Generation and characterisation of polarization entangled photon pairs using parametric downconversion

1 Introduction

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science" [1].
It may be difficult to have a more motivating introduction to this wonderful TP that you will participate in. The work that you will complete during this experiment conceptually echoes Aspect's experiment that earned him a joint Nobel prize. Indeed, in this experiment we will aim to generate a truly quantum entangled pair, and experimentally show that we have done so successfully. In order to get there, we will use an optical non linear process. During this experiment, you will learn about various techniques used in optics and general experimental physics.

2 Spontaneous parametric down conversion

In this experiment, we use spontaneous parametric down conversion to generate our entangled photon pair. SPDC is a non-linear optical process that occurs in some crystals that down converts a photon of frequency ω_p to lower frequency photons called signal and idler of frequencies ω_s, ω_i.
The idea is that we shine a laser (pump) at a crystal that can produce SPDC, and collect the new photons.

The SPDC hamiltonian is given, up to phases and scaling, by eq.1, where the a_j and $a_j^\dagger$ operators annihilate and generate a photon in mode j.

$$H_{SPDC} = \kappa \left(\hat{a}_i^\dagger \hat{a}_s^\dagger \hat{a}_p + h.c \right) \tag{1}$$

We can see that the hamiltonian has the effect of annihilating a photon in the pump mode and creating a photon in the signal and idler mode. The reverse process is also possible, and is simply sum frequency generation. This process has to respect momentum and energy conservation. Energy conservation dictates that the sum of the frequencies of the signal and idler photons equal the frequency of the pump, so $\omega_p = \omega_s + \omega_i$. If signal and idler have the same frequency, this is called degenerate down conversion, which is what we use in our setup.

Momentum must also be conserved, and in order for both these to be true, we need phase matching [2]. In short, realising the phase matching condition ensures that the non linear process happens efficiently. This condition ensures that there is no phase mismatch in the

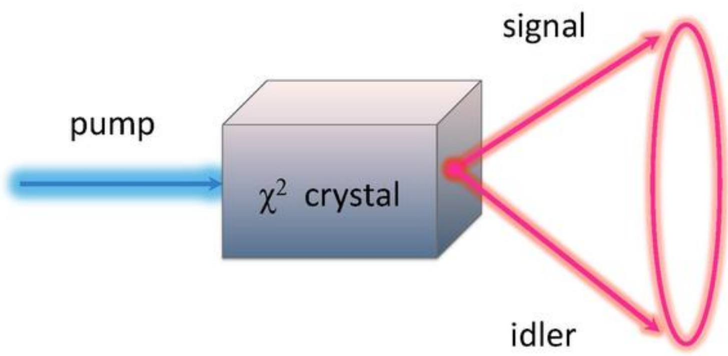

Figure 1: Scheme of the SPDC process

non-linear process, and therefore that amplitudes do not vanish. In the collinear geometry (for non collinear consider wave vectors instead of wave number), this reads:

$$k_p = k_s + k_i \implies n_p\omega_p = n_s\omega_s + n_i\omega_i \tag{2}$$

where k, n, ω are the wave number, refractive index and frequency respectively.
You will be asked to schematically show energy and momentum conservation during the SPDC process.
Depending on how we cut this crystal, we therefore change the angle at which the signal and idler are generated. For example, with the BBO that we use pumped at 405nm, the degenerate down converted photons exit at 3°. However we want to operate in the collinear geometry.
It is possible to change the angle at which the photons are emitted by physically rotating the crystal, as we do in our setup with a rotation stage (fig.2).

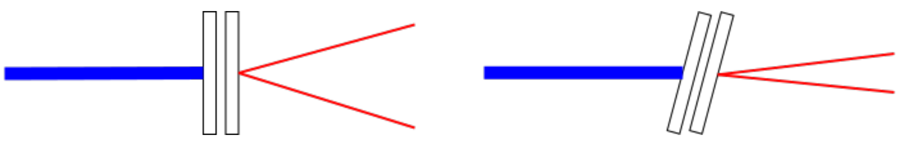

Figure 2: Changing the angle to change phase matching angle

These experiments can use two types of SPDC. In type I SDPC, the signal and idler have the same polarization, which is orthogonal to the optical axis of the crystal that generates them. In type II SPDC, the signal and idler have orthogonal polarizations. We use type I SPDC in this setup. This is achieved through the use of beta-barium-borate crystals (BBO).
You will be asked to formally write down the effect of SPDC on a quantum state by writing the input and output states of such a system.

3 CHSH inequality [3]

In the mid-sixties of the last century it was realized that the nonlocality of nature is a testable hypothesis. 1964, John Bell showed that the "locality hypothesis" with "hidden" variables leads to a conflict with quantum mechanics. He proposed a mathematical theorem containing certain inequalities. An experimental violation of his inequalities would suggest the states obeying the quantum mechanics with nonlocality. I would like to emphasize that Bell's inequalities (there are many types of Bell's inequalities, some of them were obtained later by other researchers) are classical relations [6], and they are violated only in quantum mechanics and only for some values of parameters (e.g., under some polarizer angles if we have entanglement in polarization). In many values of parameters both classical physics and quantum mechanics give the same results without violation of Bell's inequalities. The most popular form of Bell's inequality for the experimentalists is a CHSH inequality described by Clauser, Horne, Shimony and Holt, in a widely-cited paper published in 1969. The CHSH inequality for polarization entanglement can be obtained from a trivial relation that modulus of sum is less or equal to sum of moduli. And in a quantum world this classical relation can be violated. It is impossible to understand with a "common sense" mind, that inequality $|a+b+c| \leq |a|+|b|+|c|$ can be violated, but in this lab students violate this inequality.
The CHSH inequality reads:

$$|S| = E(\alpha,\beta) - E(\alpha',\beta) + E(\alpha,\beta') + E(\alpha',\beta') \leq 2 \tag{3}$$

Where α, β are detector settings and the bound is given by local hidden variable theories. Thus, violating this inequality proves Bell's theorem. We now give some definitions for the various terms. The correlation functions $E(\alpha,\beta)$ are defined as follows:

$$E(\alpha,\beta) = P_{\alpha,\beta} + P_{\alpha\perp,\beta\perp} - P_{\alpha_\perp,\beta} - P_{\alpha,\beta\perp} \tag{4}$$

Each of the correlation functions depends on two parameters α, β. These parameters correspond to detector settings. In our setup, these correspond to the choice of angle for the polarizers in front of the detectors as shown in fig.3.
The values $P_{\alpha,\beta}$ correspond to the probabilities of the photon to be detected with polarizer setting α, β. Therefore, from eq.3 and eq.4 we see that we need to choose two different settings for each polarizer, α, α' and β, β'. Then to measure a single correlation function, we need to take four separate measurements with the settings α, β and the measurements with polarizers orthogonal to these settings. The value of S is then computed easily according to eq.3. Note that we need 16 measurements to measure a single value of S.
Our setup can not measure the probabilities of eq.4 directly. Instead, we can measure the coincidences over a time interval long enough to eliminate significant noise effects and keep a record of the total coincidences. Thus, by measuring the number of coincidences over a given amount of time:

$$P_{\alpha,\beta} = \frac{N_{\alpha,\beta}}{N_{tot}} \tag{5}$$

We also define the visibility of the state as :

$$Vis = \frac{N_{max} - N_{min}}{N_{max} + N_{min}} \tag{6}$$

Where N_{max} and N_{min} correspond to the maximum number of coincidences, ie the number of coincidences with the polarizer angle that gives the highest amount of coincidences, and N_{min}

the minimum value of coincidences.
You will be asked to compute theoretically the values of the correlation functions and the maximal value of S, both in the classical and quantum case.

4 The setup

We give a brief description of the setup here. The pump shines through a HWP with which we control the polarisation. The pump emits H polarized light. It then goes through a clean up filter before the BBO (not shown here). The beam then passes through the BBO and a long pass filter eliminates the residual pump light. The non polarizing beam splitter separates signal and idler. Signal and idler are then directed to the single photon detector using mirrors. The HWP and PBS are used as polarizers to keep only a desired polarization. There are additional band pass filters at the fibre coupling.
The detector is a single photon counter, that allows us to detect when a photon reaches the detector.

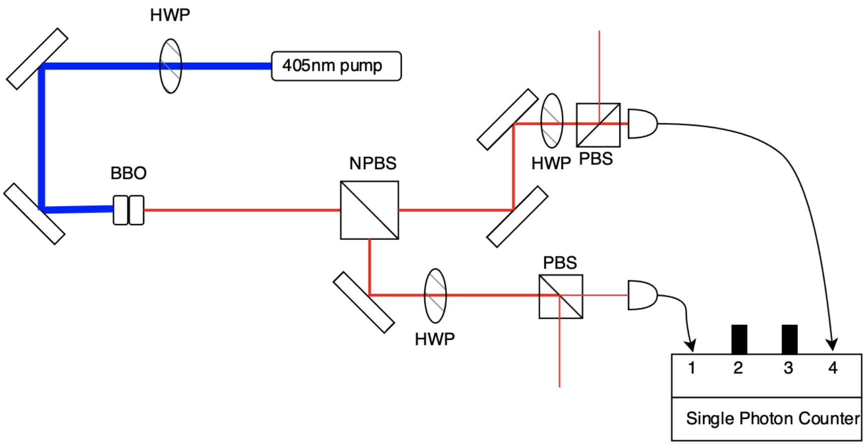

Figure 3: Scheme of the setup

The idea in this experiment is to use a single photon counter to measure simultaneous detections of signal and idler. We use this single photon counter because if we get a "click" (in optics a click means the detector detects a photon) on both channels for the signal and idler at a higher rate than clicks with other offsets, we know that these in fact correspond to the photons generated by the SPDC and not photons from other sources. By using these coincidence counts, we can measure values for the correlation functions given in eq.4.

5 Experimental procedure and exercises for your TP

WARNING: This experiment uses a single photon detector. This type of detector is very delicate and therefore you must ALWAYS ensure that: you are not sending a direct laser beam into the detector aka make sure the filters are in place when turning the detector on. When back aligning disconnect both fibres from the detector. Take all measurements with the lights in the room off.
This experiment also uses a high power laser, the assistant will repeat the safety precautions but always work above the optical table. You must wear the protective glasses at all times when the laser is on. Never adjust components without them being on the table and in their mount, you might reflect the beam into your eye. Do not use your fingers to look for the beam, and of course, never look at the beam or a reflection with your bare eyes. Also never touch the glass with your fingers, this damages the components, so you will have to wear glove whenever you are adjusting the optical components.

5.1 Day 1 : Measuring our first coincidences with 1 BBO

First steps To start getting familiar with this experiment and optics techniques, we will start with a reduced version of this epxeriment and get used to working with the various components.

- With the help of your assistant, look and understand the role of the different parts of the setup.
- With a single BBO crystal, attempt to get coincidences, you should start by back aligning the system.
- Now get the perfect alignment by tuning the mirrors and fibre coupler while looking at single photon counts.
- Make your first plot of coincidences, explain what we see and why this result is important. What are we showing with this plot?

Exercises Make sure you understand up to here the effect of the BBOs on the pump.
Show the energy and momentum conservation conditions of SPDC, in both the degenerate and non degenerate case.

5.2 Day 2 : Using and understanding polarizers, adding a second BBO

Understating how wave-plates and polarizers work Now we want to understand how to use polarizers in our setup. To do this, we will work with two half wave plates and polarising beam splitters (PBS).

- Add the wave-plates in the detection setup
- Add the polarising beam splitters (this is a delicate step and you must ensure that you don't lose too many counts when putting it in)
- Find a measurement to see the classical effect of the polarizers. Give this plot and explain the significance. Is it what you would expect?
- Add the second BBO and make the required alignments.
- Make sure you get coincidences with both BBO crystals and give both these plots.

Exercises Write down the output state of the experiment with the pump polarized H, V, and 45°, with one BBO, and with two BBOs in place. Put this result before any results with the polarizers.
Compute classically the transmission of light polarized at a certain angle through a polarizer as a function of the polarizer angle.
What is the quantum analogy to this transmission?

5.3 Day 3 : Violating the Bell inequality

We now enter the final and most interesting part of this experiment. Our end goal of the day is too successfully violate the CHSH inequality.

- Set the pump polarization to the correct angle, you should explain why this angle is the correct one.
- Keep the first polarizer fixed, and measure the coincidences as a function of the angle of the second polarizer. Do this for a few angles of the first polarizer. What would you classically expect? What do you measure? Give the plots for what you measure and what we would classically expect and give your explanations.
- Calculate the visivility of the state
- Record the single photon counts at each detector and plot them as a function of the polarizer angle. What do you notice? Give a mathematical explanation as to why we measure this result.
- Now measure the value $\mathrm{E}(\theta_1, \theta_2)$ as a function of θ_2, for two values of θ_1. What would you classically expect? What do you measure? Give both plots and explain them
- We are ready for final measurement. With the right choice of angle, measure the value of S and give your result.
- If time allows it, make the same measurement with different angles, what do you see? Is this expected?

Exercises We have computed a value of S that (hopefully) is greater than 2. Is this experiment truly a "loophole free" Bell Test? Explain why.

6 Techniques

In this section, we give the methodology on how to operate the different parts of the experiment.

6.1 Some detailed instructions on aligning

6.1.1 Aligning the pump

This first step is relatively straightforward, but once this step is completed it is crucial not to touch it again as this un-aligns the entire rest of the setup, which is then tedious to get right again.
The pump has to pass through three different optical elements. We need to be able to control the pump polarization (in our case the laser is polarized in H but this is worth checking). To control the polarization we pass the pump through a half waveplate. We should aim to pass through the center of the waveplate for maximum transmission and efficiency.
We then need to clean up the pump with the narrowband filter. Again, here the size of the filter is much bigger than the width of the pump beam, hence this should be straightforward to align.
Now the beam must pass through the two BBO crystals. These may be set up either on a single rotation stage or two separate ones. In either case, it is crucial to have good transmission through the BBO crystals. This is a bit more challenging to get right but by tuning the pump mirrors PM1 and PM2 this should be manageable. Once the pump goes through all these elements **DO NOT** touch it again.

6.1.2 Aligning the detection setup

This is now the core part and more challenging section of alignment, and may require a few tries to get right. The aim is to efficiently couple the down-converted photons to the detectors. The collinear geometry that we use makes this convenient by not having to worry about the angle of the SPDC.
We begin by giving and overview of the reasoning we use to align and then give specific techniques.
The first step is to have the right phase matching condition with our crystals. Of course these are cut at specific angles which we can not change. However, we can physically rotate the crystals to tune the phase matching angle. The BBO crystals are set on a precision rotation stage, which allows us to control the angle of these crystals to sub-degree precision. This is important as a very small change in this angle changes the path of the photons significantly.

6.1.3 Back aligning

Now comes an essential technique in aligning optics experiments. The aim is to couple to down-converted photons into the fibre. However, this can prove challenging as we can not see these down converted photons (they have visible frequency but the very low efficiency of this process means that we have a very dim flux). We therefore use a technique known as back alignment. Instead of coupling the photons from the SPDC to the fibre, we shine a laser through the fibre. To do this, carefully unscrew the fibre from the detector and screw it into the handheld laser.
VERY IMPORTANT: do this with both fibres, if you have only one laser, you can simply cover the fibre from the second detector with the protection cap. This is very important, because we will be shining powerful lasers in to the detection setup, if this light couples to the single photon detector it may destroy it. When back aligning, it is essential that no light

couples into the detector.
You will also have to remove the long pass filters on the detectors, as the lasers emit red light. Now, the aim is to align the mirrors such that the light from the lasers couples into the BBOs, and ideally reaches the pump laser output. This step is difficult, and may take some time if the user does not have experience with optics.

6.2 Software

In order to take your measurements, we will use the single photon detector as well as a CCD camera. The assistant will explain to you how the software works, but in case you want additional details you can look at the websites of the IDQ time controller. Remember that when we want to measure coincidences, you must choose time bins that are short enough to only count simultaneous clicks. As for the range of your time sweep, it does not need to be huge, just enough to clearly see coincidences and counts that drop to 0 for non simultaneous clicks.

References

[1] The nobel prize 2022

[2] https://www.fiberoptics4sale.com/blogs/wave-optics/phase-matching-for-nonlinear-optical-processes

[3] Quantum optics and nano-optics teaching laboratory for the undergraduate curriculum: teaching quantum mechanics and nano-physics with photon counting instrumentation, Svetlana G. Lukishova