Overlapping Generations Model

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1 OVERLAPPING GENERATION MODEL

The overlapping generations model (OLG) is a fundamental component of general equilibrium theory, which is the mainstream basis for economic modeling. General equilibrium theory attempts to explain how supply and demand behave within a whole economy, and how prices behave as a result. The theory is that the interaction of demand and supply with result with a “general equilibrium” in the market or markets being considered. Undergraduate students are not introduced to general equilibrium modeling or OLG until fourth year, even then only a select few who opt for certain courses will be introduced to these concepts. Our project focuses on a simple general equilibrium model, attempting to introduce a general equilibrium framework for undergraduate economic students to build upon in future studies.

Our model is directed at analyzing optimal household savings and consumption behaviour. Our main goal is to analyze optimal saving behavior, which can be understood implicitly by finding equilibrium asset demand. Our model economy is populated by overlapping generations of identical finite-lived agents, with a new cohort of agents born each period. Each agent lives for L periods, after which they die with certainty. We makes two important assumptions in our framework. First of all, we assume that agents have no bequest motives, thus the asset demand in the last period will be zero. Second, all agents are endowed with initial resources, and have autonomy over their choice of consumption and saving of this initial endowment. Each agent will maximize their lifetime utility, described by the following constrained maximization problem:

$$\max_c \sum_{l=1}^{L} \beta^{l-1} \frac{C_{l,t+l-2}^{1-\sigma}}{1-\sigma} \quad s.t. \quad y_{l,t} \equiv A_{l,t} * (P_a + D) \tag{1.1}$$

$$y_{l,t} = A_{l,t+1} * (P_a) + C_{l,t} \tag{1.2}$$

$$\max_c \sum_{l=1}^{L} \beta^{l-1} \frac{(y_{l,t+l-1} - A_{l,t+l})^{1-\sigma}}{1-\sigma} \tag{1.3}$$

Subbing equation 1.2 into equation 1.1 yields equation 1.3, which can be used to generate a system of Euler equations, which will provide optimal asset demand for each identical agent:

$$\left(y_{l,t} - P * A_{l,t+1}\right) = \beta \frac{P + D}{P} \left(y_{l+1,t+1} - P * A_{l+1,t+2}\right)^{-\sigma} \tag{1.4}$$

1.1 Steady State Equilibrium

Our model solves for a steady state equilibrium. By assumption all physical attributes of the economy are presumed to be constant over time, including human population, physical capital, and supply of natural resources. Further, we assume that in each period, all assets available must be consumed. We set the number of assets in any given period equal to 100, thus solving for resulting asset demand by each agent in each period. Solving the model produces a rate of return on assets, which is the rate of return necessary for equilibrium to occur.

2 Program User Manual

Here we will provide some details about how to use our Ox program, as well as how to interpret outputs. Our program uses the Newton-Rapshon method for solving our system of Euler equations to get the optimal asset demands and equilibrium rate of return on assets.

2.1 Setting Parameters

Discount Factor (β - beta): User can set the discounting factor by assigning an appropriate number to beta. The default setting for beta is 0.96.

Risk preference (σ - sigma): The risk preference of agents is represented by sigma. User can assign a number to sigma to represent a type of risk preference. The default setting for sigma is 1.223.

Initial Endowment (initial_a_endowment): The initial endowment is the resource available to each agent in the first period of his or her life. Since the total asset supply in each period is normalized to 100, initial endowment of 1.0 (default setting), for example, would mean each agent is endowed with 1 % of the total asset in the economy.

Dividends (D - dividend): User can set the constant dividends to dividend (default setting is 0.1). Dividend of 0.1 implies each agent receives 0.1% of the total asset supply in the economy as dividend on their asset holding.

Time Period (T): User can set the time period of the economy by assigning a positive number to T (default setting is 20). Any T<19 would result in no convergence in the Newton-Rapshon. We recommend to assign number that is greater than or equal to 19.

2.2 Understanding Outputs

Checking Euler value:
The program outputs check PASSED if all euler values are zero or close to zero meaning we have strong convergence.

Convergence value:
0: Strong convergence

1: Weak convergence
2: No convergence (max. # of iterations reached)
3: No convergence (no improvement in line search)
4: No convergence (function evaluation failed)

Optimal asset demand and consumption: First column represents a set of optimal asset demands over an agent's lifetime. The second column represents a set of consumption demand over an agent's lifetime. Since all agents are identical, user can assume that the two sets represent the optimal consumption and saving behaviour of every generation. The program also produces the graphs for optimal consumption and asset demand over T. The graphs will be saved in the same file directory.

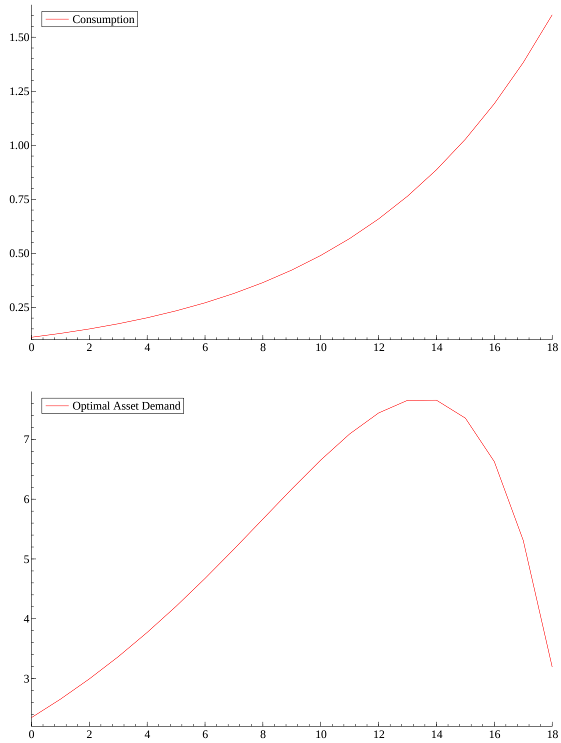

Figure 2.1: Optimal consumption and asset demand when T=20.

Sum of demands: 100.00 simply means that the sum of the total asset demand in each period must be equal to 100, the total supply of asset in each period.

Return Rate: This is the equilibrium rate of return that clears the asset market.

Price: This is the equilibrium price of asset.