Aiyagari model in niqlow

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Outline

Introduction

- Aiyagari (1994) or Bewley-Huggett-Aiyagari (BHA) is a workhorse model to study economy with heterogeneous agents in a general equilibrium setup
    - Application: Precautionary saving, liquidity constraints, distribution of wealth and income.
    - Extension: Macro labour (health, education). Fiscal policy (gov transfer)
- Key assumptions:
    - Agents face idiosyncratic shock only, not aggregate shock (Krusell & Smith 1998)
    - Incomplete market.

Outline

Model

Model details:

- Infinite horizon (extension: finite life-cycle)
- Single asset (extension: multiple assets)
- Inelastic labour supply (extension: endogenous labour choice)
- Agents face idiosyncratic labour shocks (extension: aggregate shock, or with both)
- Closed economy (haven't seen extension with open economy yet)
- Representative firm (extension: heterogeneous firms)
- No government, no central bank (extension: with either or both)
- Walrasian labour and good market (extension: decentralized search market)

Household

Solves the following infinite horizon problem:

$$\begin{aligned} &\max E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t)\right] \\ \text{s.t: } & c_t + a_{t+1} = a_t(1+r_t) + w_t\epsilon_t \\ & a_{t+1} \geq \underline{a} \end{aligned} \tag{1}$$

where $\epsilon_t \in \mathcal{E}$ is labour shocks. Assume that labour shock is AR(1) process in logs:

$$\begin{aligned} \epsilon_t^i &= \exp\left(z_t^i\right) \\ z_t^i &= \rho z_{t+1}^i + v_t^i \\ v &\sim N(0, \sigma_v^2) \end{aligned} \tag{2}$$

Household

- Focus on steady-state equilibrium where $r_t = r, w_t = w, \forall t = 0, 1 \ldots.$ Bellman equation for the problem above is:

$$\begin{aligned} V(a, \epsilon) &= \max_{a', c} \left\{ U(c) + \beta \sum_{\epsilon' \in \mathcal{E}} \Pi(\epsilon' | \epsilon) V(a', \epsilon) \right\} \\ \text{s.t: } & c + a' = a(1 + r) - w\epsilon \\ & a' \geq \underline{a} \end{aligned} \tag{3}$$

  where $\Pi(\epsilon' | \epsilon)$ describes the transition of labour shock.
- Solving the Bellman equation gives the policy function $a'(a, \epsilon), c(a, \epsilon)$ and value function $V(a, \epsilon)$.

Distribution over state-space

- Construct a probability density function $\lambda(a, \epsilon)$ over state-space, and a transition matrix $Q\big((a', \epsilon'), (a, \epsilon)\big)$
- $\lambda(a, \epsilon)$ returns the proportion of population with asset a and current labour productivity state ϵ. Thus heterogeneous agent model.
- Agents face different labour shock profile, thus heterogeneous in asset accumulation.
  - Uninsured idiosyncratic risk is the key

Distribution over state-space

The distribution λ evolves according to the following Markov chain:

$$\lambda_{t+1}(a_{t+1}, \epsilon_{t+1}) = \sum_{a_t \in \mathcal{A}} \sum_{\epsilon_t \in \mathcal{E}} Q\big((a_{t+1}, \epsilon_{t+1}), (a_t, \epsilon_t)\big)\lambda_t(a_t, \epsilon_t) \tag{4}$$

where $\mathcal{A} = [\underline{a}, \infty)$ is the action space, and:

$$Q\big((a_{t+1}, \epsilon_{t+1}), (a_t, \epsilon_t)\big) = \mathbf{1}\big(a_{t+1} = a'(a_t, \epsilon_t)\big)\Pi(\epsilon_{t+1}|\epsilon_t) \tag{5}$$

Stationary distribution satisfies:

$$\lambda_{t+1} = \lambda_t \tag{6}$$

or equivalently

$$\lambda^* = \lambda^* Q \tag{7}$$

Note that λ^* is an equilibrium object.

Firm

A representative firms solves the following (static) profit maximization problem:

$$\max AF(K_t, H_t) - w_t H_t - r_t K_t - \delta K_t \tag{8}$$

where A is an aggregate productivity level. In a stationary equilibrium, FOC gives:

$$\begin{aligned} w &= AF_H(K, H) \\ r + \delta &= AF_K(K, H) \end{aligned} \tag{9}$$

Market clearing

1. Good:

$$\sum_{a\in\mathcal{A}}\sum_{\epsilon\in\mathcal{E}} c(a,\epsilon)\lambda(a,\epsilon) + \sum_{a\in\mathcal{A}}\sum_{\epsilon\in\mathcal{E}} a'(a,\epsilon)\lambda(a,\epsilon) = AF(K,H) - \delta K \tag{10}$$

2. Labour:

$$H = \sum_{a\in\mathcal{A}}\sum_{\epsilon\in\mathcal{E}} \epsilon\lambda(a,\epsilon) \tag{11}$$

3. Capital:

$$K = \sum_{a\in\mathcal{A}}\sum_{\epsilon\in\mathcal{E}} a'(a,\epsilon)\lambda(a,\epsilon) \tag{12}$$

Stationary Recursive equilibrium (RCE)

RCE is a value function $V(a,\epsilon)$, and policy functions $c(a,\epsilon)$ and $a'(a,\epsilon)$; firm's decision for aggregate factor demand K and H; prices w and r; and a stationary measure λ^* such that:

- Given r and w, policy functions c and a' solve the household's problem, and V is the associated value function
- Given r and w, aggregate factor K and H solves the firm's profit maximization problem
- Markets for good, labour and capital clear
- Stationary measure:

$$\lambda^*(a,\epsilon) = \sum_{a\in\mathcal{A}}\sum_{\epsilon\in\mathcal{E}} Q\big((a',\epsilon'),(a,\epsilon)\big)\lambda^*(a,\epsilon) \tag{13}$$

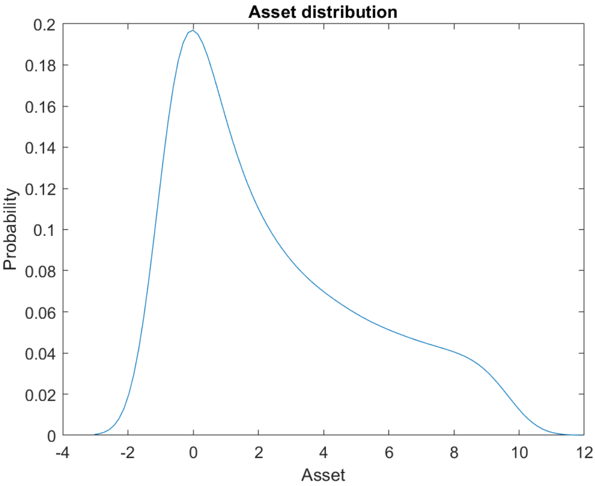
Outline

Algorithm

Nested fixed loop algorithm:

- For given prices $\hat{r}$ and $\hat{w}$, solve for an inner loop:
  - Solve for value function and policy function
  - Initialize an agent measure $\lambda(a, \epsilon)$ and construct transition matrix Q.
  - Iterate $\lambda_{t+1} = \lambda_t Q$ until convergence. Obtain stationary measure λ^*
- Outer loop:
  - Compute aggregate factor K and H implied by policy function and agent measure λ^*.
  - Compute prices r and w implied by firm's FOCs (market clearing prices)
  - If r and w are different than $\hat{r}$ and $\bar{w}$, update prices and go back to the inner loop.

Model result



Kernel plot of $a'(a, \epsilon)$ for $\mathcal{A} \in [-1, 10]$

Few notes:

- In theory, $\mathcal{A} = [\underline{a}, \infty)$. In practice, we normally discretize the state-space and experiment with the upper bound $\bar{a}$
- Value function can be solved by VFI or Policy function iteration.
- Linear interpolation between grid points in state space to reduce computational burden
- There are some alternative algorithms to compute stationary measure.
- Prices could be updated by bi-section method.

Outline

Summary for niqlow input

- Clock: Ergodic
- Action variable: $a'(\theta) \in \mathcal{A} = [\underline{a}, \bar{a}]$
- States: $\theta = (a, \epsilon)$
- Transition:

$$\begin{aligned} a' &= a(1+r) - w\epsilon - c \\ \log\big(\epsilon'\big) &= \rho \log(\epsilon) + v, v \sim N(0, \sigma_v^2) \end{aligned} \tag{14}$$

- Utility:

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \sigma > 0 \tag{15}$$

- Stationary distribution:

$$\lambda^*(a, \epsilon) = \sum_{a \in \mathcal{A}} \sum_{\epsilon \in \mathcal{E}} Q\big((a', \epsilon'), (a, \epsilon)\big) \lambda^*(a, \epsilon) \tag{16}$$

- How about solve for equilibrium prices w and r?