# Computer Graphics

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**Fall 2016**

http://www.cs.pdx.edu/~fliu/courses/cs447/

**10/26/2016**

# Last time

- Clipping

# Today

- Rasterization
- In-class Mid-term
  - November 2
  - Close-book exam
  - Notes on 1 page of A4 or Letter size paper

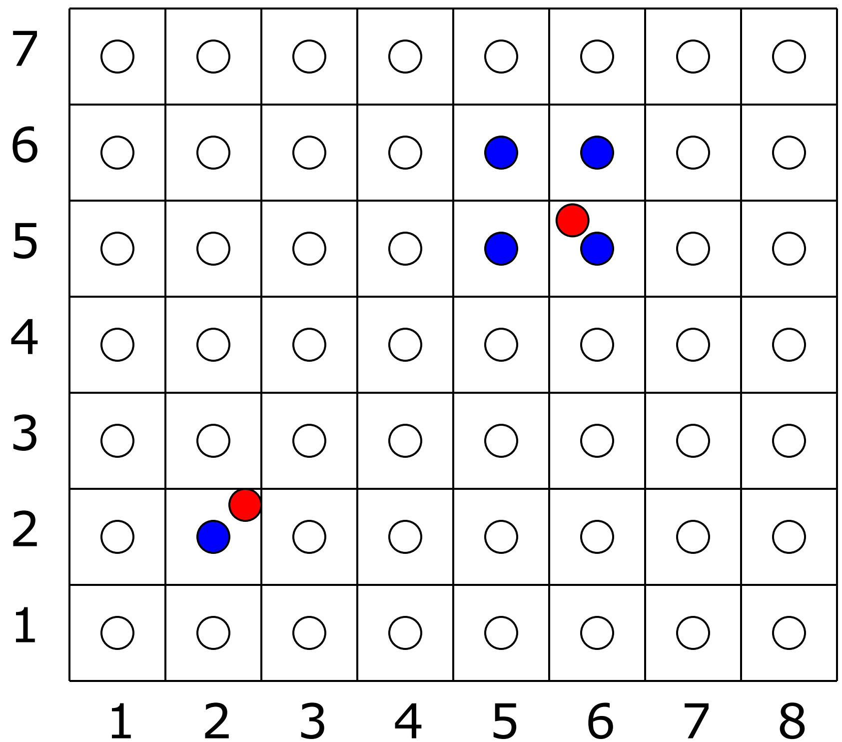
# Where We Stand

- At this point we know how to:
  - Convert points from local to screen coordinates
  - Clip polygons and lines to the view volume
- Next thing:
  - Determine which pixels to fill for any given point, line or polygon

# Drawing Points

- When points are mapped into window coordinates, they could land anywhere – not just at a pixel center
- Solution is the simple, obvious one
  - Map to window space
  - Fill the closest pixel
  - Can also specify a radius – fill a square of that size, or fill a circle
    - Square is faster

# Drawing Points (2)



# Drawing Lines

- Task: Decide which pixels to fill (samples to use) to represent a line
- We know that all of the line lies inside the visible region (clipping gave us this!)

# Line Drawing Algorithms

- Consider lines of the form $y = m\,x + c$, where $m = \Delta y / \Delta x$, $0 < m < 1$, integer coordinates
  - All others follow by symmetry, modify for real numbers
- Variety of slow algorithms (Why slow?):
  - step x, compute new y at each step by equation, rounding:
  - step x, compute new y at each step by adding m to old y, rounding:

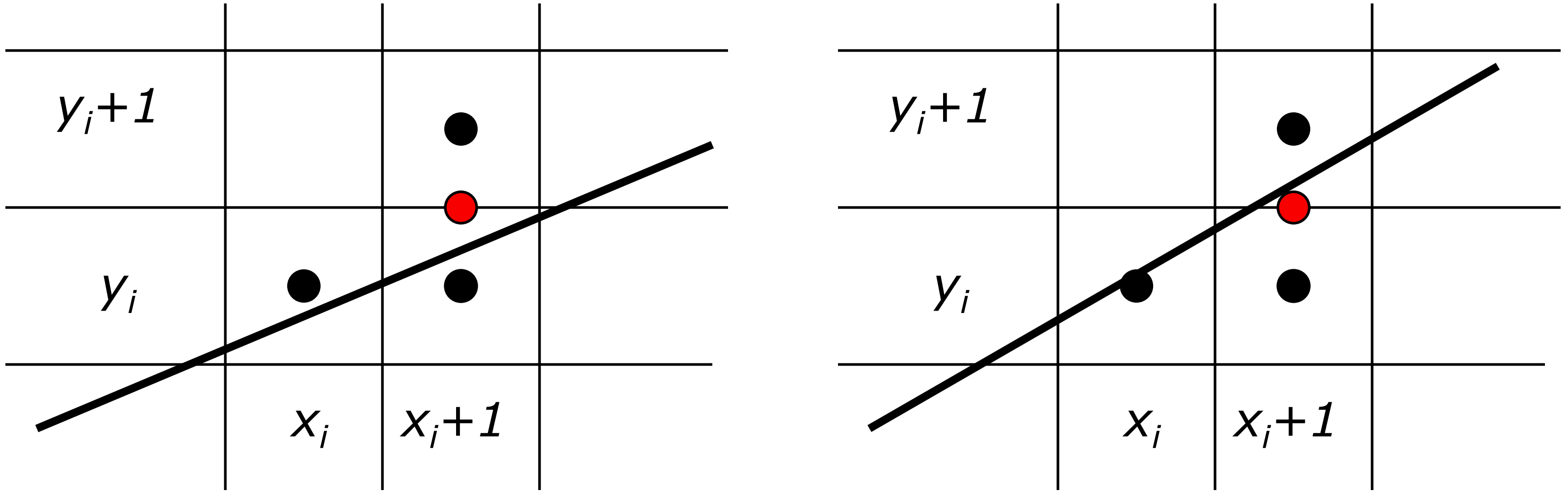
$$x_{i+1} = x_i + 1, \qquad y_{i+1} = round(mx_{i+1} + b)$$
$$x_{i+1} = x_i + 1, \qquad y_{i+1} = round(y_i + m)$$

# Bresenham's Algorithm Overview

- Aim: For each *x*, plot the pixel whose *y*-value is closest to the line
- Given $(x_i, y_i)$, must choose from either $(x_i+1, y_i+1)$ or $(x_i+1, y_i)$
- Idea: compute a *decision variable*
  - Value that will determine which pixel to draw
  - Easy to update from one pixel to the next
- Bresenham's algorithm is the *midpoint algorithm* for lines
  - Other midpoint algorithms for conic sections (circles, ellipses)

# Midpoint Methods

- Consider the midpoint between $(x_i+1, y_i+1)$ and $(x_i+1, y_i)$
- If it's above the line, we choose $(x_i+1, y_i)$, otherwise we choose $(x_i+1, y_i+1)$




Choose $(x_i+1, y_i)$ Choose $(x_i+1, y_i+1)$

# Midpoint Decision Variable

- Write the line from $(x_1, y_1)$ to $(x_2, y_2)$ in *implicit form:*
  $$F(x,y) = ax + by + c = \Delta x \cdot y - \Delta y \cdot x + (\Delta y \cdot x_1 - \Delta x \cdot y_1)$$
  - Assume $x_1 <= x_2$
  - $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$
- The value of *F(x,y)* tells us where points are with respect to the line
  - *F(x,y)=0*: the point is on the line
  - *F(x,y)>0*: The point is above the line
  - *F(x,y)<0*: The point is below the line
- **The decision variable is the value of $d_i = 2F(x_i+1, y_i+0.5)$**
  - The factor of two makes the math easier

# What Can We Decide?

$$d_i = 2\Delta x y_i - 2\Delta y(x_i + 1) + 2\left(\Delta y \cdot x_1 - \Delta x \cdot y_1\right) + \Delta x$$

- $d_i$ positive=> next point at $(x_i+1, y_i)$
- $d_i$ negative => next point at $(x_i+1, y_i+1)$
- At each point, we compute $d_i$ and decide which pixel to draw
- How do we update it? What is $d_{i+1}$?

# Updating The Decision Variable

- $d_{k+1}$ is the old value, $d_k$, plus an increment:

$$d_{k+1} = d_k + (d_{k+1} - d_k)$$

- If we chose $y_{i+1}=y_i+1$: $$d_{k+1} = d_k - 2\Delta y + 2\Delta x$$

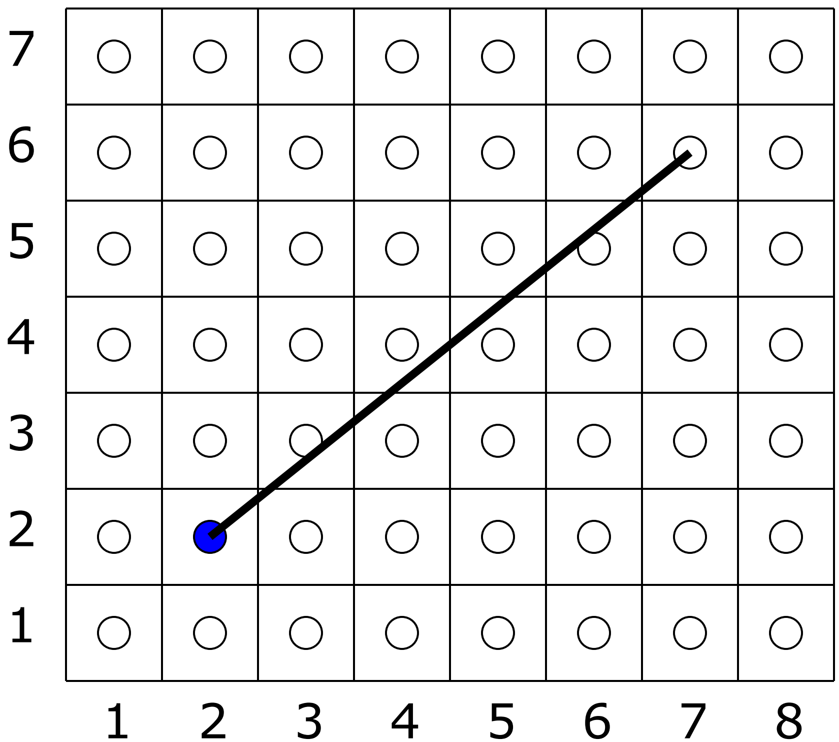
- If we chose $y_{i+1}=y_i$: $$d_{k+1} = d_k - 2\Delta y$$

- What is $d_1$ (assuming integer endpoints)? $$d_1 = \Delta x - 2\Delta y$$
- Notice that we don't need $c$ any more

# Bresenham's Algorithm

- For integers, slope between 0 and 1:
  - $x=x_1$, $y=y_1$, $d= dx\ -2dy$, draw $(x, y)$
  - until $x=x_2$
    - $x=x+1$
    - If $d<0$ then { $y=y+1$, draw $(x, y)$, $d=d-2\Delta y + 2\Delta$x }
    - If $d>0$ then { $y=y$, draw $(x, y)$, $d=d-2\Delta y$ }
- Compute the constants ($2\Delta y-2\Delta$x and $2\Delta y$ ) once at the start
  - Inner loop does only adds and comparisons
- For floating point, initialization is harder, $\Delta x$ and $\Delta y$ will be floating point, **but still no rounding required**

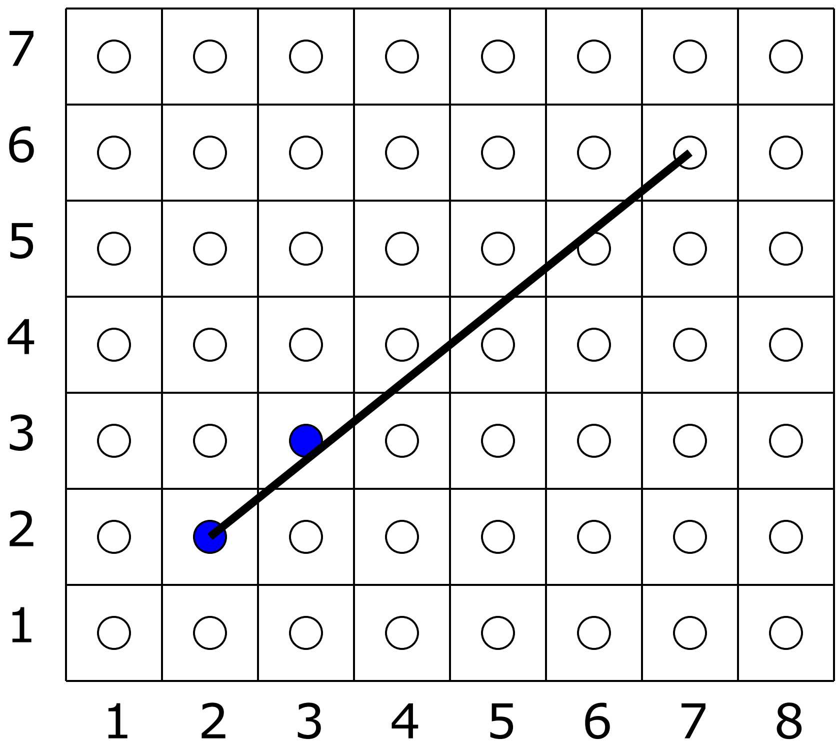
# Example: (2,2) to (7,6)




$\Delta x=5$, $\Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |

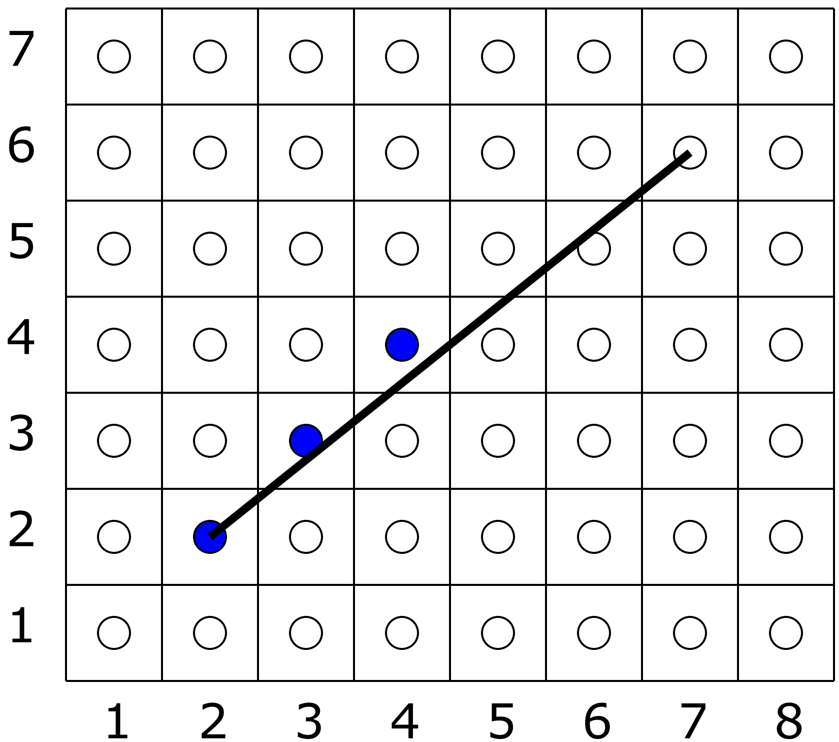
# Example: (2,2) to (7,6)




$\Delta x=5$, $\Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |
| 3 | 3 | -1 |

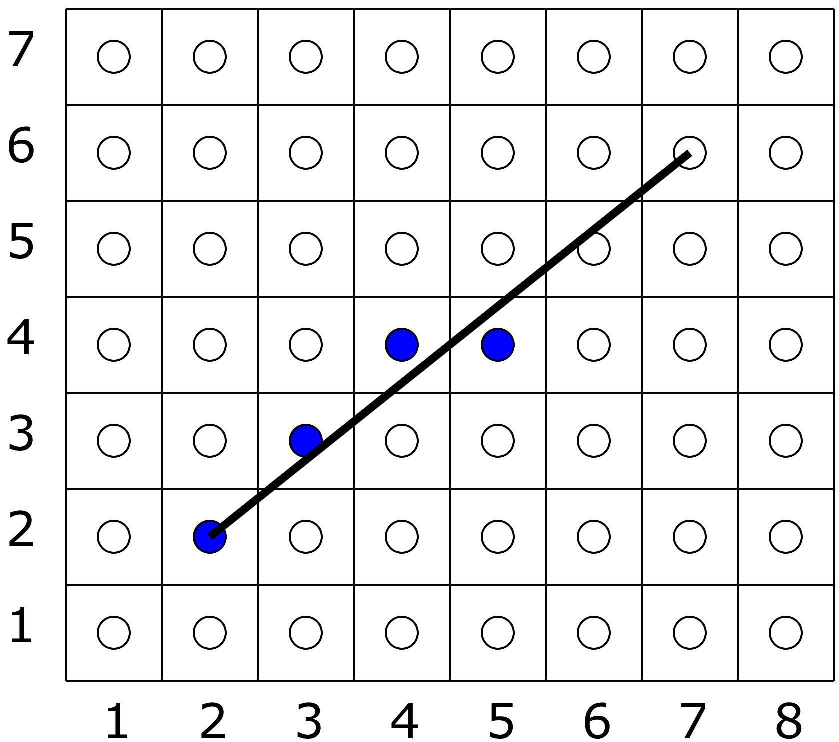
# Example: (2,2) to (7,6)




$\Delta x=5,\ \Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |
| 3 | 3 | -1 |
| 4 | 4 | 1 |

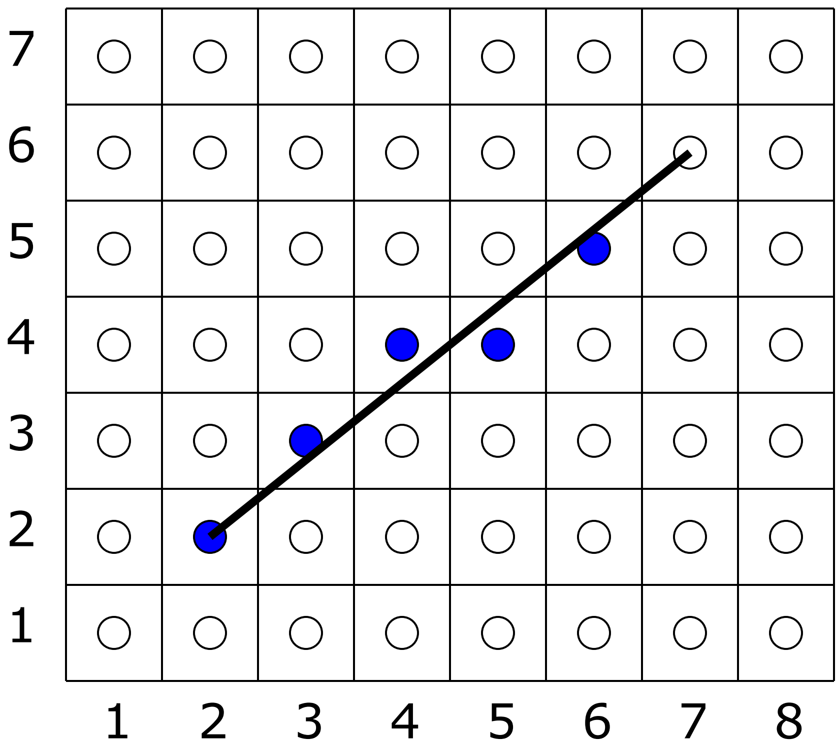
# Example: (2,2) to (7,6)




$\Delta x=5$, $\Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |
| 3 | 3 | -1 |
| 4 | 4 | 1 |
| 5 | 4 | -7 |

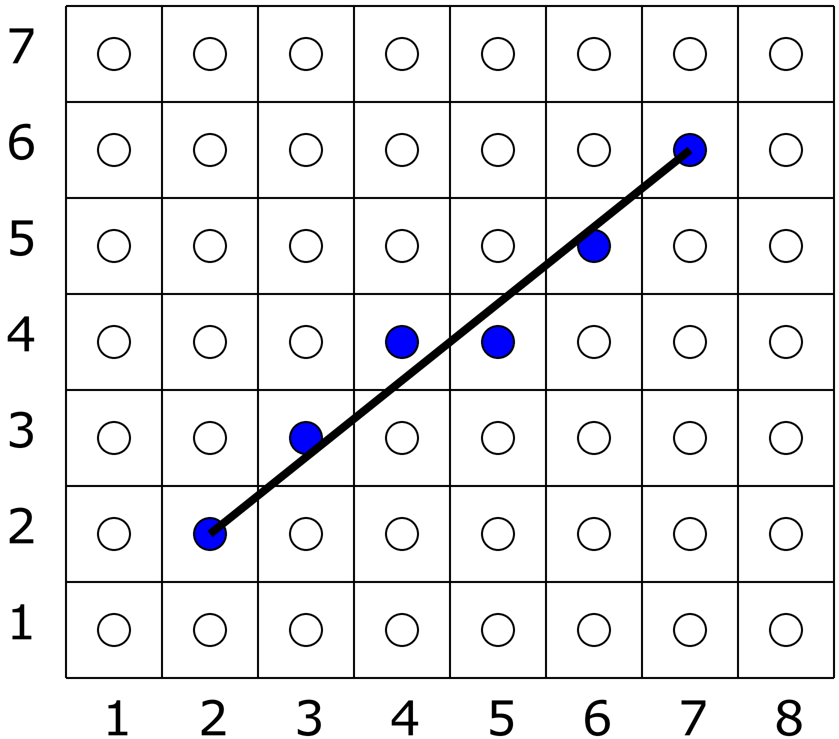
# Example: (2,2) to (7,6)




$\Delta x=5,\ \Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |
| 3 | 3 | -1 |
| 4 | 4 | 1 |
| 5 | 4 | -7 |
| 6 | 5 | -5 |

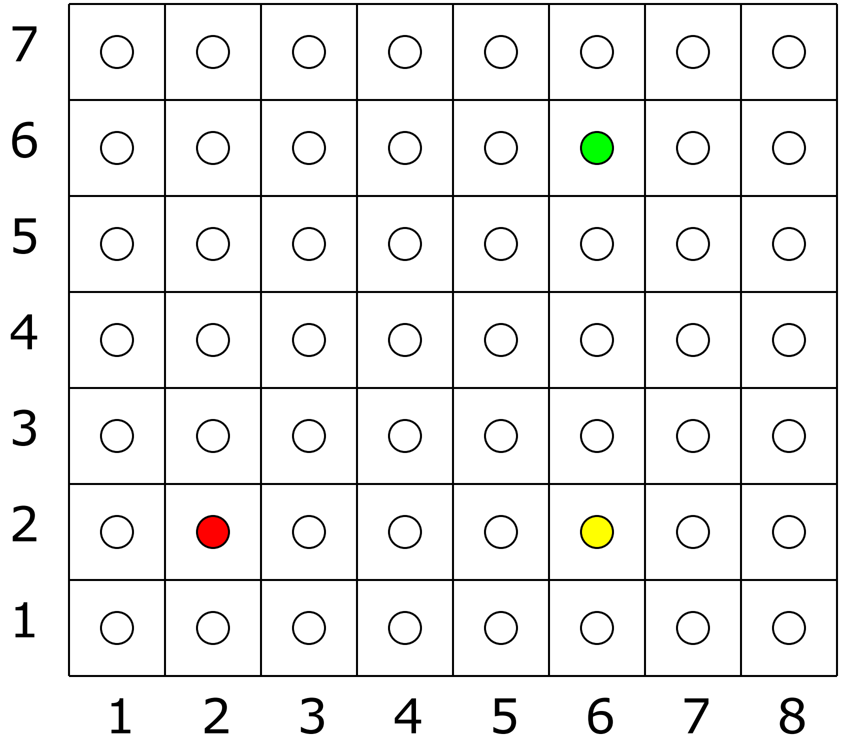
# Example: (2,2) to (7,6)



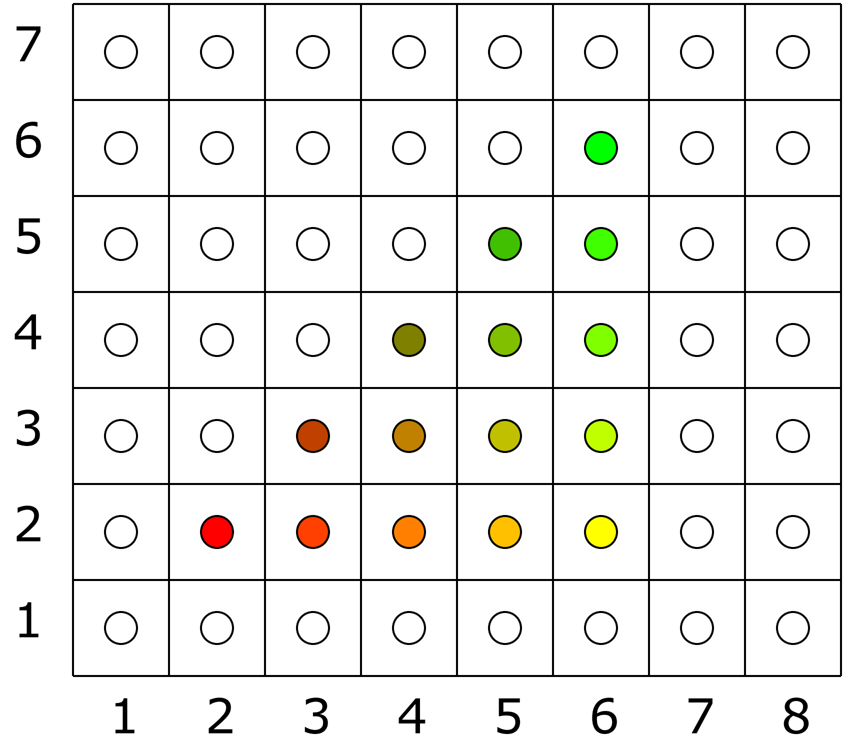

$\Delta x=5$, $\Delta y=4$

| x | y | d |
|---|---|---|
| 2 | 2 | -3 |
| 3 | 3 | -1 |
| 4 | 4 | 1 |
| 5 | 4 | -7 |
| 6 | 5 | -5 |
| 7 | 6 | -3 |

# Filling Triangles



# Filling Triangles



# Algorithm

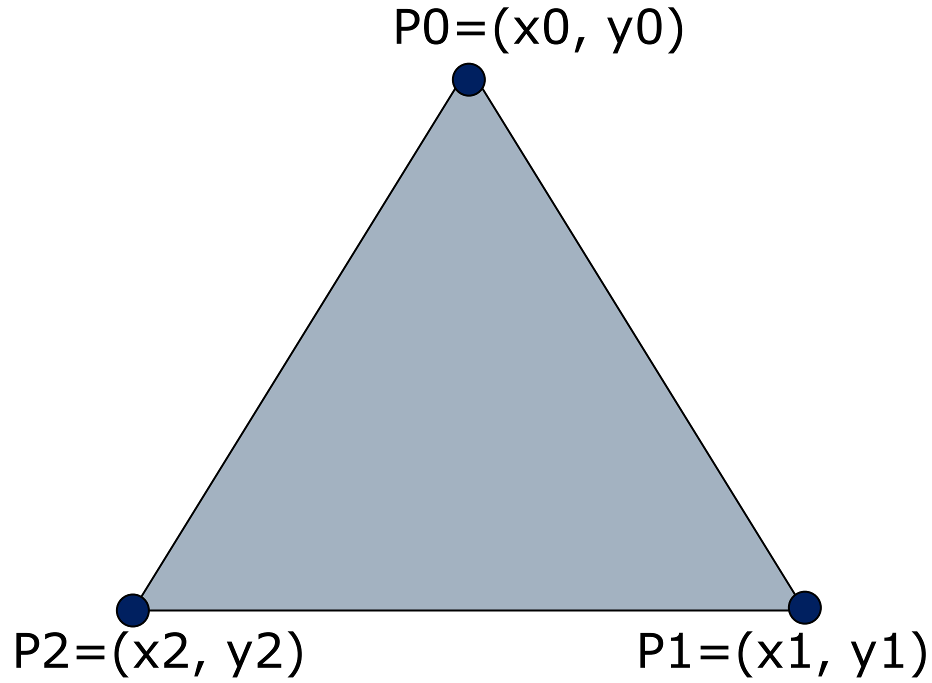
- Decide which pixels to fill (samples to use) to represent a triangle?
- Calculate the color for each pixel?

# Barycentric coordinates

$$P = \alpha P_0 + \beta P_1 + \lambda P_2$$

$$\alpha + \beta + \lambda = 1$$

$$0 \le \alpha, \beta, \lambda \le 1$$



# Barycentric coordinates

$$\alpha = f_{12}(x, y) / f_{12}(x_0, y_0)$$
$$\beta = f_{20}(x, y) / f_{20}(x_1, y_1)$$
$$\lambda = f_{01}(x, y) / f_{01}(x_2, y_2)$$

$$f_{12}(x, y) = (y_1 - y_2)x + (x_2 - x_1)y + x_1 y_2 - x_2 y_1$$
$$f_{20}(x, y) = (y_2 - y_0)x + (x_0 - x_2)y + x_2 y_0 - x_0 y_2$$
$$f_{01}(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0$$

# Rasterizing Triangle

- $y_{min}=\min(y_0, y_1, y_2)$, $y_{max}=\max(y_0, y_1, y_2)$
- $x_{min}=\min(x_0, x_1, x_2)$, $x_{max}=\max(x_0, x_1, x_2)$
- for $y=y_{min}$ to $y_{max}$
  - for $x=x_{min}$ to $x_{max}$
    - calculate $\alpha, \beta$, and $\lambda$
    - if $0 \le \alpha, \beta$, and $\lambda \le 1$

      $c = \alpha c_0 + \beta c_1 + \lambda c_2$

      draw (x, y) with color c

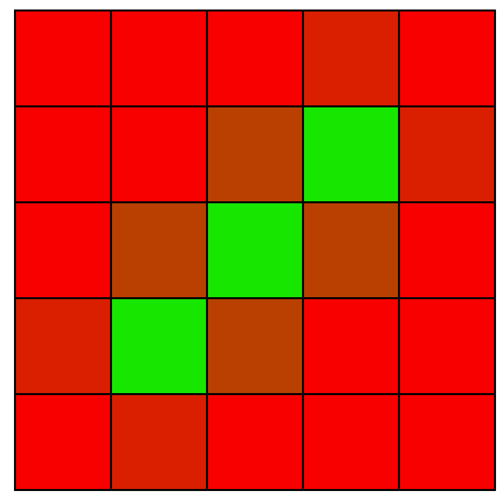
# Anti-Aliasing

- Recall: We can't sample and then accurately reconstruct an image that is not band-limited
  - Infinite Nyquist frequency
  - Attempting to sample sharp edges gives "jaggies", or stair-step lines
- Solution: Band-limit by filtering (pre-filtering)
  - What sort of filter will give a band-limited result?
- In practice, difficult to do for graphics rendering

# Alpha-based Anti-Aliasing

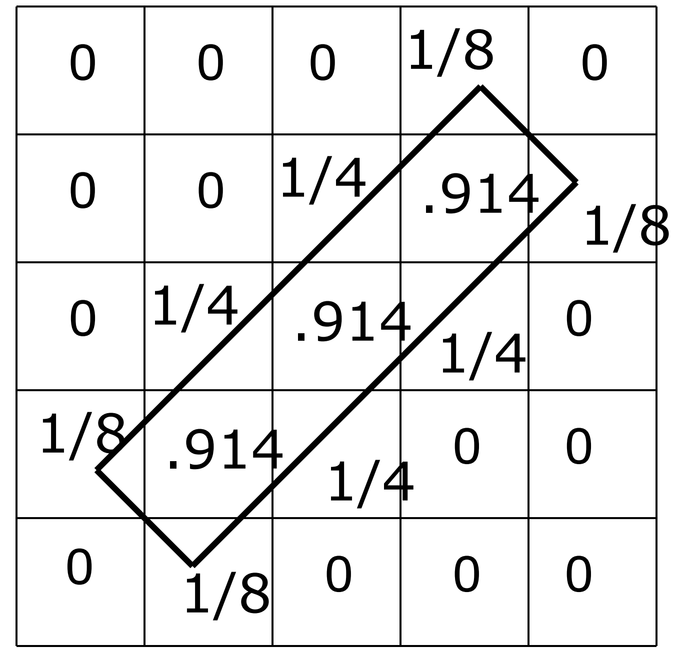
- Set the $\alpha$ of a pixel to simulate a thick line
  - The pixel gets the line color, but with $\alpha$<=1
- This supports the correct drawing of primitives one on top of the other
  - Draw back to front, and **composite** each primitive *over* the existing image
  - Only some hidden surface removal algorithms support it

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1/8 | 0 |
| 0 | 0 | 1/4 | .914 | 1/8 |
| 0 | 1/4 | .914 | 1/4 | 0 |
| 1/8 | .914 | 1/4 | 0 | 0 |
| 0 | 1/8 | 0 | 0 | 0 |



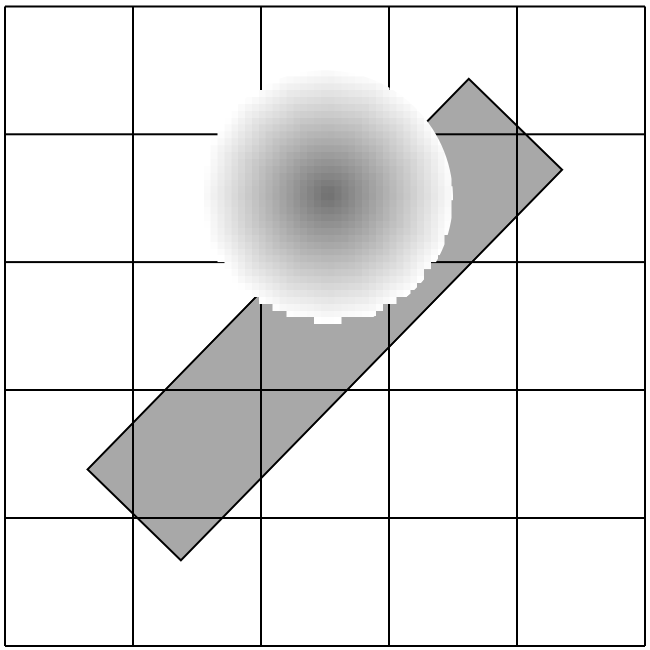
# Calculating $\alpha$

- Consider a line as having thickness (all good drawing programs do this)
- Consider pixels as little squares
- Set $\alpha$ according to the proportion of the square covered by the line
- The sub-pixel coverage interpretation of $\alpha$

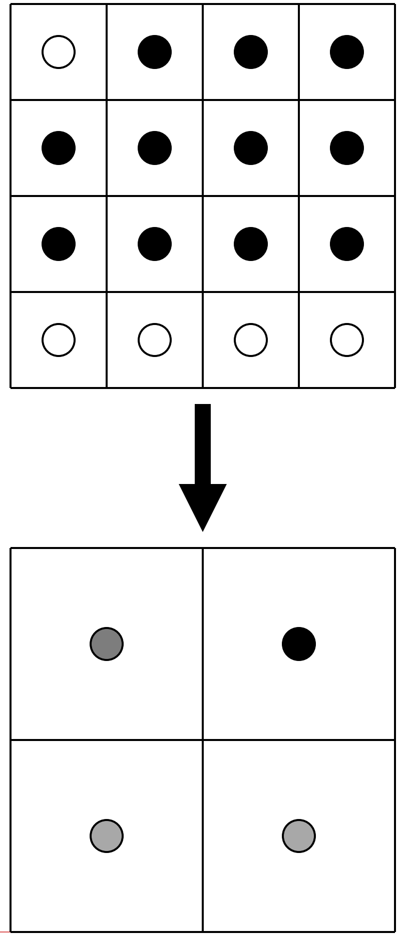


# Weighted Sampling

- Instead of using the proportion of the area covered by the line, use convolution to do the sampling
  - Equivalent to filtering the line then point sampling the result
- Place the “filter” at each pixel, and integrate product of pixel and line
- Common filters are cones (like Bartlett) or Gaussians



# Post-Filtering (Supersampling)



- Sample at a higher resolution than required for display, and filter image down
  - Easy to implement in hardware
  - Typical is 2x2 sampling per pixel, with simple averaging to get final
    - What kind of filter?
- More advanced methods generate different samples (eg. not on regular grid) and filter properly
  - Issues of which samples to take, and how to filter them

# Next Time

- Hidden Surface Removal
- 