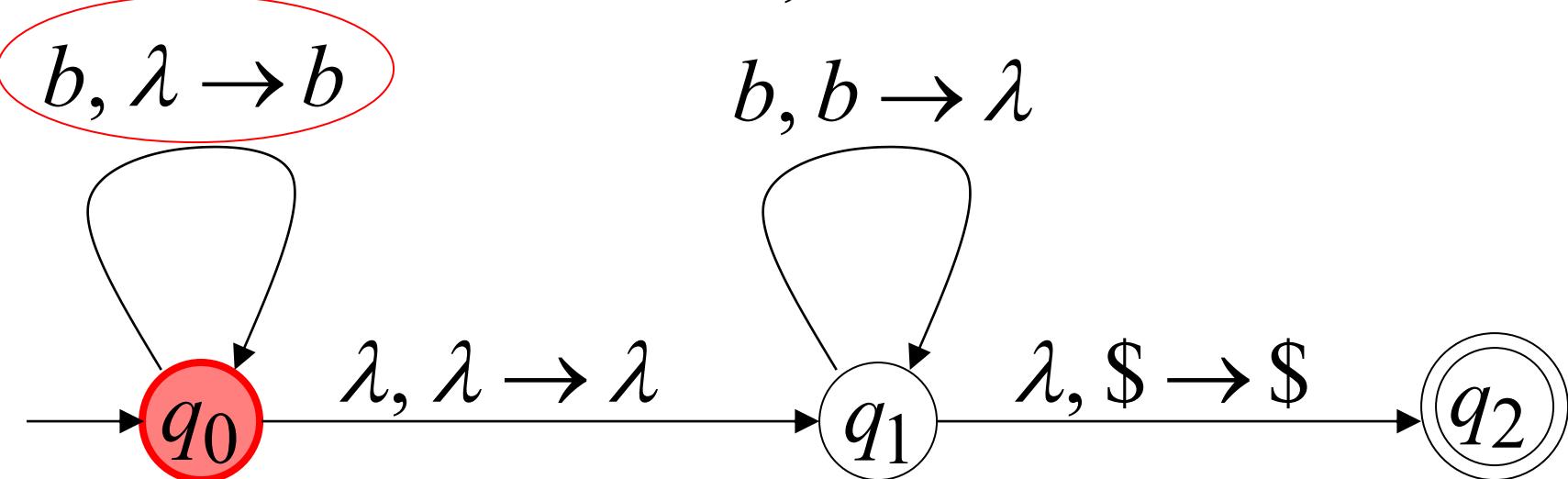
$$b, \lambda \rightarrow b$$

$$a, a \rightarrow \lambda$$

$$b, b \rightarrow \lambda$$

$$\lambda, \lambda \rightarrow \lambda$$

$$\lambda, \$ \rightarrow \$$$



Input

a	b	b	b
-----	-----	-----	-----

Time 4

b
b
b
a
\$

Stack

$$a, \lambda \rightarrow a$$

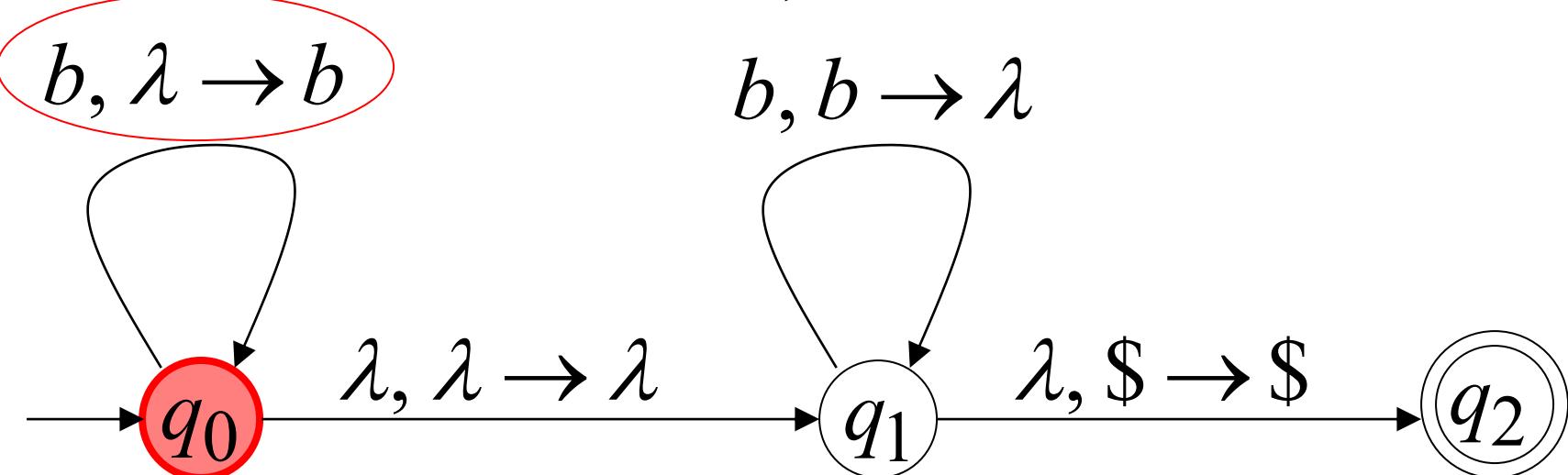
$$b, \lambda \rightarrow b$$

$$a, a \rightarrow \lambda$$

$$b, b \rightarrow \lambda$$

$$\lambda, \lambda \rightarrow \lambda$$

$$\lambda, \$ \rightarrow \$$$



Input

a	b	b	b
---	---	---	---

Time 5

No final state
is reached

b
b
b
a
\$

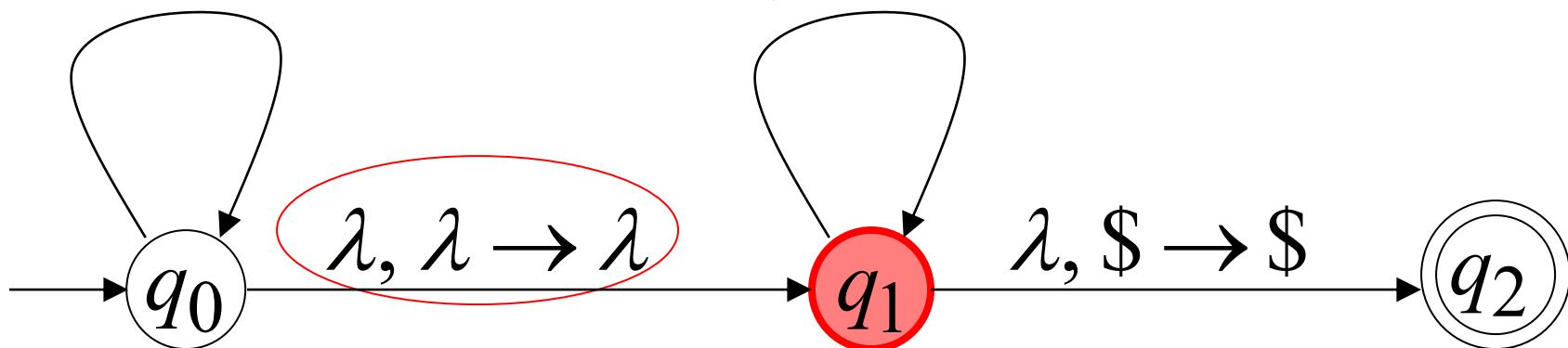
Stack

$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



There is no computation
that accepts string $abbb$ මෙයින්වාගැනීම්

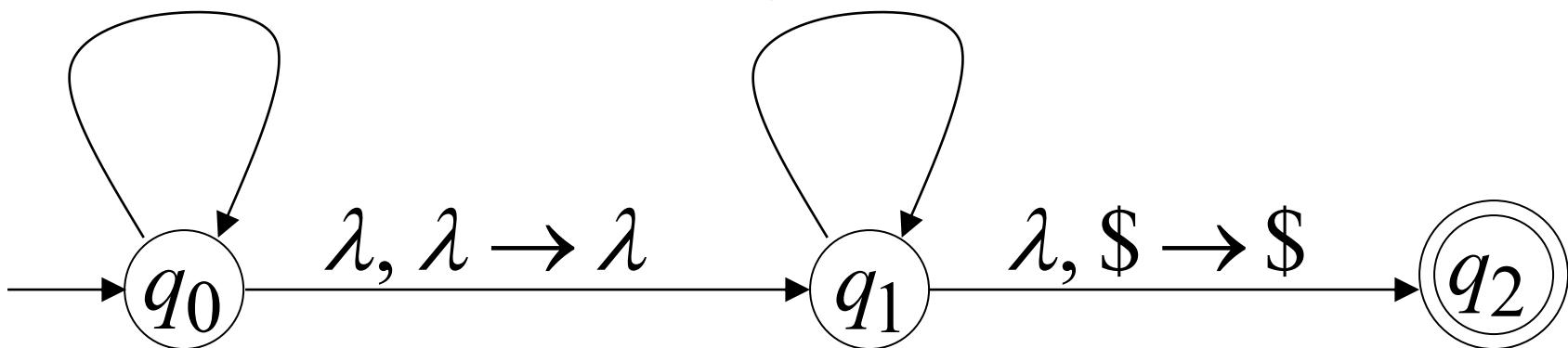
$$abbb \notin L(M)$$

$$a, \lambda \rightarrow a$$

$$a, a \rightarrow \lambda$$

$$b, \lambda \rightarrow b$$

$$b, b \rightarrow \lambda$$



A string is rejected if there is no computation such that:

All the input is consumed

AND

The last state is a final state

At the end of the computation,
we do not care about the stack contents

In other words, a string is rejected if in every computation with this string:

စိမ်များ

The input cannot be consumed

OR

နောက်

ဆုတေယာ

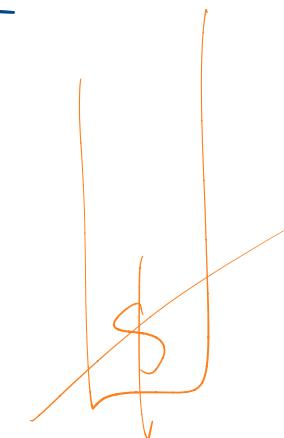
The input is consumed but the last state is not a final state

သူမရန်

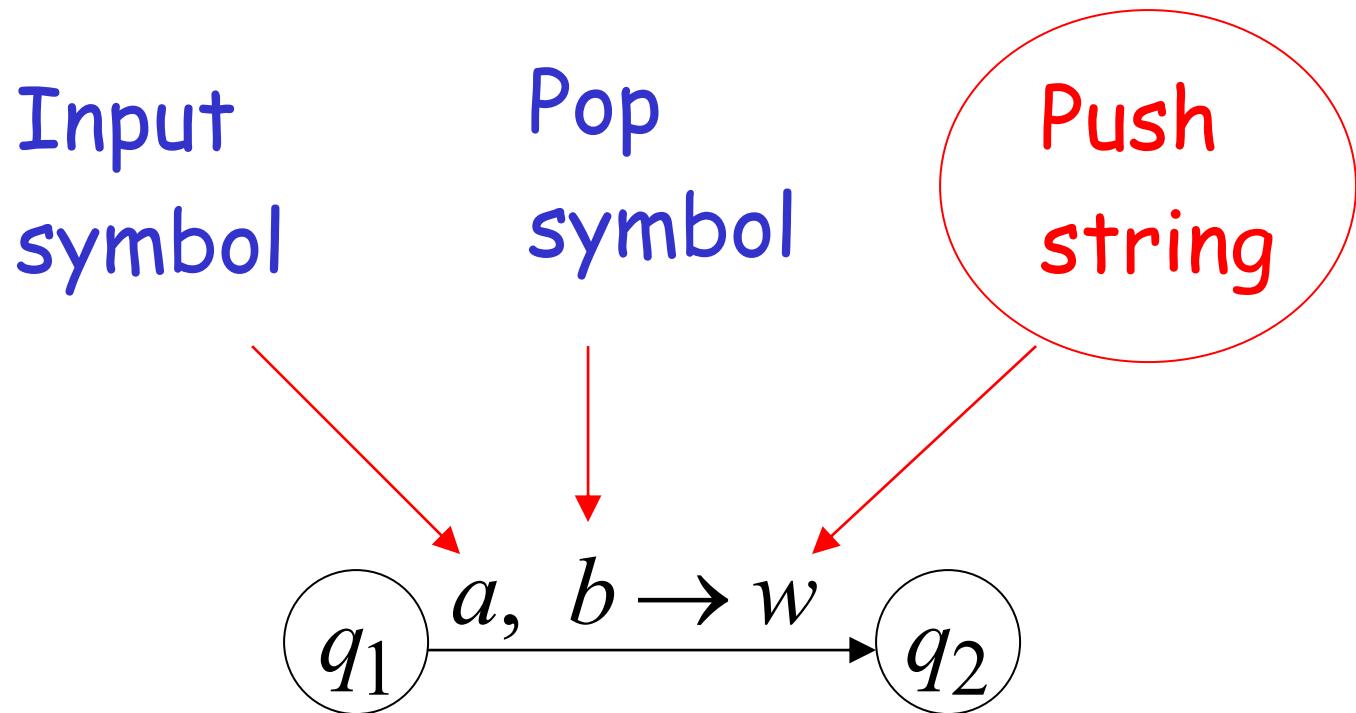
OR

စားစွဲစံအတွက်အမြတ် \$

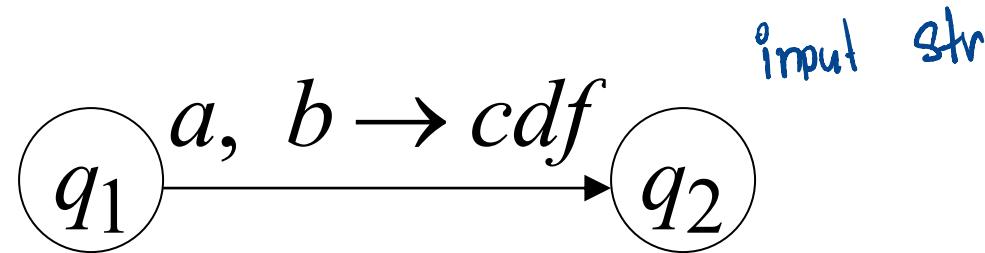
The stack head moves below the bottom of the stack



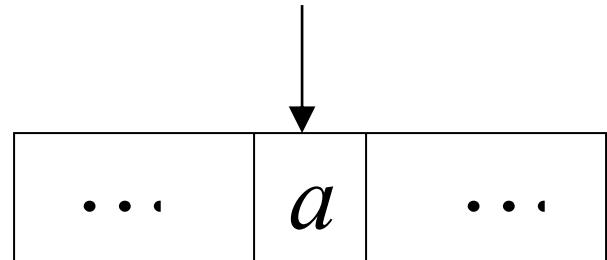
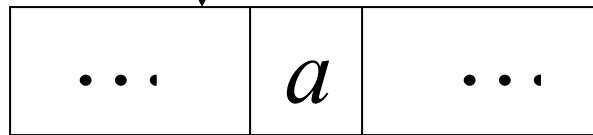
Pushing Strings



Example:



input

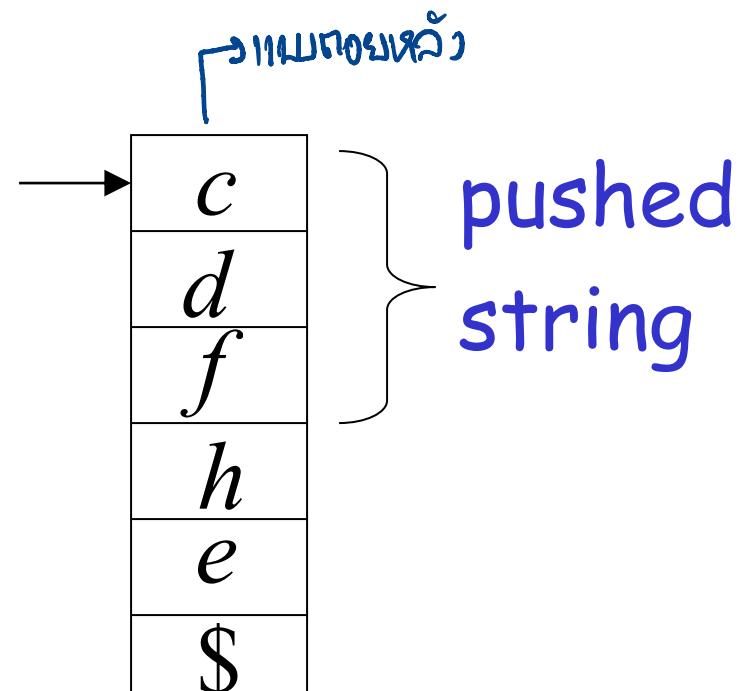


stack

top

b
h
e
\$

Push



Another NPDA example

NPDA M

$$w \in \{a, b\}^*$$

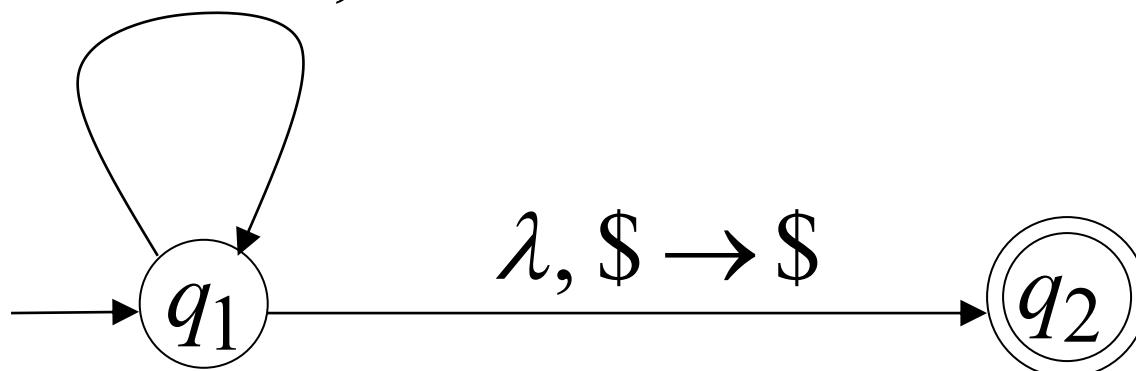
$$L(M) = \{w : n_a = n_b\}$$

$a=0$ $b=1$

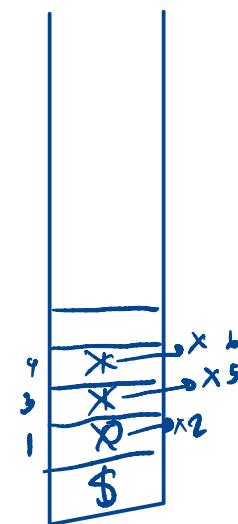
$$a, \$ \rightarrow 0\$ \quad b, \$ \rightarrow 1\$$$

$$a, 0 \rightarrow 00 \quad b, 1 \rightarrow 11$$

$$a, 1 \rightarrow \lambda \quad b, 0 \rightarrow \lambda$$



input
abbbbaa
↑↑↑↑↓↓
1 2 3 4 5 6



Execution Example:

Time 0

Input

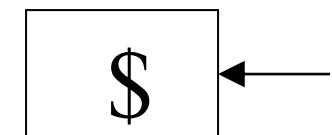
a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



$a, \$ \rightarrow 0\$$ $b, \$ \rightarrow 1\$$

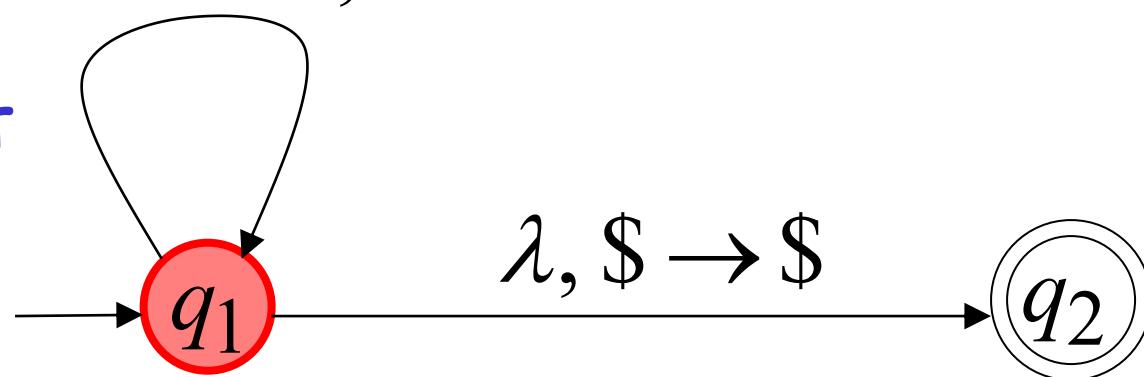
$a, 0 \rightarrow 00$ $b, 1 \rightarrow 11$

$a, 1 \rightarrow \lambda$ $b, 0 \rightarrow \lambda$



Stack

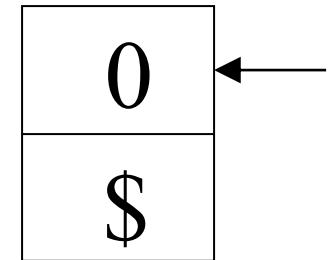
current
state



Time 1

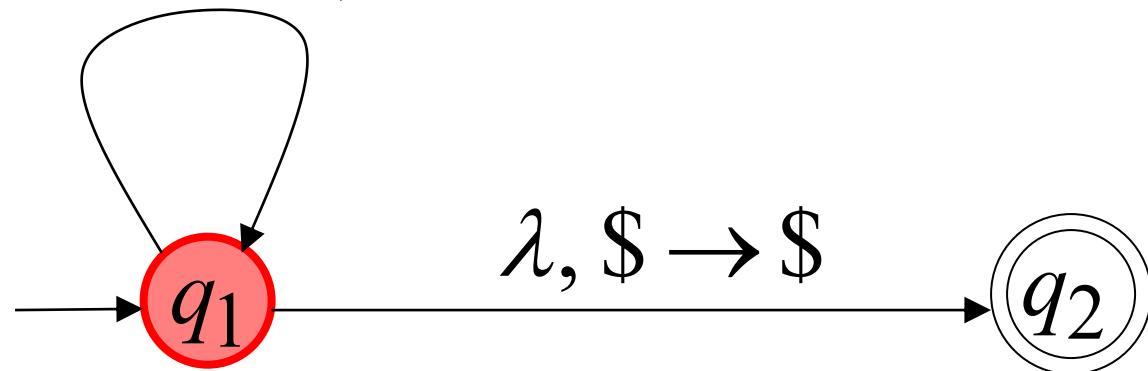
Input

a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



Stack

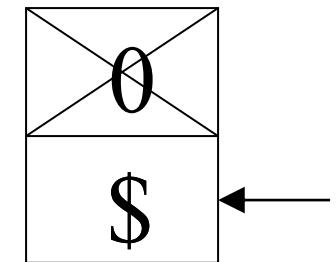
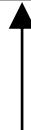
$$\begin{array}{ll} a, \$ \rightarrow 0\$ & b, \$ \rightarrow 1\$ \\ a, 0 \rightarrow 00 & b, 1 \rightarrow 11 \\ a, 1 \rightarrow \lambda & b, 0 \rightarrow \lambda \end{array}$$



Time 3

Input

a	b	b	b	a	a
-----	-----	-----	-----	-----	-----

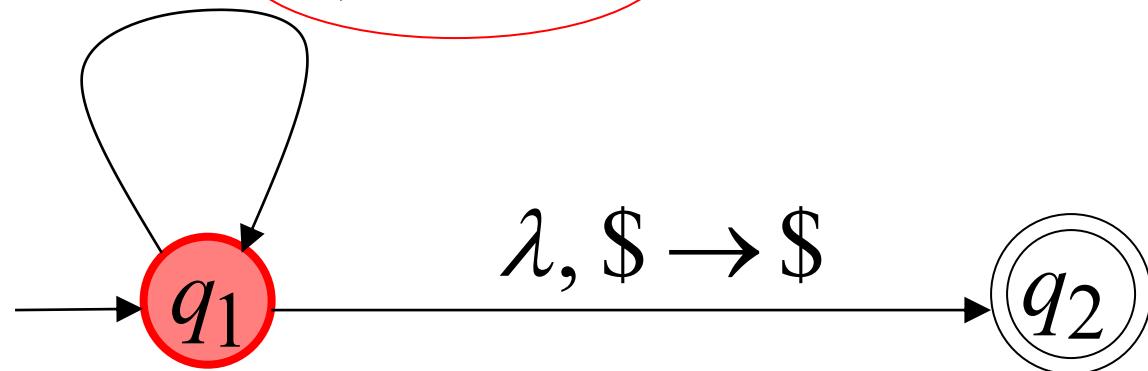


Stack

$$a, \$ \rightarrow 0\$ \quad b, \$ \rightarrow 1\$$$

$$a, 0 \rightarrow 00 \quad b, 1 \rightarrow 11$$

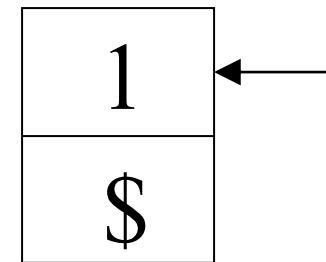
$$a, 1 \rightarrow \lambda \quad b, 0 \rightarrow \lambda$$



Time 4

Input

a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



Stack

$a, \$ \rightarrow 0\$$

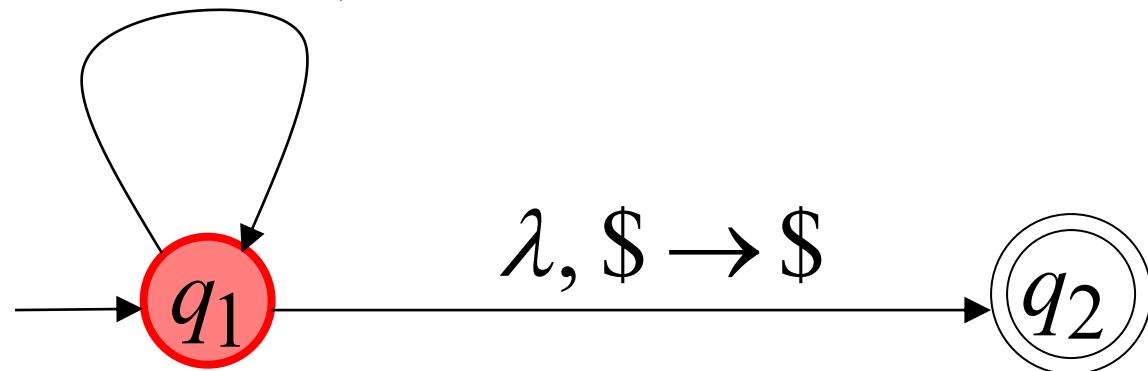
$a, 0 \rightarrow 00$

$a, 1 \rightarrow \lambda$

$b, \$ \rightarrow 1\$$

$b, 1 \rightarrow 11$

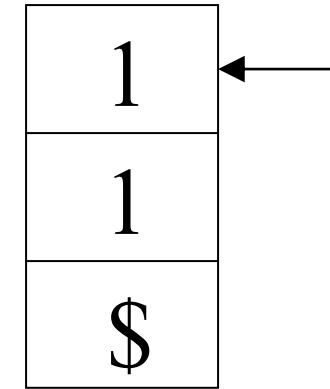
$b, 0 \rightarrow \lambda$



Time 5

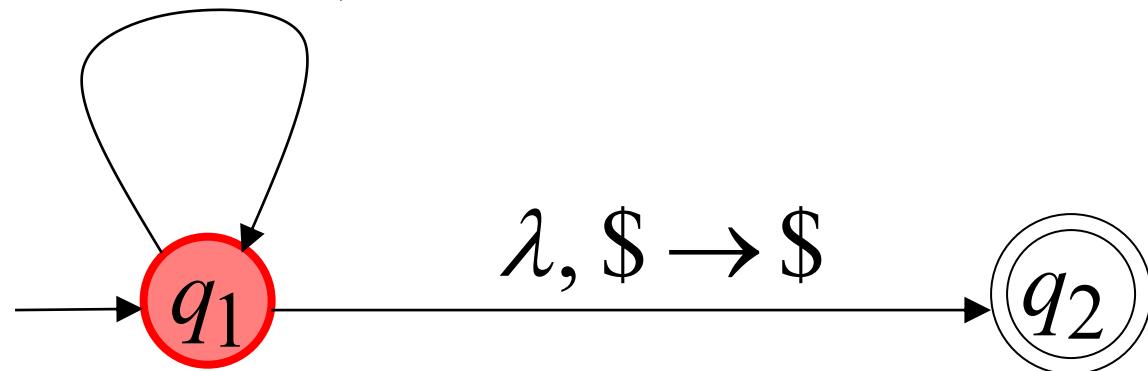
Input

a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



Stack

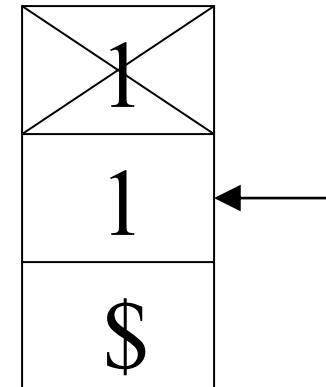
$$\begin{array}{ll} a, \$ \rightarrow 0\$ & b, \$ \rightarrow 1\$ \\ a, 0 \rightarrow 00 & b, 1 \rightarrow 11 \\ a, 1 \rightarrow \lambda & b, 0 \rightarrow \lambda \end{array}$$



Time 6

Input

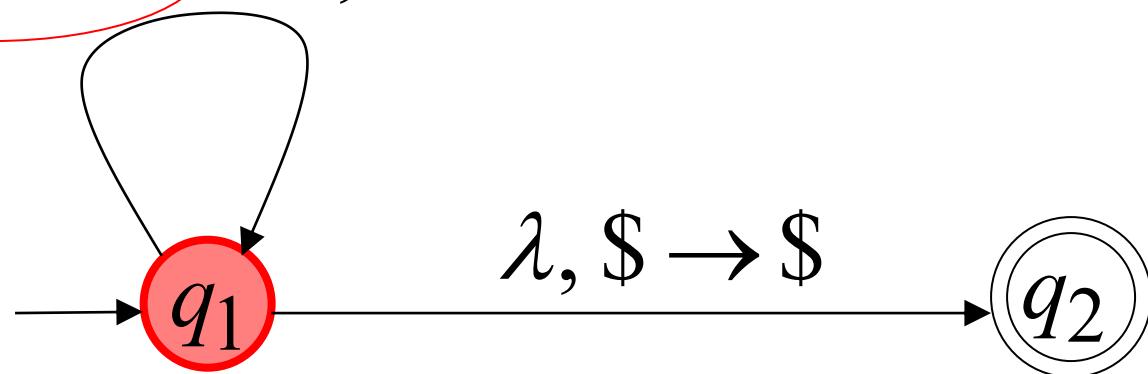
a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



Stack

תפקיד
הගדר

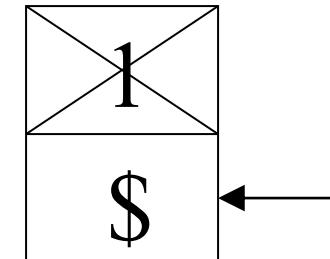
$$\left\{ \begin{array}{ll} a, \$ \rightarrow 0\$ & b, \$ \rightarrow 1\$ \\ a, 0 \rightarrow 00 & b, 1 \rightarrow 11 \\ a, 1 \rightarrow \lambda & b, 0 \rightarrow \lambda \end{array} \right\} \text{תפקיד הולך}$$



Time 7

Input

a	b	b	b	a	a
-----	-----	-----	-----	-----	-----

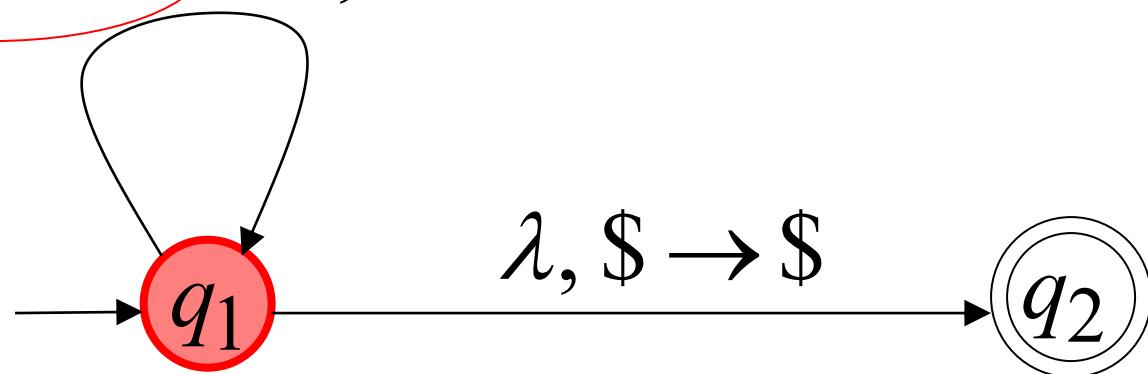


Stack

$$a, \$ \rightarrow 0\$ \quad b, \$ \rightarrow 1\$$$

$$a, 0 \rightarrow 00 \quad b, 1 \rightarrow 11$$

$$a, 1 \rightarrow \lambda \quad b, 0 \rightarrow \lambda$$



Time 8

Input

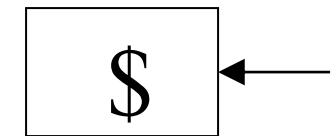
a	b	b	b	a	a
-----	-----	-----	-----	-----	-----



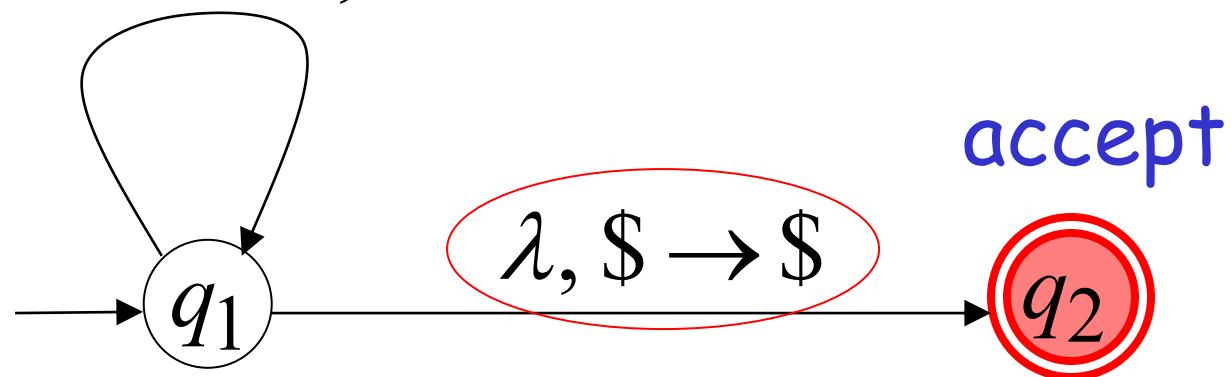
$$a, \$ \rightarrow 0\$ \quad b, \$ \rightarrow 1\$$$

$$a, 0 \rightarrow 00 \quad b, 1 \rightarrow 11$$

$$a, 1 \rightarrow \lambda \quad b, 0 \rightarrow \lambda$$



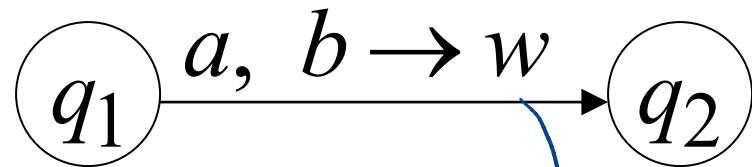
Stack



Formalities for NPDAs

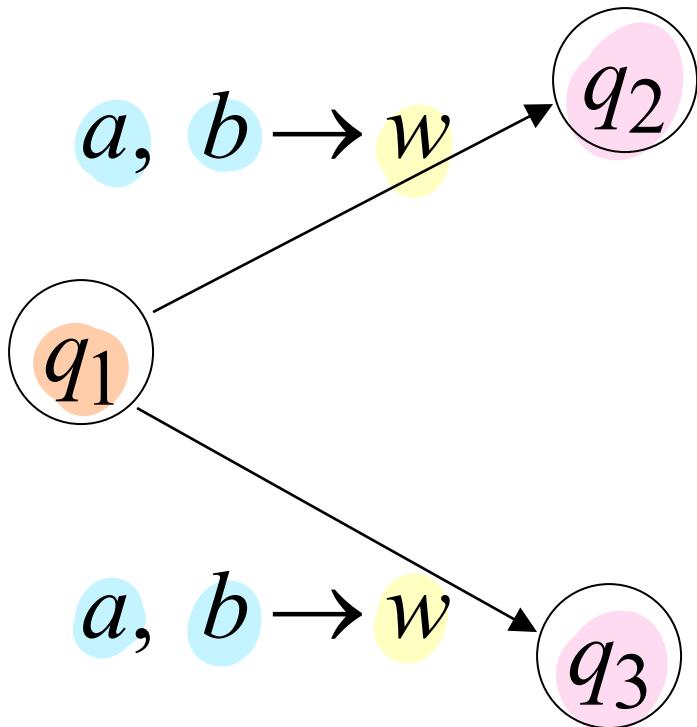
Text mode

formal



Transition function:

$$\delta(q_1, a, b) \xrightarrow{\text{state input pop / top stack}} \{(q_2, w)\}$$



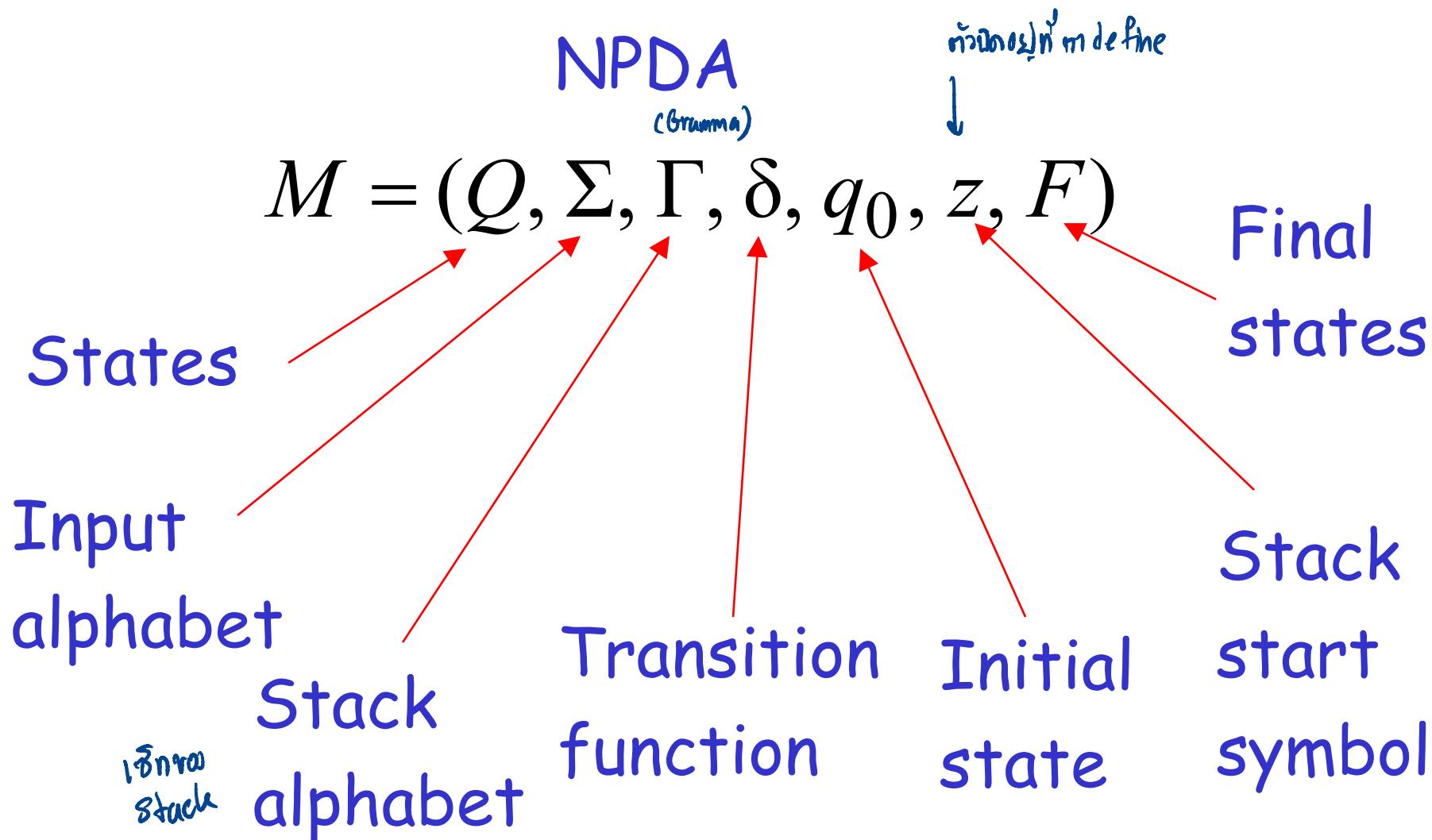
Transition function:

$$\delta(q_1, a, b) = \{(q_2, w), (q_3, w)\}$$

សាខាបីយោ រឿងខ្លួន

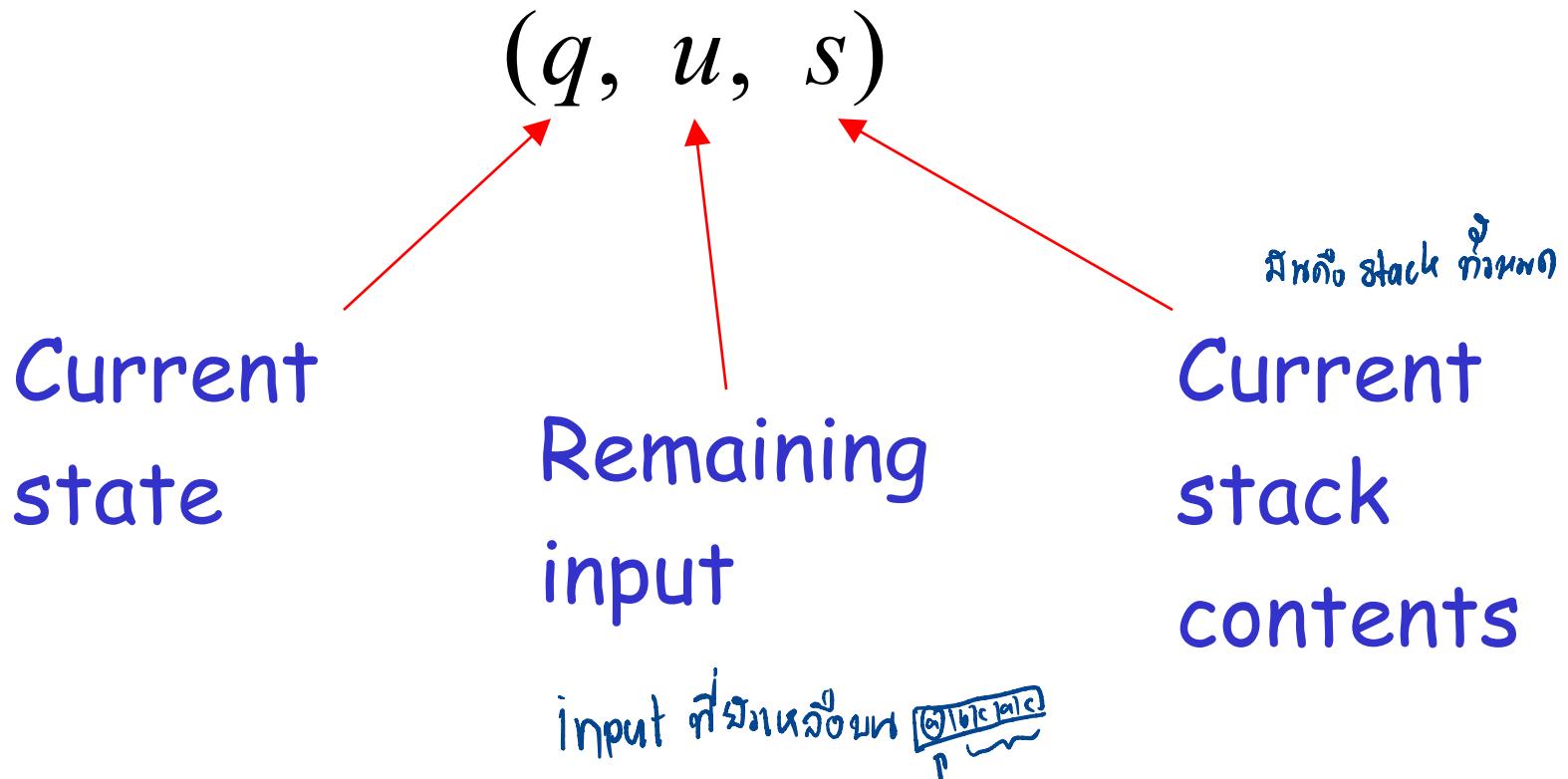
Formal Definition

Non-Deterministic Pushdown Automaton



Instantaneous Description

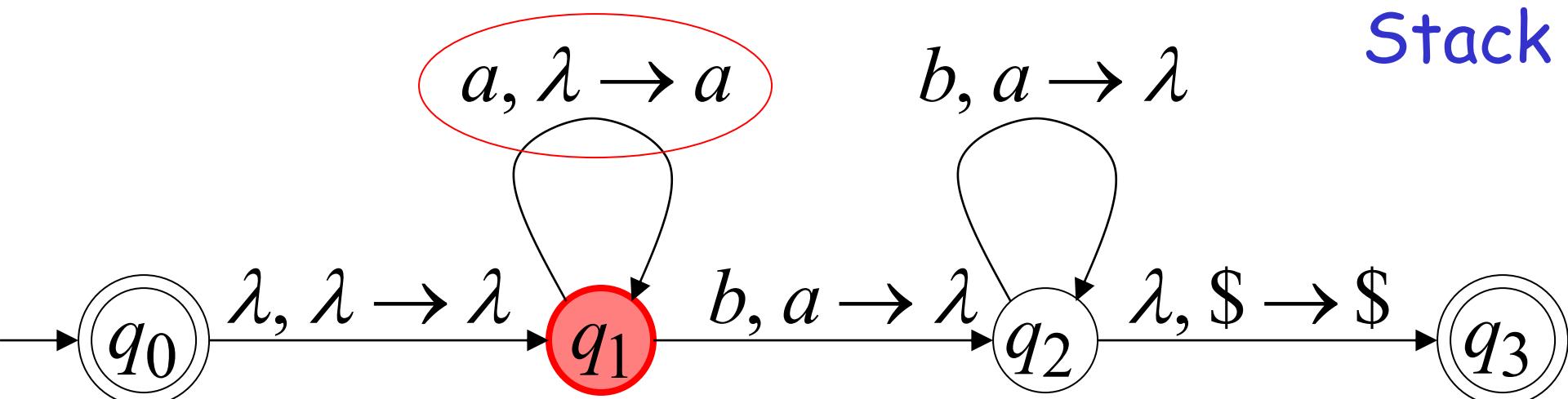
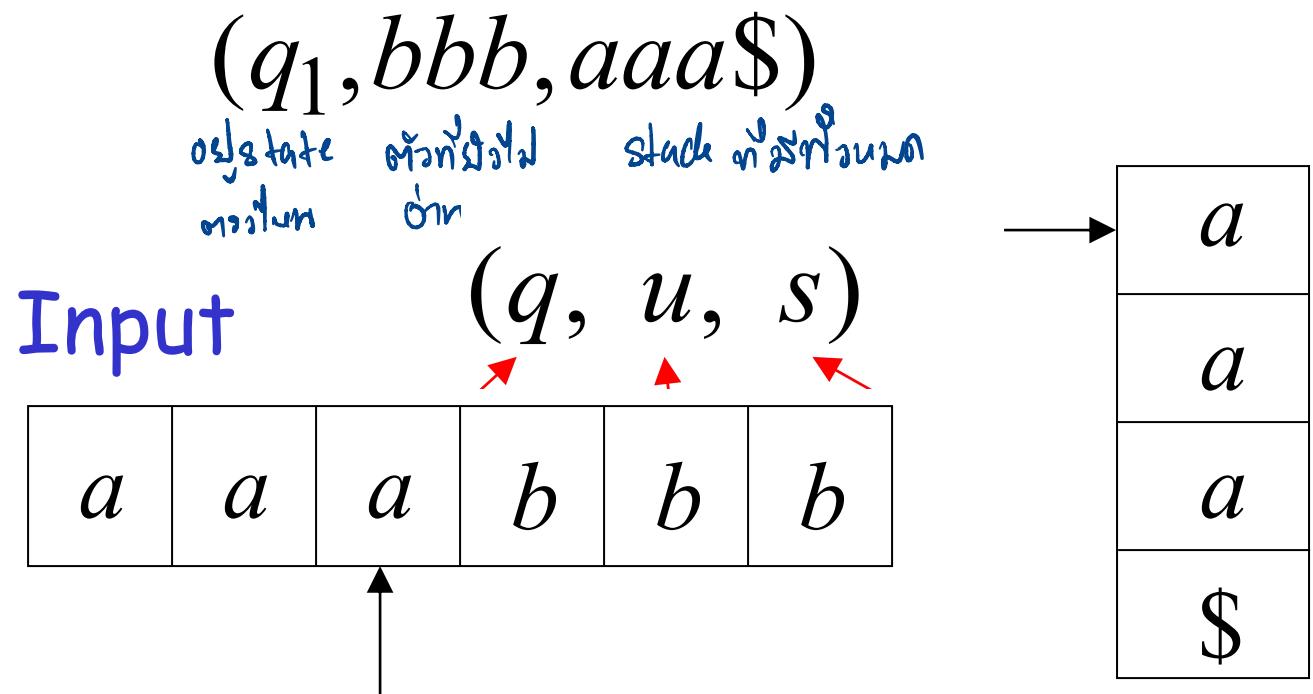
ဝန်ယူ Time step



Example:

Instantaneous Description

Time 4:



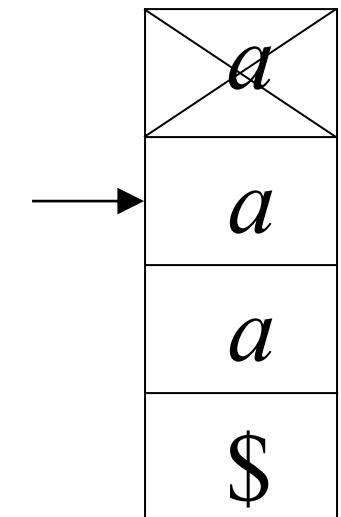
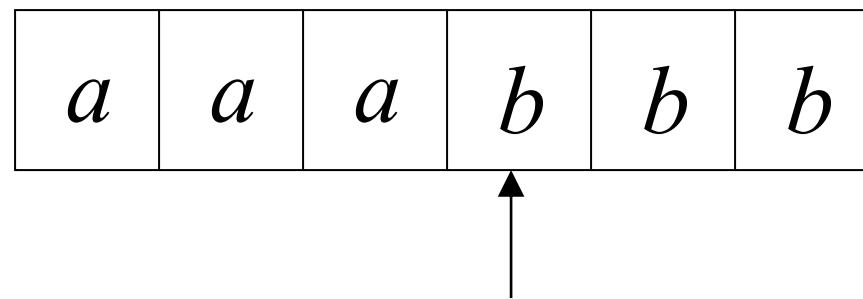
Example:

Instantaneous Description

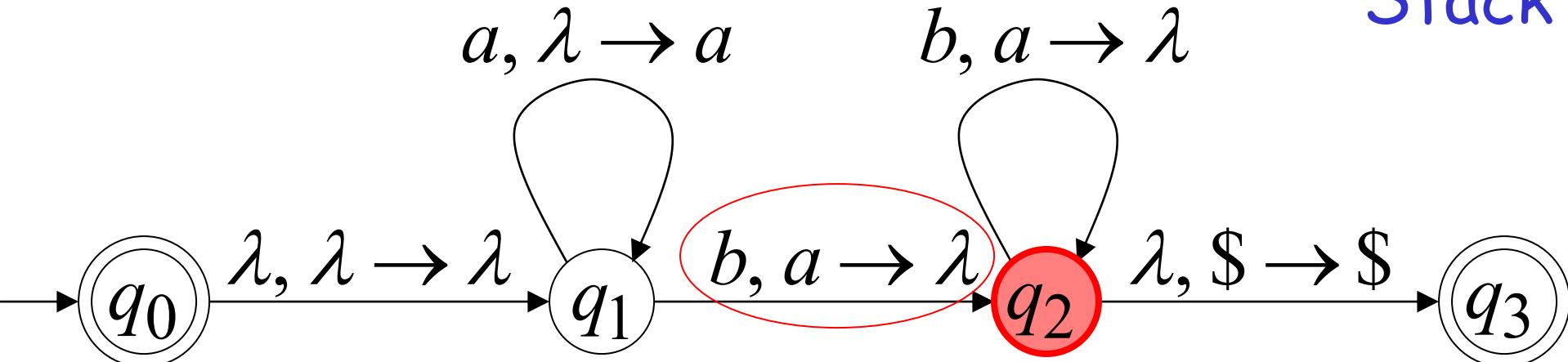
$(q_2, bb, aa\$)$

Time 5:

Input



Stack



We write:

$$(q_1, bbb, aaa\$) \xrightarrow{\text{remove } b} (q_2, bb, aa\$)$$

Time 4

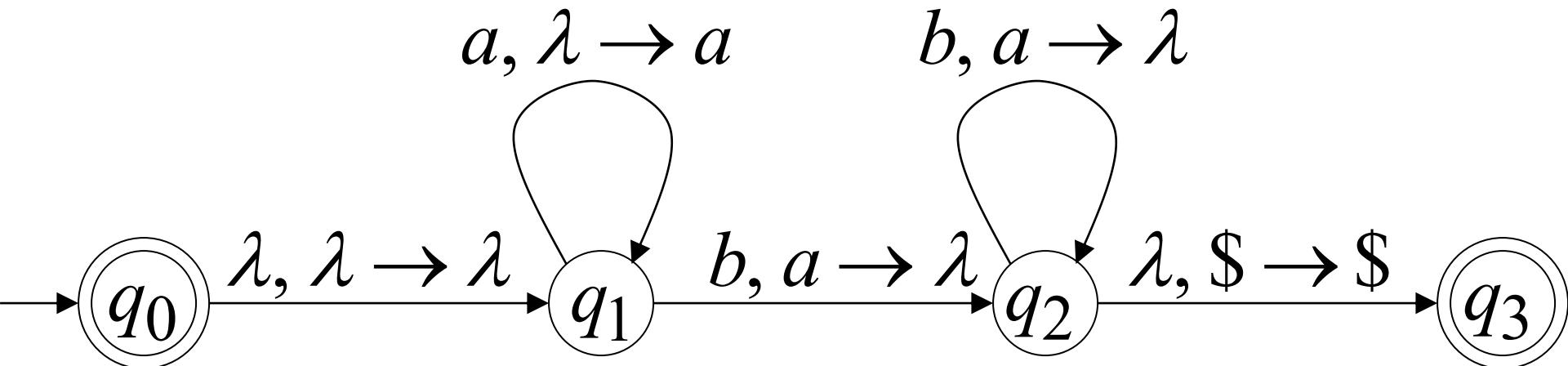
Time 5

A computation:

$(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ$

$(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ$

$(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$)$



$$(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ$$
$$(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ$$
$$(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$)$$

For convenience we write:

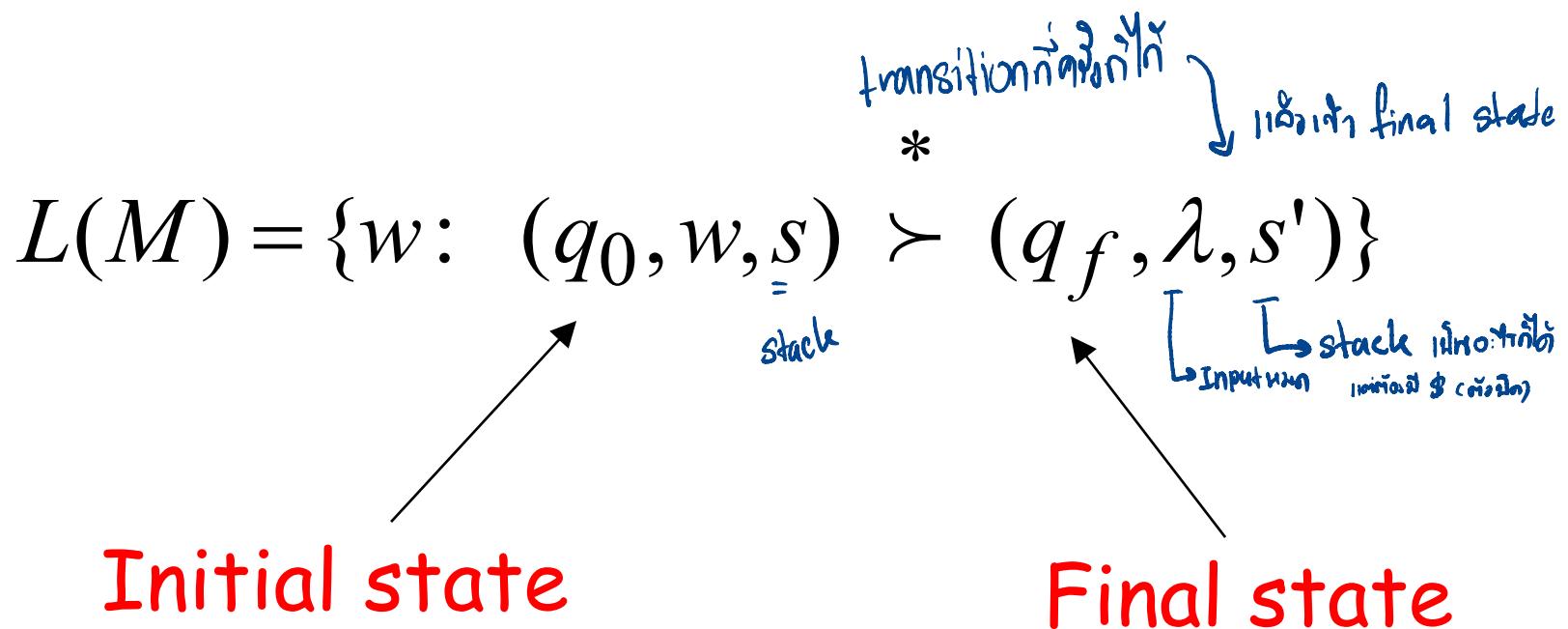
▷ ផ្លាស់បន្ថែម ឱ្យគឺជានុវត្តន៍របស់វា

*

$$(q_0, aaabbb, \$) \succ (q_3, \lambda, \$)$$

Formal Definition

Language $L(M)$ of NPDA M :



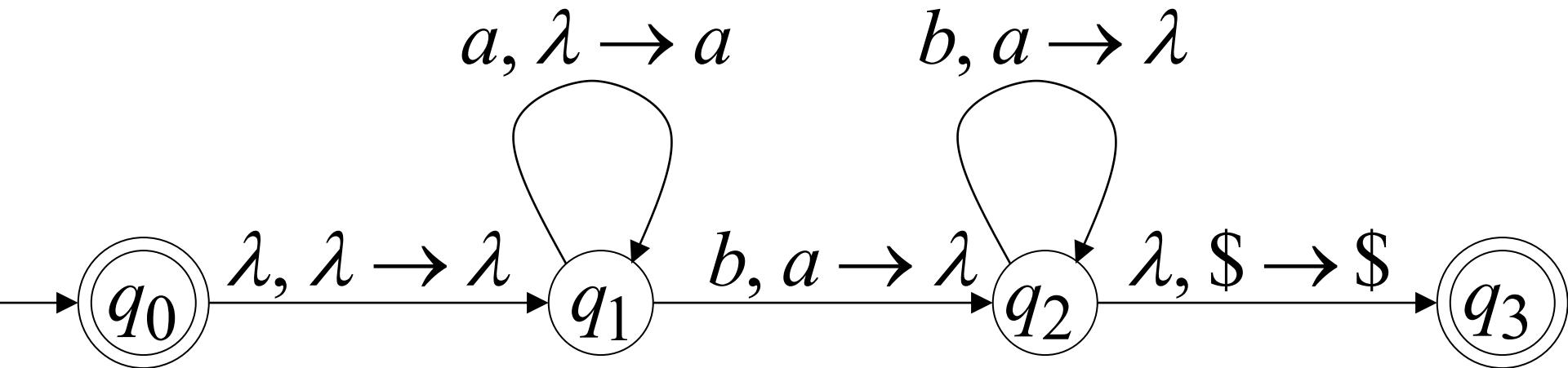
Example:

$$(q_0, aaabbb, \$) \xrightarrow{*} (q_3, \lambda, \$)$$

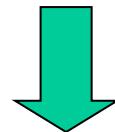


$$aaabbb \in L(M)$$

NPDA M :

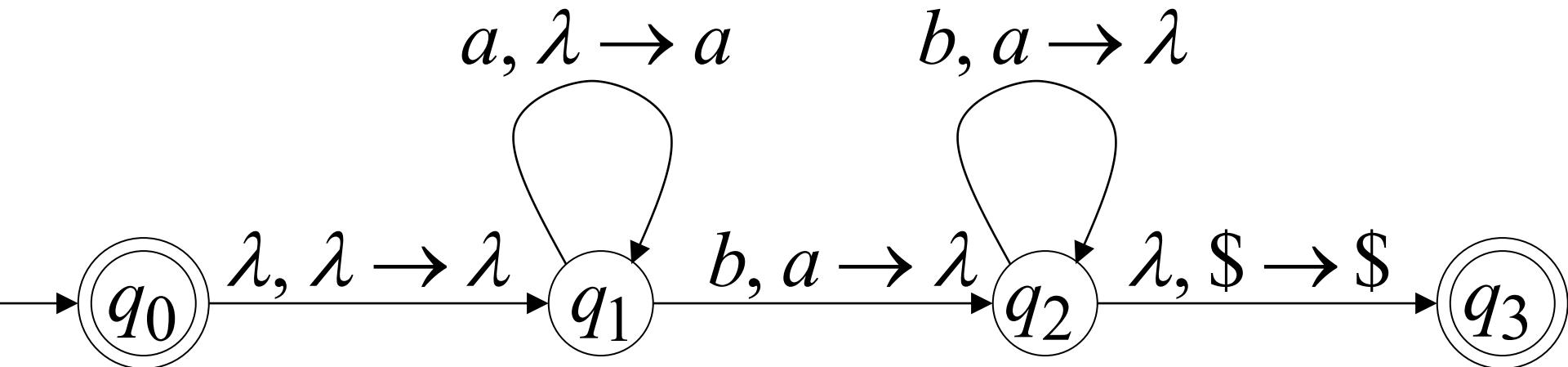


$$(q_0, a^n b^n, \$) \xrightarrow{*} (q_3, \lambda, \$)$$



$$a^n b^n \in L(M)$$

NPDA M :



Therefore: $L(M) = \{a^n b^n : n \geq 0\}$

NPDA M :

