NPDAs Accept Context-Free Languages

Theorem: แบ่งการพิสูจน์ได้ 2 ส่วน

$$\left\{ \begin{array}{c} \text{Context-Free} \\ \text{Languages} \\ \text{(Grammars)} \end{array} \right\} = \left\{ \begin{array}{c} \text{Languages} \\ \text{Accepted by} \\ \text{NPDAs} \end{array} \right\}$$

Proof - Step 1: 1

$$\left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{(Grammars)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Languages}\\ \text{Accepted by}\\ \text{NPDAs}\end{array}\right\}$$

Convert any context-free grammar G to a NPDA M with: $L(G) = L(M)$

Proof - Step 2: 2

$$\left\{\begin{array}{c}\text{Context-Free}\\\text{Languages}\\\text{(Grammars)}\end{array}\right\} \supseteq \left\{\begin{array}{c}\text{Languages}\\\text{Accepted by}\\\text{NPDAs}\end{array}\right\}$$

NPDA → context-free G.

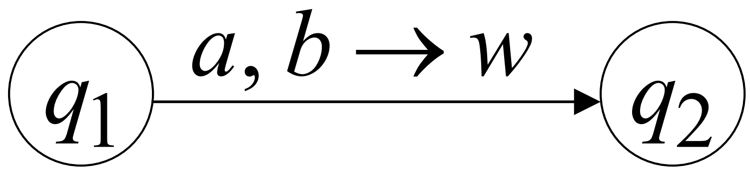
Convert any NPDA M to a context-free grammar G with: $L(G) = L(M)$

Deterministic PDA

DPDA

Deterministic PDA: DPDA

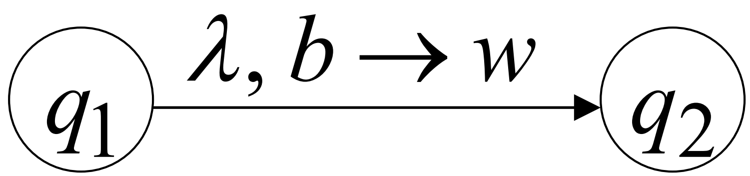
Allowed transitions:



คาดเดาได้

รู้ state ปัจจุบัน
input ปัจจุบัน
Top stack

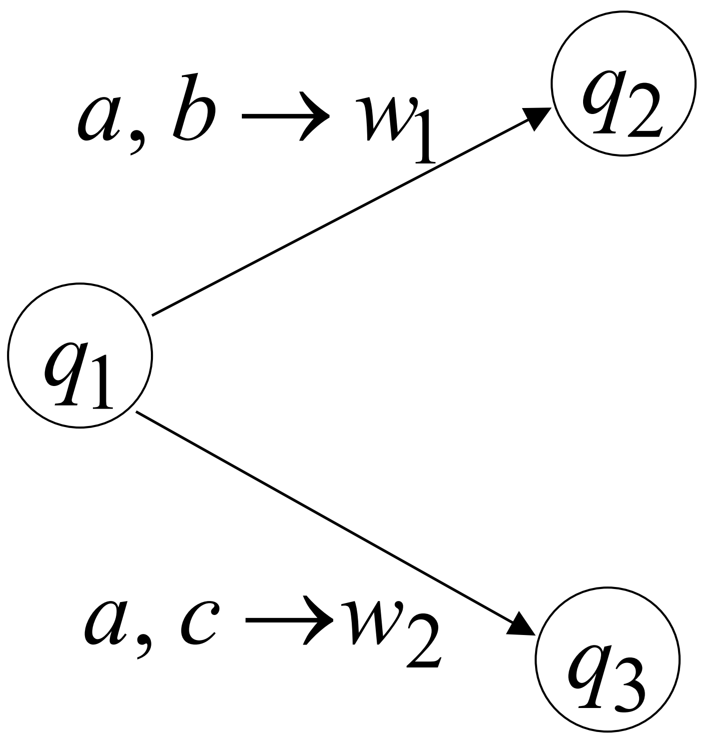
คาดเดาได้ว่าหยุดอยู่ state ไหน

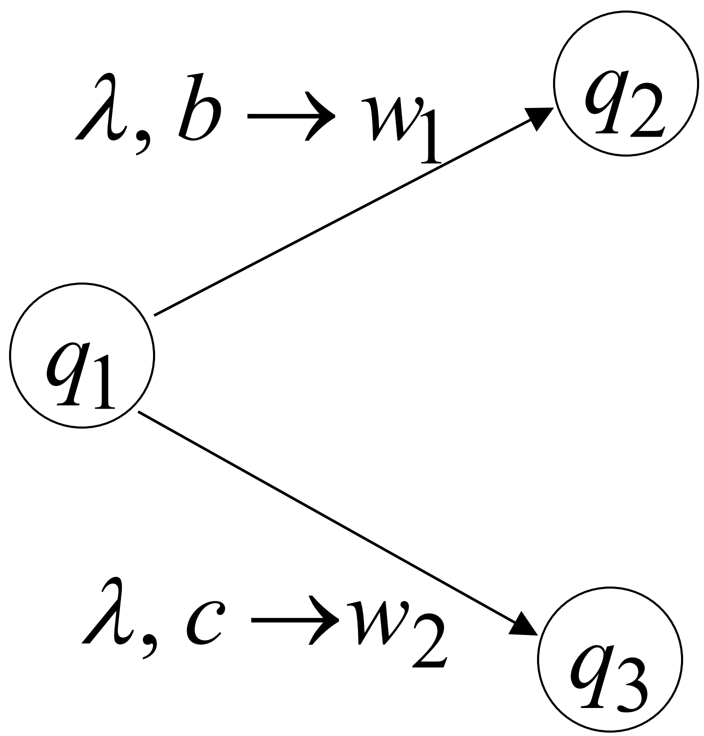


(deterministic choices)

Allowed transitions:

Top stack ต่าง

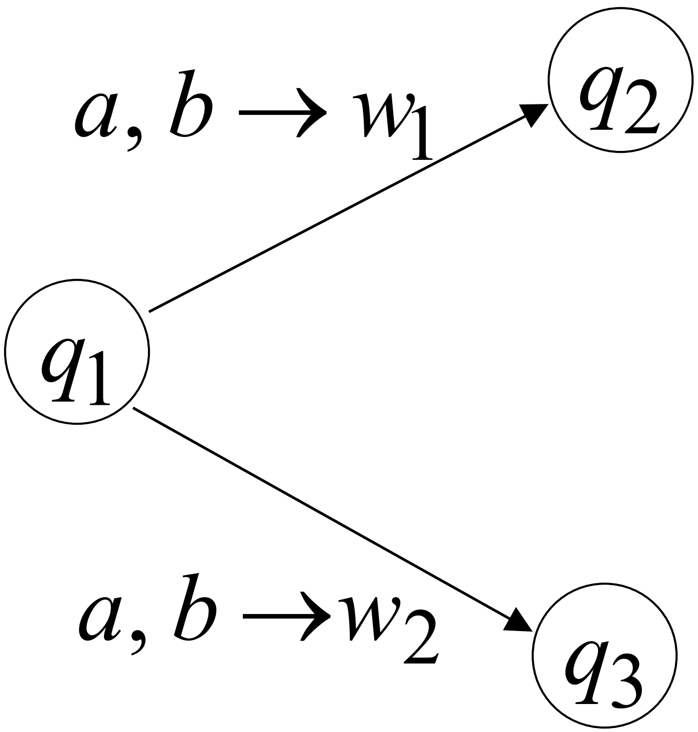


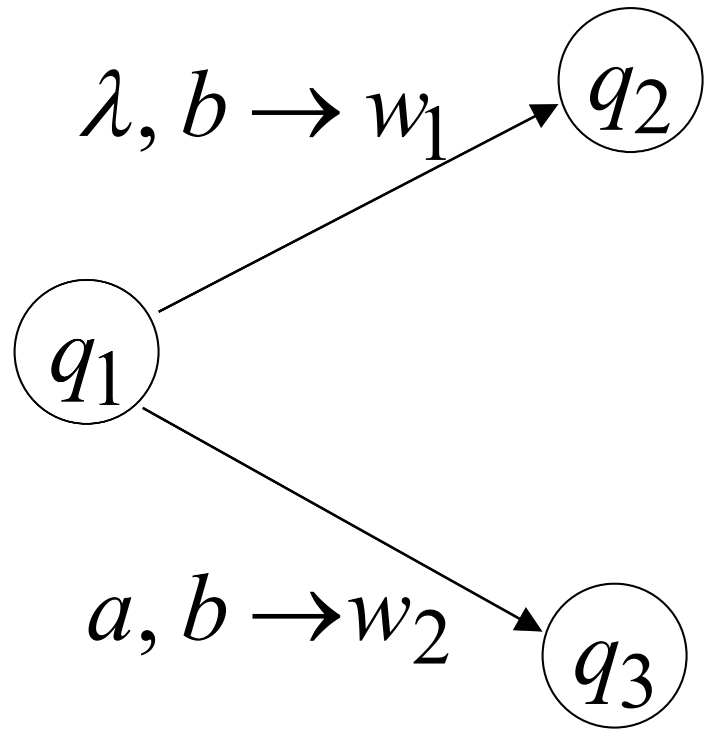


(deterministic choices)

Not allowed:

Top stack เหมือน

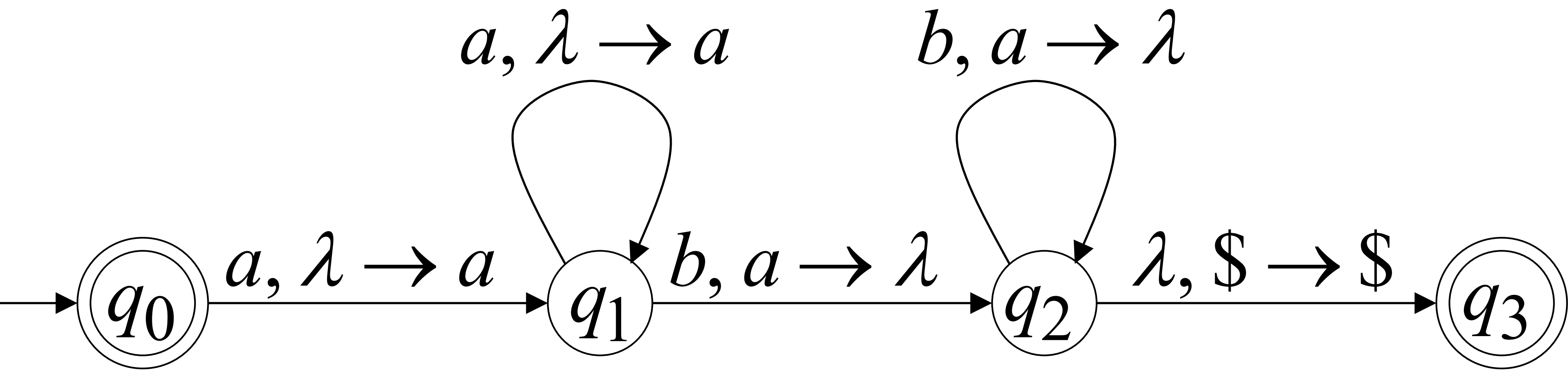




คือ → (non-deterministic choices)

DPDA example

$$L(M) = \{a^n b^n : n \geq 0\}$$



DPDA รองรับ $\{a^n b^n : n > 0\}$

The language $L(M) = \{a^n b^n : n \geq 0\}$

is **deterministic context-free**

Definition:

A language L is **deterministic context-free**
if there exists some DPDA that accepts it

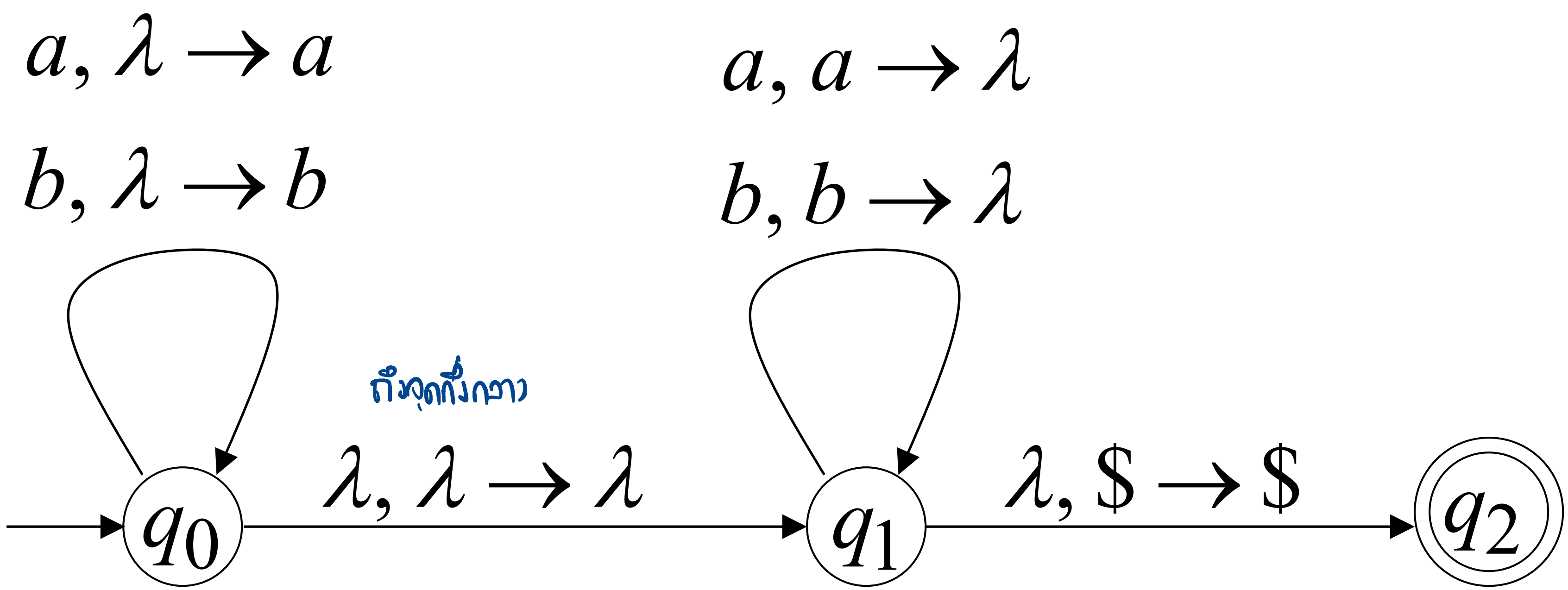
ถ้ามี ที่ยอมรับมัน

Example of Non-DPDA (NPDA)

$$L(M) = \{ww^R\}$$

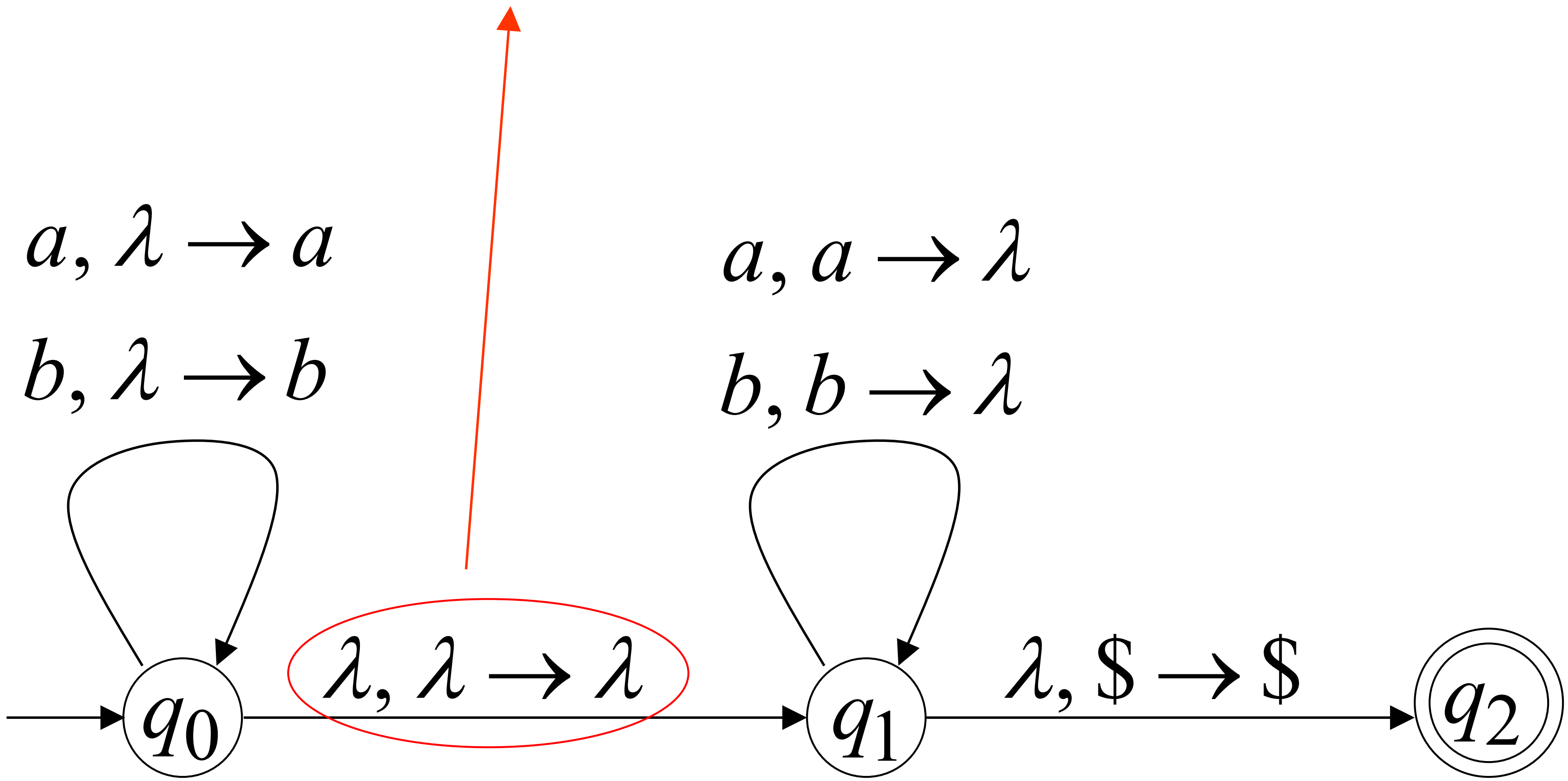
เป็น NPDA อยู่แล้ว

คาดเดากึ่งกลางไม่ได้ = non PDA



* สำคัญ

Not allowed in DPDAs



NPDAs

มีประสิทธิภาพ มากกว่า

Have More Power than

DPDAs

It holds that:

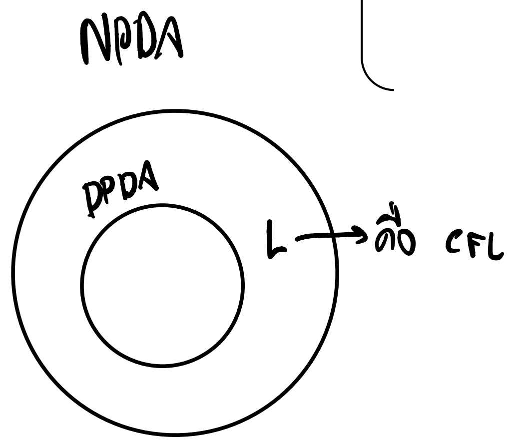
$$\left\{\begin{array}{c}\text{Deterministic}\\ \text{Context-Free}\\ \text{Languages}\\ \text{(DPDA)}\end{array}\right\} \subseteq \left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{NPDAs}\end{array}\right\}$$

Since every DPDA is also a NPDA

We will actually show:

$$\left\{\begin{array}{c}\text{Deterministic}\\ \text{Context-Free}\\ \text{Languages}\\ \text{(DPDA)}\end{array}\right\} \subset \left\{\begin{array}{c}\text{Context-Free}\\ \text{Languages}\\ \text{(NPDA)}\end{array}\right\}$$

$L \notin$ $\qquad$ $L \in$

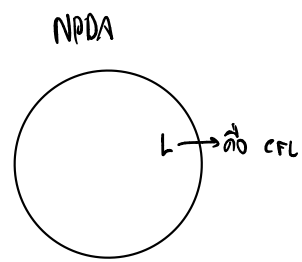


concept

We will show that there exists a context-free language L which is not accepted by any DPDA

The language is:

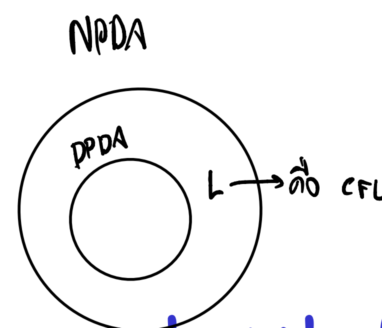
$$L = \{a^n b^n\} \cup \{a^n b^{2n}\} \qquad n \geq 0$$



We will show:

2 step

1 • L is context-free



2 • L is **not** deterministic context-free

1

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

Language L is context-free

สร้าง

Context-free grammar for L :

พิสูจน์แล้ว

$$S \to S_1 \mid S_2 \qquad \{a^n b^n\} \cup \{a^n b^{2n}\}$$

การ union ($\cup$)

$$S_1 \to aS_1b \mid \lambda \qquad \{a^n b^n\}$$

Gramma 1

$$S_2 \to aS_2bb \mid \lambda \qquad \{a^n b^{2n}\}$$

Gramma 2

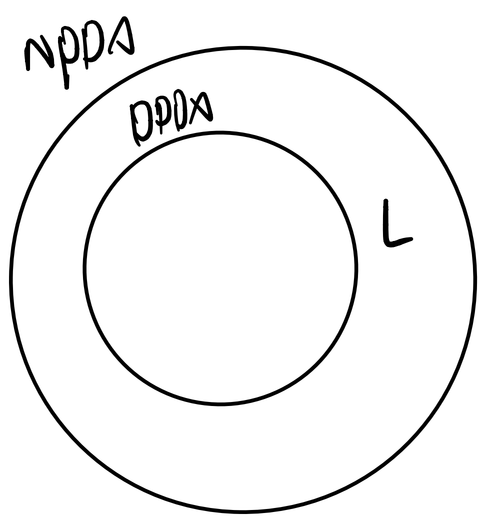
Theorem: 2

The language $L = \{a^n b^n\} \cup \{a^n b^{2n}\}$

is **not** deterministic context-free

DPDA

(there is **no** DPDA that accepts L)



พิสูจน์ว่า L ไม่อยู่ใน DPDA

สมมติ L คือ DPDA

Proof: Assume for contradiction that

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

is deterministic context free

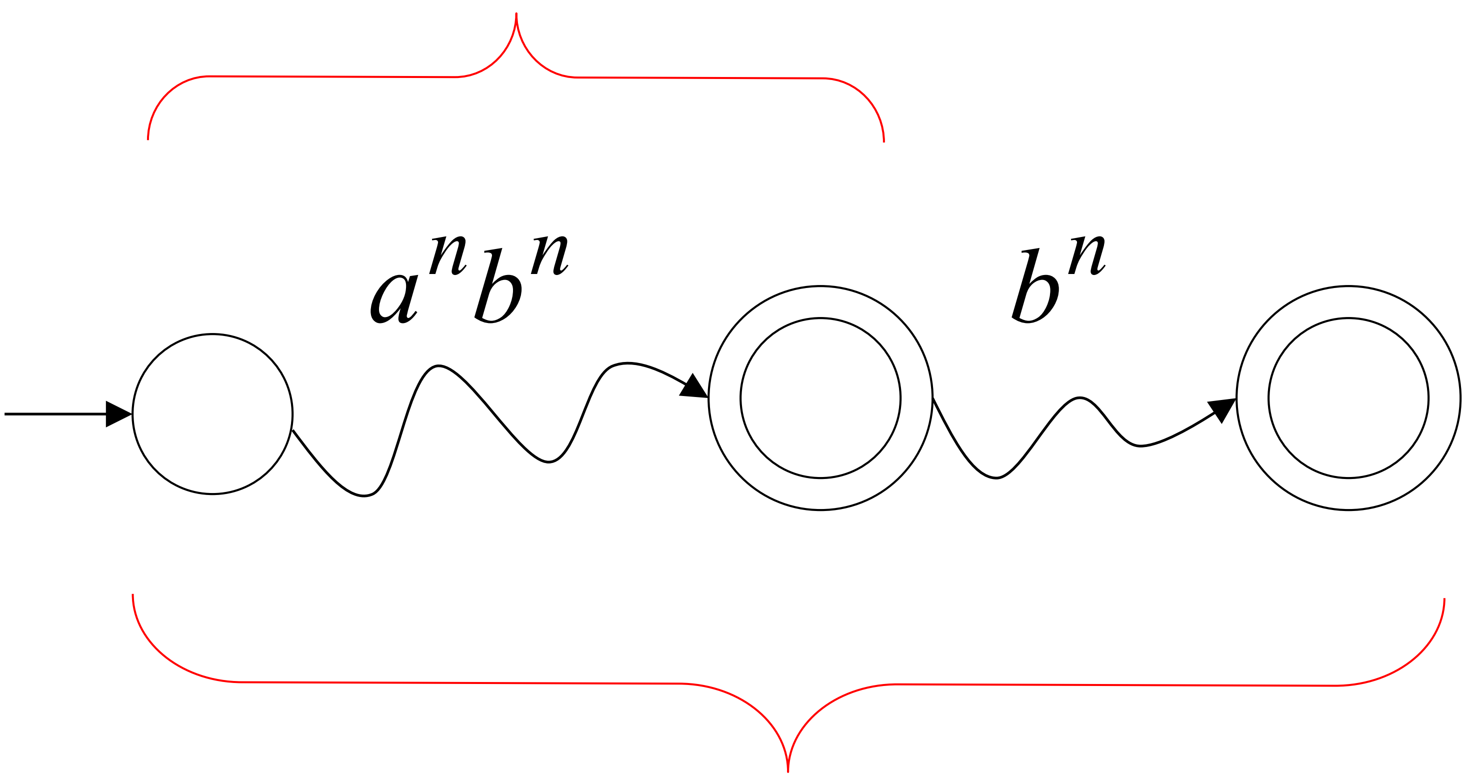
Therefore:

there is a DPDA M that accepts L

DPDA M with $L(M)=\{a^nb^n\}\cup\{a^nb^{2n}\}$

สร้าง DPDA ในนั้นมิน

accepts a^nb^n



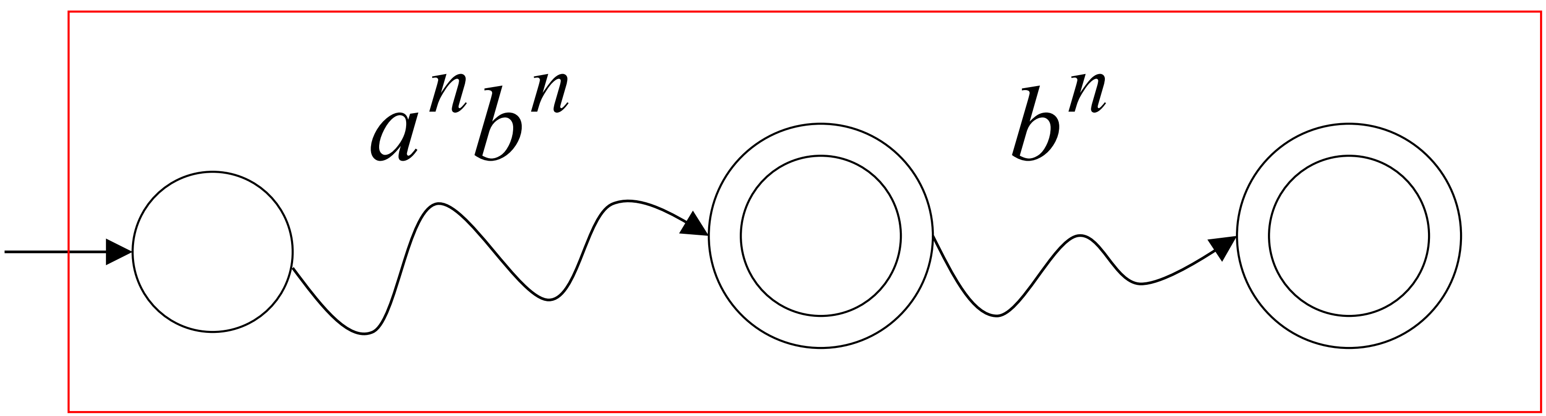
accepts a^nb^{2n}

DPDA M with $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$

Such a path exists because of the determinism

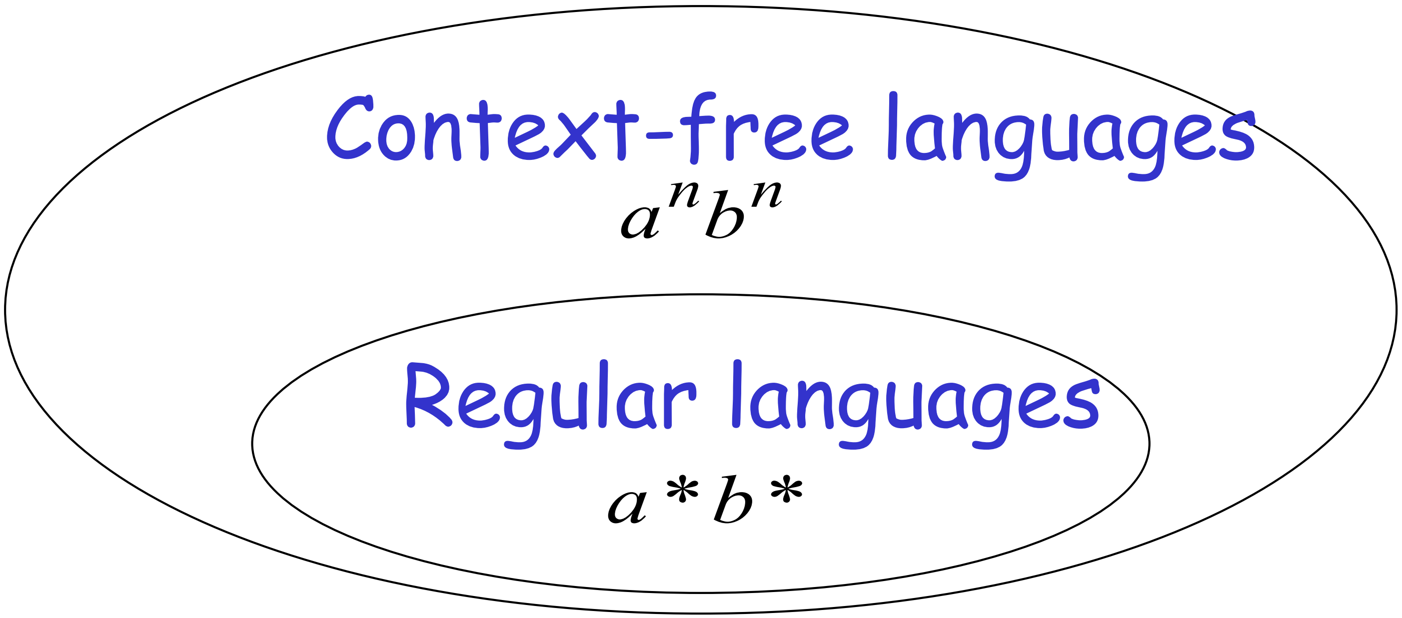
DPDA สำหรับภาษา L

M



Fact 1: The language $\{a^n b^n c^n\}$

is **not** context-free



(we will prove this at a later class using pumping lemma for context-free languages)

ภาษาใดๆ ก็ตามที่ $\cup$ กับ non CFL = non CFL

Fact 2: The language $L \cup \{a^n b^n c^n\}$

is **not** context-free

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

(we can prove this using pumping lemma
for context-free languages)

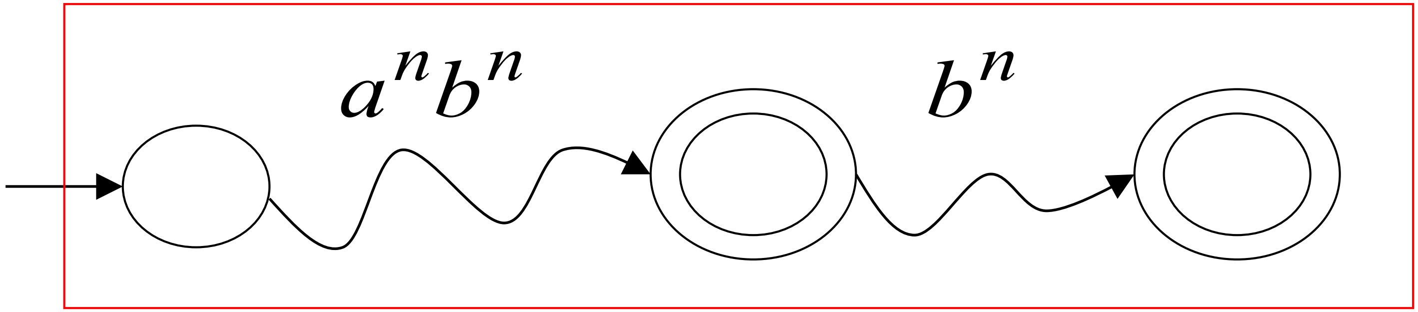
We will construct a NPDA that accepts:

$$L \cup \{a^n b^n c^n\}$$

$$(L = \{a^n b^n\} \cup \{a^n b^{2n}\})$$

which is a contradiction!

M $L(M) = \{a^n b^n\} \cup \{a^n b^{2n}\}$



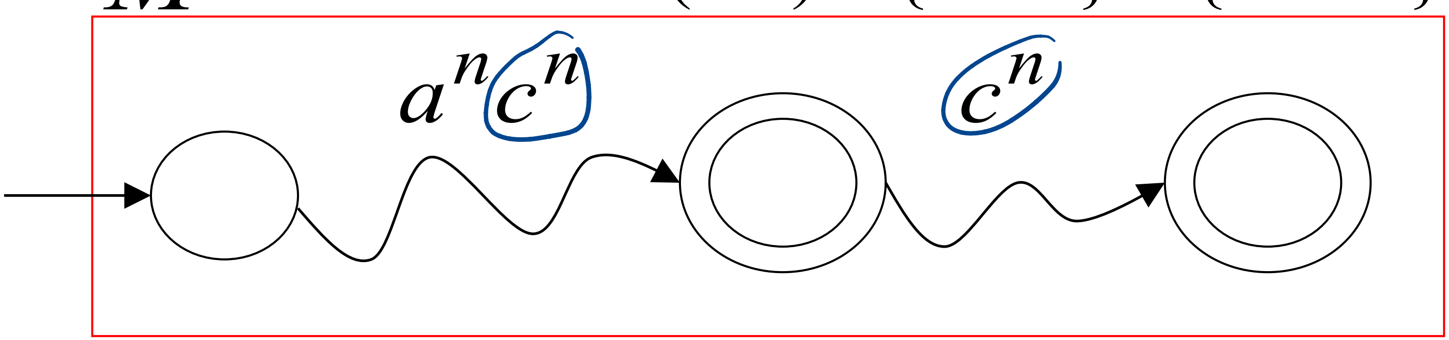
DPDA

Modify M

Replace b with c

เปลี่ยนจาก b → c

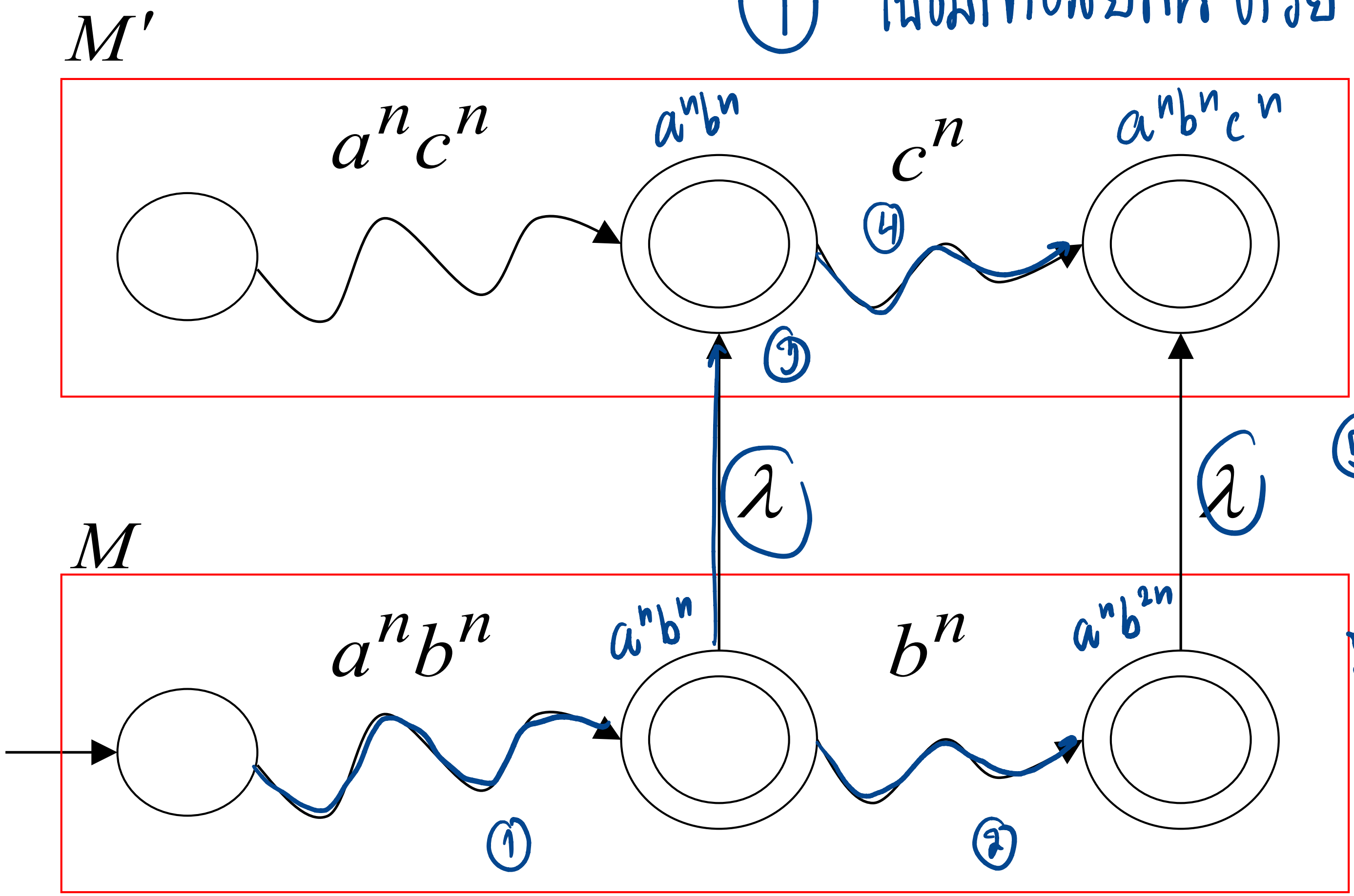
M' $L(M') = \{a^n c^n\} \cup \{a^n c^{2n}\}$



The NPDA that accepts $L \cup \{a^n b^n c^n\}$

Connect final states of M'
with final states of M

① เชื่อมเข้าด้วยกัน ด้วย λ



⑤ $\{a^n b^n \cup a^n b^{2n}\}$ (= L)
$\cup$
$a^n b^n c^n$

! สร้าง NPDA ที่รองรับได้
แต่จริงๆไม่ควรได้ เพราะเป็น
non CFL

ดังนั้นจึงโทษสมมติฐานที่ตั้งไว้ ว่า

สมมติ L คือ DPDA

Proof: Assume for contradiction that

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

is deterministic context free

สรุปได้ว่า $L(\text{DPDA}) \subset L(\text{NPDA})$

Since $L \cup \{a^n b^n c^n\}$ is accepted by a NPDA

it is context-free

Contradiction!

(since $L \cup \{a^n b^n c^n\}$ is not context-free)

Therefore:

Not deterministic context free

$$L = \{a^n b^n\} \cup \{a^n b^{2n}\}$$

There is **no** DPDA that accepts

End of Proof

Supplementary proof : https://goo.gl/zoPKmY

สรุปเป็นหน้าเดียว

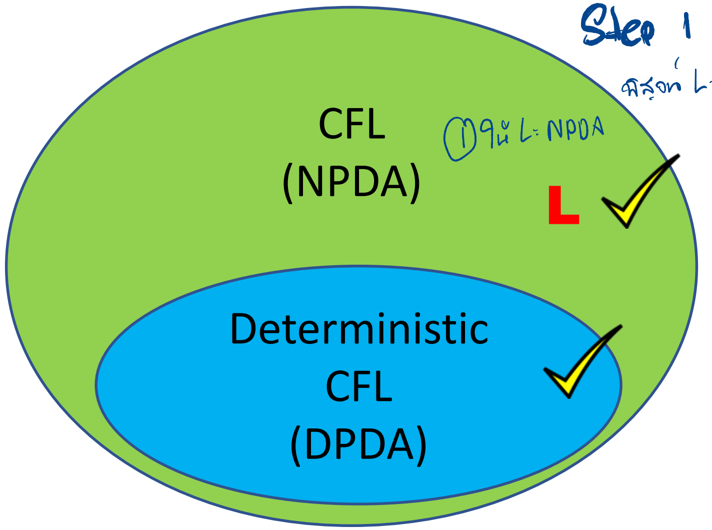
ไม่ออกสอบ เพราะยากเกินไป แต่ต้องเข้าใจ

We will prove that:

"There is a language **L** that is in L(NPDA) but is not in L(DPDA)."

Step 1

พิสูจน์ L: CFL



② Let **L** = $\{a^n b^n\} \cup \{a^n b^{2n}\}$; n≥ 0

③ **L** is in CFL because there is a CFG for **L**.

สร้าง CFG

$$S \rightarrow S_1 \mid S_2 \qquad \{a^n b^n\} \cup \{a^n b^{2n}\}$$

$$S_1 \rightarrow aS_1b \mid \lambda \qquad \{a^n b^n\}$$

$$S_2 \rightarrow aS_2bb \mid \lambda \qquad \{a^n b^{2n}\}$$

Therefore, **L** $\in L(NPDA)$.

Step 2

ไม่อยู่ใน

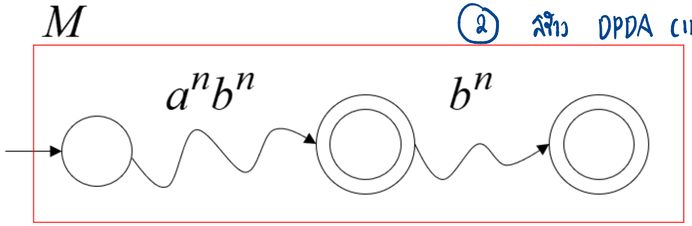
Next, we will show that "**L** is not in L(DPDA)."

Proof by contradiction:

Assume for contradiction that ① สมมติให้ขัดแย้ง

L = $\{a^n b^n\} \cup \{a^n b^{2n}\}$ is in L(DPDA).

② สร้าง DPDA (แบบสมมติ)



This DPDA M exists because of our assumption.

FACT: ③ CFL ∪ non-CFL = CFL

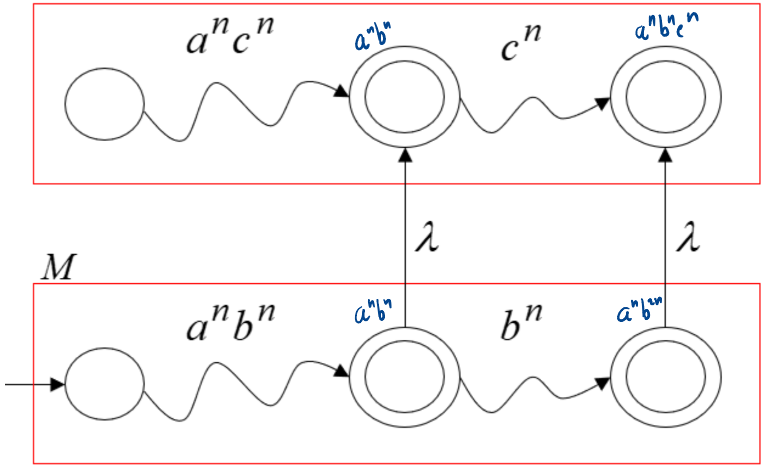
$$\underbrace{\{a^n b^n\} \cup \{a^n b^{2n}\}}_{L} \cup \underbrace{\{a^n b^n c^n\}}_{\text{Non-CFL}} \text{ is not CFL.}$$

If we can construct NPDA that accepts the above language, we will reach a contradiction.

We construct NPDA from M to accept L.

M′ ④ สร้าง PDA <NPDA / DPDA / ถ้าสร้างได้ = ขัดแย้ง or สร้างไม่ได้ = ไม่ขัดแย้ง

สมมติฐาน



This NPDA accepts the non-CFL $\{a^n b^n\} \cup \{a^n b^{2n}\} \cup \{a^n b^n c^n\}$.

Contradiction !!!

Our assumption is wrong.

Therefore, L is not in L(DPDA).