CS 447/547: Computer Graphics

Homework 1

This homework must be done individually. Submission date is October 5, 2016, in class.

Question 1: Vectors are very important to computer graphics and they are used to represent both locations in space (points) and directions. Assume you have three points in 2D space, represented by $\mathbf{a} = [a_x, a_y]$, $\mathbf{b} = [b_x, b_y]$, and $\mathbf{c} = [c_x, c_y]$.

a. How do you find the direction vector **v** that points *from* **a** *toward* **b**?

b. How is the length, $\|\mathbf{v}\|$, of **v** computed?

c. A *unit vector*, $\hat{\mathbf{v}}$, in the direction **v** is a vector in the same direction as **v** but with length 1. How do you compute $\hat{\mathbf{v}}$? Computing $\hat{\mathbf{v}}$ is also referred to as *normalizing* **v**.

Question 2: Consider two vectors in 3D, **a** and **b**.

a. How is the dot product $\mathbf{a} \cdot \mathbf{b}$ computed?

b. What is the relationship between $\mathbf{a} \cdot \mathbf{b}$ and the angle between **a** and **b**?

c. How is the cross product vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ computed?

d. What is the geometric relationship between **a**, **b** and **c**?

e. What is the geometric relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$?

f. What is the relationship between $\mathbf{a} \times \mathbf{b}$ and the angle between **a** and **b**?

Question 3: What is the solution to the following quadratic equation?

$x^2 + 3x + 2 = 0$

Question 4: What is the distance from a 2D point $\mathbf{p} = [p_x, p_y]$ to a line $ax + by + c = 0$?

Question 5: This question concerns the definition of a 3D parametric line.

a. What is the minimum number of points needed to define a unique line in 3D that passes through all the points? What other conditions must the points satisfy for the line to be unique?

b. Given more than the minimum number of points, is it in general possible to find one line that passes through all of them?

c. A 3D parametric line is usually defined as $\mathbf{p} = \mathbf{o} + t\mathbf{d}$. Label your points $\mathbf{p}_1$, $\mathbf{p}_2$, etc. Find two vectors **o** and **d** in terms of the points.

Question 6: What is the result of the following matrix multiplication of a vector?

$$\begin{bmatrix} 1 & 2 & 5 \\ 4 & 1 & 12 \\ 3 & 1 & 15 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$