

# CS 447/547: Computer Graphics

## Homework 2

This homework must be done individually. Submission date is October 19, 2016, in class.

**Question 1:** This question concerns human's intensity perception. Humans are tuned to the *ratio* of intensities, not their absolute difference. If we want to make a perceptually uniform intensity system with intensities  $l_1 = 1, l_2, l_3, l_4, l_5, l_6 = 32$ . What are the values of  $l_2, l_3, l_4$ , and  $l_5$  ?

**Question 2:** CIE L\*a\*b\* color space is often considered approximately perceptually uniform. We can convert RGB into L\*a\*b\* in two steps:

**Step 1:** Convert RGB to XYZ using the formula in our lecture 2.

**Step 2:** Convert XYZ to L\*a\*b\*.

L\*a\*b\* is not a linear color space, so converting XYZ to L\*a\*b\* is more complicated than RGB to XYZ. We will use the following formulas to the conversion.

$$L^* = \begin{cases} 116 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 & \frac{Y}{Y_n} > 0.008856 \\ 903.3 * \left(\frac{Y}{Y_n}\right) & \text{else} \end{cases}$$
$$a^* = 500 * \left( f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right)$$
$$b^* = 200 * \left( f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right)$$

where

$$f(t) = \begin{cases} t^{\frac{1}{3}} & t > 0.008856 \\ 7.787 * t + \frac{16}{116} & \text{else} \end{cases}$$

Here  $Y_n = 1.0$  is the luminance, and  $X_n = 0.950455, Z_n = 1.088753$ .

Suppose we have two colors in RGB color space: (0.5, 0, 0) and (1, 1, 1).

**a.** What are the coordinates for these two colors in L\*a\*b\* color space?

In computer graphics, we often need to perform linear interpolation between two colors. The linear interpolation from  $(r_1, g_1, b_1)$  and  $(r_2, g_2, b_2)$  can be implemented as follows.

$$r(u) = (1 - u)r_1 + ur_2$$
$$g(u) = (1 - u)g_1 + ug_2$$
$$b(u) = (1 - u)b_1 + ub_2$$

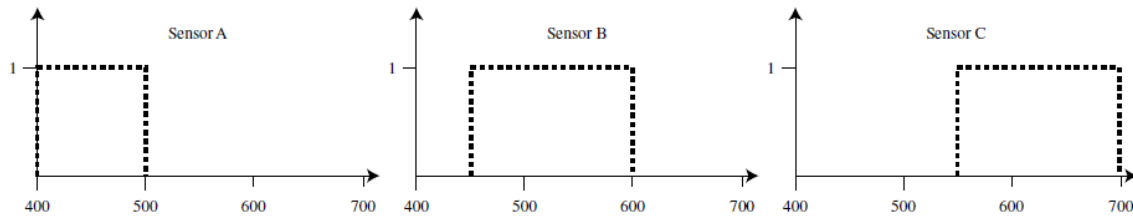
**b.** We want to interpolate from (0.5, 0, 0) to (1, 1, 1) in 5 steps, which can be achieved by using  $u=0, u=0.25, u=0.5, u=0.75, u=1$ , respectively. Compute the 5 RGB colors.

**c.** Compute the corresponding coordinates in L\*a\*b\* color space of the above 5 RGB colors.

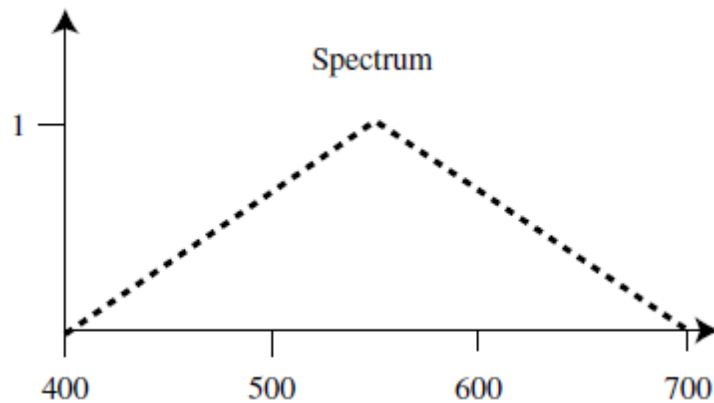
d. Plot two graphs: one showing  $L^*$  as a function of  $\lambda$  and the other showing  $a^*$  as a function of  $\lambda$ .

Here we can see that  $L^*$  and  $a^*$  are not a linear function of  $\lambda$ .

**Question 3:** Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.



What is the response of each of these three sensors to the following spectrum? You need to give actual points.



**Question 4:** The **Sobel operator** is used in image processing for edge detection. The following shows a  $3 \times 3$  Sobel filter mask for a vertical edge detector.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

a. What is the response of this filter to the following  $6 \times 6$  image? Ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a  $4 \times 4$  image.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

b. What is the response of this filter to the following  $6 \times 6$  image? Again, ignore the boundary pixels that do not have all the pixel values for the filter, so we will get a  $4 \times 4$  image.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

c. Can you design a  $3 \times 3$  filter that can detect the horizontal edge in the image shown in (a)?

**Question 5:** Gaussian is one of the most popular filters in computer graphics. What is the  $9 \times 9$  2D gaussian filter mask? (Use the method described in Lecture 4: first construct a 1d filter mask, and then construct the corresponding 2D filter mask. You are not required to fill in the actual number for the 2D filter mask, and a correct formula is good enough.).