Positive Properties of Context-Free languages

Union

Context-free languages
are closed under: **Union**

L_1 is context free

L_2 is context free

$\Longrightarrow$ $L_1 \cup L_2$ is context-free

Example

Language | **Grammar**

$$L_1 = \{a^n b^n\}$$

$$S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\}$$

$$S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Union

$$L = \{a^n b^n\} \cup \{ww^R\} = \quad S \rightarrow S_1 \mid S_2$$

In general:

For context-free languages $L_1,\ L_2$
with context-free grammars $G_1,\ G_2$
and start variables $S_1,\ S_2$

The grammar of the **union** $L_1 \cup L_2$
has new start variable S
and additional production $S \to S_1 \mid S_2$

Concatenation

Context-free languages are closed under: **Concatenation**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 L_2$ is context-free

Example

Language | **Grammar**

$$L_1 = \{a^n b^n\}$$

$$S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\}$$

$$S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Concatenation

$$L = \{a^n b^n\}\{ww^R\}$$

$$S \rightarrow S_1 S_2$$

In general: สูบทั่วไป

For context-free languages $L_1,\ L_2$

with context-free grammars $G_1,\ G_2$

and start variables $S_1,\ S_2$

The grammar of the **concatenation** L_1L_2

has new start variable S

and additional production $S \rightarrow S_1S_2$

Star Operation

Context-free languages are closed under: **Star-operation**

L is context free $\Rightarrow$ L^* is context-free

Example

Language

$$L = \{a^n b^n\}$$

Grammar

$$S \to aSb \mid \lambda$$

Star Operation

$$L = \{a^n b^n\}^*$$

$$S_1 \to SS_1 \mid \lambda$$

In general:

For context-free language L

with context-free grammar G

and start variable S

The grammar of the **star operation** L^*

has new start variable S_1

and additional production $S_1 \to SS_1 \mid \lambda$

คุณสมบัติ แง่ลบ

ข้อยกเว้น

$CFL \cap Reg\ L.$

1. $\cap$
2. $\overline{}$ or $'$

Negative Properties of Context-Free Languages

Intersection $\cap$

Context-free languages
are **not** closed under: **intersection**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 \cap L_2$

ไม่จำเป็นที่เป็น

not necessarily context-free

บางครั้งก็ได้
แต่ไม่ใช่ทุกครั้ง

Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \rightarrow AC$$

$$A \rightarrow aAb \mid \lambda$$

$$C \rightarrow cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow bBc \mid \lambda$$

Intersection

$L_1 \cap L_2 = \{a^n b^n c^n\}$ **NOT** context-free

Complement

Context-free languages
are **not** closed under: **complement**

L is context free $\Rightarrow$ $\overline{L}$ not necessarily context-free

Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \to AC$$

$$A \to aAb \mid \lambda$$

$$C \to cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \to AB$$

$$A \to aA \mid \lambda$$

$$B \to bBc \mid \lambda$$

Complement

$$\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \{a^n b^n c^n\}$$

NOT context-free

ข้อยกเว้น

CFL $\cap$ Reg L.

Intersection of Context-free languages and Regular Languages

The intersection of

a context-free language and

a regular language

is a context-free language

L_1 context free

L_2 regular

$$\Rightarrow \quad L_1 \cap L_2$$

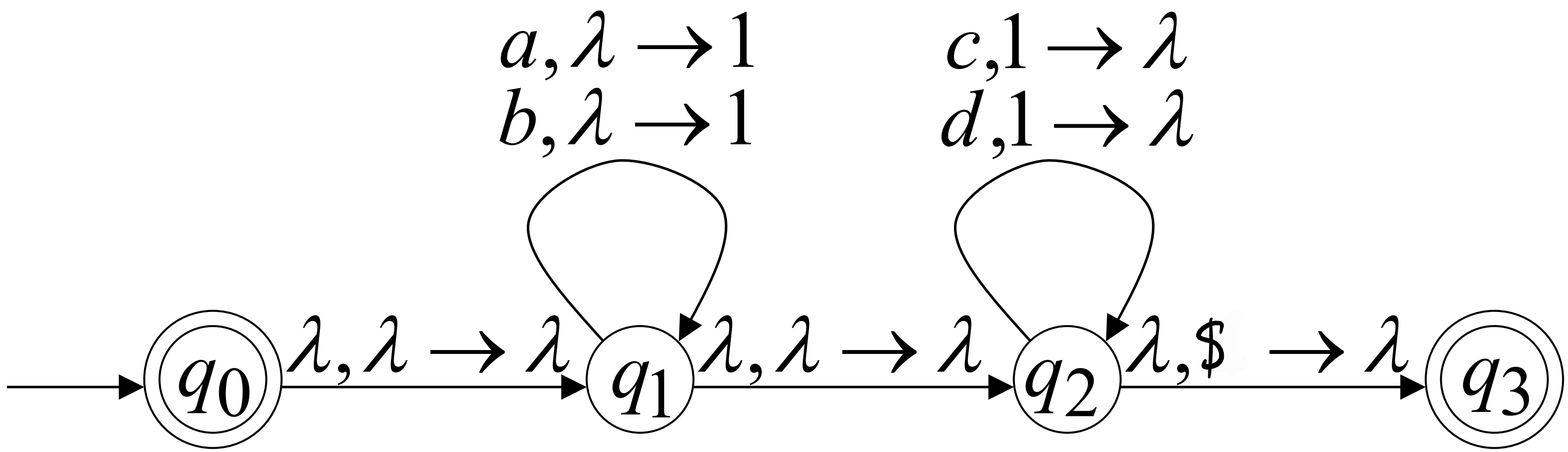
context-free

Example:

context-free

$$L_1 = \{w_1 w_2 : |w_1| = |w_2|, w_1 \in \{a,b\}^*, w_2 \in \{c,d\}^*\}$$

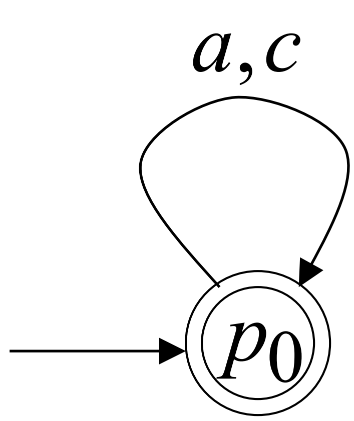
NPDA M_1



regular

$$L_2 = \{a, c\}^*$$

DFA M_2

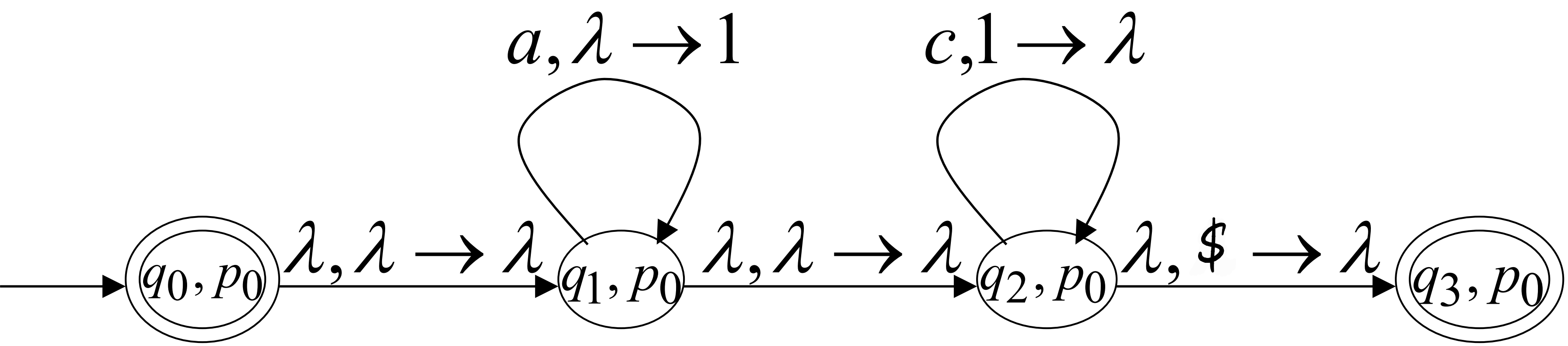


context-free

Automaton for: $L_1 \cap L_2 = \{a^n c^n : n \geq 0\}$

NPDA M

↳ใช้ DPDA ก็ได้เหมือนกัน ขอแค่เป็น CFL



Applications of Regular Closure

The intersection of

a context-free language and

a regular language

is a context-free language

Regular Closure

L_1 context free

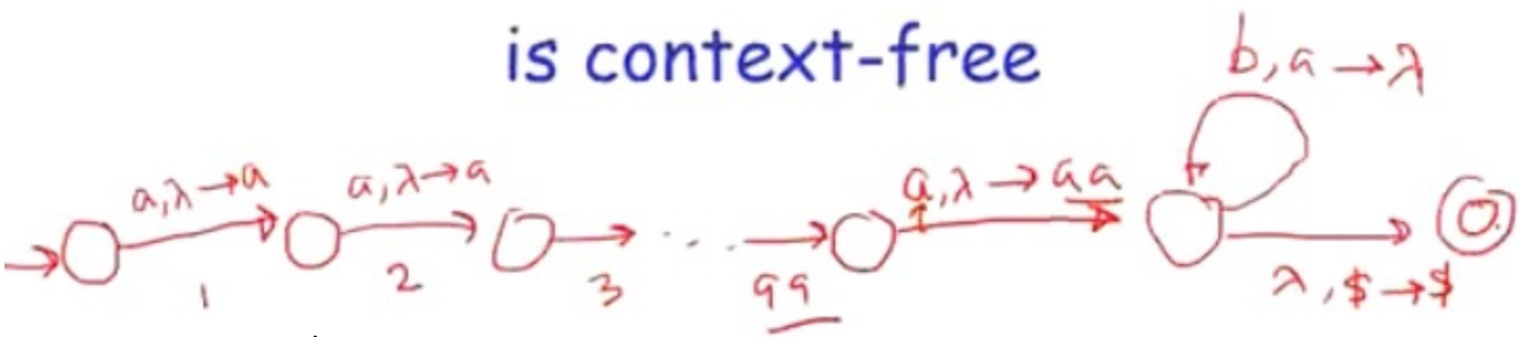
L_2 regular

$$\left.\begin{array}{l} L_1 \text{ context free} \\ L_2 \text{ regular} \end{array}\right\} \Rightarrow L_1 \cap L_2 \text{ context-free}$$

An Application of Regular Closure

Prove that: $L = \{a^n b^n : \ n \neq 100, n \geq 0\}$

is context-free



มีวิธีที่ง่ายกว่านี้

We know:

$\{a^n b^n : n \geq 0\}$ is context-free

We also know:

เพาะ: มีหะ:พุลขตาว, มีจุดจบที่แน่นอน

$$L_1 = \{a^{100}b^{100}\}$$ is regular

ไม่มี

$$\overline{L_1} = \{(a+b)^*\} - \{a^{100}b^{100}\}$$ is regular

คุณสมบัติปิด
ของ reg L.

$\{a^n b^n\}$ $\overline{L_1} = \{(a+b)^*\} - \{a^{100} b^{100}\}$

context-free regular

(regular closure) $\{a^n b^n\} \cap \overline{L_1}$ context-free

ผลลัพธ์ คือ $n \neq 100$ เพราะตัดทิ้งไปแล้ว

$$\{a^n b^n\} \cap \overline{L_1} = \{a^n b^n : \ n \neq 100, n \geq 0\} = L$$

is context-free

Another Application of Regular Closure

สลับที่ได้จำนวนเท่ากัน

Prove that: $L = \{w: \quad n_a = n_b = n_c\}$

is **not** context-free

If $L = \{w: \ n_a = n_b = n_c\}$ is context-free

(regular closure)

Then $L \cap \{a^*b^*c^*\} = \{a^n b^n c^n\}$

context-free regular context-free

* ถ้า L เป็น CFL เวลา ∩ กับ reg L. ต้องเป็น CFL.
แต่ที่ ∩ กับ reg L. → Non-CFL สรุปได้ว่า

Impossible!!!

Therefore, L is **not** context free