

# Computer Graphics

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**Prof. Feng Liu**

**Fall 2016**

<http://www.cs.pdx.edu/~fliu/courses/cs447/>

**10/05/2016**

# Last Time

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- ☐ Image file formats
- ☐ Color quantization

# Today

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- ☐ Dithering
- ☐ Signal Processing
- ☐ Homework 1 due today in class
- ☐ Homework 2 available, due in class on October 19

# Dithering to Black-and-White

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- Black-and-white is still the preferred way of displaying images in many areas
  - Black ink is cheaper than color
  - Printing with black ink is simpler and hence cheaper
  - Paper for black inks is not special

# Dithering to Black-and-White

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- ❑ To get color to black and white, first turn into grayscale:  
 $I = 0.299R + 0.587G + 0.114B$ 
  - This formula reflects the fact that green is more representative of perceived brightness than blue is
  - NOTE that it is **not** the equation implied by the RGB->XYZ color space conversion matrix
- ❑ **For all dithering we will assume that the image is gray and that intensities are represented as a value in [0, 1.0)**
  - Define new array of floating point numbers
  - `new_image[i] = old_image[i] / (float)256;`
  - To get back:  
`output[i] = (unsigned char) floor(new_image[i] * 256)`

# Sample Images

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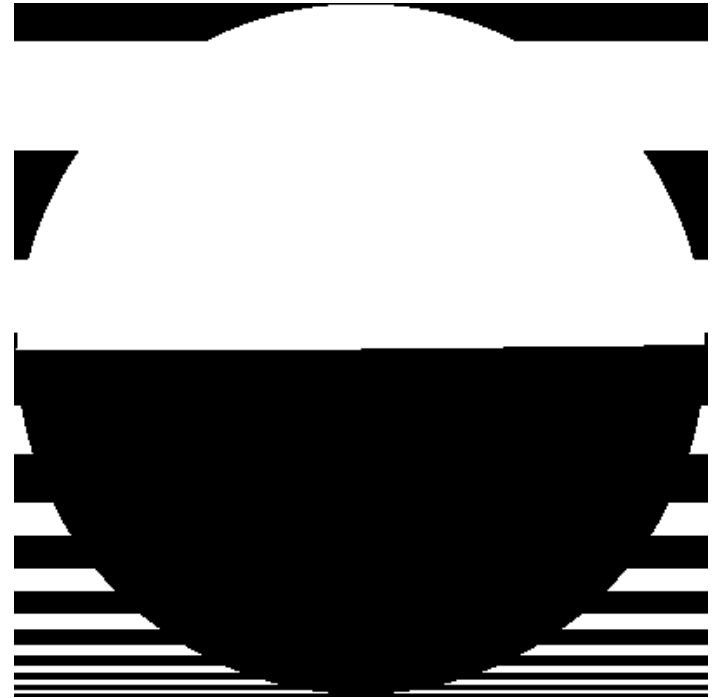
# Threshold Dithering

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- For every pixel: If the intensity  $< 0.5$ , replace with black, else replace with white
  - 0.5 is the threshold
  - This is the naïve version of the algorithm

# Naïve Threshold Algorithm

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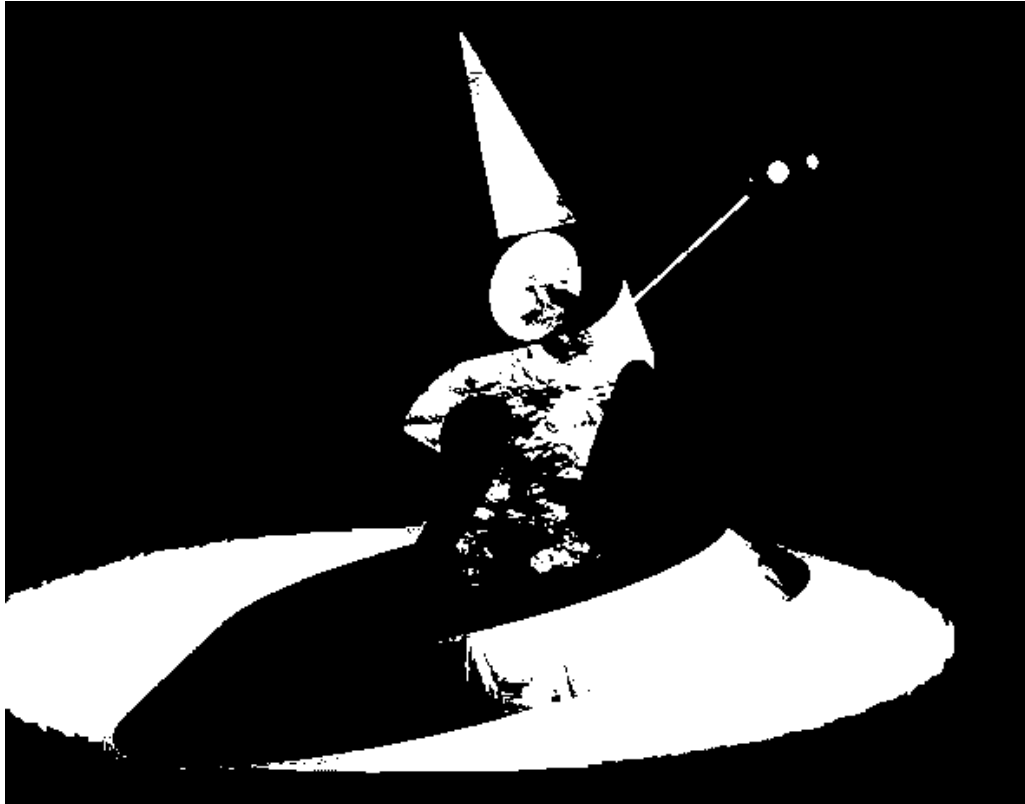
# Threshold Dithering

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- For every pixel: If the intensity  $< 0.5$ , replace with black, else replace with white
  - 0.5 is the threshold
  - This is the naïve version of the algorithm
- To keep the overall image brightness the same, you should:
  - Compute the average intensity over the image
  - Use a threshold that gives that average
  - For example, if the average intensity is 0.6, use a threshold that is higher than 40% of the pixels, and lower than the remaining 60%

# Brightness Preserving Algorithm

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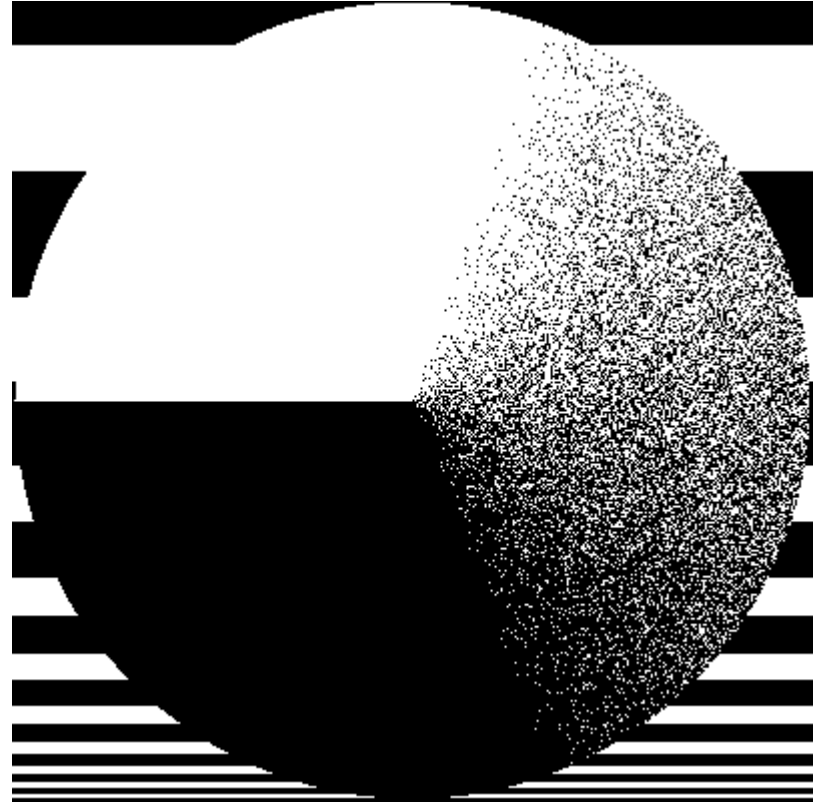
# Random Modulation

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- Add a random amount to each pixel *before* thresholding
  - Typically add *uniformly* random amount from  $[-a, a]$
- Pure addition of noise to the image
  - For better results, add better quality noise
  - For instance, use Gaussian noise (random values sampled from a normal distribution)
- Should use same procedure as before for choosing threshold
- Not good for black and white, but OK for more colors
  - Add a small random color to each pixel before finding the closest color in the table

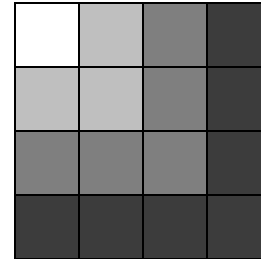
# Random Modulation

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# Ordered Dithering

- Break the image into small blocks
- Define a *threshold matrix*
  - Use a different threshold for each pixel of the block
  - Compare each pixel to its own threshold
- The thresholds can be clustered, which looks like newsprint
- The thresholds can be “random” which looks better

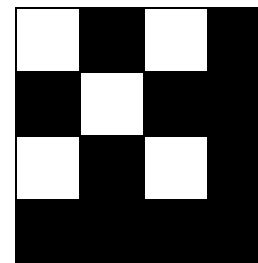


Threshold  
matrix

$$\begin{bmatrix} 1 & 0.75 & 0.5 & 0.25 \\ 0.75 & 0.75 & 0.5 & 0.25 \\ 0.5 & 0.5 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

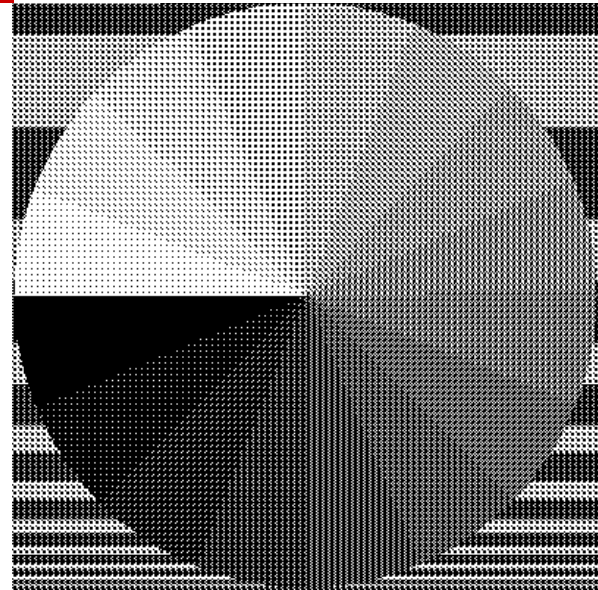
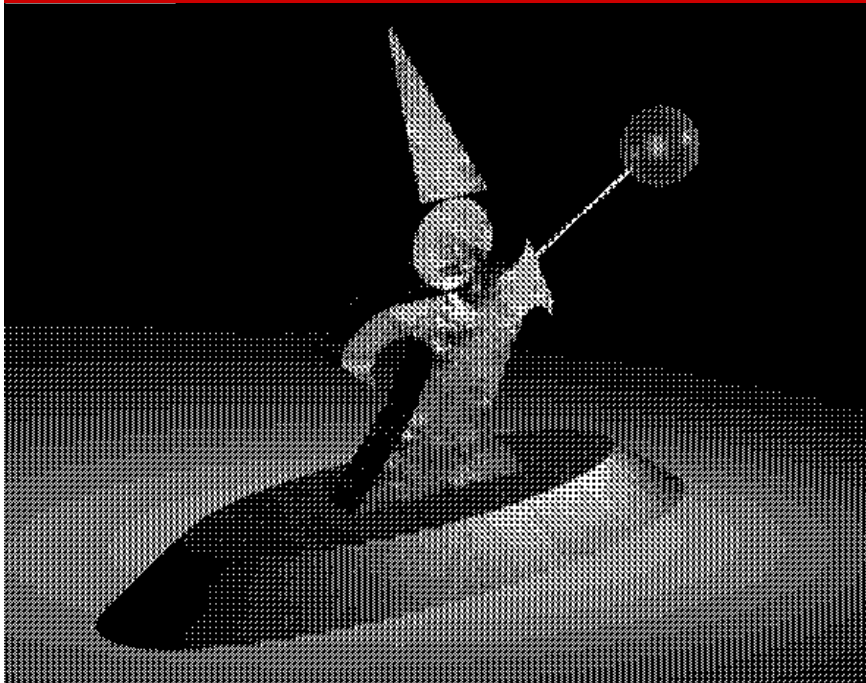
$$\frac{1}{16} \begin{bmatrix} 2 & 16 & 3 & 13 \\ 10 & 6 & 11 & 7 \\ 4 & 14 & 1 & 15 \\ 12 & 8 & 9 & 5 \end{bmatrix}$$

Result



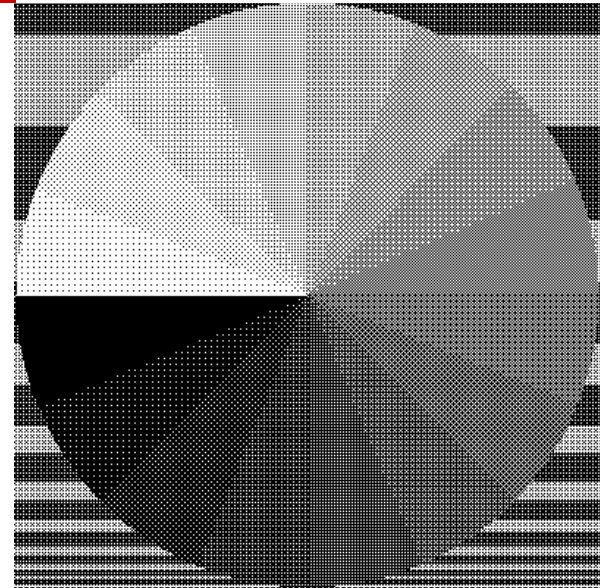
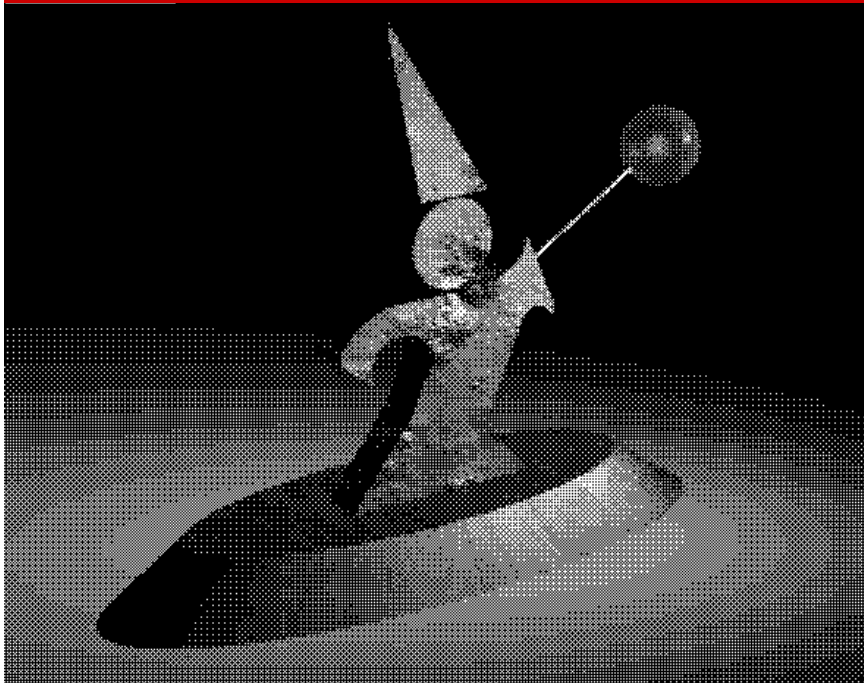
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

# Clustered Dithering



.75	.375	.625	.25
.0625	1	.875	.4375
.5	.8125	.9375	.125
.1875	.5625	.3125	.6875

# Dot Dispersion

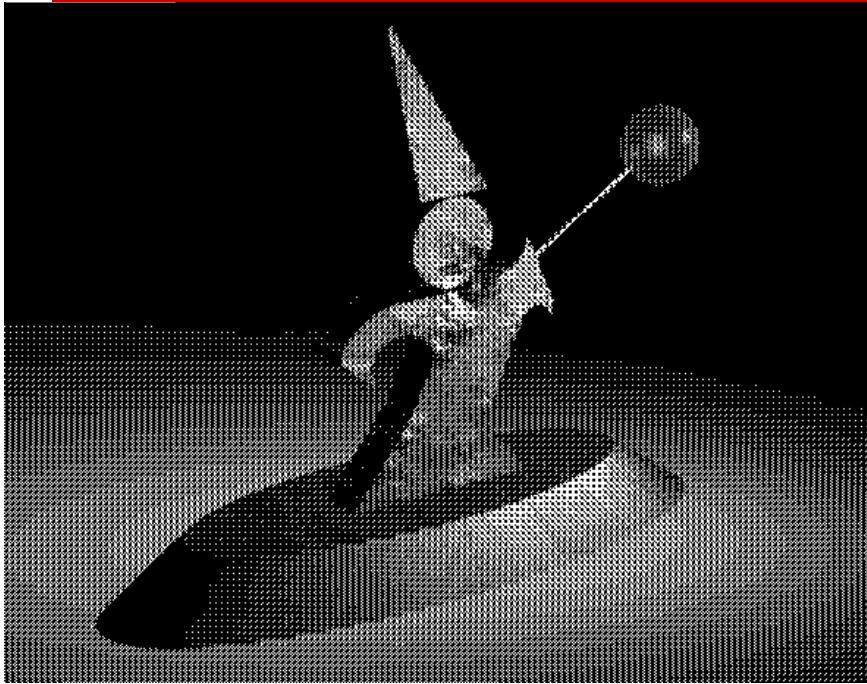


.125	1	.1875	.8125
.625	.375	.6875	.4375
.25	.875	.0625	.9375
.75	.5	.5625	.3125

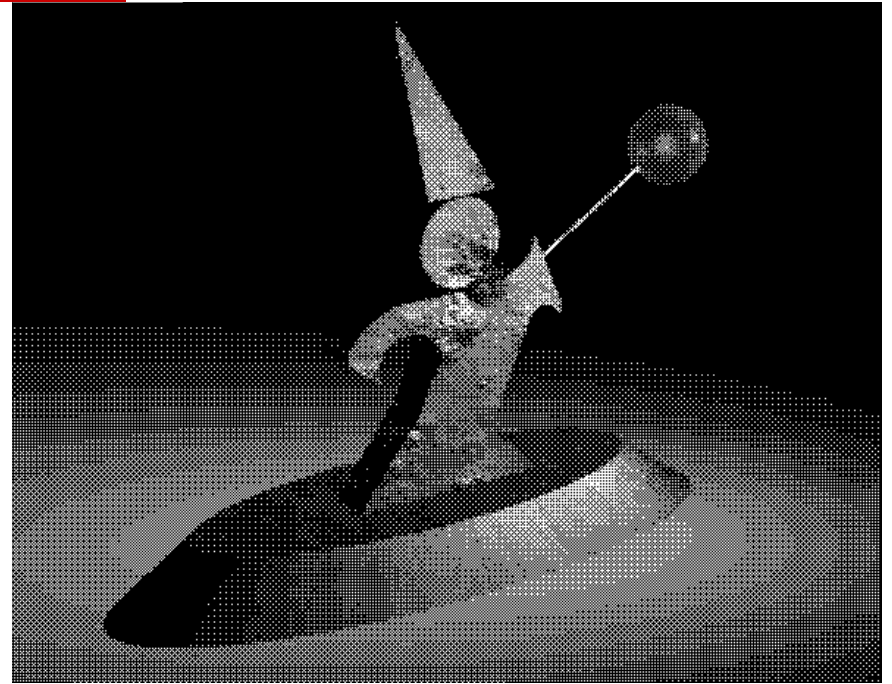


# Comparison

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Clustered



Dot Dispersion

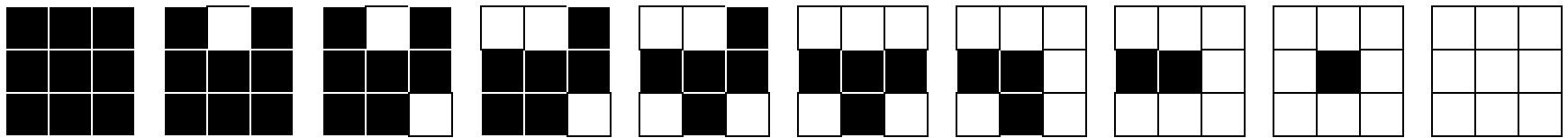
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# Pattern Dithering

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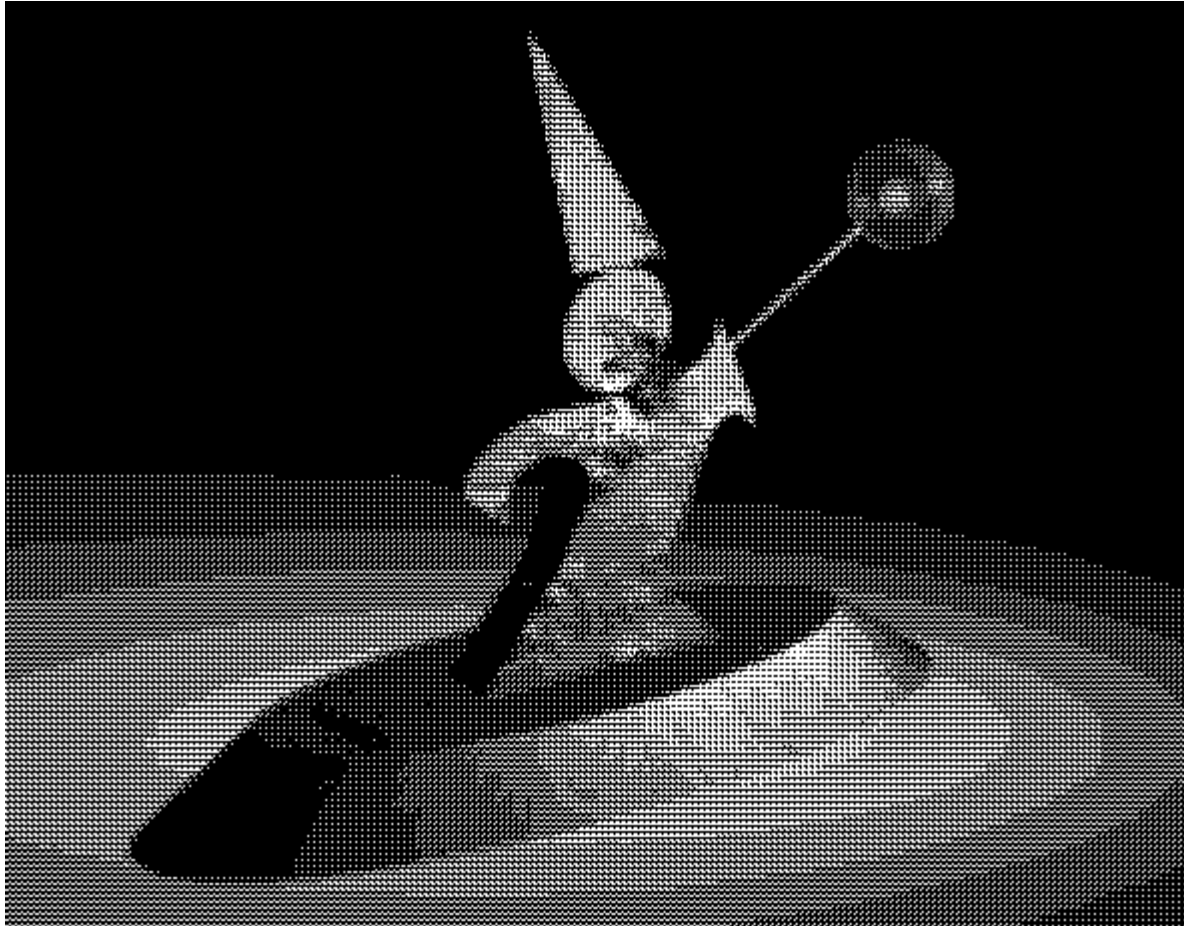
- Compute the intensity of each sub-block and index a pattern



- NOT the same as before
  - Here, each sub-block has one of a fixed number of patterns - pixel is determined only by average intensity of sub-block
  - In ordered dithering, each pixel is checked against the dithering matrix before being turned on
- Used when display resolution is higher than image resolution - not uncommon with printers
  - Use 3x3 output for each input pixel

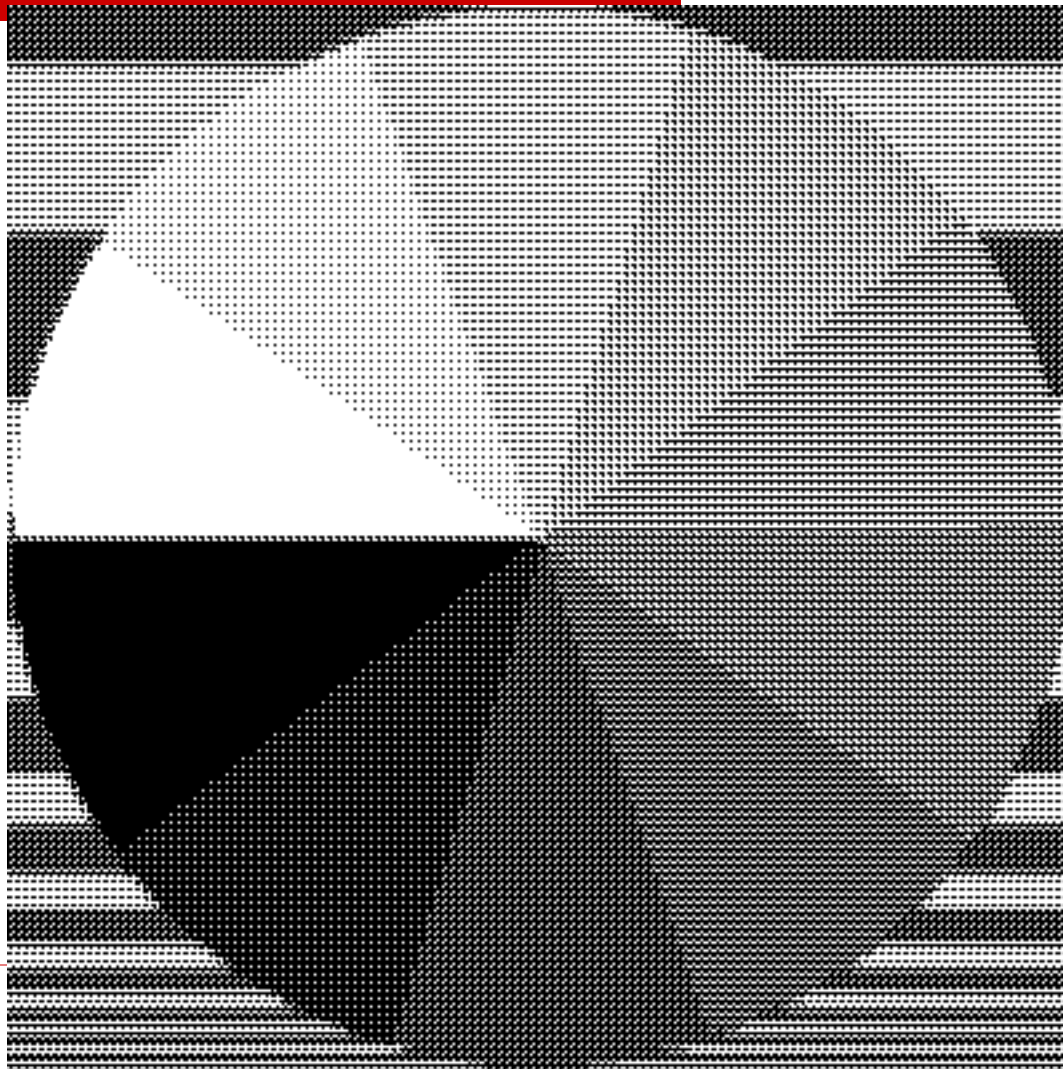
# Pattern Dither (1)

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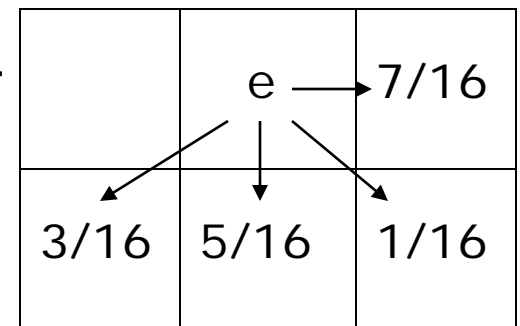
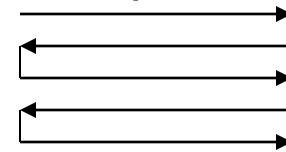
# Pattern Dither (2)

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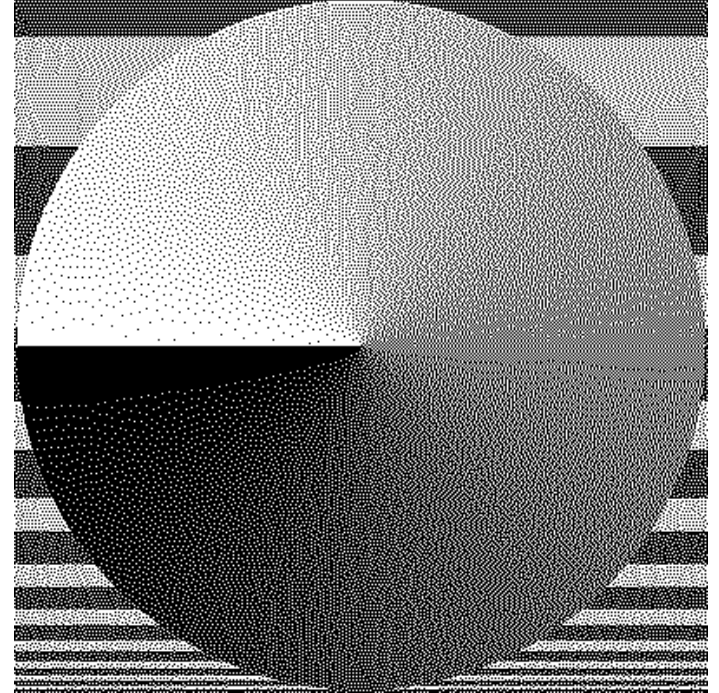
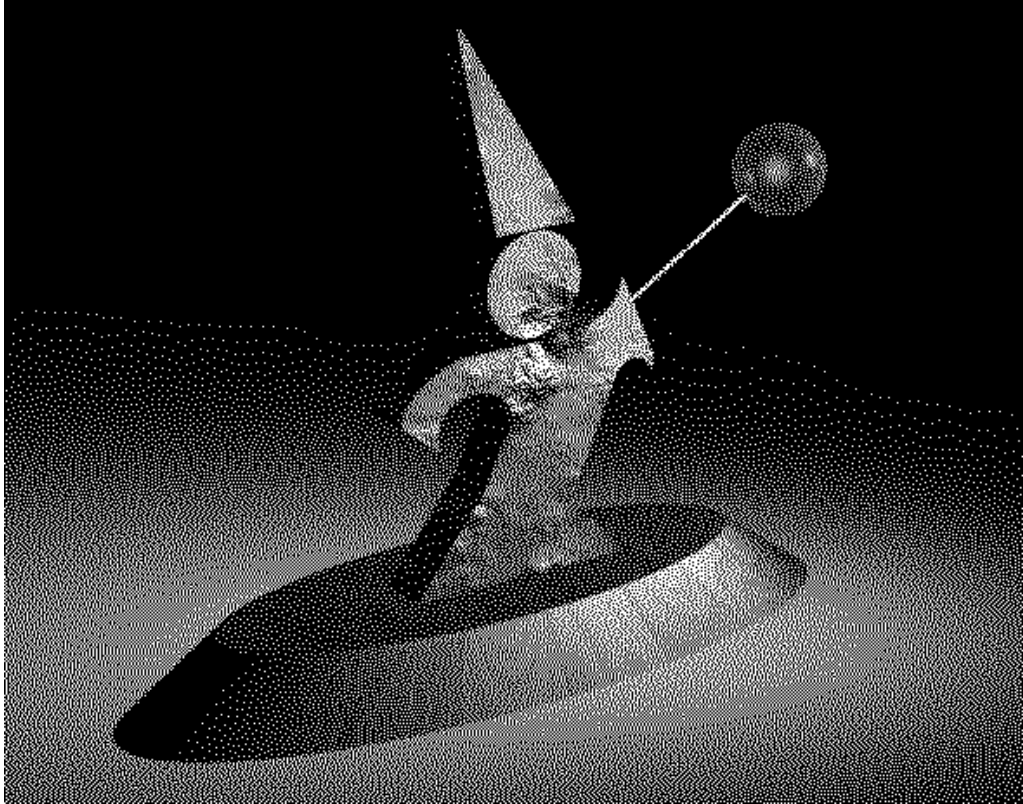
# Floyd-Steinberg Dithering

- Start at one corner and work through image pixel by pixel
  - Usually scan top to bottom in a zig-zag
- Threshold each pixel
- Compute the error at that pixel: The difference between what should be there and what you did put there
  - If you made the pixel 0,  $e = \text{original}$ ; if you made it 1,  $e = \text{original} - 1$
- Propagate error to neighbors by adding some proportion of the error to each unprocessed neighbor
  - A *mask* tells you how to distribute the error
- Easiest to work with floating point image
  - Convert all pixels to 0-1 floating point
- More detail in class reading materials



# Floyd-Steinberg Dithering

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# Color Dithering

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- All the same techniques can be applied, with some modification
- Example is Floyd-Steinberg:
  - Uniform color table
  - Error is difference from nearest color in the color table
  - Error propagation same as that for greyscale
    - Each color channel treated independently



# Color Dithering

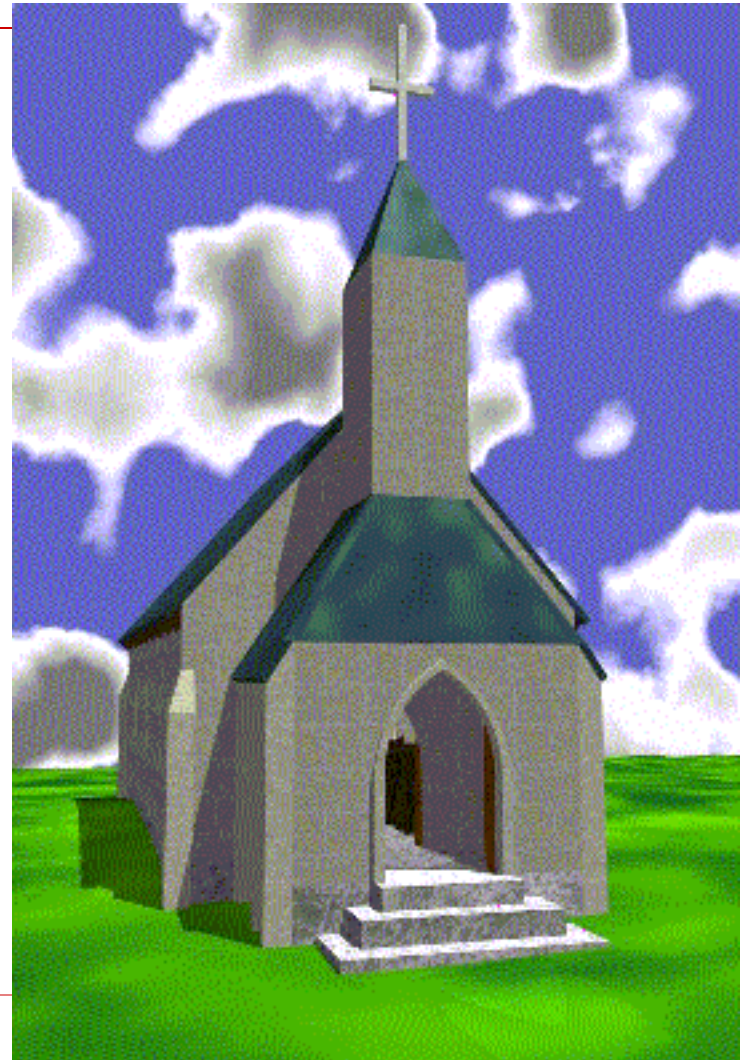
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# Comparison to Uniform Quant.



Same color  
table!





# Today

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- ☐ Dithering
- ☐ Signal Processing
- ☐ Homework 2 available, due October 19

# Image Manipulation

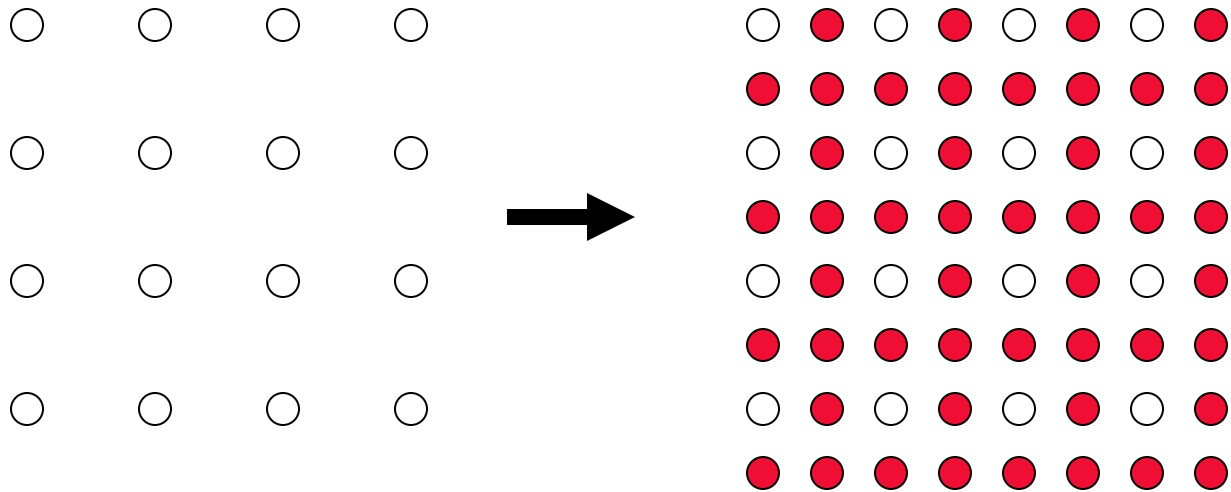
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- We have now looked at basic image formats and color, including transformations of color
- Next, operations involving image *resampling*
  - Scaling, rotating, morphing, ...
- But first, we need some signal processing
  - Also important for *anti-aliasing*, later in class

# Enlarging an Image

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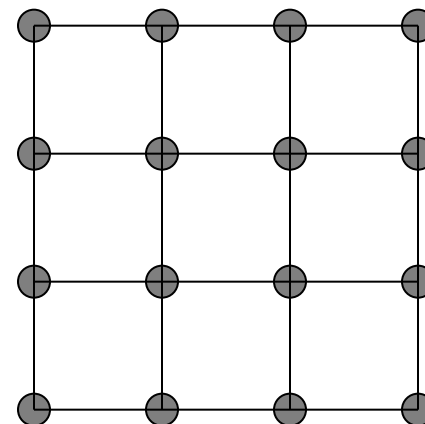
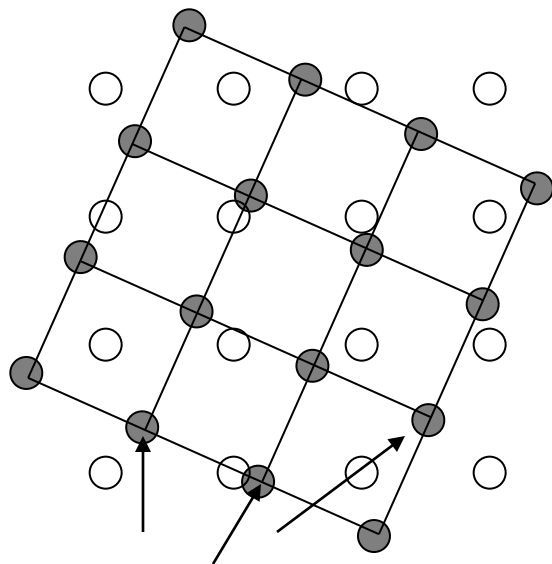
- ❑ To enlarge an image, you have to add pixels between the old pixels
- ❑ What values do you choose for those pixels?



# Rotating an Image

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- ❑ Pixels in the new image come from their rotated positions in the original image
- ❑ These rotated locations might not be in “nice” places



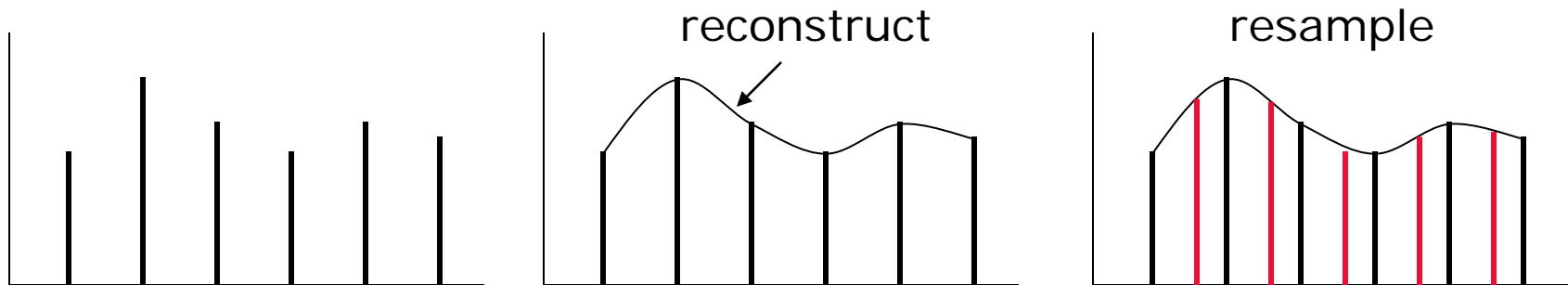
New image

The positions of the new pixels in the original image

# Images as Samples of Functions

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- We can view an image as a set of *samples* from an ideal function
- If we knew what the function was, we could enlarge the image by *resampling* the function
- Failing that, we can *reconstruct* the function and then resample it



# Why Signal Processing?

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- ☐ Signal processing provides the tools for understanding sampling and reconstruction

# Function representations

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- A function can be represented as a sum of sin's and cos's of (possibly) all frequencies:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega$$
$$e^{i\omega x} = \cos \omega x + i \sin \omega x$$

- $F(\omega)$  is the *spectrum* of the function  $f(x)$ 
  - The spectrum is how much of each frequency is present in the function
  - We're talking about functions, not colors, but the idea is the same

# Fourier Transform

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- $F(\omega)$  is computed from  $f(x)$  by the *Fourier Transform*:

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$$



# Example: Box Function

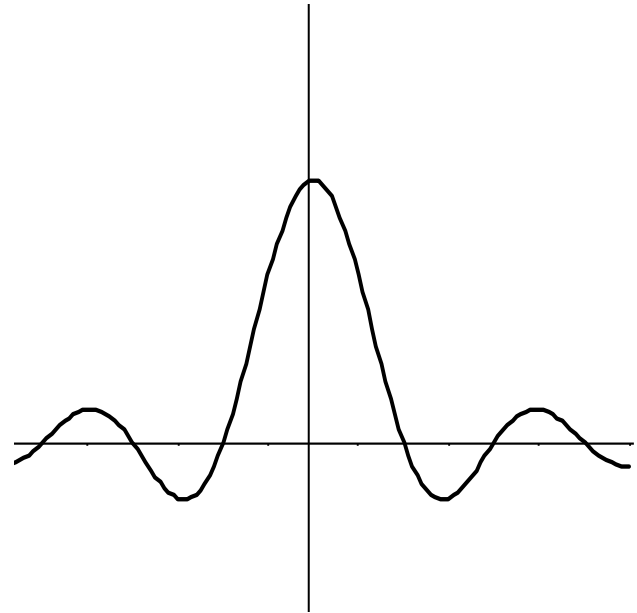
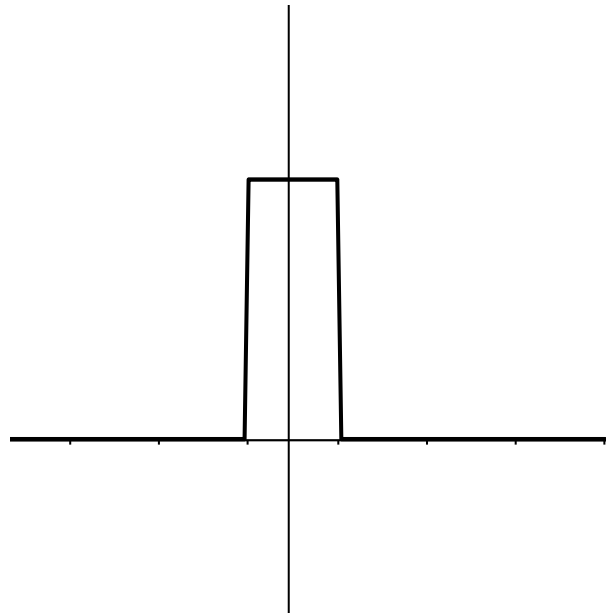
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$$f(x) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & |x| > \frac{1}{2} \end{cases}$$

$$\begin{aligned} F(\omega) &= \frac{\sin \pi f}{\pi f} & f &= \frac{\omega}{2\pi} \\ &= \text{sinc } f \end{aligned}$$

# Box Function and Its Transform

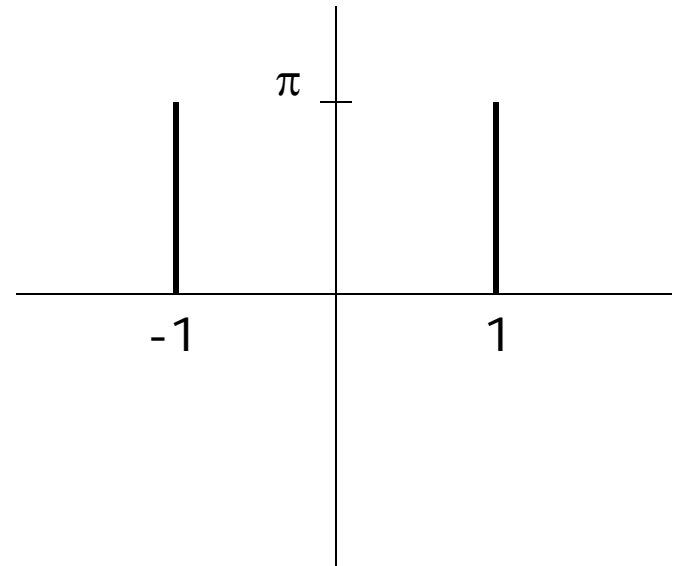
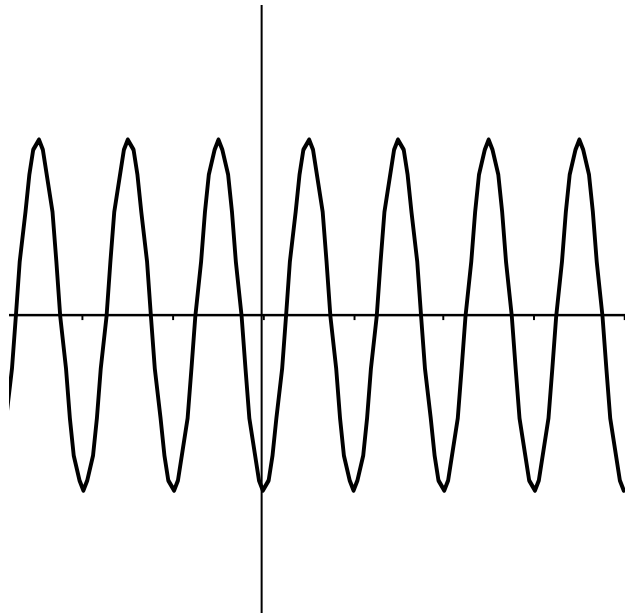
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- Two different *representations* of the *same* function
  - $f(x)$  spatial domain
  - $F(\omega)$  frequency domain

# Cosine and Its Transform

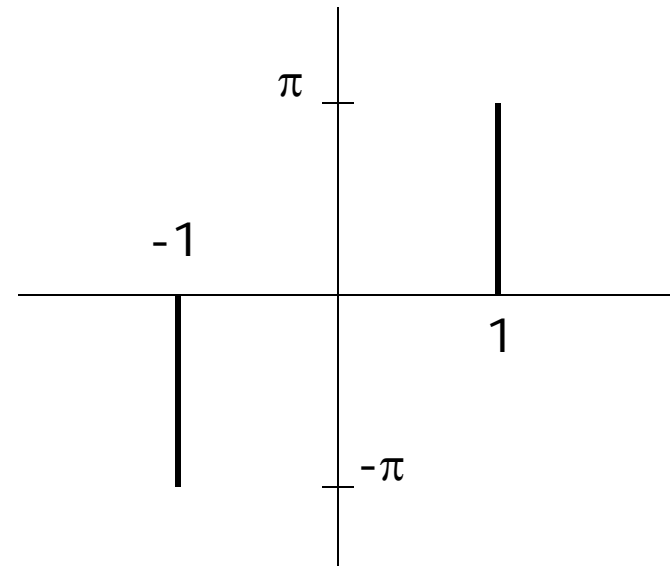
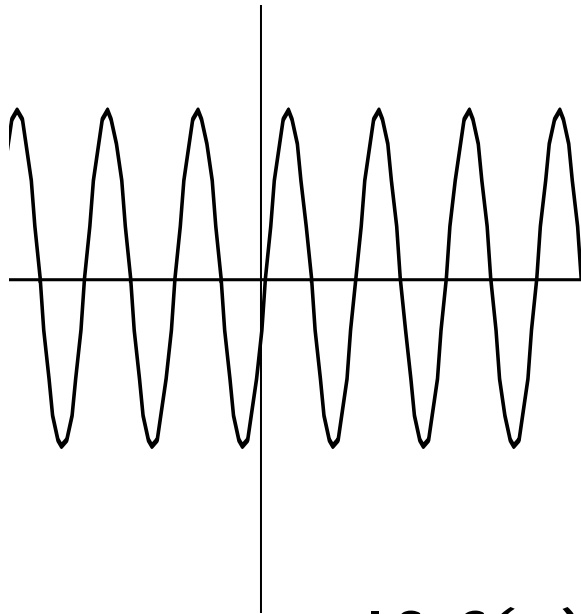
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If  $f(x)$  is even, so is  $F(\omega)$

# Sine and Its Transform

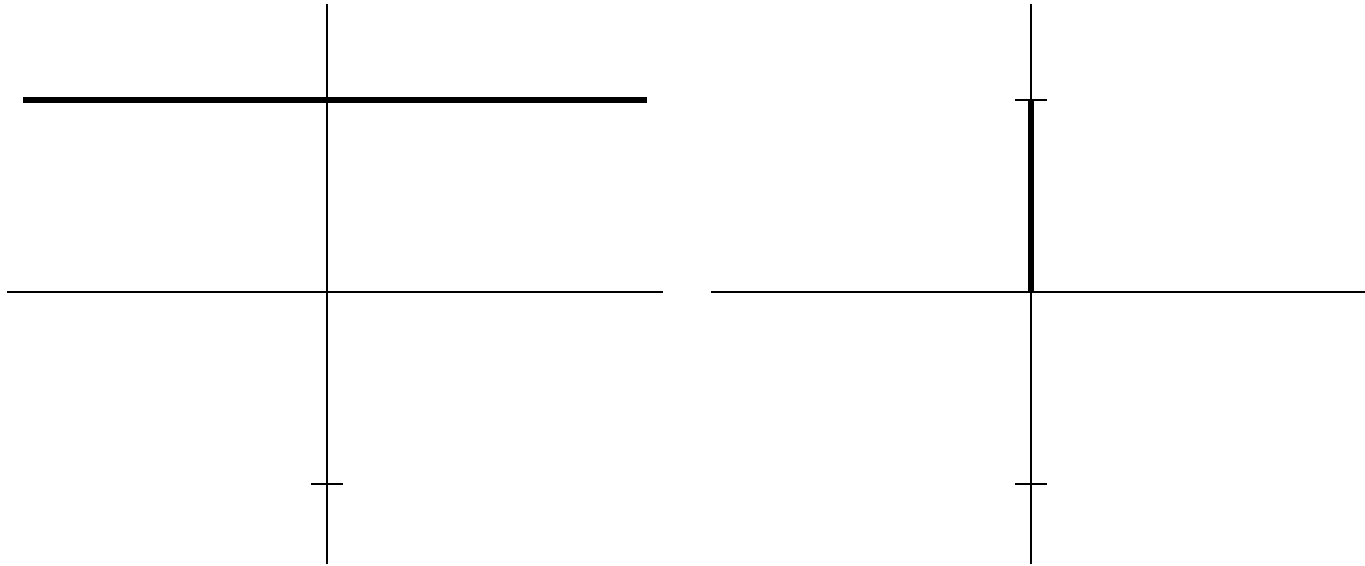
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If  $f(x)$  is odd, so is  $F(\omega)$

# Constant Function and Its Transform

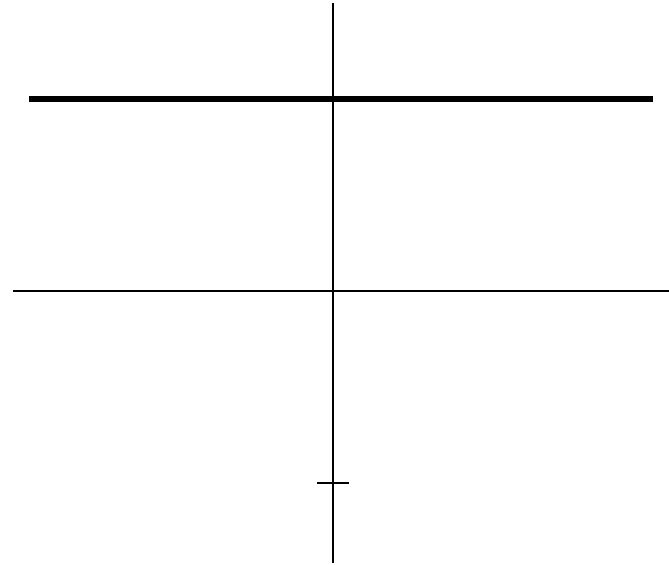
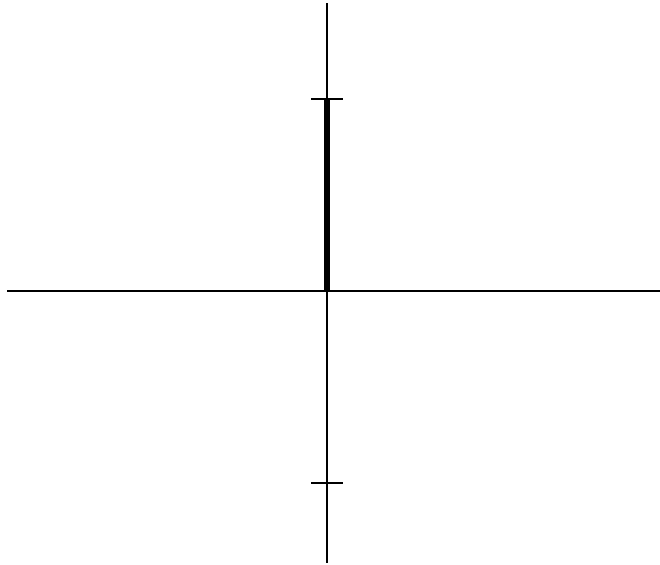
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The constant function only contains the 0<sup>th</sup> frequency – it has no wiggles

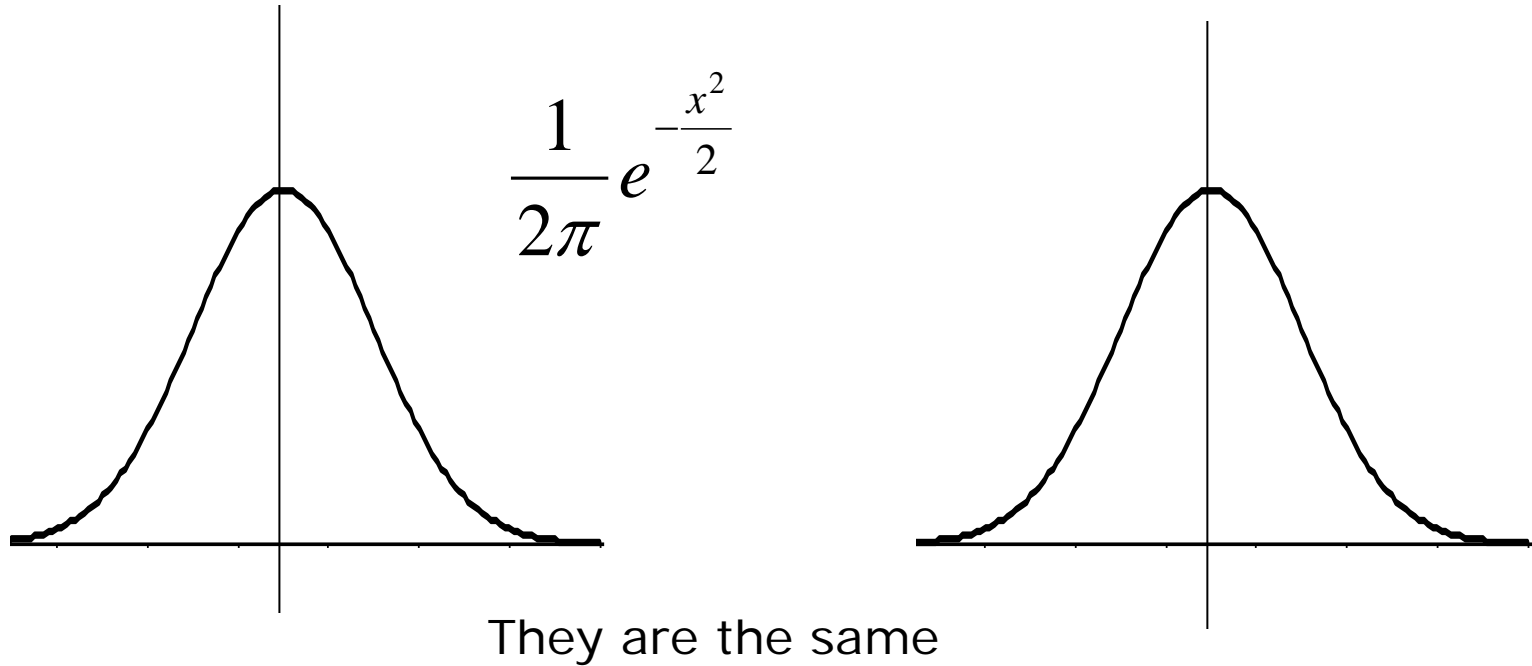
# Delta Function and Its Transform

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# Gaussian and Its Transform

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# Qualitative Properties

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- The spectrum of a function tells us the relative amounts of high and low frequencies
  - Sharp edges give high frequencies
  - Smooth variations give low frequencies
- A function is *bandlimited* if its spectrum has no frequencies above a maximum limit
  - sin, cos are band limited
  - Box, Gaussian, etc are not



# Functions to Images

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- ❑ Images are 2D, discrete functions
- ❑ 2D Fourier transform uses product of sin's and cos's
- ❑ Fourier transform of a discrete, quantized function will only contain discrete frequencies in quantized amounts
  - In particular, we can store the Fourier transform of a discrete image in the same amount of space as we can store the image
- ❑ Numerical algorithm: Fast Fourier Transform (FFT) computes discrete Fourier transforms

# Next Time

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- ☐ Filtering
- ☐ Resampling
- ☐ Aliasing
- ☐ Compositing