Computer Graphics

Prof. Feng Liu

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http://www.cs.pdx.edu/~fliu/courses/cs447/

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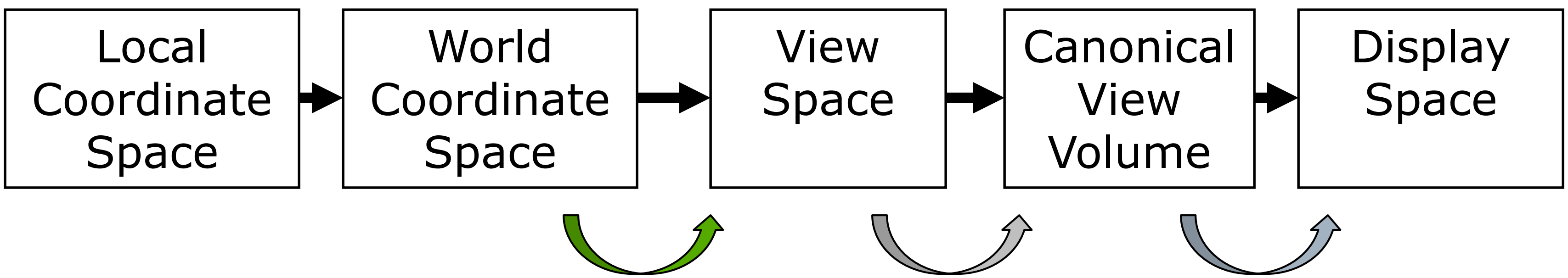
Last time

- More 2D Transformations
- Homogeneous Coordinates
- 3D Transformations
- The Viewing Pipeline

Today

- Perspective projection
- Homework 2 due in class today
- In-class Middle-Term
  - Wednesday, Nov. 2
  - Close-book exam
  - Notes on 1 page of A4 or Letter size paper
  - To-know list available online

Graphics Pipeline



Defining Cameras

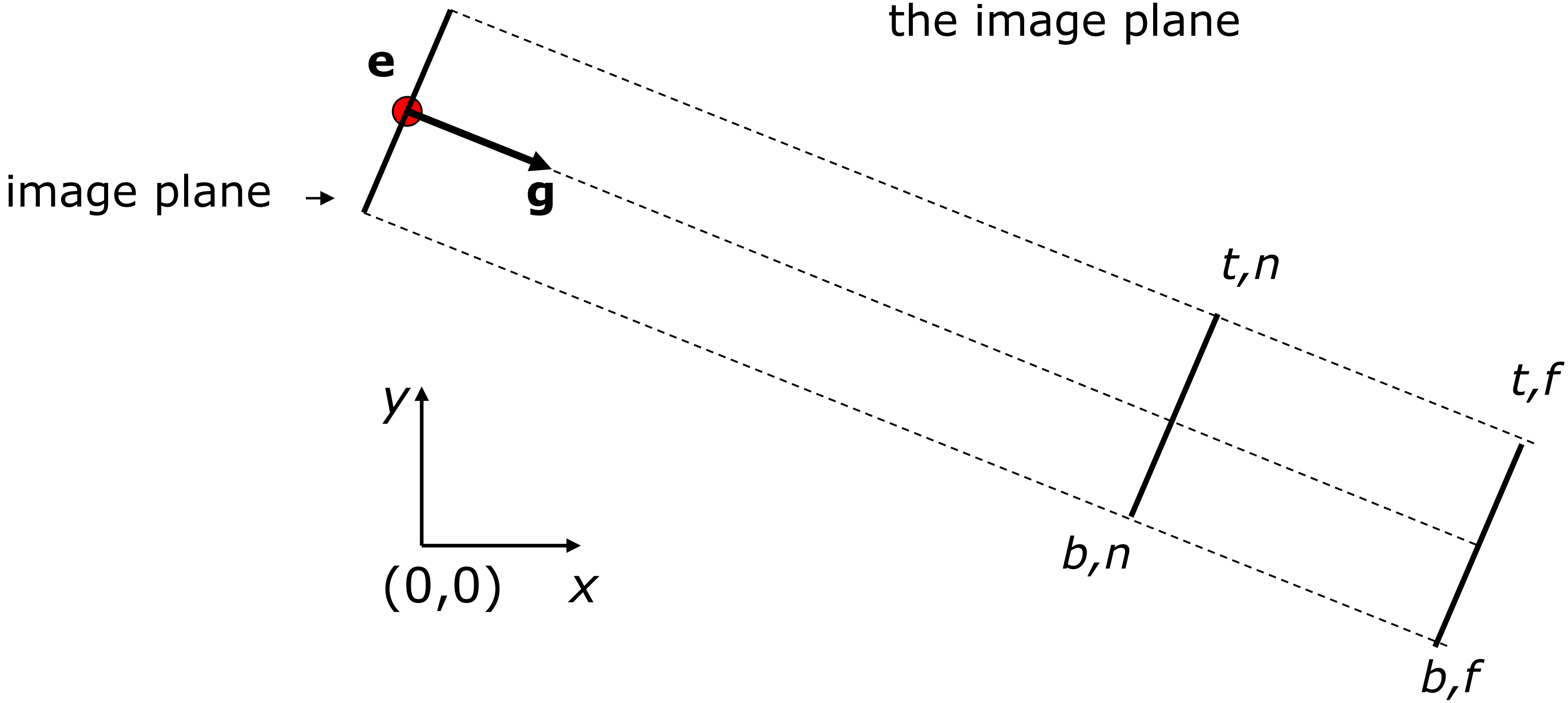
- View Space is the *camera's* local coordinates
  - The camera is in some location
  - The camera is looking in some direction
  - It is tilted in some orientation
- It is inconvenient to model everything in terms of View Space
  - Biggest problem is that the camera might be moving – we don't want to have to explicitly move every object too
- We specify the camera, and hence View Space, with respect to World Space
  - How can we specify the camera?

Specifying a View

- The **location** of View Space with respect to World Space
  - A point in World Space for the origin of View Space, (e_x, e_y, e_z)
- The **direction** in which we are looking: gaze direction
  - Specified as a vector: (g_x, g_y, g_z)
  - This vector will be normal to the image plane
- A **direction** that we want to *appear up* in the image
  - (up_x, up_y, up_z), this vector does not have to be perpendicular to $\boldsymbol{g}$
- We also need the size of the view volume – l, r, t, b, n, f
  - Specified **with respect to the eye and image plane**, not the world

General Orthographic

Subtle point: it doesn't precisely matter where we put the image plane



Getting there…

- We wish to end up in View Space, so we need a coordinate system with:
  - A vector toward the viewer, View Space z
  - A vector pointing right in the image plane, View Space x
  - A vector pointing up in the image plane, View Space y
  - The origin at the eye, View Space (0,0,0)
- We must:
  - Say what each of these vectors are in **World Space**
  - Transform points from the World Space into View Space
  - We can then apply the orthographic projection to get to Canonical View Space, and so on

View Space in World Space

- Given our camera definition, in World Space:
  - Where is the origin of view space? It will transform into $(0,0,0)_{view}$
  - What is the normal to the view plane, *w*? It will become z_{view}
  - How do we find the right vector, *u*? It will become x_{view}
  - How do we find the up vector, *v*? It will become y_{view}
- Given these points, how do we do the transformation?

View Space

- The origin is at the eye: (e_x, e_y, e_z)
- The normal vector is the normalized viewing direction

$$\mathbf{w} = -\hat{\mathbf{g}}$$

- We know which way up should be, and we know we have a right handed system, so $u = up \times w$, normalized: $\hat{\mathbf{u}}$
- We have two vectors in a right handed system, so to get the third: $v = w \times u$

World to View

- We must translate so the origin is at (e_x, e_y, e_z)
- To complete the transformation we need to do a rotation
- After this rotation:
  - The direction *u* in world space should be the direction (1,0,0) in view space
  - The vector *v* should be (0,1,0)
  - The vector *w* should be (0,0,1)
- The matrix that does the rotation is:
  - It's a "change of basis" matrix

$$\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

All Together

- We apply a translation and then a rotation, so the result is:

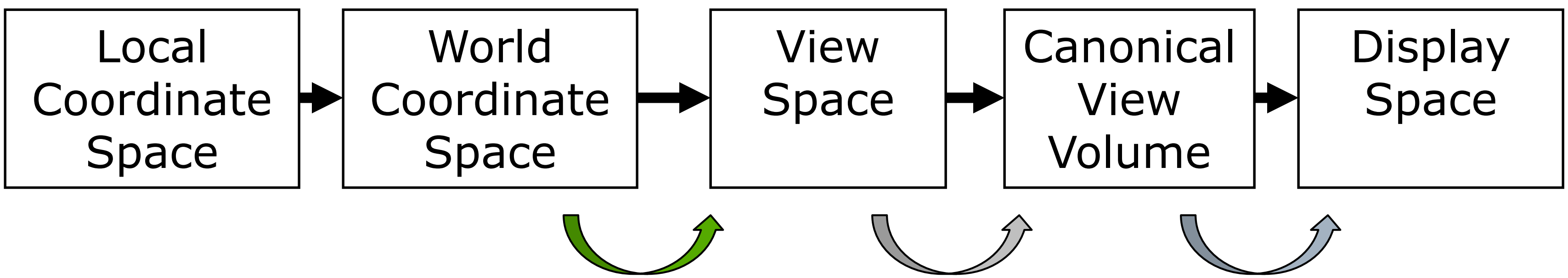
$$\mathbf{M}_{world->view} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathrm{u} \bullet \mathbf{e} \\ v_x & v_y & v_z & -\mathrm{v} \bullet \mathbf{e} \\ w_x & w_y & w_z & -\mathrm{w} \bullet \mathbf{e} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- And to go all the way from world to screen:

$$\mathbf{M}_{world->canonical} = \mathbf{M}_{view->canonical} \mathbf{M}_{world->view}$$

$$\mathbf{x}_{canonical} = \mathbf{M}_{world->canonical} \mathbf{x}_{world}$$

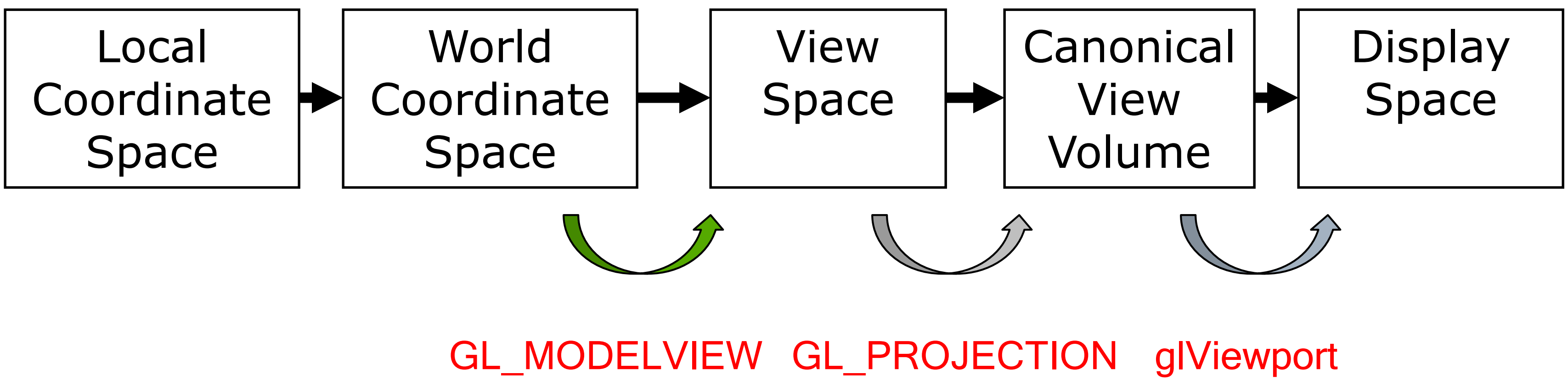
Graphics Pipeline



OpenGL and Transformations

- OpenGL internally stores two matrices that control viewing of the scene
  - The `GL_MODELVIEW` matrix is intended to capture all the transformations up to view space
  - The `GL_PROJECTION` matrix captures the view to canonical conversion
- You also specify the mapping from the canonical view volume into window space
  - Directly through a `glViewport` function call
- Matrix calls, such as `glRotate`, multiply some matrix M onto the current matrix C, resulting in CM
  - Set view transformation first, then set transformations from local to world space – **last one set is first one applied**
  - This is the convenient way for modeling, as we will see

Graphics Pipeline



OpenGL Camera

- The **default** OpenGL image plane has u aligned with the x axis, v aligned with y, and n aligned with z
  - Means the default camera looks along the negative z axis
  - Makes it easy to do 2D drawing (no need for any view transformation)
- `glOrtho(…)` sets the view->canonical matrix
  - Modifies the `GL_PROJECTION` matrix
- `gluLookAt(…)` sets the world->view matrix
  - Takes an image center point, a point along the viewing direction and an up vector
  - Multiplies a world->view matrix **onto the current `GL_MODELVIEW` matrix**
  - You could do this yourself, using `glMultMatrix(…)` with the matrix from the previous slides

Typical Usage

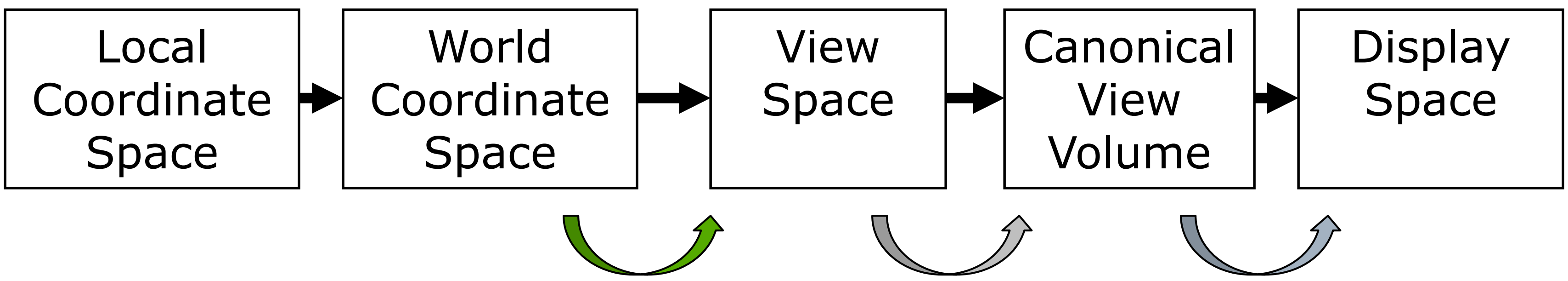
```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(l, r, b, t, n, f);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,cx,cy,cx,ux,uy,uz);
```

- GLU functions, such as `gluLookAt(…)`, are not part of the core OpenGL library
  - They can be implemented with other core OpenGL commands
  - For example, `gluLookAt(…)` uses `glMultMatrix(…)` with the matrix from the previous slides
  - They are not dependent on a particular graphics card

Left vs Right Handed View Space

- You can define **u** as right, **v** as up, and **n** as toward the viewer: a right handed system $u \times v = w$
  - Advantage: Standard mathematical way of doing things
- You can also define **u** as right, **v** as up and **n** as into the scene: a left handed system $v \times u = w$
  - Advantage: Bigger *n* values mean points are further away
- OpenGL is right handed
- Many older systems, notably the Renderman standard developed by Pixar, are left handed

Graphics Pipeline

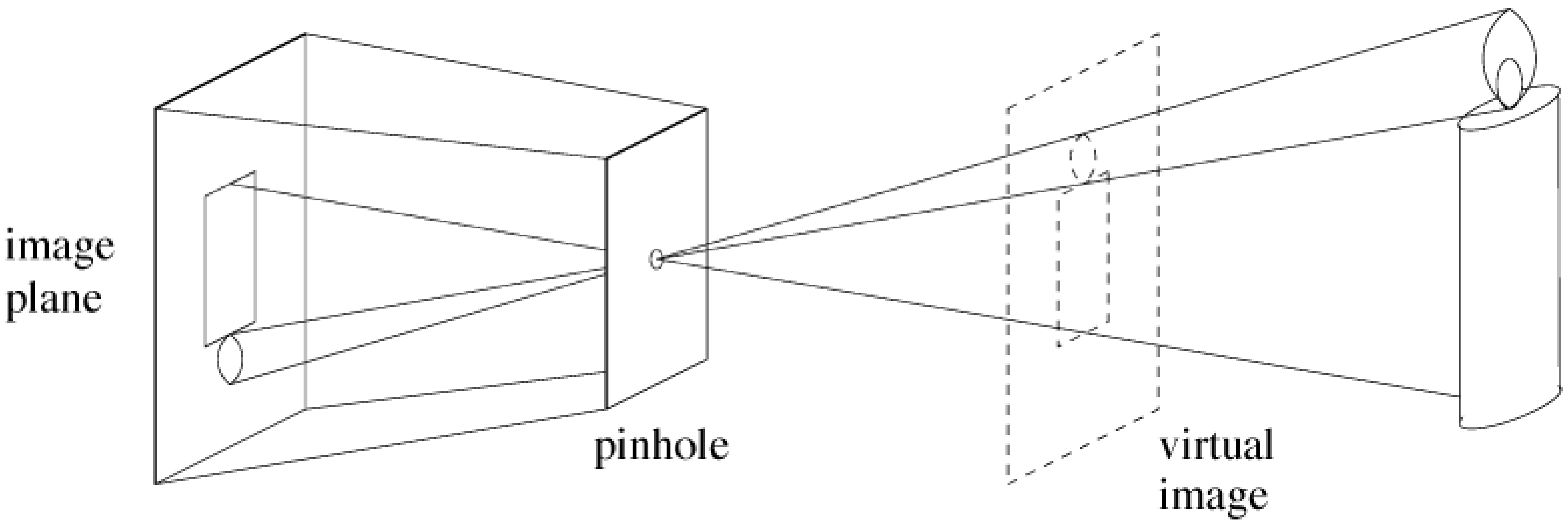


Review

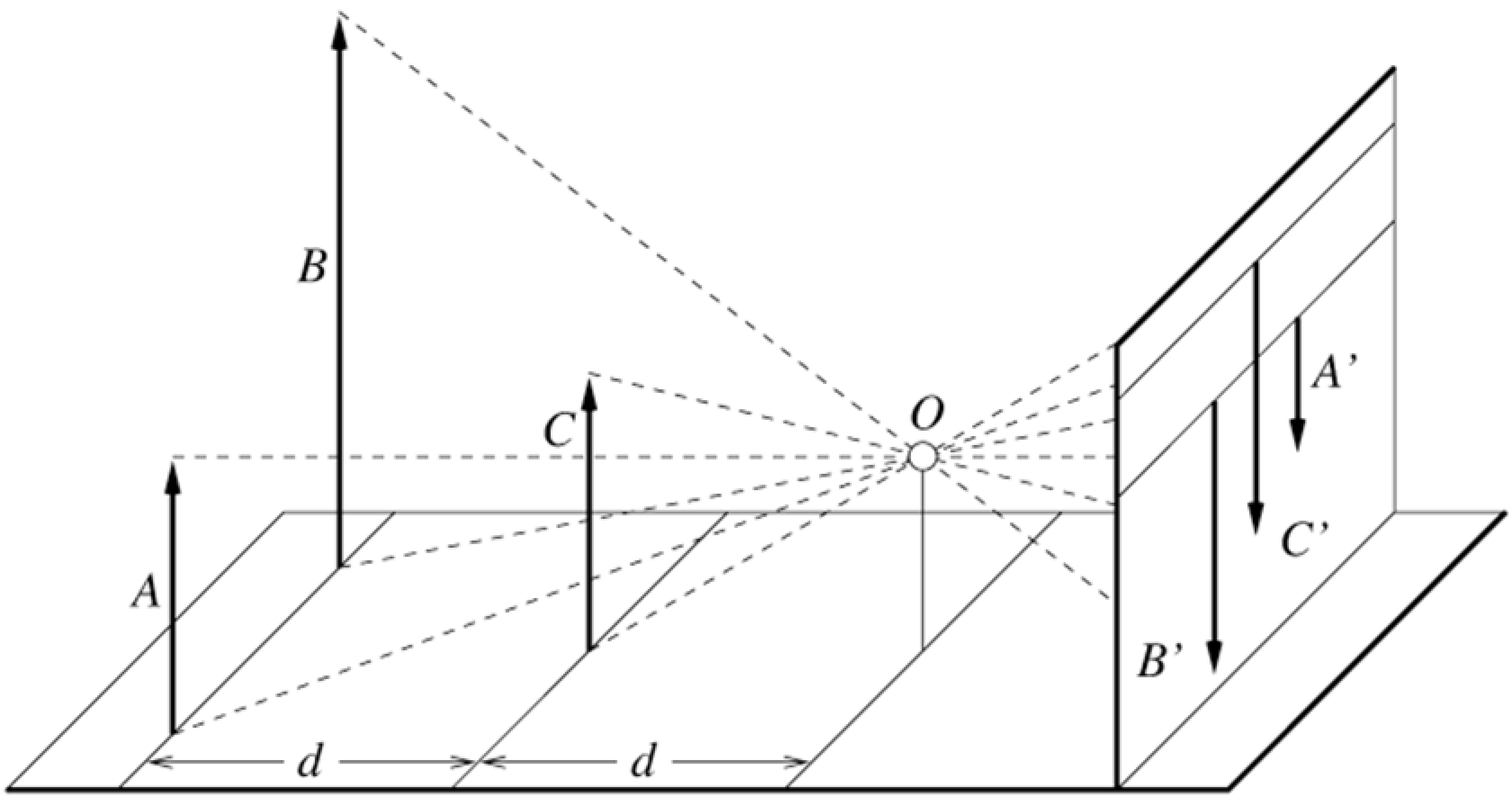
- **View Space** is a coordinate system with the viewer looking down the $-z$ axis, with x to the right and y up
- The World->View transformation takes points in world space and converts them into points in view space
- The Projection matrix, or View->Canonical matrix, takes points in view space and converts them into points in Canonical View Space
  - **Canonical View Space** is a coordinate system with the viewer looking along $-z$, x to the right, y up, and everything to be drawn inside the cube [-1,1]x[-1,1]x[-1,1] using parallel projection

Perspective Projection

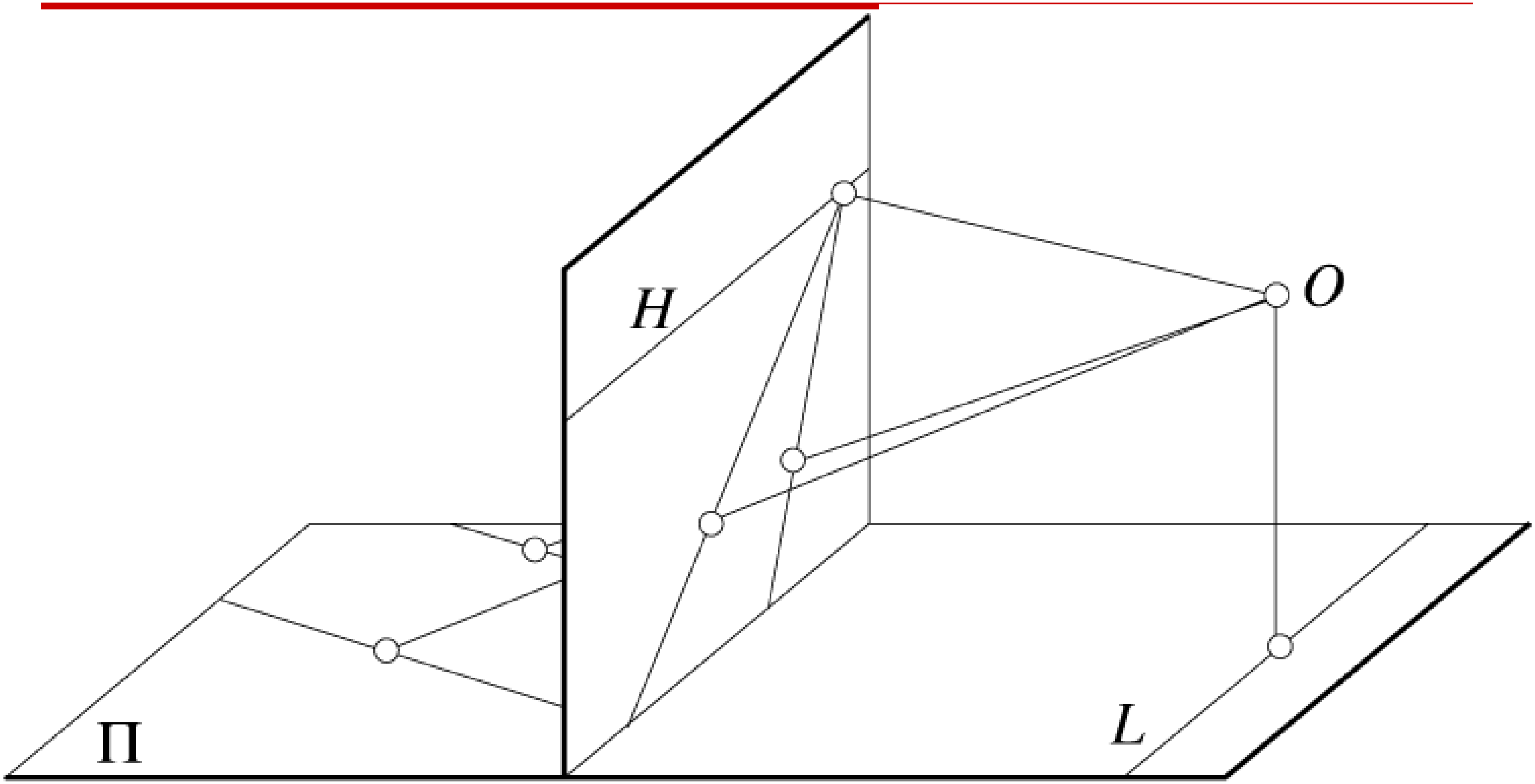
- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice



Distant Objects Are Smaller

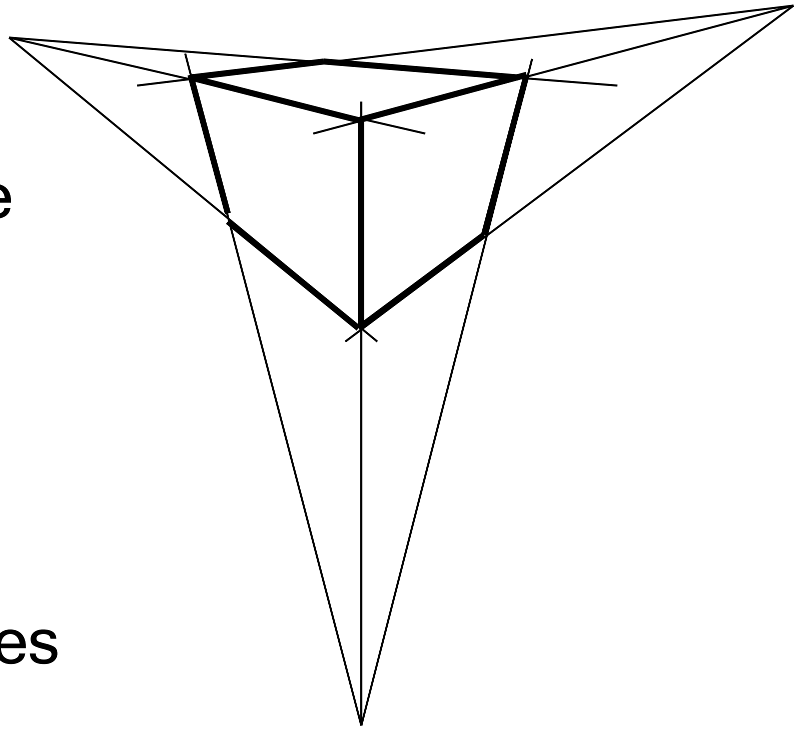


Parallel lines meet



Vanishing points

- Each set of parallel lines (=direction) meets at a different point: The vanishing point for this direction
  - Classic artistic perspective is 3-point perspective
- Sets of parallel lines on the same plane lead to collinear vanishing points: the horizon for that plane
- Good way to spot faked images

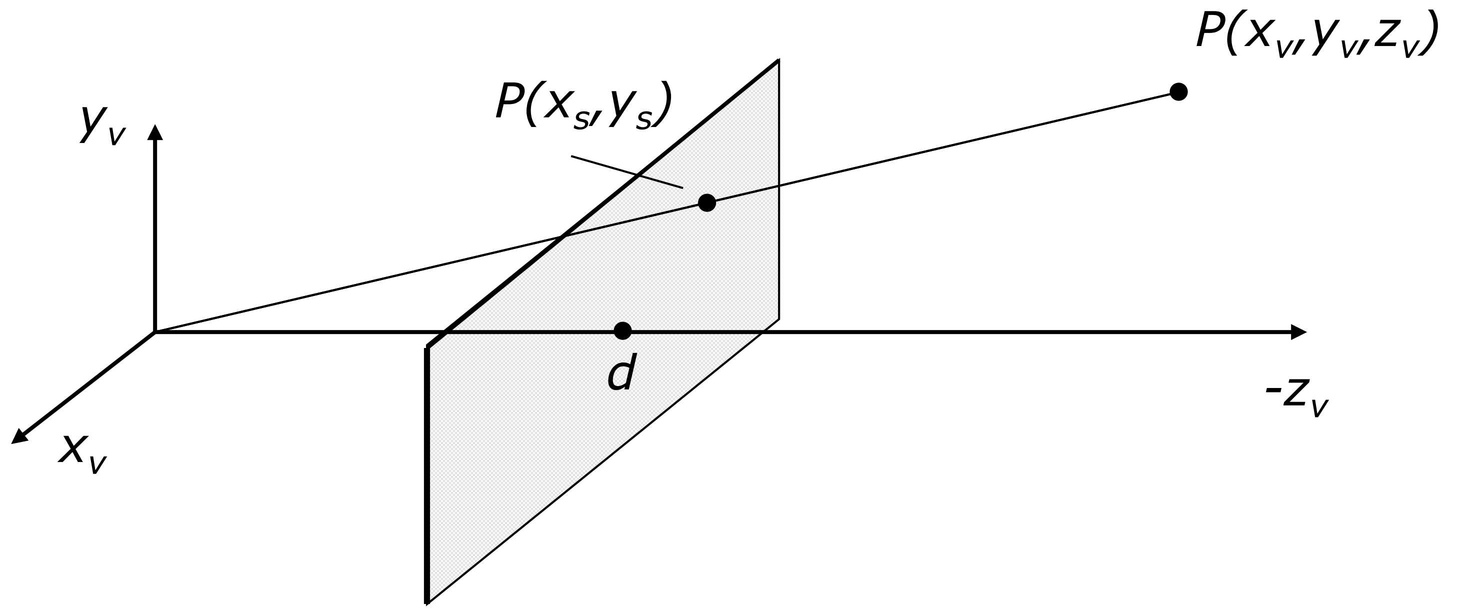


Basic Perspective Projection

- We are going to temporarily ignore canonical view space, and go straight from view to window
- Assume you have transformed to view space, with x to the right, y up, and z back toward the viewer
- Assume the origin of view space is at the center of projection (the eye)
- Define a focal distance, d, and put the image plane there (note d is negative)
  - You can define d to control the size of the image

Basic Perspective Projection

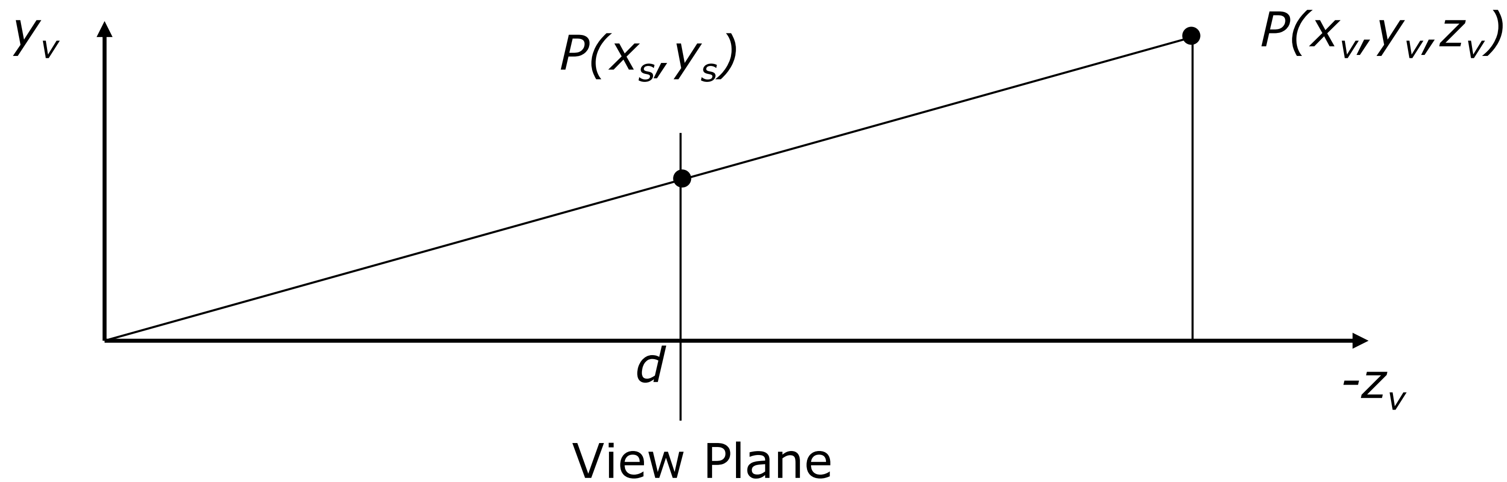
- If you know $P(x_v,y_v,z_v)$ and d, what is $P(x_s,y_s)$?
  - Where does a point in view space end up on the screen?



Basic Case

Similar triangles gives:

$$\frac{x_s}{d} = \frac{x_v}{z_v} \qquad \frac{y_s}{d} = \frac{y_v}{z_v}$$



Simple Perspective Transformation

- Using homogeneous coordinates we can write:
  - Our next big advantage to homogeneous coordinates

$$\begin{bmatrix} x_s \\ y_s \\ d \end{bmatrix} \equiv \begin{bmatrix} x_v \\ y_v \\ z_v \\ z_v/d \end{bmatrix} \qquad \mathbf{P}_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \mathbf{P}_v$$

Parallel Lines Meet?

- Parallel lines are of the form: $\mathbf{x} = \mathbf{x}_0 + t\mathbf{d}$
  - Parametric form: $\mathbf{x}_0$ is a point on the line, t is a scalar (distance along the line from $\mathbf{x}_0$) and $\mathbf{d}$ is the direction of the line (unit vector)
  - Different $\mathbf{x}_0$ give different parallel lines
- Transform and go from homogeneous to regular:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x_d \\ y_d \\ z_d \\ 0 \end{bmatrix} = f \begin{bmatrix} \dfrac{x_0 + tx_d}{z_0 + tz_d} \\ \dfrac{y_0 + ty_d}{z_0 + tz_d} \\ 1 \end{bmatrix}$$

- Limit as $t \to \infty$ is $\begin{bmatrix} fx_d/z_d & fy_d/z_d & f \end{bmatrix}$

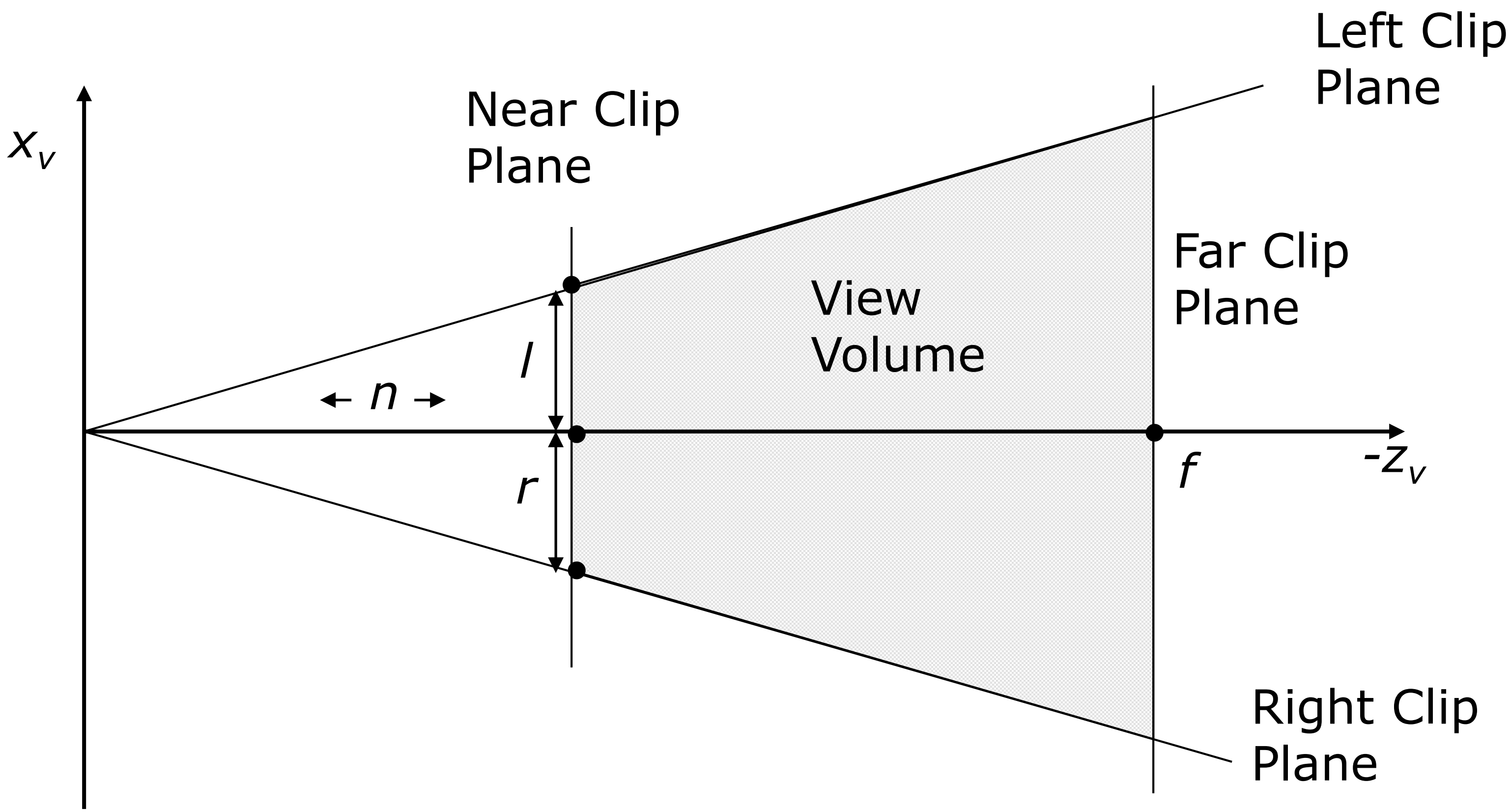
General Perspective

- The basic equations we have seen give a flavor of what happens, but they are insufficient for all applications
- They do not get us to a Canonical View Volume
- They make assumptions about the viewing conditions
- To get to a Canonical Volume, we need a Perspective Volume …

Perspective View Volume

- Recall the orthographic view volume, defined by a near, far, left, right, top and bottom plane
- The perspective view volume is also defined by near, far, left, right, top and bottom planes – the *clip planes*
  - **Near and far planes are parallel to the image plane**: $z_v=n$, $z_v=f$
  - Other planes all pass through the center of projection (the origin of view space)
  - The left and right planes intersect the image plane in vertical lines
  - The top and bottom planes intersect in horizontal lines

Clipping Planes

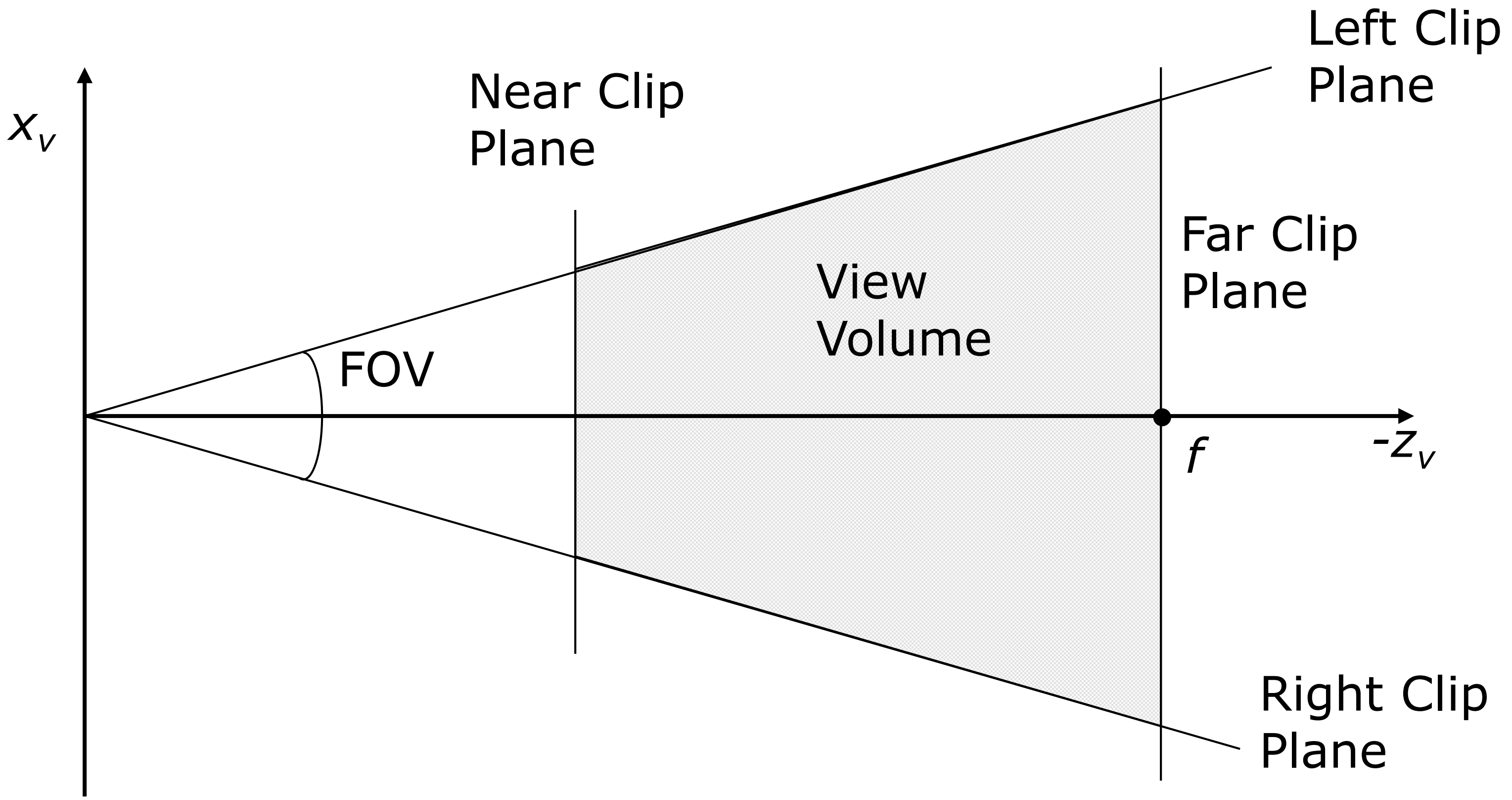


Where is the Image Plane?

- Notice that it doesn't really matter where the image plane is located, once you define the view volume
  - You can move it forward and backward along the z axis and still get the same image, only scaled
- The left/right/top/bottom planes are defined **according to where they cut the near clip plane**
- Or, define the left/right and top/bottom clip planes by the *field of view*

Field of View

Assumes a *symmetric* view volume

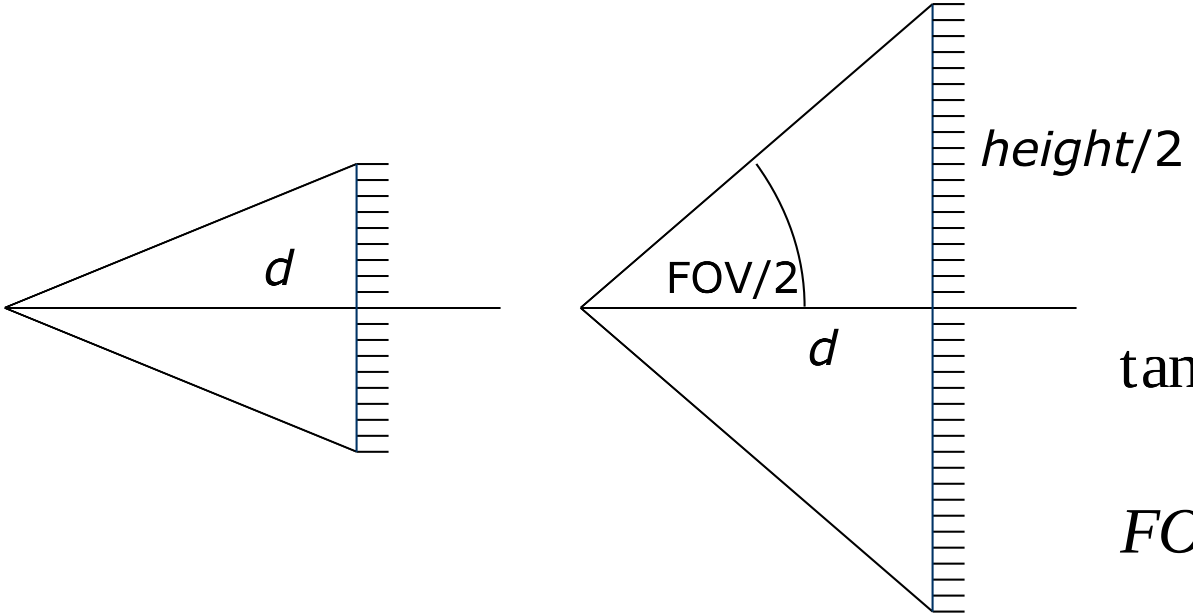


Perspective Parameters

- We have seen several different ways to describe a perspective camera
  - Focal distance, Field of View, Clipping planes
- The most general is clipping planes – they directly describe the region of space you are viewing
- For most graphics applications, field of view is the most convenient
  - It is *image size invariant* – having specified the field of view, what you see does not depend on the image size
- You can convert one thing to another

Focal Distance to FOV

- You must have the image size to do this conversion
  - Why? Same *d*, different image size, different FOV

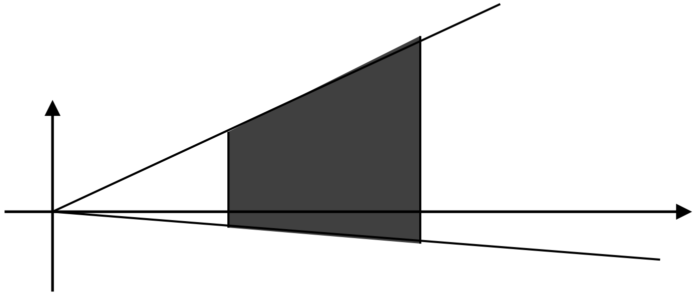


$$\tan\left(\frac{FOV}{2}\right) = \frac{height}{2d}$$

$$FOV = 2\tan^{-1}\left(\frac{height}{2d}\right)$$

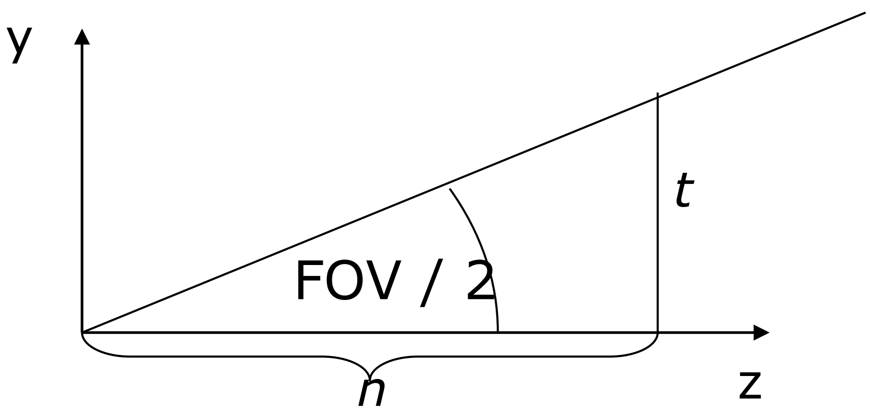
OpenGL

- `gluPerspective(…)`
  - Field of view in the *y* direction, *FOV*, (vertical field-of-view)
  - Aspect ratio, *a,* **should match window aspect ratio**
  - Near and far clipping planes, *n* and *f*
  - Defines a symmetric view volume
- `glFrustum(…)`
  - Give the near and far clip plane, and places where the other clip planes cross the near plane
  - Defines the general case
  - Used for stereo viewing, mostly



`gluPerspective` to `glFrustum`

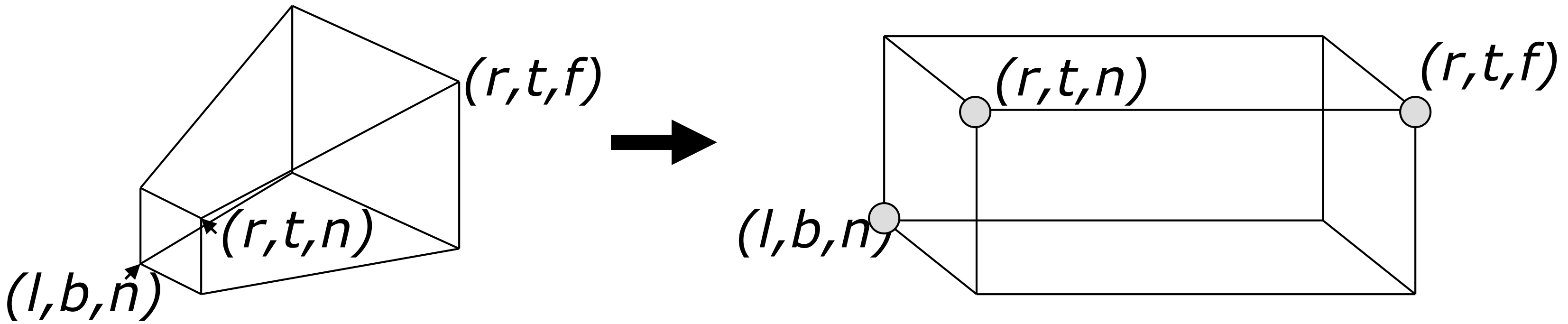
- As noted previously, `glu` functions don't add basic functionality, they are just more convenient
  - So how does `gluPerspective` convert to `glFrustum`?
  - Symmetric, so only need *t* and *l*



?

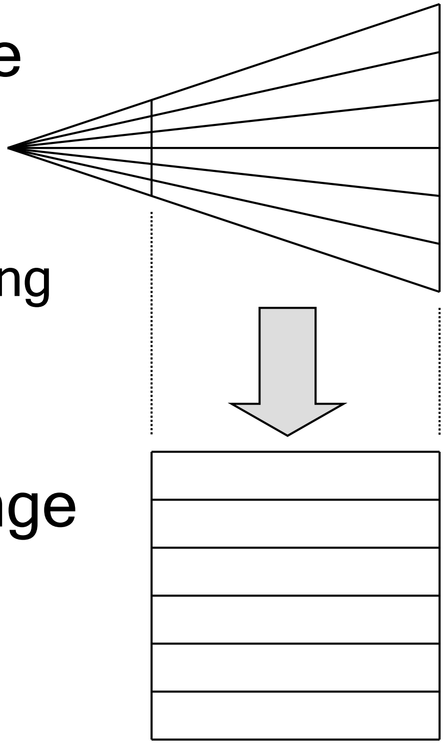
Perspective Projection Matrices

- We want a matrix that will take points in our perspective view volume and transform them into the orthographic view volume
  - This matrix will go in our pipeline before an orthographic projection matrix



Mapping Lines

- We want to map all the lines through the center of projection to parallel lines
  - This converts the perspective case to the orthographic case, we can use all our existing methods
- The relative intersection points of lines with the near clip plane should not change
- The matrix that does this looks like the matrix for our simple perspective case



General Perspective

$$\mathbf{M}_P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & (n+f)/n & -f \\ 0 & 0 & 1/n & 0 \end{bmatrix} \equiv \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- This matrix leaves points with $z=n$ unchanged
- It is just like the simple projection matrix, but it does some extra things to z to map the depth properly
- We can multiply a homogenous matrix by any number without changing the final point, so the two matrices above have the same effect

Complete Perspective Projection

- After applying the perspective matrix, we map the orthographic view volume to the canonical view volume:

$$\mathbf{M}_{view->canonical} = \mathbf{M}_O \mathbf{M}_P = \begin{bmatrix} \frac{2}{(r-l)} & 0 & 0 & \frac{-(r+l)}{(r-l)} \\ 0 & \frac{2}{(t-b)} & 0 & \frac{-(t+b)}{(t-b)} \\ 0 & 0 & \frac{2}{(n-f)} & \frac{-(n+f)}{(n-f)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & (n+f) & -nf \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_{world->canonical} = \mathbf{M}_{view->canonical} \mathbf{M}_{world->view}$$

$$\mathbf{x}_{canonical} = \mathbf{M}_{world->canonical} \mathbf{x}_{world}$$

Near/Far and Depth Resolution

- It may seem sensible to specify a very near clipping plane and a very far clipping plane
  - Sure to contain entire scene
- But, a bad idea:
  - OpenGL only has a finite number of bits to store screen depth
  - Too large a range reduces resolution in depth - wrong thing may be considered “in front”
  - See Shirley for a more complete explanation
- **Always place the near plane as far from the viewer as possible, and the far plane as close as possible**

OpenGL Perspective Projection

- For OpenGL you give the distance to the near and far clipping planes
- The total perspective projection matrix resulting from a `glFrustum` call is:

$$\mathbf{M}_{OpenGL} = \begin{bmatrix} \frac{2|n|}{(r-l)} & 0 & \frac{(r+l)}{(r-l)} & 0 \\ 0 & \frac{2|n|}{(t-b)} & \frac{(t+b)}{(t-b)} & 0 \\ 0 & 0 & \frac{(|n|+|f|)}{(|n|-|f|)} & \frac{2|f||n|}{(|n|-|f|)} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Next Time

- Clipping
- Rasterization
-