Computer Graphics

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http://www.cs.pdx.edu/~fliu/courses/cs447/

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Last time

- Graphics Pipeline

Today

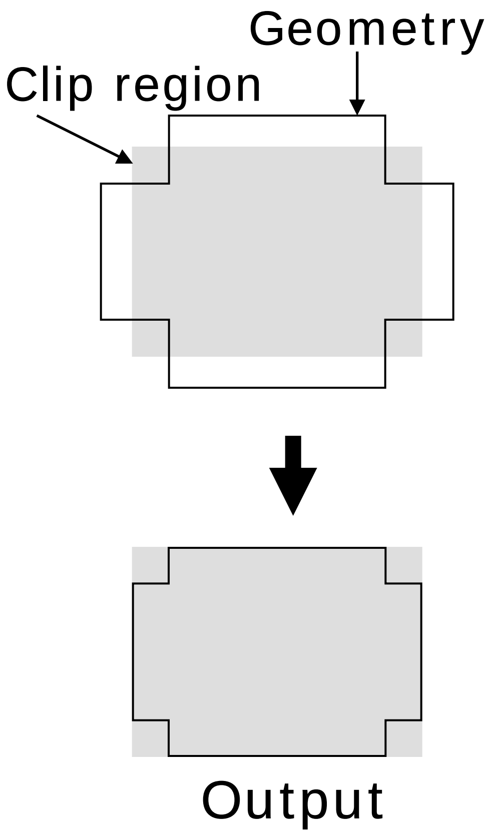
- Clipping
- In-class Middle-Term
  - Wednesday, Nov. 2
  - Close-book exam
  - Notes on 1 page of A4 or Letter size paper
  - To-know list available online

Clipping

- Parts of the geometry to be rendered may lie outside the view volume
- *Clipping* removes parts of the geometry that are outside the view
- Best done in canonical space *before perspective divide*
  - Before dividing out the homogeneous coordinate

Clipping Terminology

- Clip region: the region we wish to restrict the output to
- Geometry: the thing we are clipping
  - Only those parts of the geometry that lie inside the clip region will be output
- Clipping edge/plane: an infinite line or plane and we want to output only the geometry on one side of it
  - Frequently, one edge or face of the clip region



Clipping

- In hardware, clipping is done in canonical space *before perspective divide*
  - Before dividing out the homogeneous coordinate
- Clipping is useful in many other applications
  - Building BSP trees for visibility and spatial data structures
  - Hidden surface removal algorithms
  - Removing hidden lines in line drawings
  - Finding intersection/union/difference of polygonal regions
  - 2D drawing programs: cropping, arbitrary clipping
- We will make explicit assumptions about the geometry and the clip region
  - Assumption depend on the algorithm

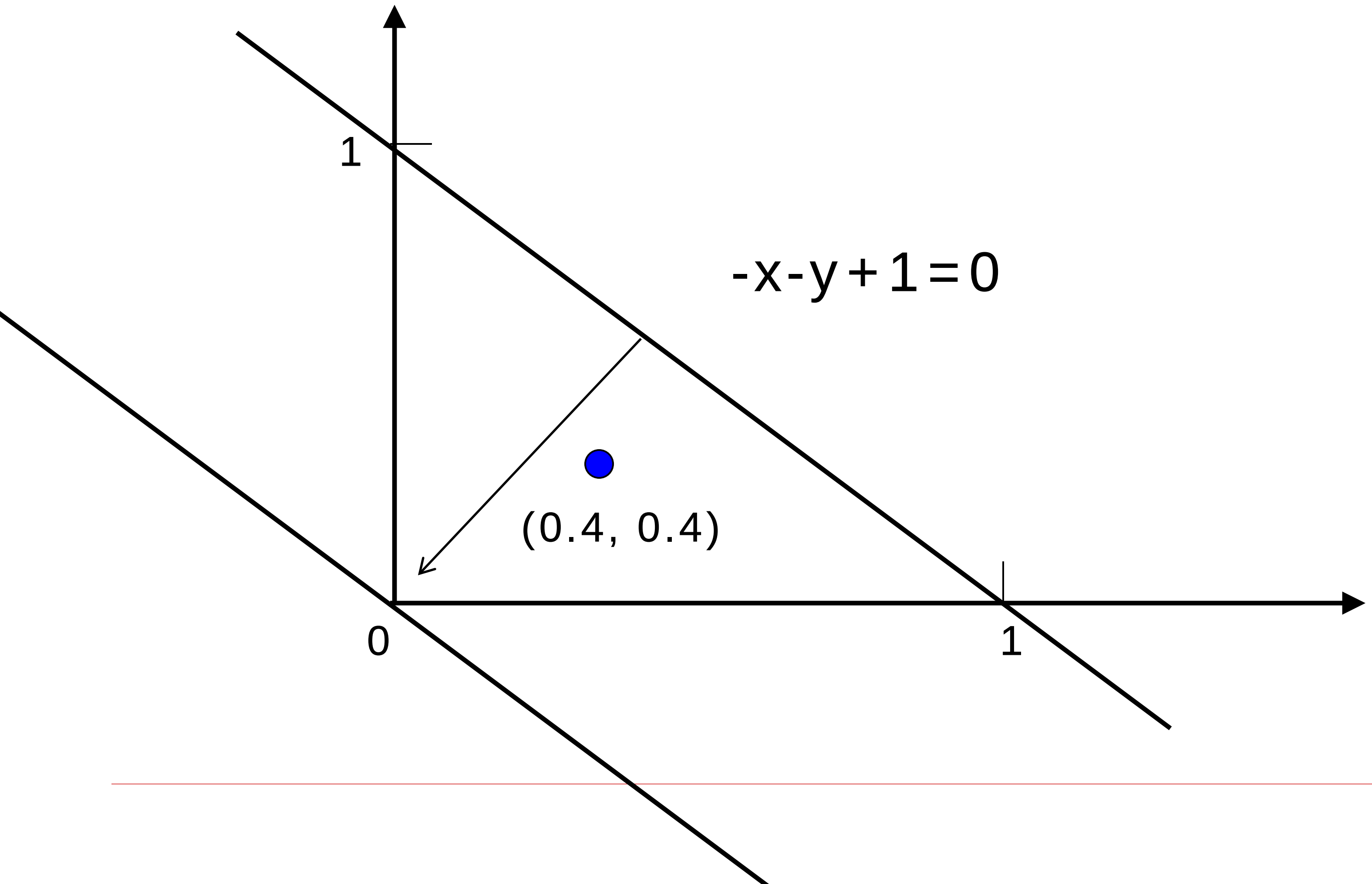
Types of Geometry

- *Points* are clipped via inside/outside tests
  - Many algorithms for this task, depending on the clip region
- Two main algorithms for clipping polygons exist
  - Sutherland-Hodgman
  - Weiler that we will not talk about in our class

Clipping Points to View Volume

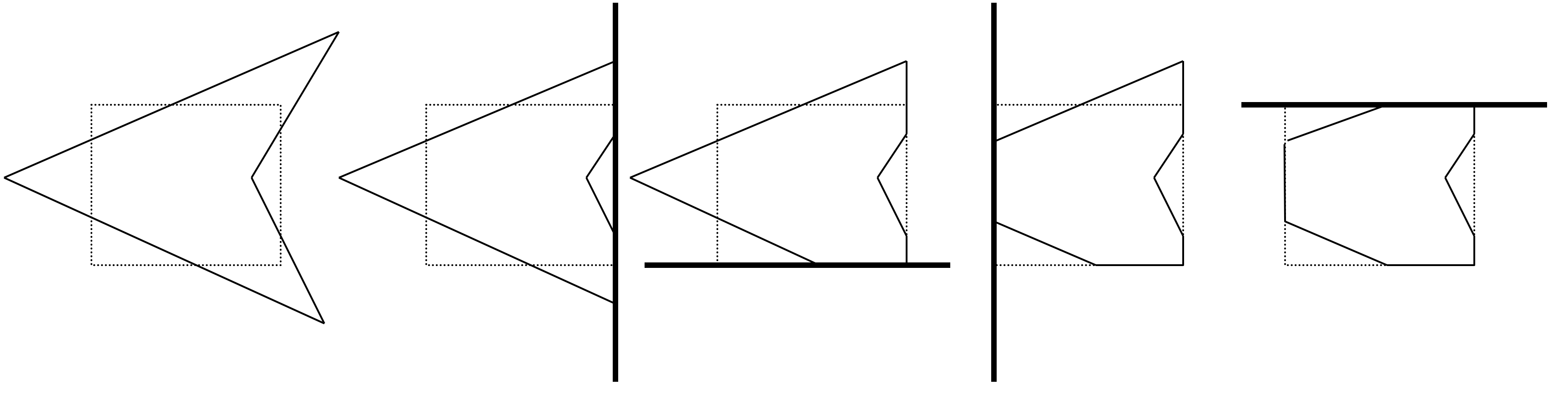
- A point is inside the view volume if it is on the "inside" of all the clipping planes
  - The normals to the clip planes are considered to point inward, toward the visible region
- Now we see why clipping is done in canonical view space
  - For instance, to check against the left plane:
  - X coordinate in 3D must be > -1
  - In homogeneous screen space, same as: $x_{screen} > -w_{screen}$
- In general, a point, p, is "inside" a plane if:
  - You represent the plane as $n_x x+n_y y+n_z z+d=0$, with (n_x,n_y,n_z) pointing inward
  - And $n_x p_x+n_y p_y+n_z p_z+d>0$

Clipping Point to Line



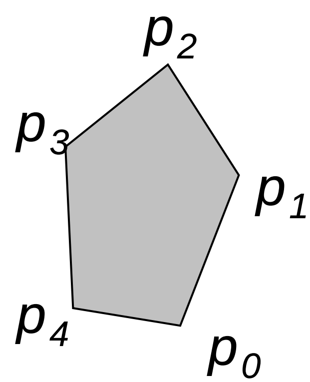
Sutherland-Hodgman Clip

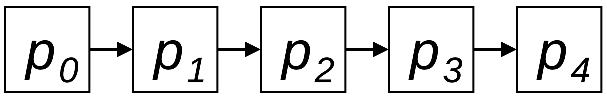
- Clip polygons to convex clip regions
- Clip the polygon against each edge of the clip region in turn
  - Clip polygon each time to line containing edge
  - Only works for convex clip regions (Why? Example that breaks?)

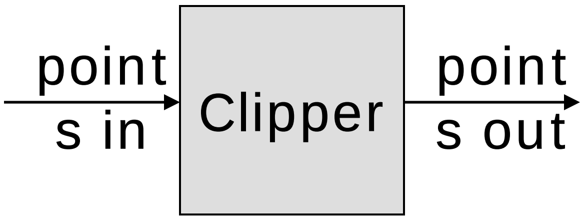


Sutherland-Hodgman Clip (2)

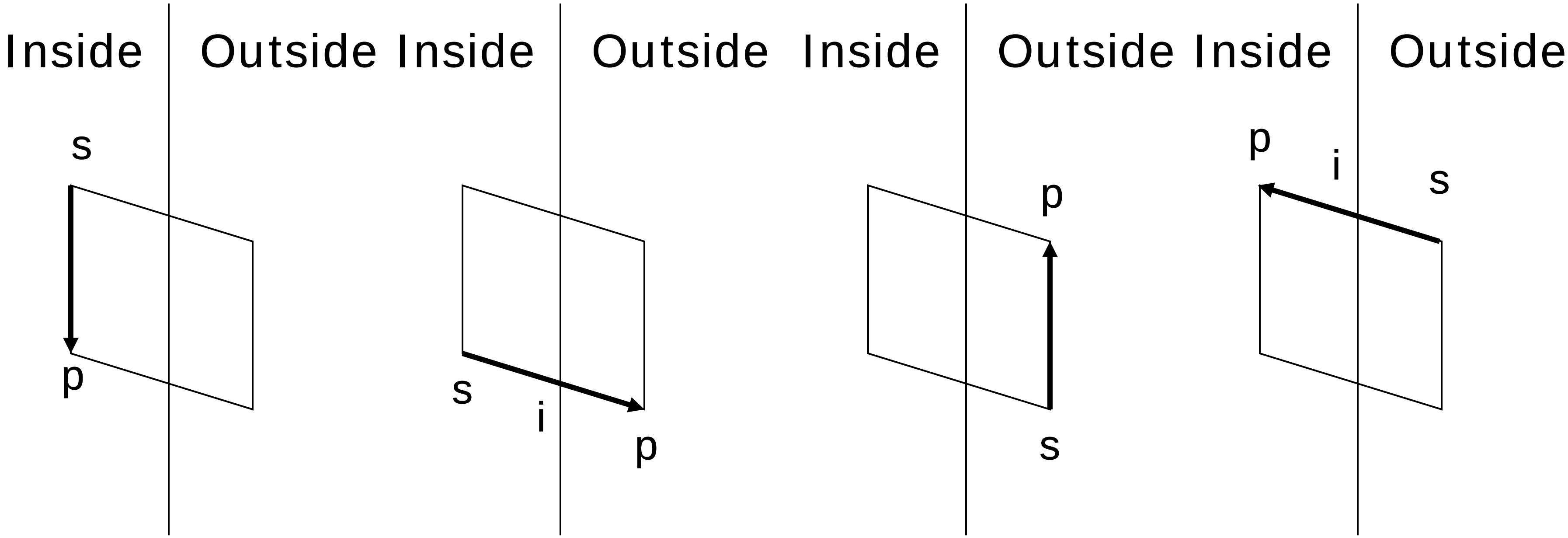
- To clip a polygon to a line/plane:
  - Consider the polygon as a list of vertices
  - One side of the line/plane is considered inside the clip region, the other side is outside
  - We are going to rewrite the polygon one vertex at a time – the rewritten polygon will be the polygon clipped to the line/plane
  - Check start vertex: if “inside”, *emit* it, otherwise ignore it
  - Continue processing vertices as follows…







Sutherland-Hodgman (3)



Output p Output i No output Output i and p

Sutherland-Hodgman (4)

- Look at the next vertex in the list, **p**, and the edge from the last vertex, **s**, to **p**. If the…
  - polygon edge crosses the clip line/plane going from out to in: emit crossing point, **i**, next vertex, **p**
  - polygon edge crosses clip line/plane going from in to out: emit crossing, **i**
  - polygon edge goes from out to out: emit nothing
  - polygon edge goes from in to in: emit next vertex, **p**

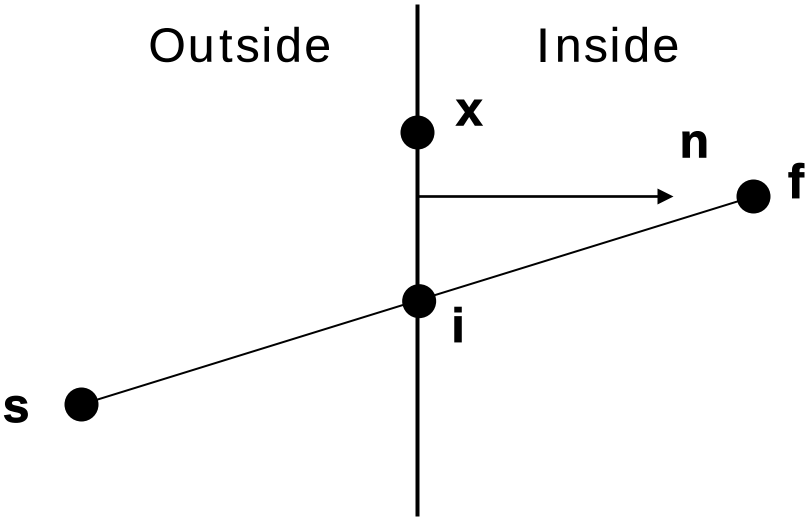
Inside-Outside Testing

- Lines/planes store a vector pointing toward the inside of the clip region - the inward pointing normal
  - Could re-define for outward pointing
- Dot products give inside/outside information
- Note that **x** (a vector) is any point on the clip line/plane

$$\mathbf{n} \bullet (\mathbf{s} - \mathbf{x}) < 0$$

$$\mathbf{n} \bullet (\mathbf{i} - \mathbf{x}) = 0$$

$$\mathbf{n} \bullet (\mathbf{f} - \mathbf{x}) > 0$$



Finding Intersection Pts

- Use the parametric form for the edge between two points, $\mathbf{x}_1$ and $\mathbf{x}_2$:

$$\mathbf{x}(t) = \mathbf{x}_1 + (\mathbf{x}_2 - \mathbf{x}_1)t \qquad 0 \le t \le 1$$

- For planes of the form *x=a*:

$$\boldsymbol{x}_i = \left(a, y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)}(a - x_1), z_1 + \frac{(z_2 - z_1)}{(x_2 - x_1)}(a - x_1)\right)$$

- Similar forms for *y=a, z=a*
- Solution for general plane can also be found

Inside/Outside in Screen Space

- In canonical view space, clip planes are $x_s=\pm1$, $y_s=\pm1$, $z_s=\pm1$
- Inside/Outside reduces to comparisons before perspective divide

$$-w_s \le x_s \le w_s$$

$$-w_s \le y_s \le w_s$$

$$-w_s \le z_s \le w_s$$

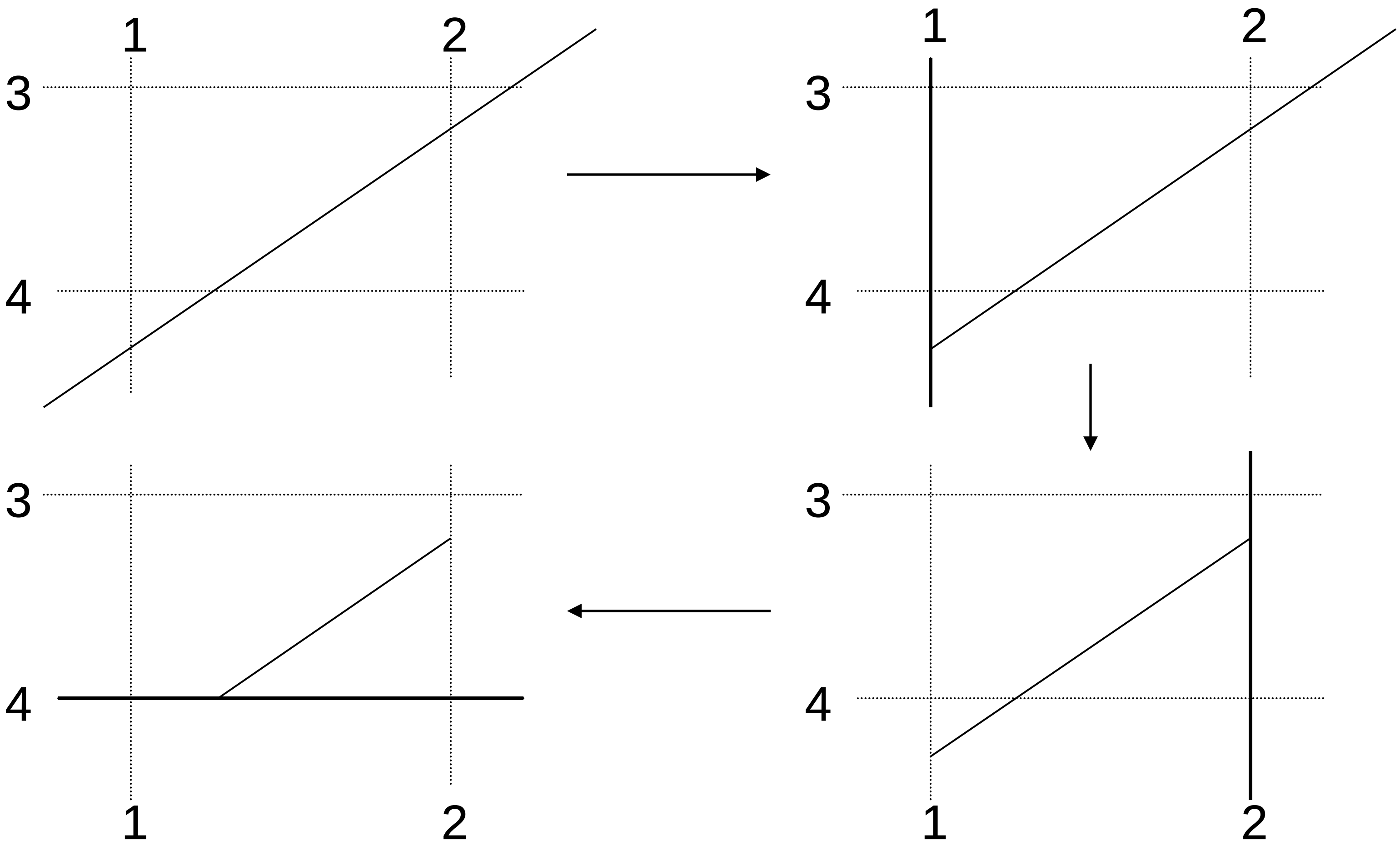
Clipping Lines

- Lines can also be clipped by Sutherland-Hodgman
  - Slower than necessary, unless you already have hardware
- Better algorithms exist
  - Cohen-Sutherland
  - Liang-Barsky
  - Nicholl-Lee-Nicholl (we won’t cover this one – only good for 2D)

Cohen-Sutherland (1)

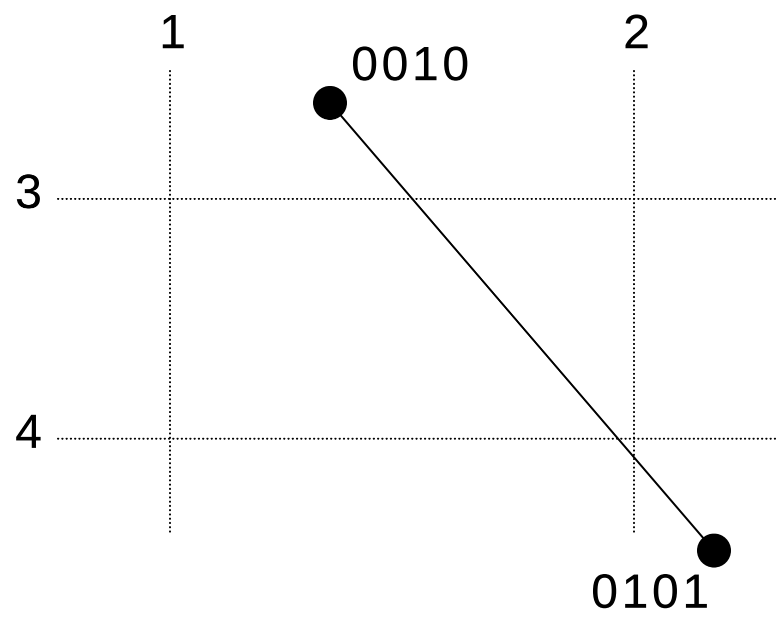
- Works basically the same as Sutherland-Hodgman
  - Was developed earlier
- Clip line against each edge of clip region in turn
  - If both endpoints outside, discard line and stop
  - If both endpoints in, continue to next edge (or finish)
  - If one in, one out, chop line at crossing pt and continue
- Works in both 2D and 3D for convex clipping regions

Cohen-Sutherland (2)



Cohen-Sutherland - Details

- Only need to clip line against edges where one endpoint is out
- Use *outcode* to record endpoint in/out wrt each edge. One bit per edge, 1 if out, 0 if in.
- Trivial reject:
  - outcode(x1) & outcode(x2)!=0
- Trivial accept:
  - outcode(x1) | outcode(x2)==0
- Which edges to clip against?
  - outcode(x1) ^ outcode(x2)

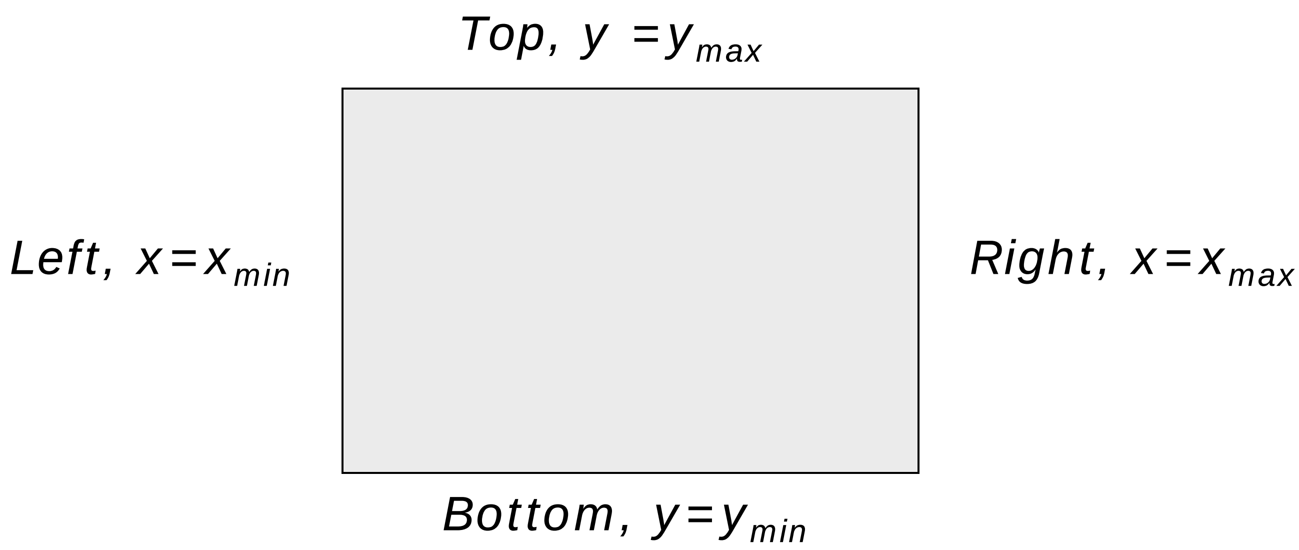


Liang-Barsky Clipping

- Parametric clipping - view line in parametric form and reason about the parameter values
  - Parametric form: $\mathbf{x} = \mathbf{x}_1+(\mathbf{x}_2-\mathbf{x}_1)t$
  - $t\in[0,1]$ are points between $\mathbf{x}_1$ and $\mathbf{x}_2$
- Liang-Barsky is more efficient than Cohen-Sutherland
  - Computing intersection vertices is most expensive part of clipping
  - Cohen-Sutherland may compute intersection vertices that are later clipped off, and hence don't contribute to the final answer
- Works for convex clip regions in 2D or 3D

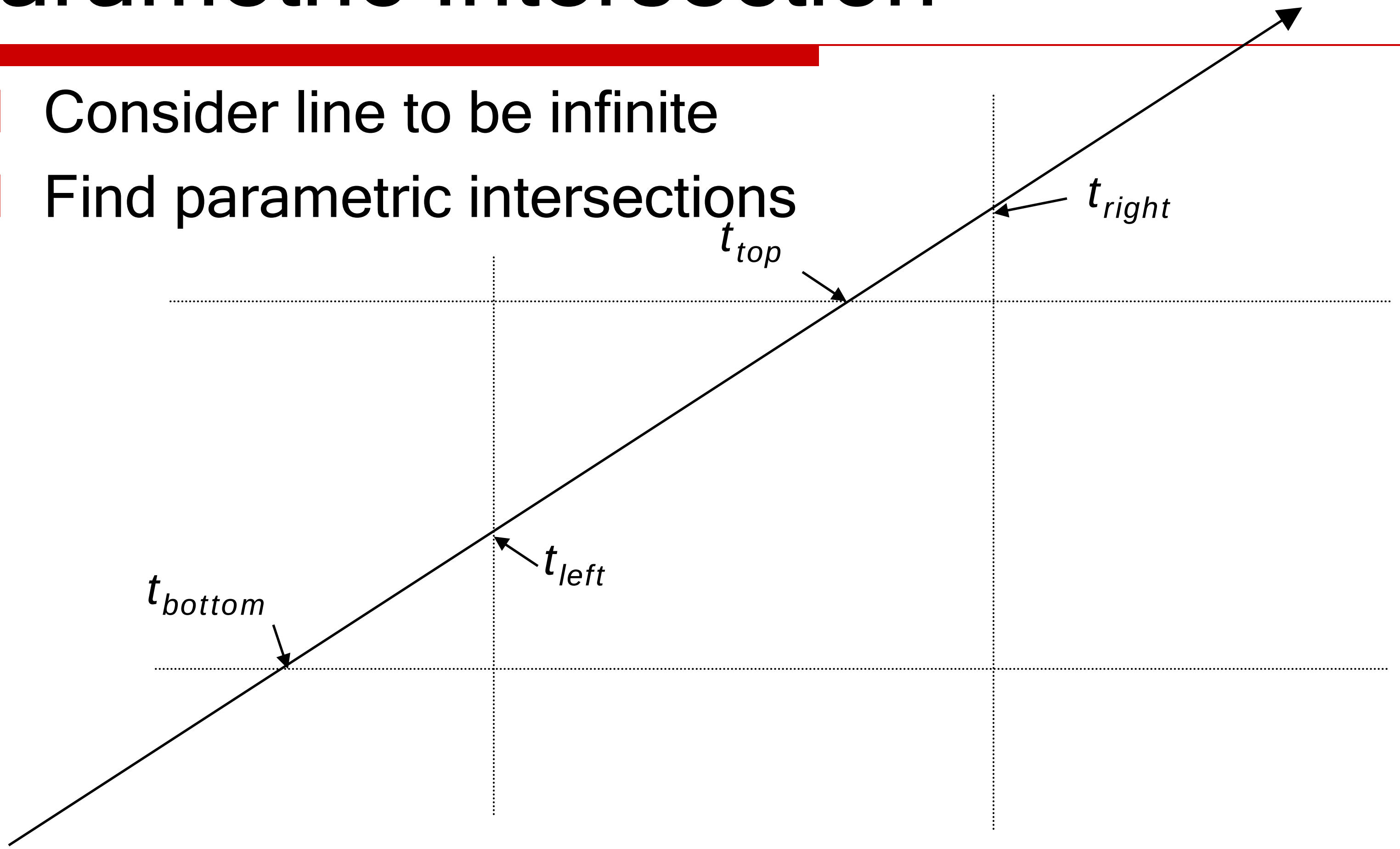
Parametric Clipping

- Recall, points inside a convex region are inside all clip planes
- Parametric clipping finds the values of *t*, the parameter, that correspond to points inside the clip region
- Consider a rectangular clip region



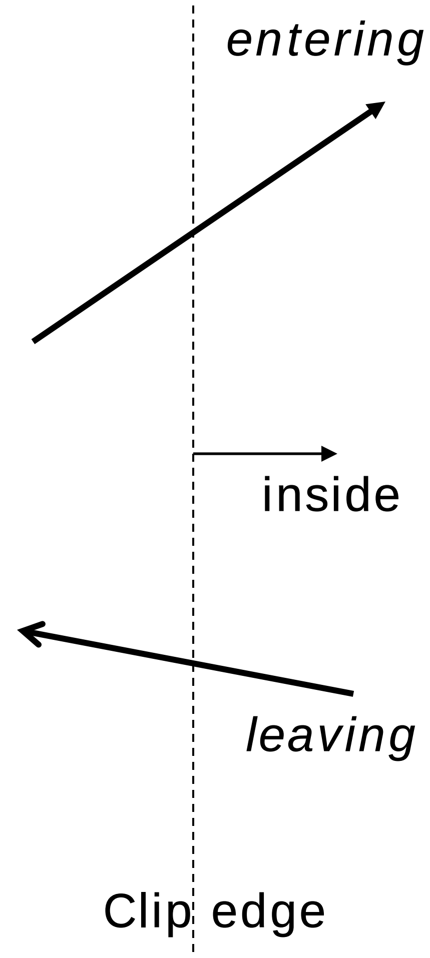
Parametric Intersection

- Consider line to be infinite
- Find parametric intersections



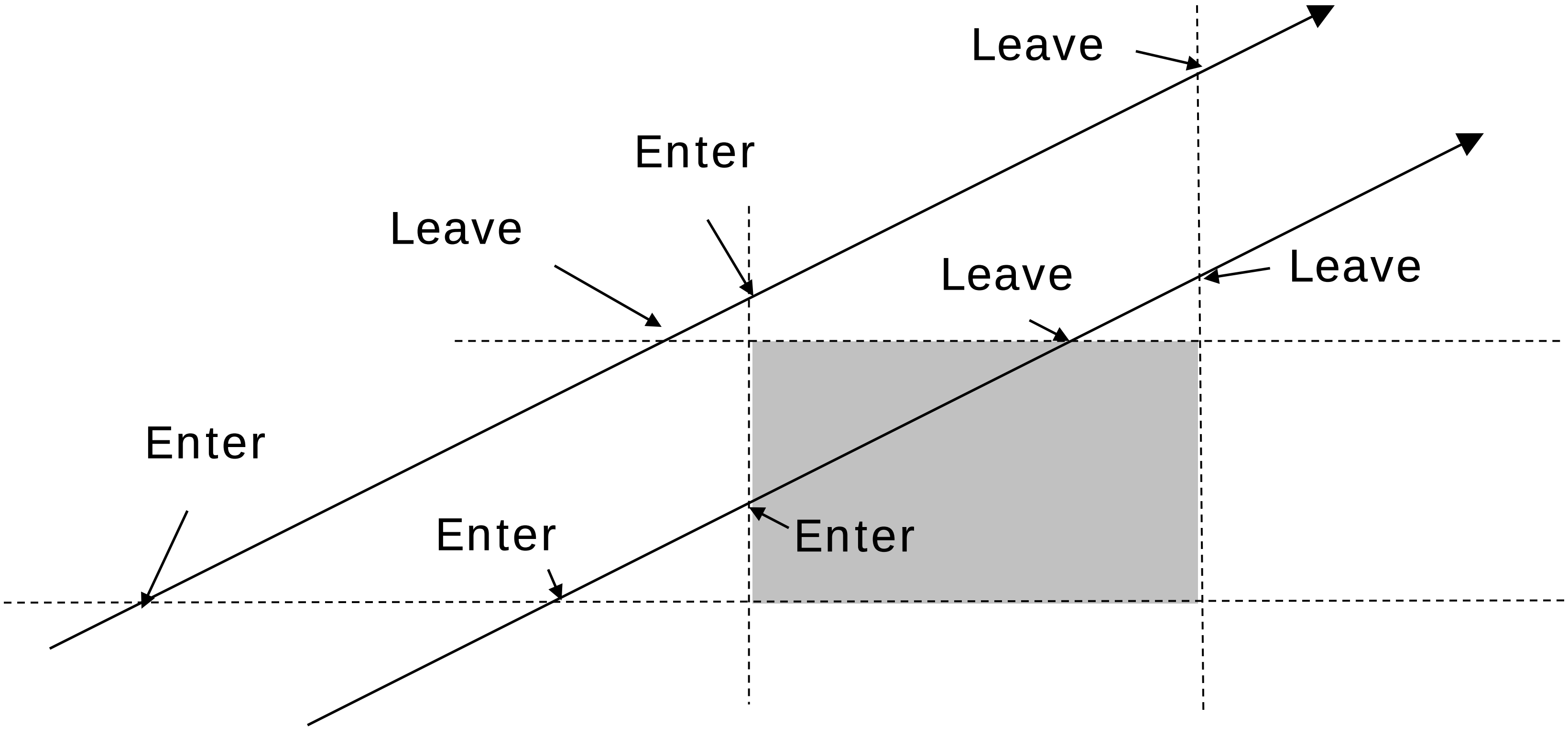
Entering and Leaving

- Recall, a point is inside a view volume if it is on the inside of every clip edge/plane
- Consider the left clip edge and the infinite line. Two cases:
  - $t<t_{left}$ is inside, $t>t_{left}$ is outside $\rightarrow$ *leaving*
  - $t<t_{left}$ is outside, $t>t_{left}$ is inside $\rightarrow$ *entering*
- To be inside a clip plane we either:
  - Started inside, and have not left yet
  - Started outside, and have entered



Entering/Leaving Example

- To be inside the clip region, you must have entered every clip edge before you have left any clip edge



When are we Inside?

- We want parameter values that are inside *all* the clip planes
- Any clip plane that we started inside we must not have left yet
  - First parameter value to leave is the end of the visible segment
- Any clip plane that we started outside we must have already entered
  - Last parameter value to enter is the start of the visible segment
- If we leave some clip plane before we enter another, we cannot see any part of the line
- All this leads to an algorithm – Liang-Barsky

Liang-Barsky Sub-Tasks

1. Find parametric intersection points
   - Parameter values where line crosses each clip edge/plane
2. Find entering/leaving flags
   - For every clip edge/plane, are either entering or leaving
3. Find last parameter to enter, and first one to leave
   - Check that enter before leave
4. Convert these into endpoints of clipped segment

1. Parametric Intersection

- Segment goes from (x_1, y_1) to (x_2, y_2): $\Delta x = x_2 - x_1$
  $\Delta y = y_2 - y_1$
- Rectangular clip region with x_{min}, x_{max}, y_{min}, y_{max}
- **Infinite** line intersects **rectangular** clip region edges when:

$$t_k = \frac{q_k}{p_k} \quad \text{where} \quad \begin{aligned} p_{left} &= -\Delta x & q_{left} &= x_1 - x_{\min} \\ p_{right} &= \Delta x & q_{right} &= x_{\max} - x_1 \\ p_{bottom} &= -\Delta y & q_{bottom} &= y_1 - y_{\min} \\ p_{top} &= \Delta y & q_{top} &= y_{\max} - y_1 \end{aligned}$$

2. Entering or Leaving?

- When p_k<0, as t increases line goes from outside to inside - entering
- When p_k>0, line goes from inside to outside - leaving
- When p_k=0, line is parallel to an edge
  - Special case: one endpoint outside, no part of segment visible, otherwise, ignore this clip edge and continue

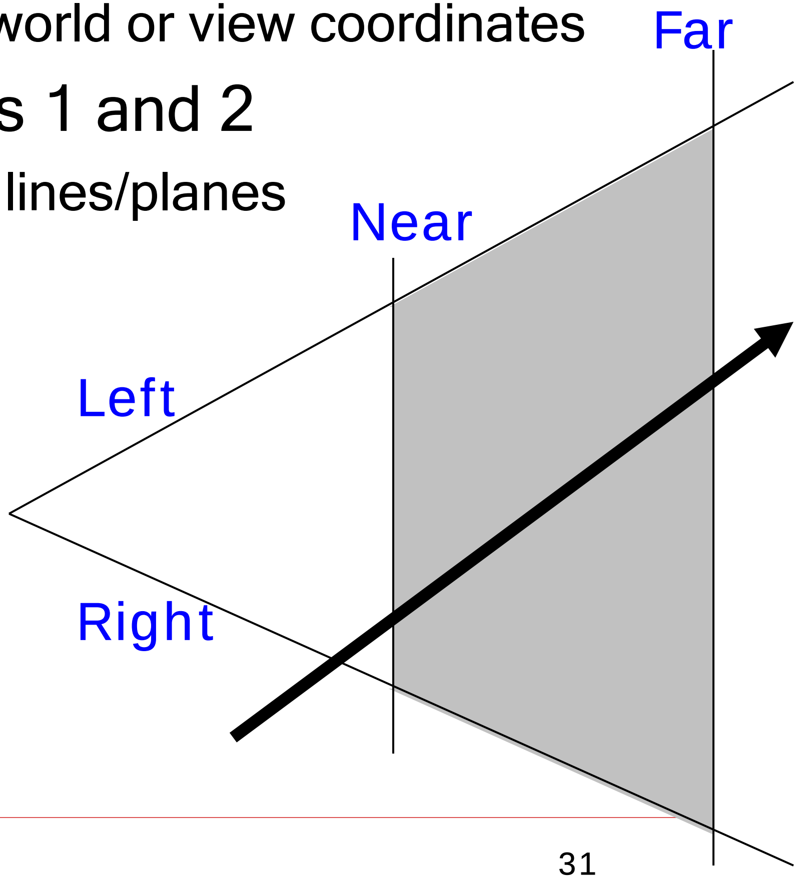
$$p_{left} = -\Delta x$$
$$p_{right} = \Delta x$$
$$p_{bottom} = -\Delta y$$
$$p_{top} = \Delta y$$

Find Visible Segment *t*s

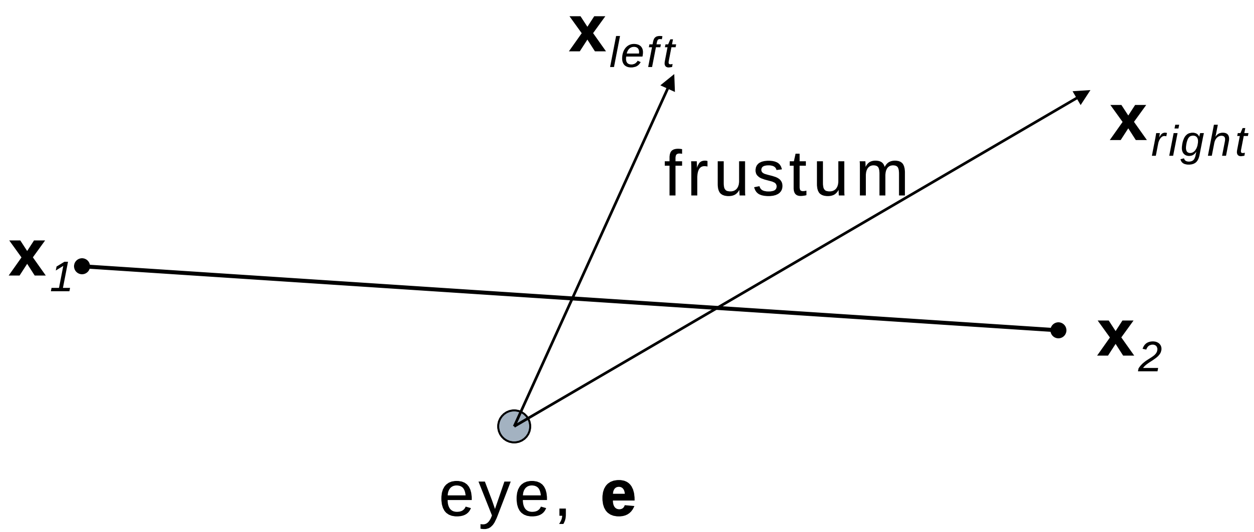
- Last parameter is enter is t_{small}=max(0, entering *t*'s)
- First parameter is leave is t_{large}=min(1, leaving *t*'s)
- If $t_{small}>t_{large}$, there is no visible segment
- If $t_{small}<t_{large}$, there is a line segment
  - Compute endpoints by substituting *t* values into parametric equation for the line segment
- Improvement (and actual Liang-Barsky):
  - compute *t*'s for each edge in turn (some rejects occur earlier like this)

General Liang-Barsky

- Liang-Barsky works for any convex clip region
  - E.g. Perspective view volume in world or view coordinates
- Require a way to perform steps 1 and 2
  1. Compute intersection *t* for all clip lines/planes
  2. Label them as entering or exiting



In View Space



First Step

- Compute inside/outside for endpoints of the line segment
  - Determine which side of each clip plane the segment endpoints lie
  - Use the cross product
  - What do we know if $(\mathbf{x}_1 - \mathbf{e}) \times (\mathbf{x}_{left} - \mathbf{e}) > 0$?
  - Other cross products give other information
- What can we say if both segment endpoints are **outside** one clip plane?
  - Stop here if we can, otherwise…

Finding Parametric Intersection

- Left clip edge: $\mathbf{x} = \mathbf{e} + (\mathbf{x}_{left} - \mathbf{e})\ t$
- Line: $\mathbf{x} = \mathbf{x}_1 + (\mathbf{x}_2 - \mathbf{x}_1)\ s$
- Solve simultaneous equations in t and s:

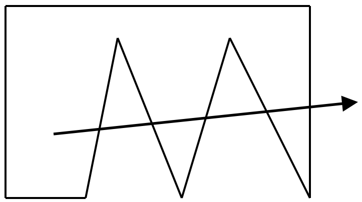
$$\mathbf{e}_x + (\mathbf{x}_{left,x} - \mathbf{e}_x)t = \mathbf{x}_{1,x} + (\mathbf{x}_{2,x} - \mathbf{x}_{1,x})s$$

$$\mathbf{e}_y + (\mathbf{x}_{left,y} - \mathbf{e}_y)t = \mathbf{x}_{1,y} + (\mathbf{x}_{2,y} - \mathbf{x}_{1,y})s$$

- **Use endpoint inside/outside information to label as entering or leaving**
- Now we have general Liang-Barsky case

General Clipping

- Liang-Barsky can be generalized to clip line segments to arbitrary polygonal clip regions
  - Consider clip edges as non-infinite segments
  - Look at all intersecting *t*s between 0 and 1
- Clipping general polygons against general clip regions is quite hard: Weiler-Atherton algorithm



Next Time

- Rasterization
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