Single Final State for NFAs

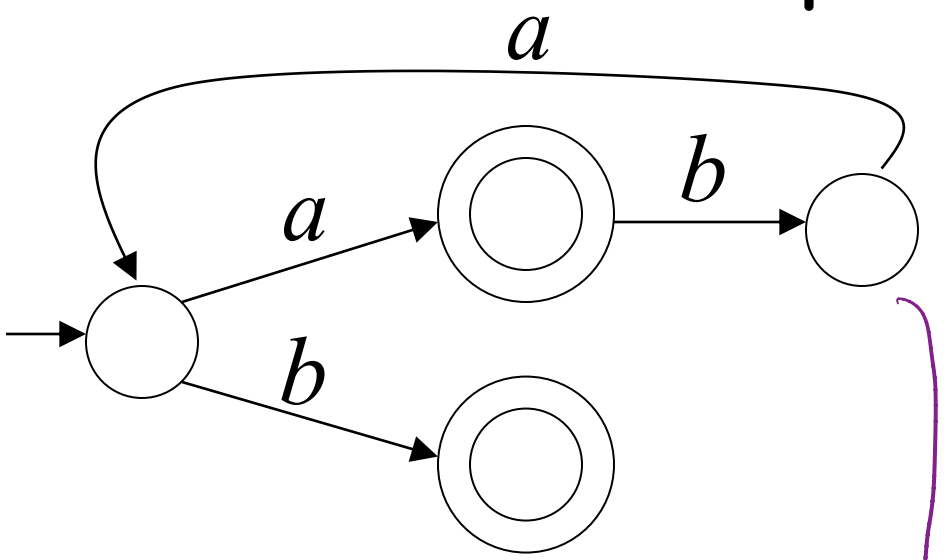
แปลง NFA หลาย final state
เป็น NFA 1 final state ยังไง

Any NFA can be converted

to an equivalent NFA

with a single final state

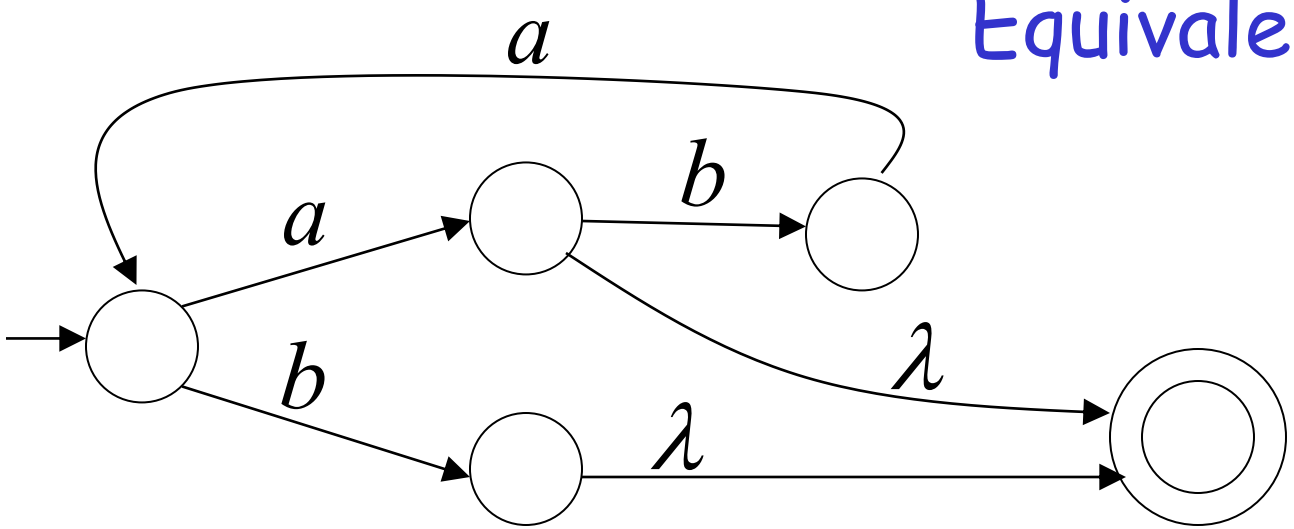
Example



NFA

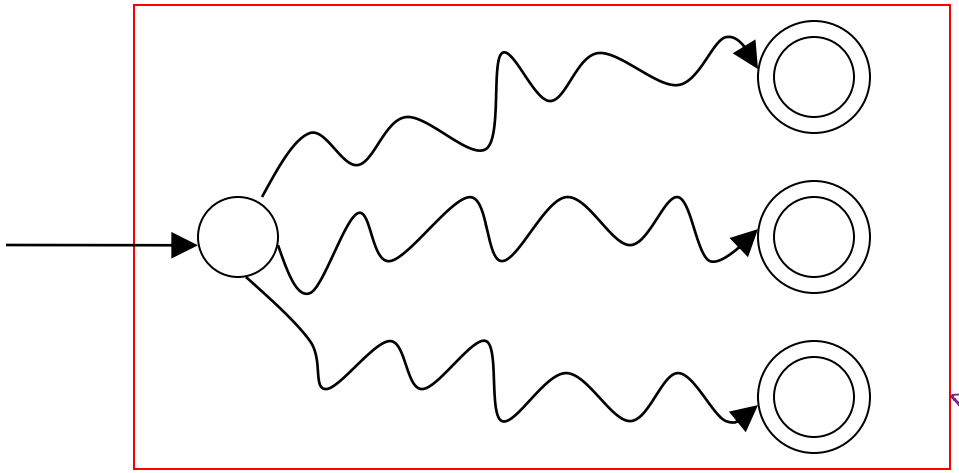
ลาก final state ใหม่

Equivalent NFA



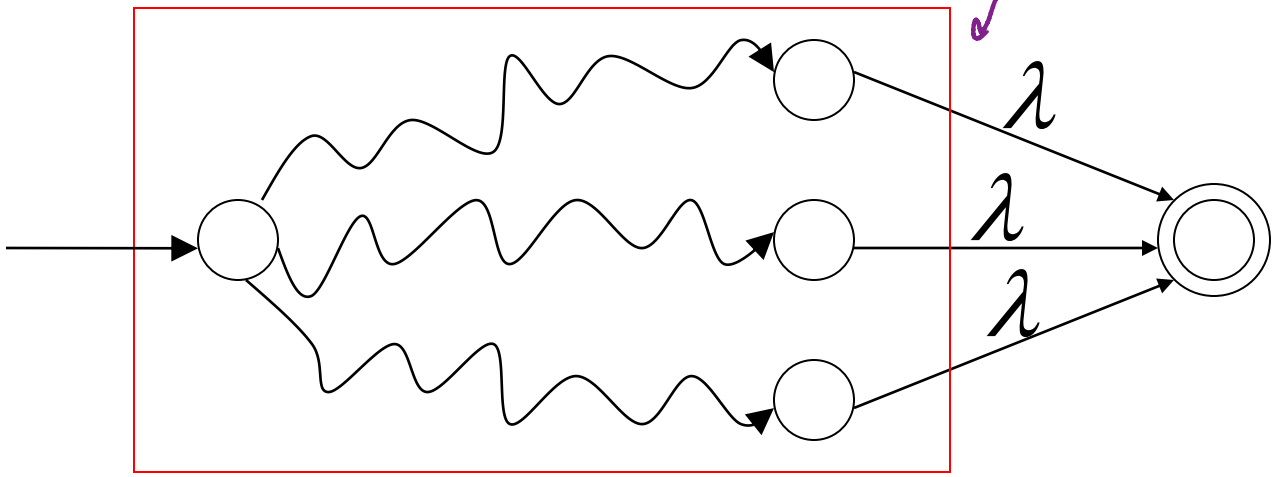
In General

NFA



สร้าง final state ขึ้นมาใหม่

Equivalent NFA

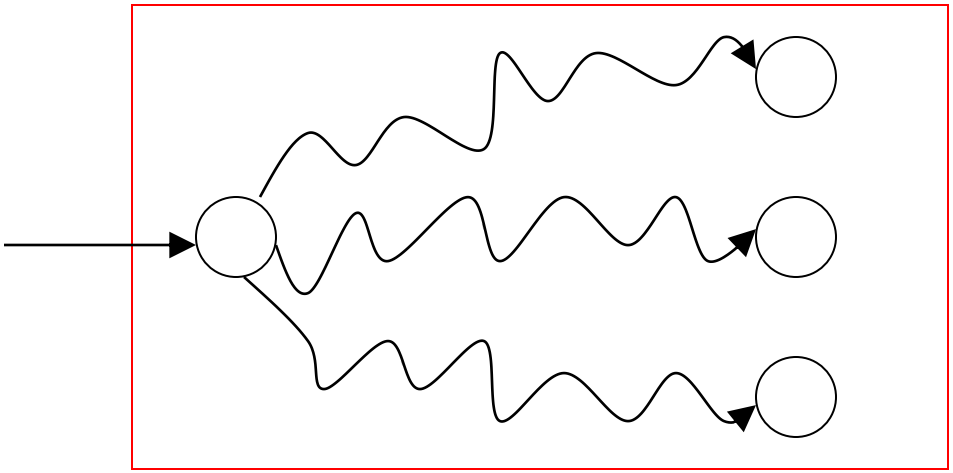


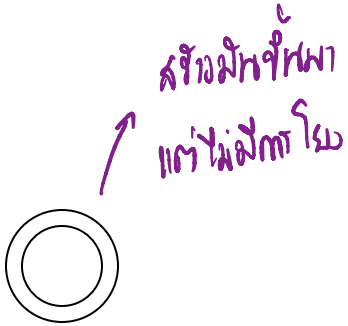
Single final state

Extreme Case

ตัวอย่าง

NFA without final state





Add a final state

Without transitions

คุณสมบัติ ปิดของ Regular

Properties of Regular Languages

For regular languages L_1 and L_2 we will prove that:

ดุกรณีปิดมี 6 ข้อ

1. Union: $L_1 \cup L_2$
2. Concatenation: $L_1 L_2$
3. Star: L_1^*
4. Reversal: L_1^R
5. Complement: $\overline{L_1}$
6. Intersection: $L_1 \cap L_2$

ยังคงเป็นภาษา regular ทั้งหมด

Are regular Languages

We say: Regular languages are **closed under**

เป็น operation ที่เวลา กระทำกับ regular

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1{}^*$

Reversal: $L_1{}^R$

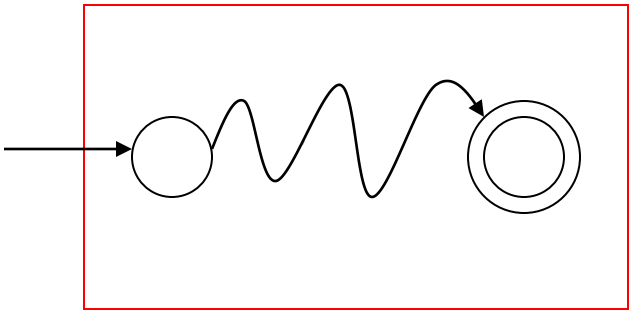
Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

Regular language L_1

$L(M_1) = L_1$

NFA M_1

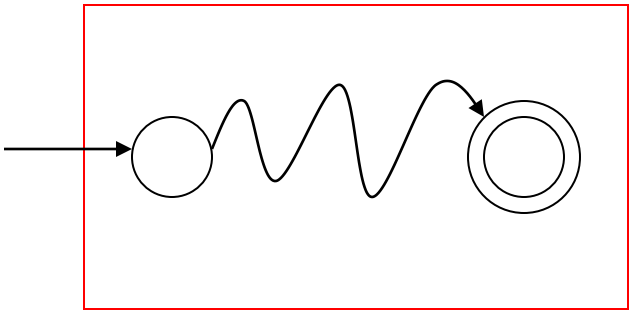


Single final state

Regular language L_2

$L(M_2) = L_2$

NFA M_2



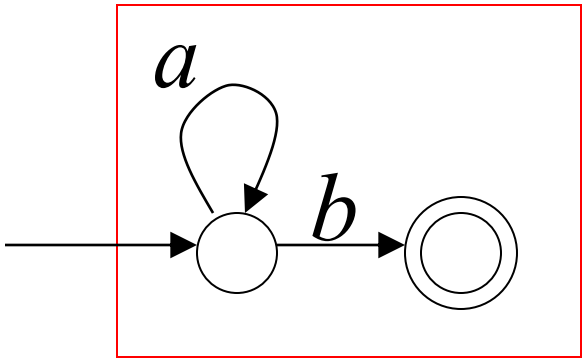
Single final state

Example

$n \geq 0$

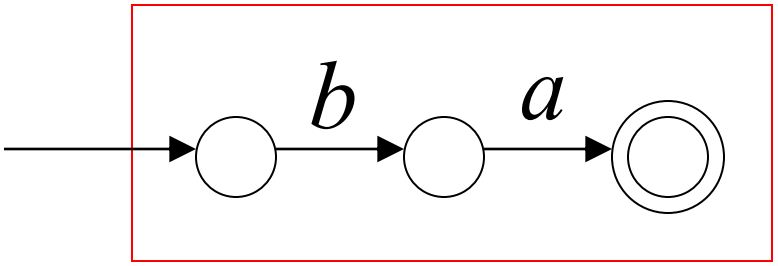
$L_1 = \{a^n b\}$

M_1



$L_2 = \{ba\}$

M_2

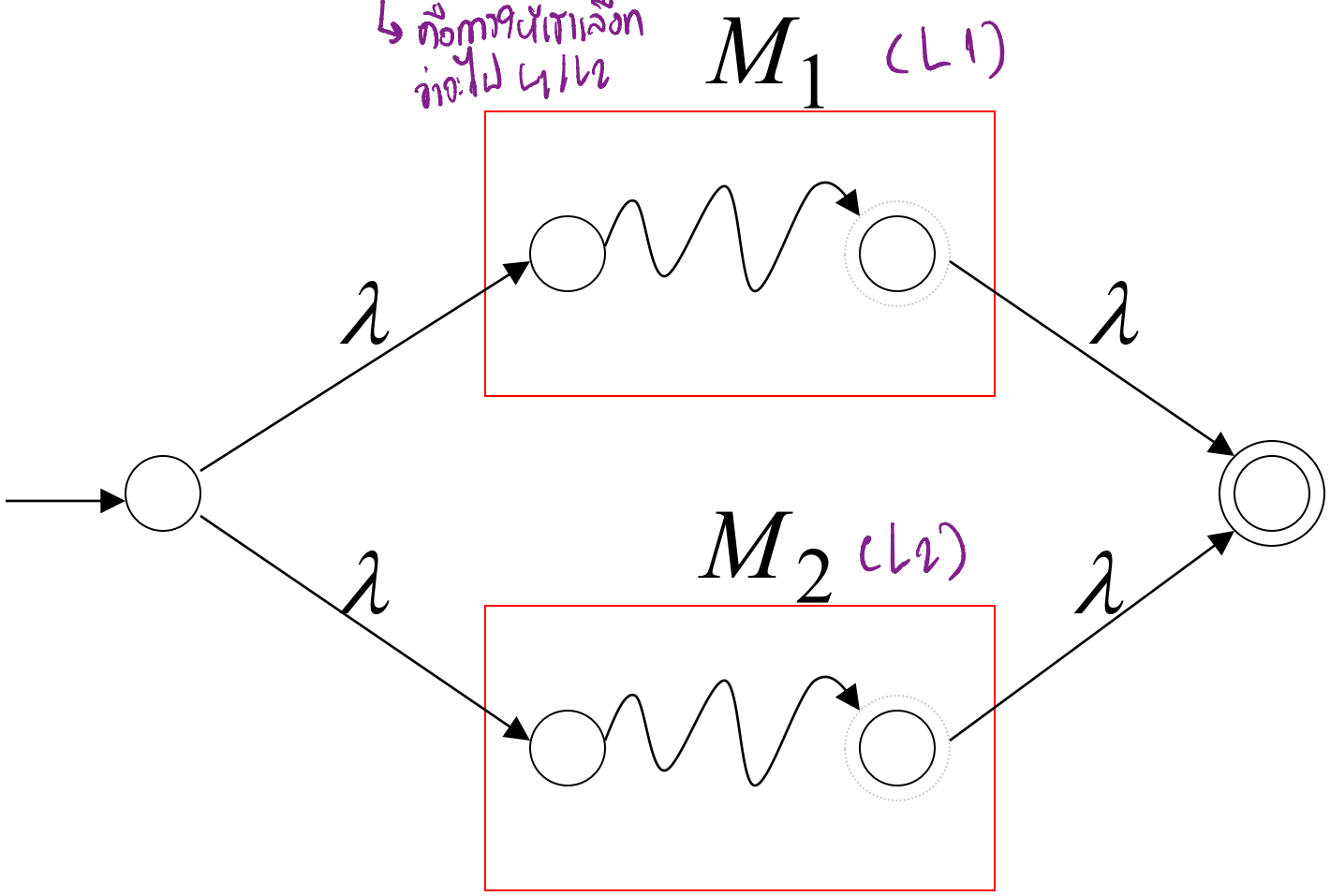


Union

NFA for $L_1 \cup L_2$

ผลจาก การ ∪ ต้องเป็น Regular ดังนั้น สร้าง NFA ได้

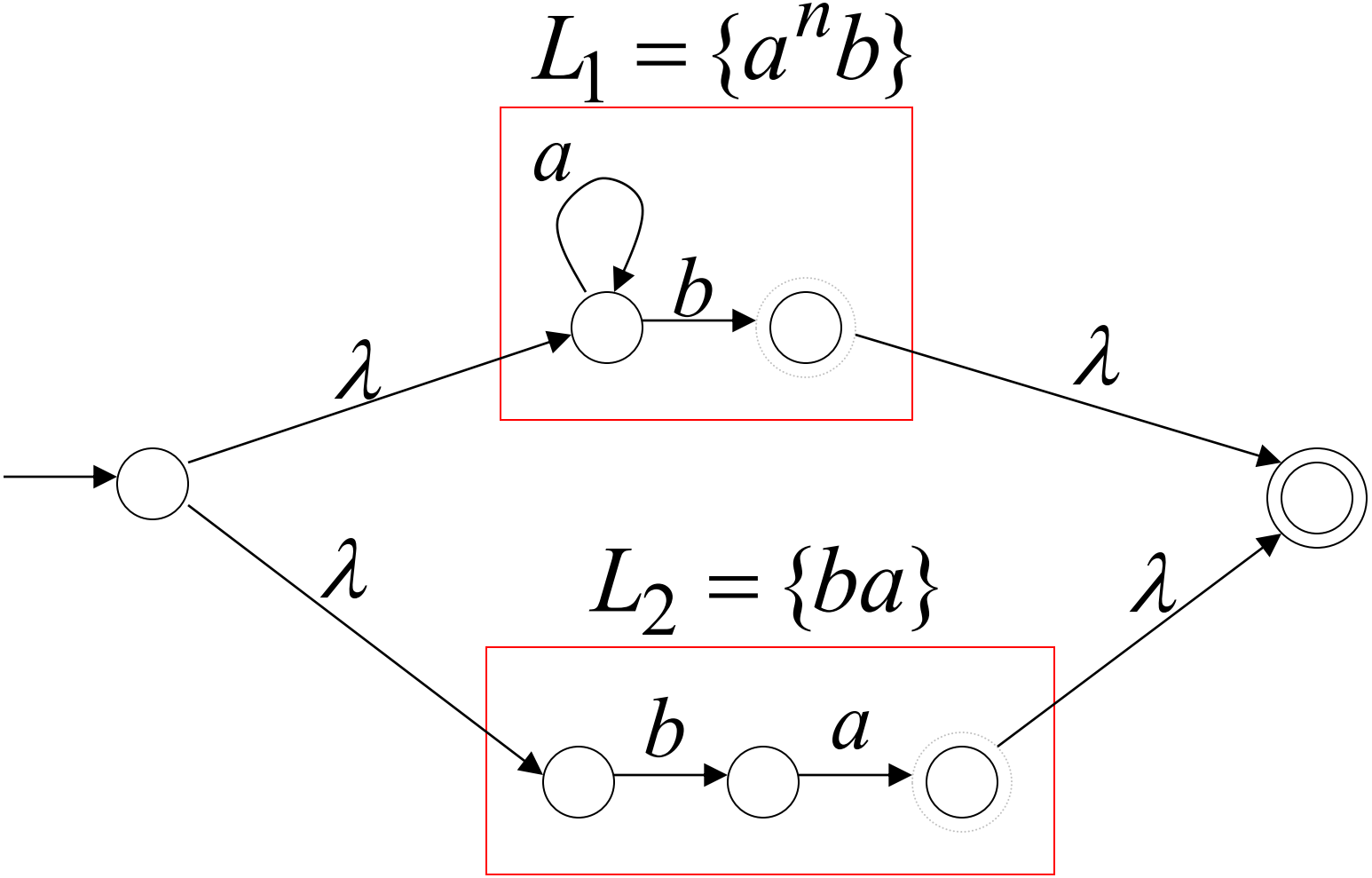
↳ คือการที่เราเลือกว่าจะไป L_1 หรือ



นี่คือ NFA ของการ ∪

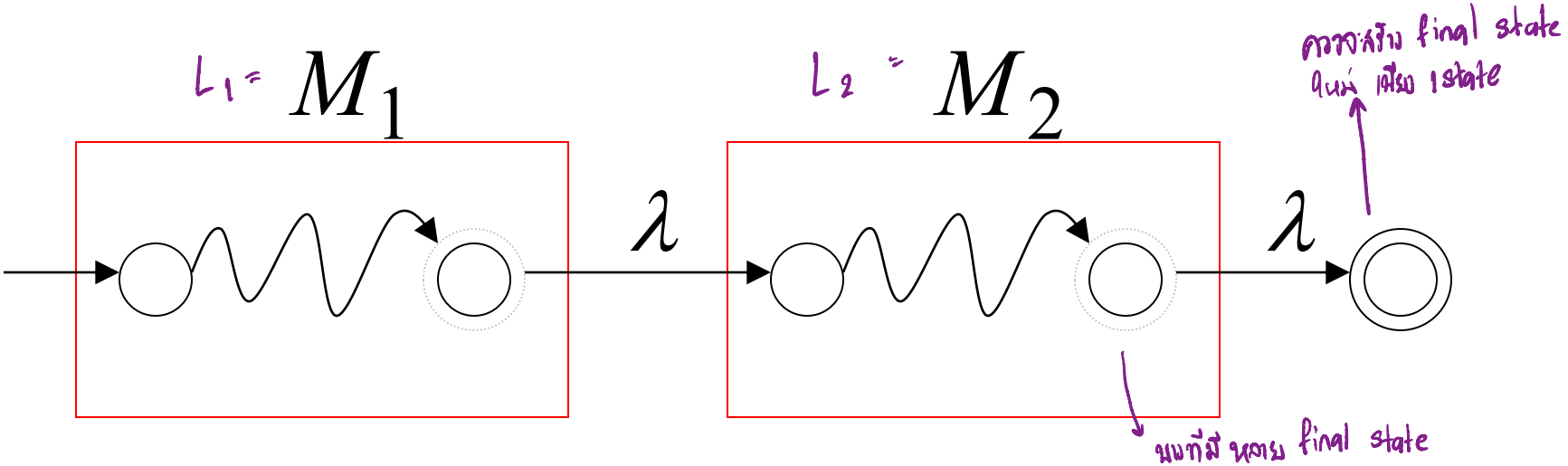
Example

NFA for $L_1 \cup L_2 = \{a^n b\} \cup \{ba\}$



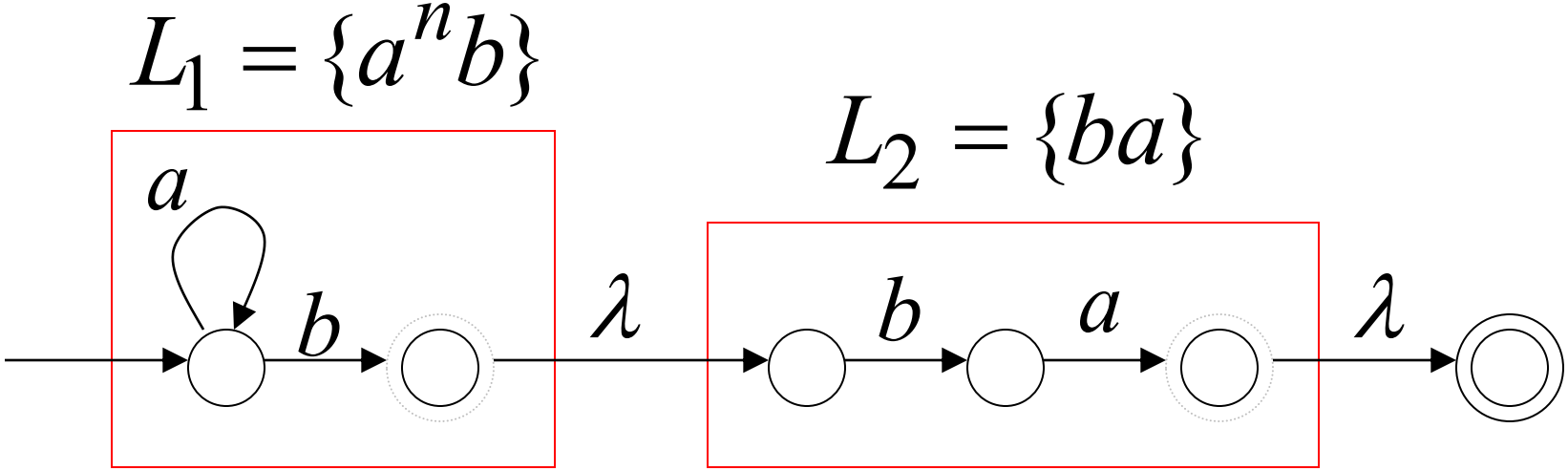
Concatenation

NFA for $L_1 L_2$



Example

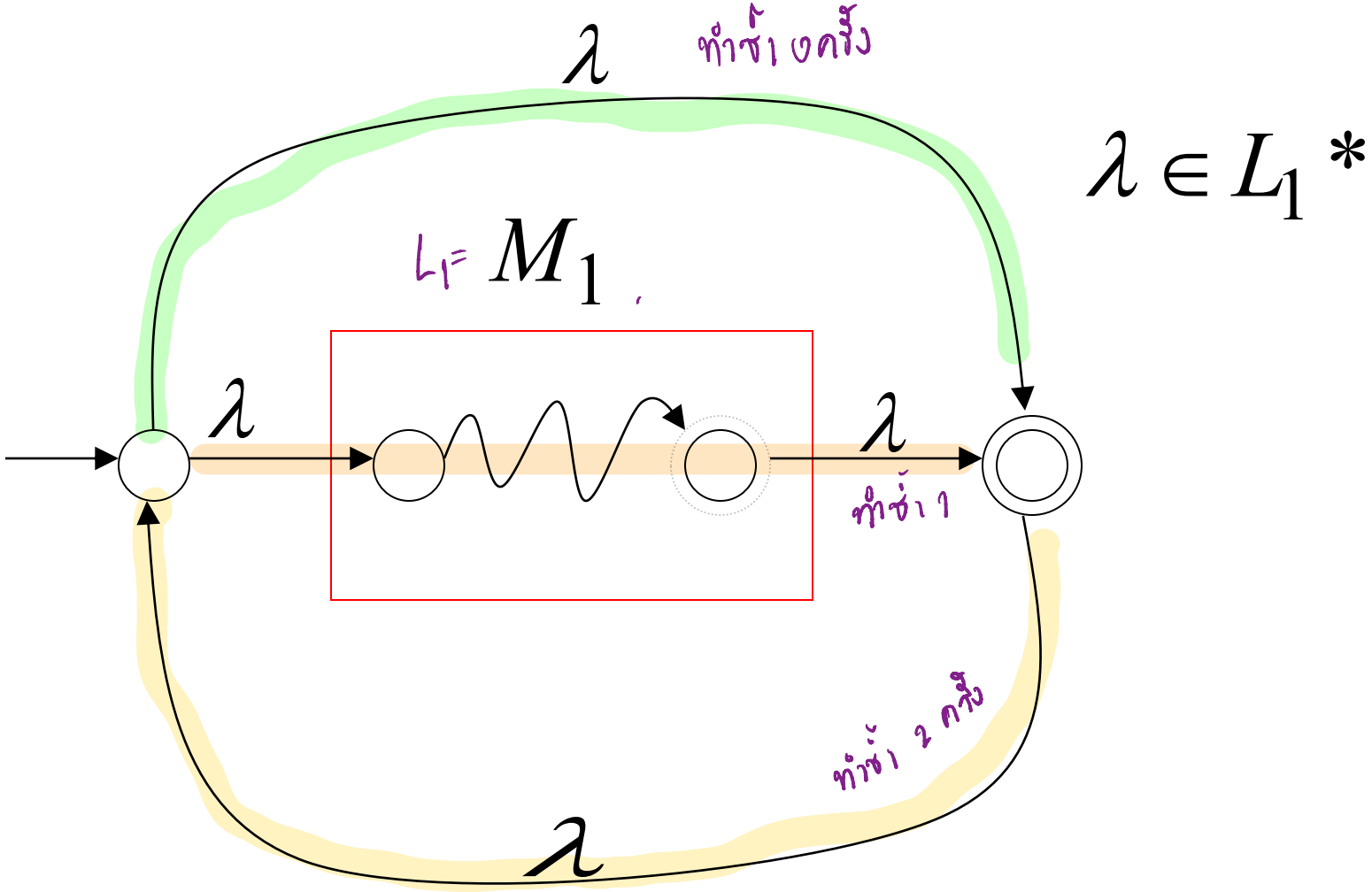
NFA for $L_1 L_2 = \{a^n b\}\{ba\} = \{a^n bba\}$



Star Operation

ทำตั้งแต่ 0 ครั้งขึ้นไป

NFA for $L_1{}^*$

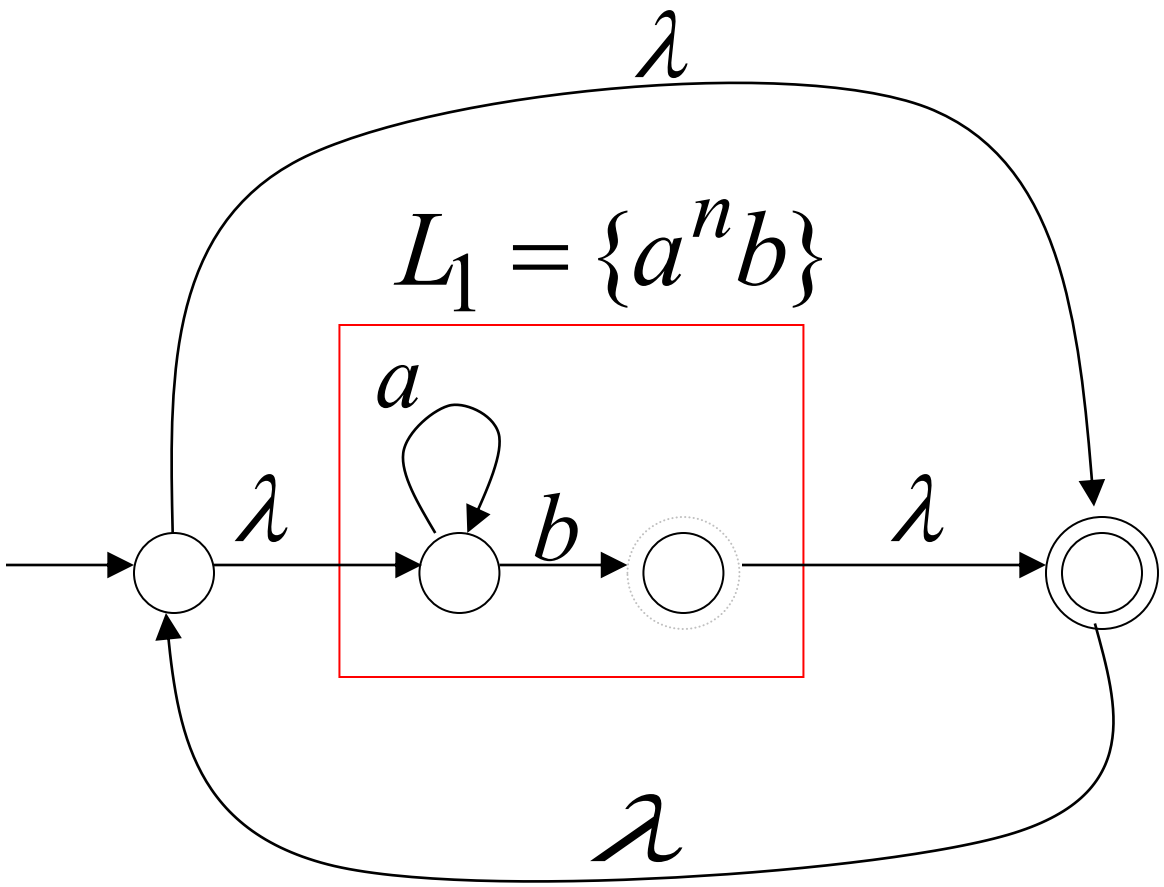


Example

NFA for $L_1^* = \{a^n b\}^*$

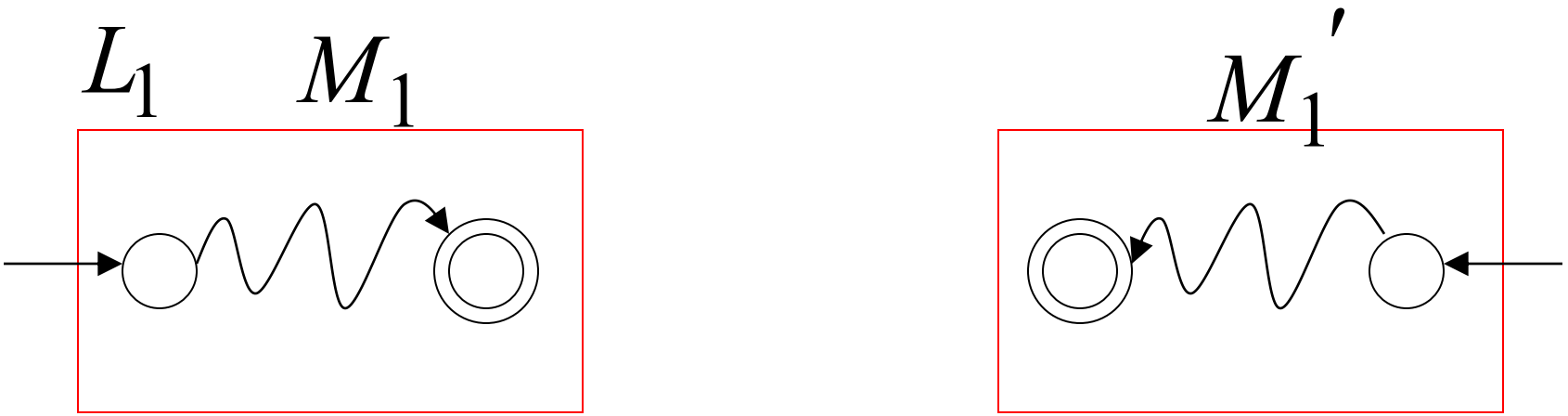
$w = w_1 w_2 \cdots w_k$

$w_i \in L_1$



Reverse

NFA for L_1^R



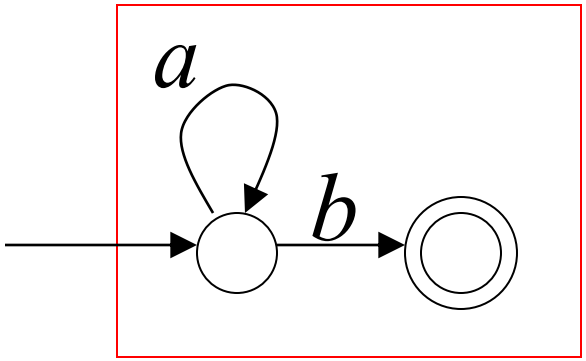
1. Reverse all transitions เปลี่ยนทิศทางเป็นตรงข้าม

2. Make initial state final state สลับ initial state กับ final state
   and vice versa

Example

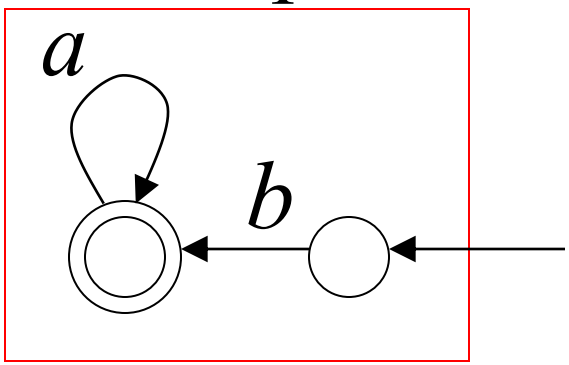
$L_1 = \{a^n b\}$

M_1

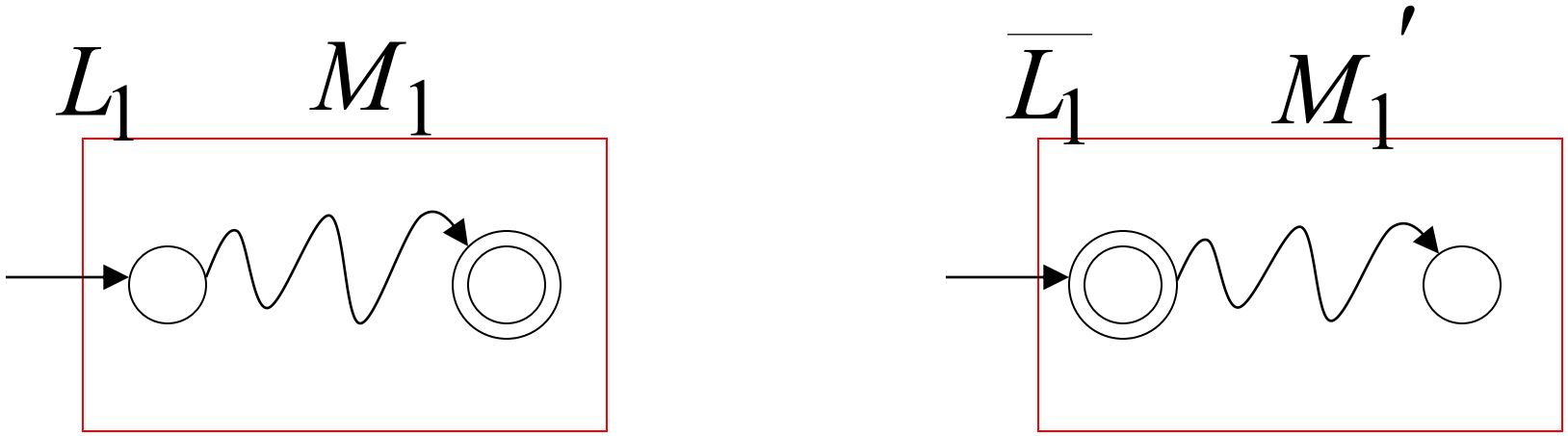


$L_1^R = \{b a^n\}$

M_1'



Complement

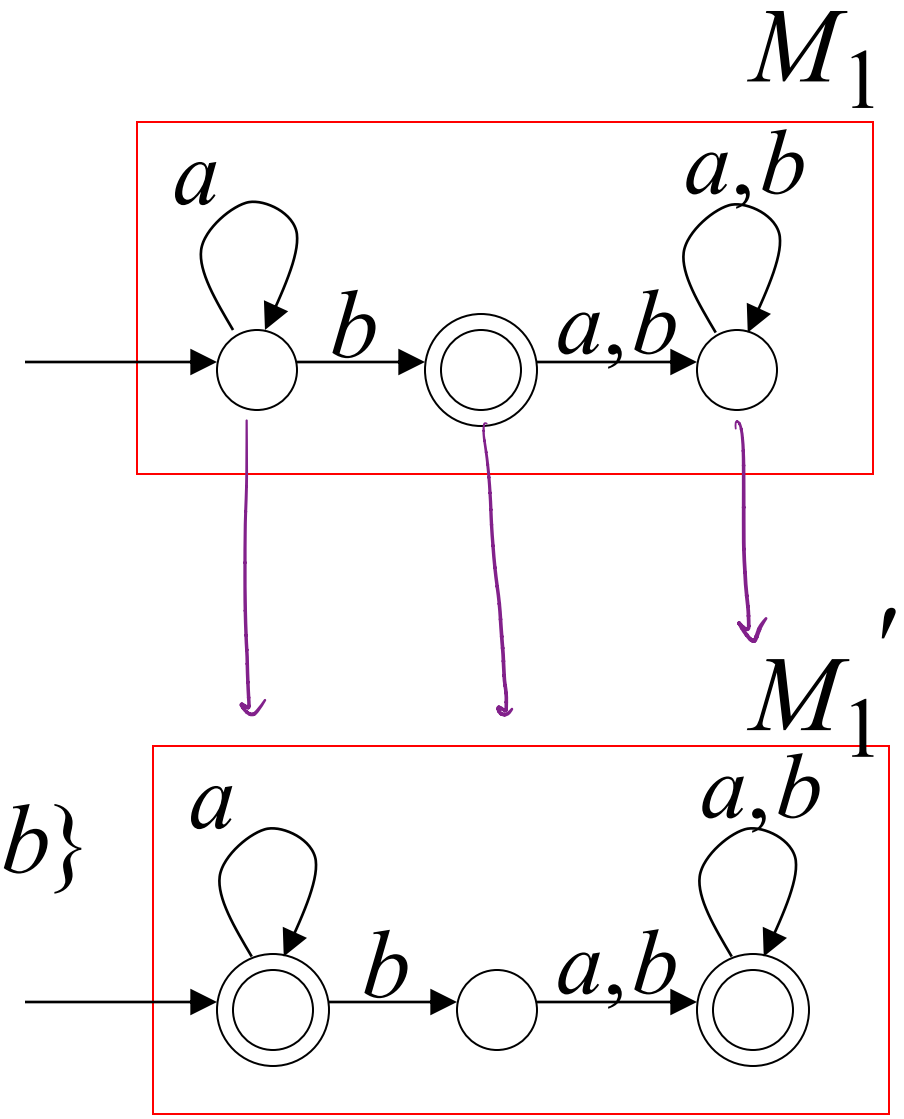


1. Take the **DFA** that accepts L_1

2. Make final states non-final, and vice-versa

Example

$L_1 = \{a^n b\}$



$\overline{L_1} = \{a,b\}^* - \{a^n b\}$

Intersection

DeMorgan's Law: $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$

$= \overline{\overline{L_1}} \cap \overline{\overline{L_2}}$

$= L_1 \cap L_2$

L_1 , L_2 regular

$\Rightarrow$ $\overline{L_1}$, $\overline{L_2}$ regular

$\Rightarrow$ $\overline{L_1} \cup \overline{L_2}$ regular

$\Rightarrow$ $\overline{\overline{L_1} \cup \overline{L_2}}$ regular

$\Rightarrow$ $L_1 \cap L_2$ regular

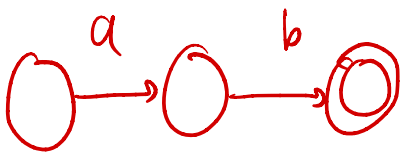
Example

$L_1 = \{a^n b\}$ regular

$L_2 = \{ab, ba\}$ regular

$L_1 \cap L_2 = \{ab\}$

regular



เครื่องมืออธิบายภาษา

Regular Expressions

Regular Expressions

Regular expressions

describe regular languages

Example: $(a+b\cdot c)^*$

describes the language

$$\{a,bc\}^* = \{\lambda, a, bc, aa, abc, bca, \ldots\}$$

Why do we need Regular Expressions ? <<Click>>

Recursive Definition

alphabet

Primitive regular expressions: $\emptyset, \quad \lambda, \quad \alpha$

Base case

Given regular expressions r_1 and r_2

Union

$r_1 + r_2$

$r_1 \cdot r_2$

r_1^*

(r_1)

Are regular expressions

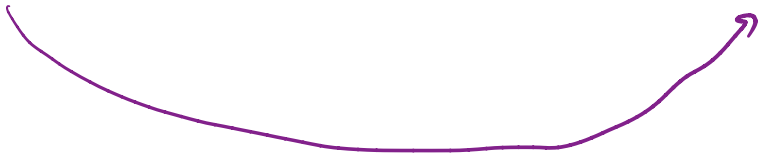
Examples

A regular expression: $(a+b\cdot c)^*\cdot(c+\varnothing)$

Not a regular expression: $(a+b+)$ ✗ reg ที่สมบูรณ์

Languages of Regular Expressions

$L(r)$: language of regular expression r



Example

$$L((a+b\cdot c)^*)=\{\lambda,a,bc,aa,abc,bca,\ldots\}$$

r

Definition

For primitive regular expressions:

$$L(\emptyset) = \emptyset$$

$$L(\lambda) = \{\lambda\}$$

$$L(a) = \{a\}$$

Definition (continued)

For regular expressions r_1 and r_2

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) = L(r_1).L(r_2)$$

↳ มี/ไม่ ก็ได้

$$L(r_1^*) = (L(r_1))^*$$

$$L((r_1)) = L(r_1)$$

Example

Regular expression: $(a+b)\cdot a^*$

$$
\begin{aligned}
L((a+b)\cdot a^*) &= L((a+b))\,L(a^*) \\
&= L(a+b)\,L(a^*) \\
&= (L(a)\cup L(b))\,(L(a))^* \\
&= (\{a\}\cup\{b\})\,(\{a\})^* \\
&= \{a,b\}\,\{\lambda,a,aa,aaa,\ldots\} \\
&= \{a,aa,aaa,\ldots,b,ba,baa,\ldots\}
\end{aligned}
$$

Example

$\{a,b\}^* \cdot \{a, bb\}^*$

Regular expression $r = (a+b)^*(a+bb)$

$\{\{a\} \cup \{b\}\}^* \{a\} \cup \{bb\}$

$\{a,b\}^* \cdot \{a, bb\}$

$L(r) = \{a, bb, aa, abb, ba, bbb, \ldots\}$

Example

ไม่จำเป็นต้องตัวเลขเท่ากัน

Regular expression $r = (aa)^*(bb)^* b$

$\{aa\}^*$ $\{bb\}^*$ b

$$L(r) = \{a^{2n} b^{2m} b : \quad n, m \geq 0\}$$

คู่ปกติ ไม่

Example

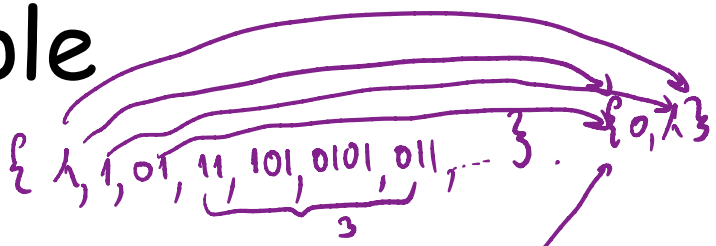
Regular expression $r = (0+1)^* 00 (0+1)^*$

$(01)^n 00 (01)^m$

ที่มีเลข 0 อยู่ติดกัน อย่างน้อย 2 ตัวขึ้นไป

$L(r)$ = { all strings with at least two consecutive 0 }

Example



Regular expression $r = (1+01)^*(0+\lambda)$

$\{\lambda, 0, 1, 10, 01, 010, \ldots\}$

$\{1, 01\}^* \cdot \{0\}^*$

ที่ไม่มีเลข 0 ติดกันสองตัว

$L(r)$ = { all strings without two consecutive 0 }

Equivalent Regular Expressions

Definition:

Regular expressions r_1 and r_2

ก็คือ

are **equivalent** if $L(r_1) = L(r_2)$

Example

L = { all strings without two consecutive 0 }

$$r_1 = (1+01)^*(0+\lambda)$$

ง่ายและสั้นที่สุด

$$r_2 = (1^*011^*)^*(0+\lambda)+1^*(0+\lambda)$$

$$L(r_1) = L(r_2) = L \Rightarrow$$

r_1 and r_2 are equivalent regular expr.

Regular Expressions and Regular Languages

Theorem

พิสูจน์ว่าเท่ากัน

{ Languages Generated by Regular Expressions } = { Regular Languages }

Theorem - Part 1

subset

$$\left\{\begin{array}{l}\text{Languages}\\ \text{Generated by}\\ \text{Regular Expressions}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Regular}\\ \text{Languages}\end{array}\right\}$$

set fun

1. For any regular expression r the language $L(r)$ is regular

Theorem - Part 2

set equ

$$\left\{\begin{array}{l}\text{Languages}\\ \text{Generated by}\\ \text{Regular Expressions}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Regular}\\ \text{Languages}\end{array}\right\}$$

2. For any regular language L there is a regular expression r with $L(r) = L$

Proof - Part 1

1. For any regular expression r the language $L(r)$ is regular

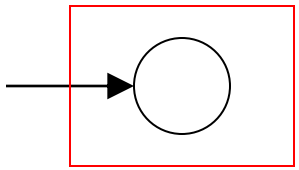
Proof by induction on the size of r

Induction Basis

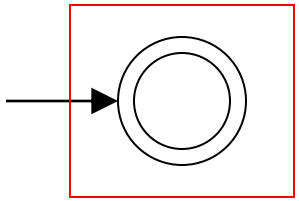
Case

Primitive Regular Expressions: $\emptyset$, λ, α

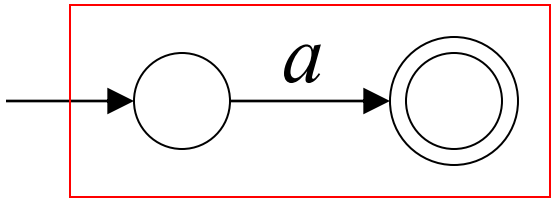
NFAs



$$L(M_1) = \emptyset = L(\emptyset)$$



$$L(M_2) = \{\lambda\} = L(\lambda)$$



$$L(M_3) = \{a\} = L(a)$$

regular languages

Inductive Hypothesis

Assume

for regular expressions r_1 and r_2

that

$L(r_1)$ and $L(r_2)$ are regular languages

Inductive Step

We will prove:

$$L(r_1 + r_2)$$

$$L(r_1 \cdot r_2)$$

$$L(r_1 *)$$

$$L((r_1))$$

Are regular Languages

By definition of regular expressions:

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) = L(r_1)\, L(r_2)$$

$$L(r_1^*) = (L(r_1))^*$$

$$L((r_1)) = L(r_1)$$

จากคุณสมบัติที่เป็น regular

By inductive hypothesis we know:

$L(r_1)$ and $L(r_2)$ are regular languages

สำเอามาทำ

We also know:

Regular languages are closed under:

Union	$L(r_1) \cup L(r_2)$
Concatenation	$L(r_1)\,L(r_2)$
Star	$(L(r_1))^*$

เป็น reg ทั้งหมด

Therefore:

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) = L(r_1)\, L(r_2)$$

$$L(r_1 *) = (L(r_1))*$$

Are regular languages

And trivially:

$L((r_1))$ is a regular language

Proof – Part 2

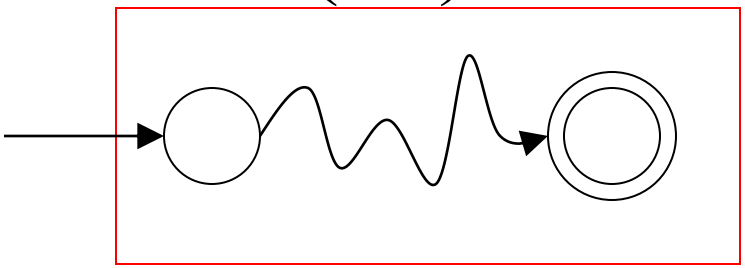
2. For any regular language L there is a regular expression r with $L(r) = L$

Proof by construction of regular expression

Since L is regular take the NFA M that accepts it

ต้อง NFA นี้

$$L(M) = L$$



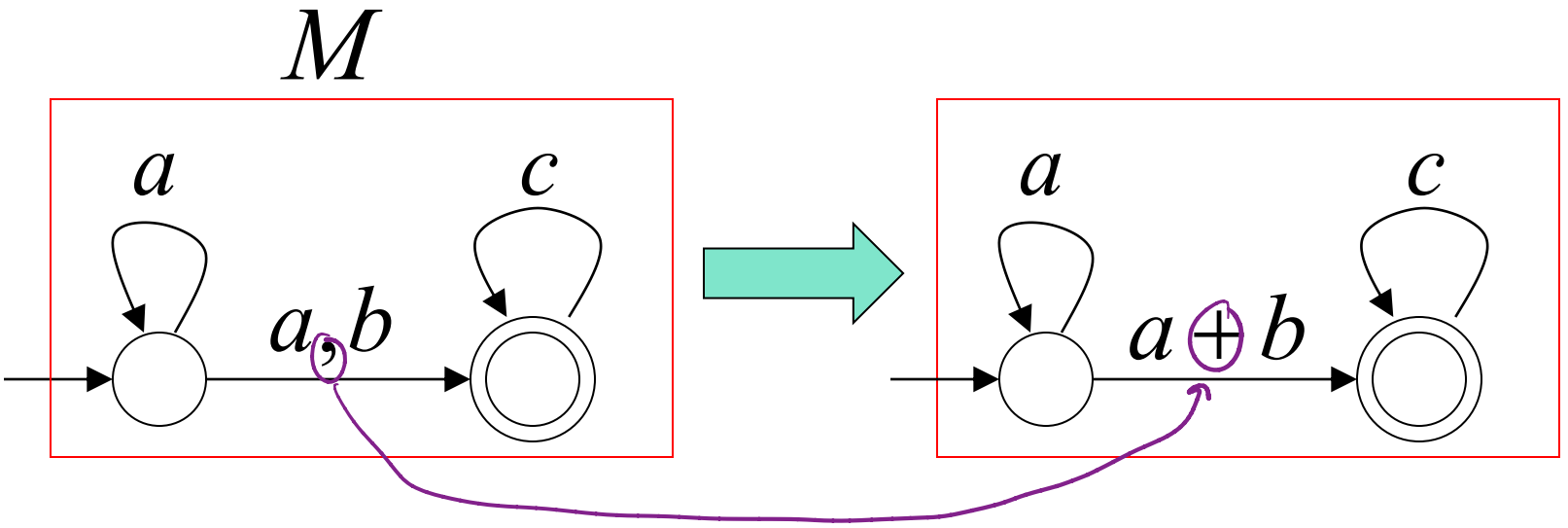
Single final state

From M construct the equivalent

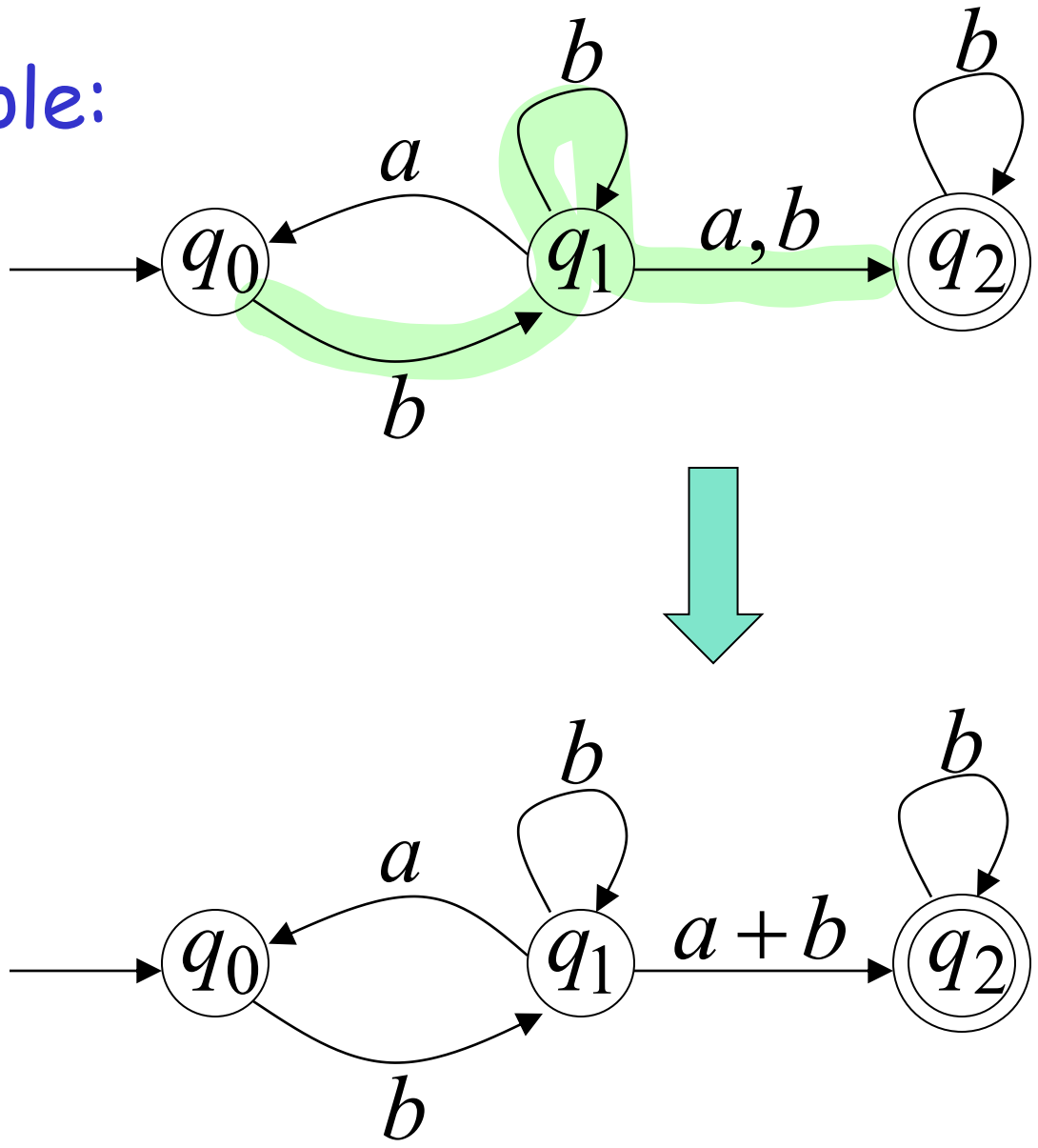
Generalized Transition Graph

in which transition labels are regular expressions

Example:

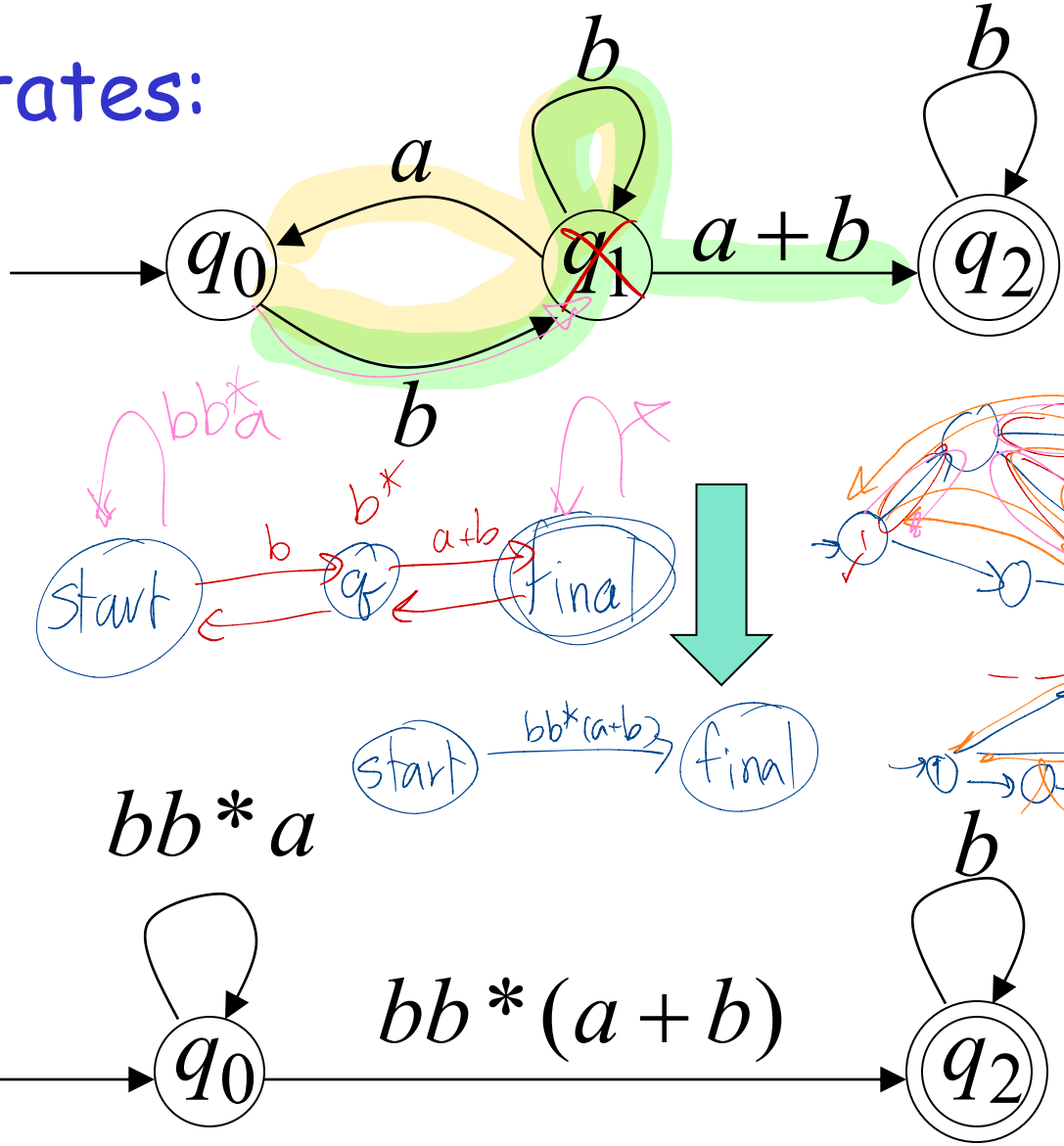


Another Example:

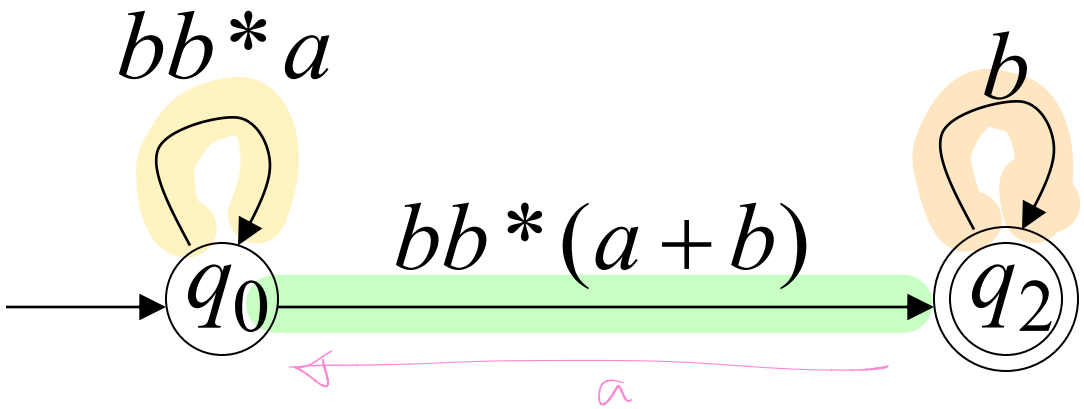


หาเส้นทางที่ยาวที่สุด

Reducing the states:



Resulting Regular Expression:



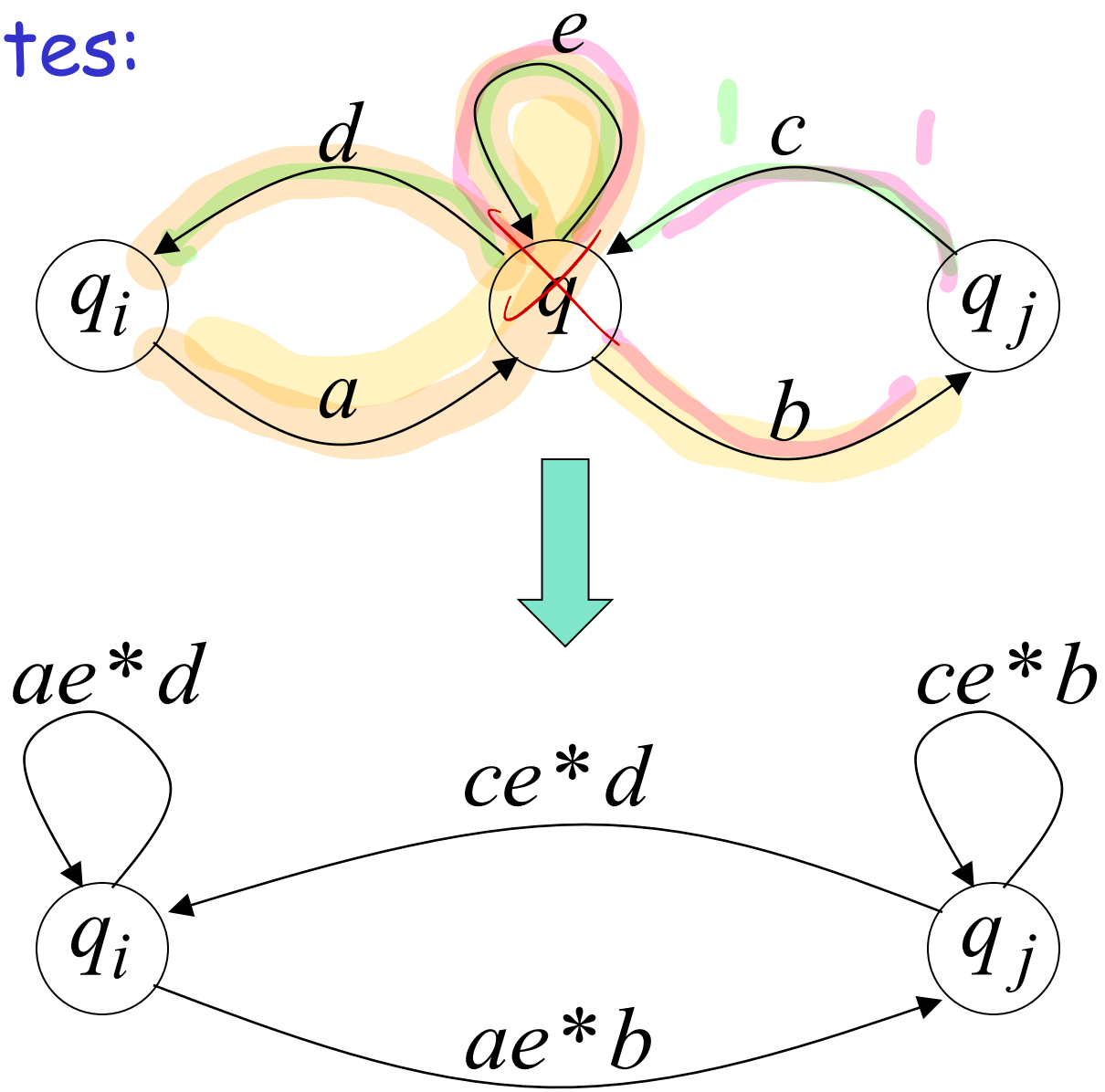
$(bb^*a)^*\, bb^*(a+b)\,(b+a(bb^*a)^*bb^*(a+b))^*$

$$r = (bb^*a)^*bb^*(a+b)b^*$$

$$L(r) = L(M) = L$$

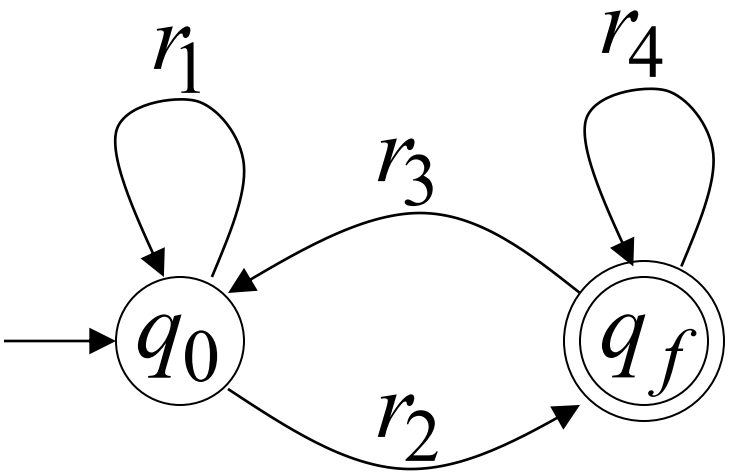
In General

Removing states:



หาที่ยาวที่สุดไป final state

The final transition graph:



The resulting regular expression:

มี 2 ทางเลือก

$$r = r_1 * r_2 (r_4 + r_3 r_1 * r_2) *$$

มีหรือไม่มี loop

$$L(r) = L(M) = L$$

Why do we need Regular Expressions ?

Let's say you want to find a phone number in a string. You know the pattern: three numbers, a hyphen, three numbers, a hyphen, and four numbers.

Here's an example: 415-555-4242.

Without RE, your Python code may look lengthy like this.

```
def isPhoneNumber(text):
    if len(text) != 12:
        return False
    for i in range(0, 3):
        if not text[i].isdecimal():
            return False
    if text[3] != '-':
        return False
    for i in range(4, 7):
        if not text[i].isdecimal():
            return False
    if text[7] != '-':
        return False
    for i in range(8, 12):
        if not text[i].isdecimal():
            return False
    return True
```

Why do we need Regular Expressions ?

With Regular Expression, your Python code will be compact.

regular Expression

```
import re
phoneNumRegex = re.compile(r'\d\d\d-\d\d\d-\d\d\d\d')
mo = phoneNumRegex.search('My number is 415-555-4242.')
print('Phone number found: ' + mo.group())
```

ใส่ text ลงไป มันจะ
หา format เอง

Output:
Phone number found: 415-555-4242

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