

# Gene Regulatory Circuits Exercises

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Ferran Pegenaute, [ferran.pegenaute01@estudiant.edu.upf](mailto:ferran.pegenaute01@estudiant.edu.upf)

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## 0.0.2 Exercise 1

Consider a gene whose expression is affected by a direct positive feedback, with Hill coefficient 2, maximum expression rate 1 nM/s, activation threshold 100 nM, and degradation rate 0.001 s<sup>-1</sup>. Integrate the differential equation that represents the dynamics of the concentration of the expressed protein, and determine the threshold value of the initial condition that separates the basins of attraction of the two coexisting equilibrium states of the system.

```
[2]: import numpy as np
      from scipy.integrate import odeint

      import matplotlib.pyplot as plt
      from IPython.display import Image
```

First, store the values given in the exercise in the corresponding variables:

```
[2]: # Hill coefficient
      n=2
      # Maximum expression rate
      b=1
      # Activation threshold
      K = 100
      # Degradation rate
      y=0.001
```

Let's define the function which return  $dx/dt$ :

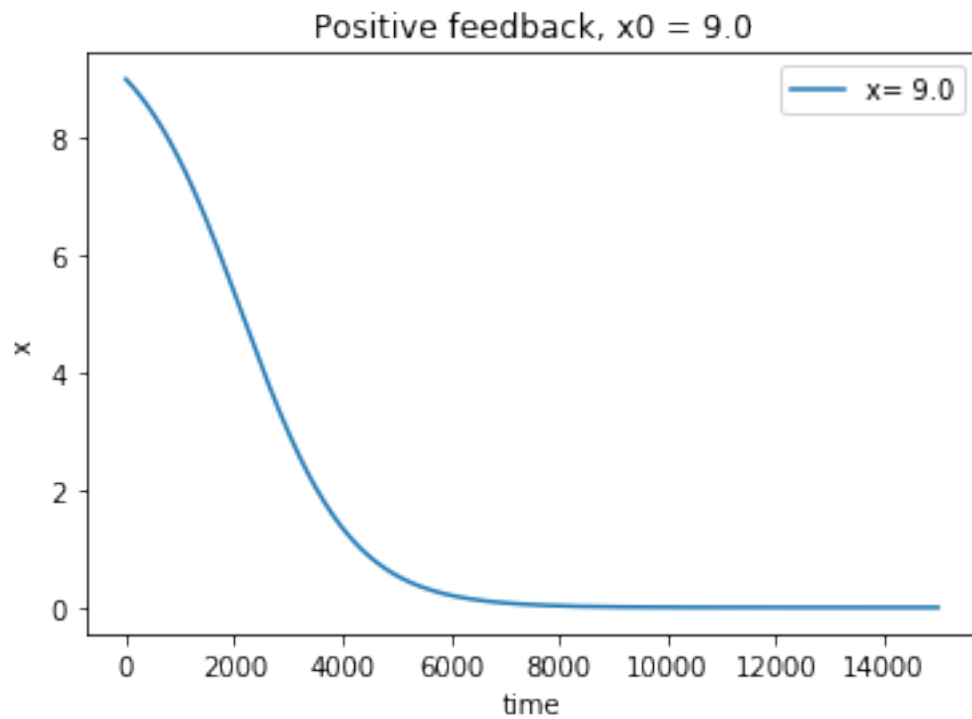
```
[3]: def dx_dt(x,t):
      """
      Positive feedback model, Hill Cooperativity. Ex1
      """
      dxdt = ((b*x**n)/(K**n+ x**n))-y*x
      return dxdt
```

Now define the time intervals and the initial conditios X0.

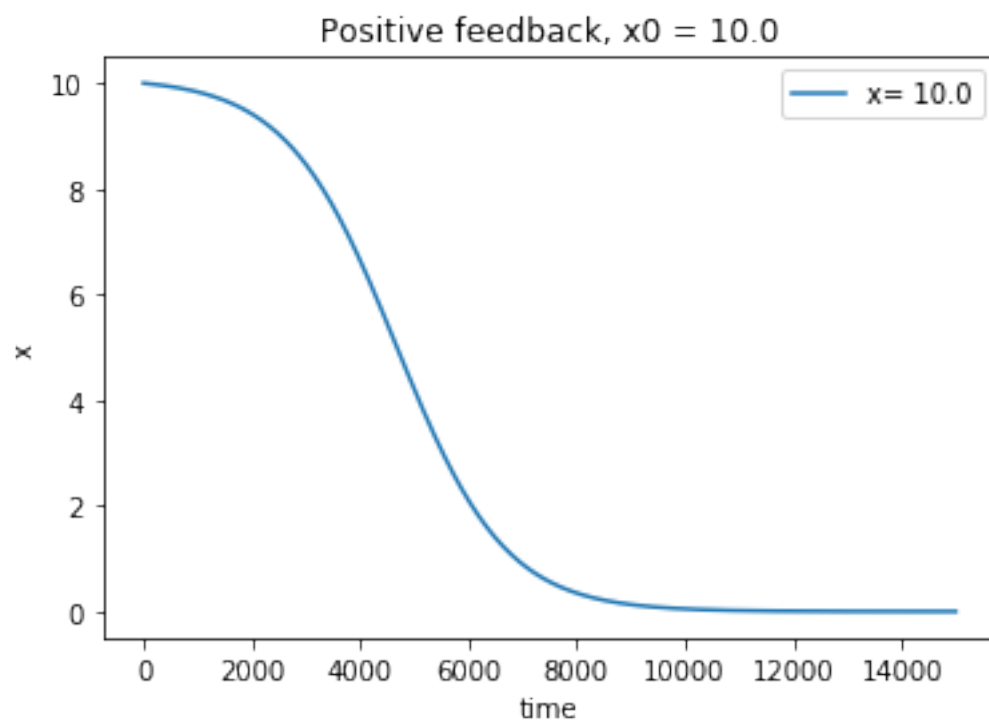
```
[4]: t = np.linspace(0, 15000, 1000)
      x0=[9, 10, 11,1000]
```

Plot x in function of the time for each initial condition. In this exercises, the integration will be done with `scipy.integrate.odeint`, which takes three arguments in this occasion: a function name that returns derivative values at requested y and t values (`dx_dt`), the initial conditions of the differential states (x), and the time points at which the solution should be reported (t):

```
[5]: for x in x0:
      y0 = odeint(dx_dt, x, t)
      plt.plot(t, y0)
      plt.xlabel('time')
      plt.ylabel('x')
      plt.legend(["x= %.1f" %x])
      if x == 11:
          print("\n **Now look what happens**")
      plt.title('Positive feedback, x0 = %.1f'%x)
      plt.show()
      len_y0=len(y0)
      print("The function tends to the value: ", round(float(y0[len_y0-1]),4))
```

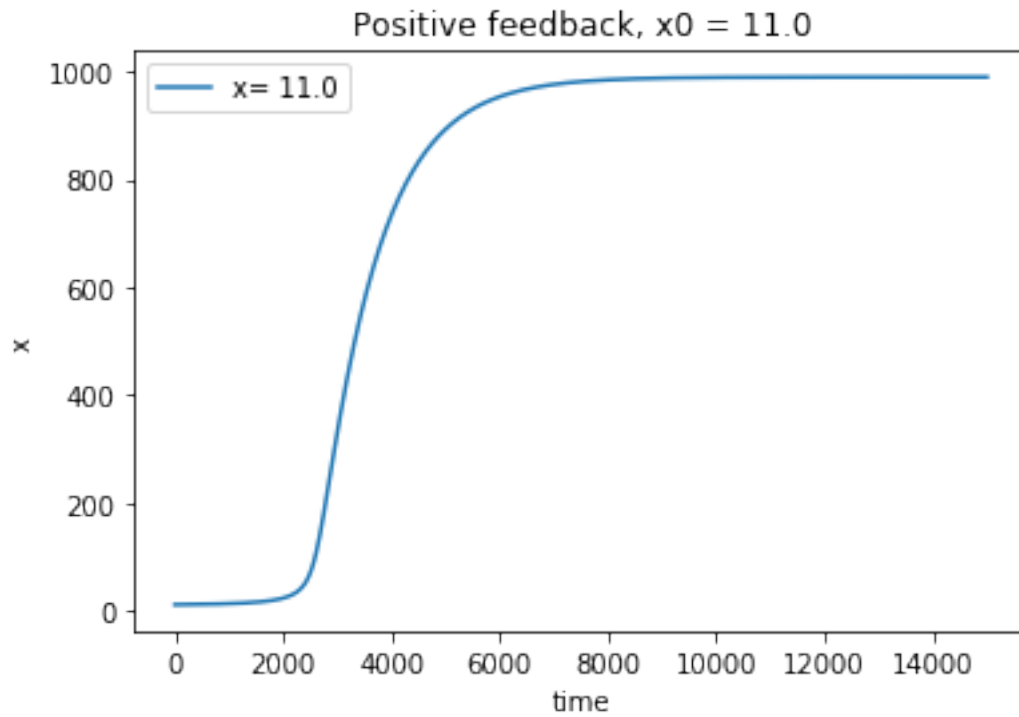


The function tends to the value: 0.0

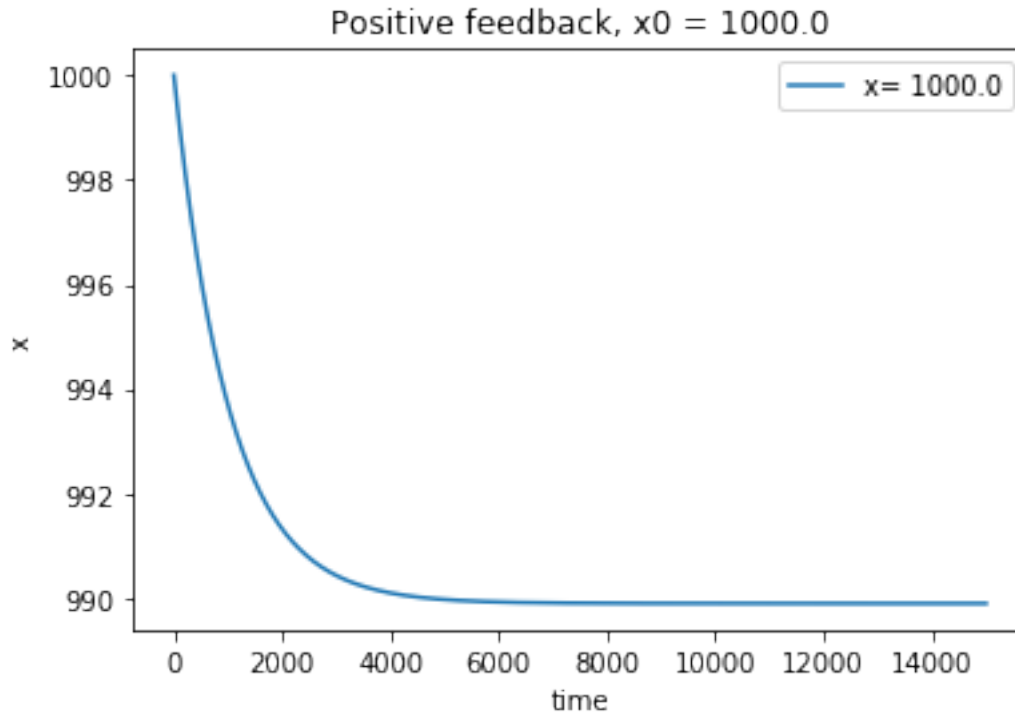


The function tends to the value: 0.0003

**\*\*Now look what happens\*\***



The function tends to the value: 989.8926

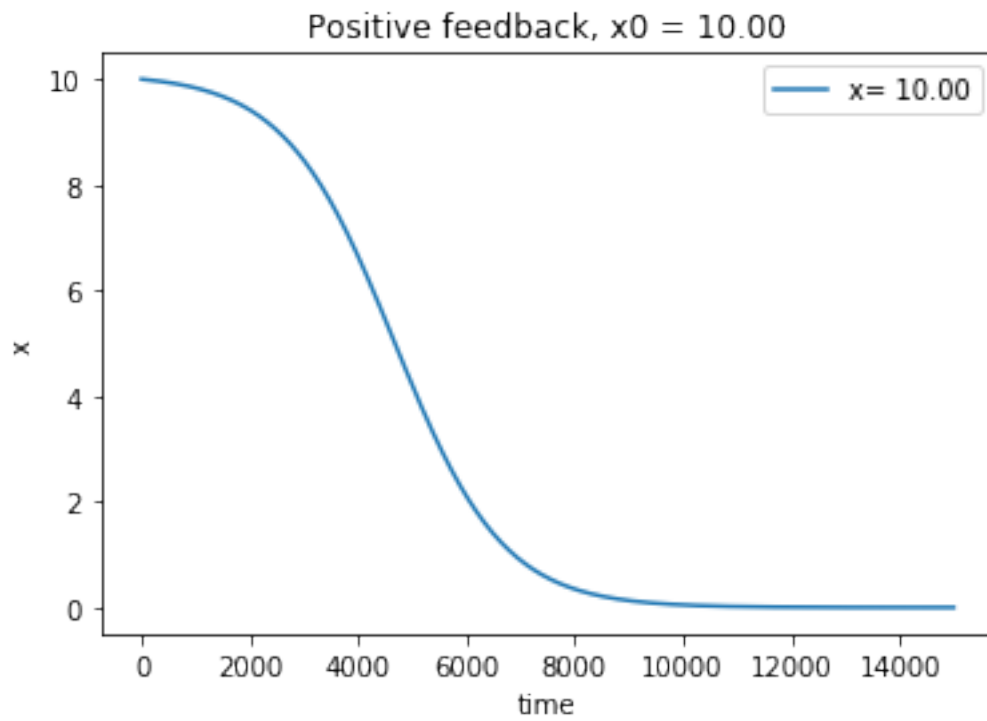


The function tends to the value: 989.898

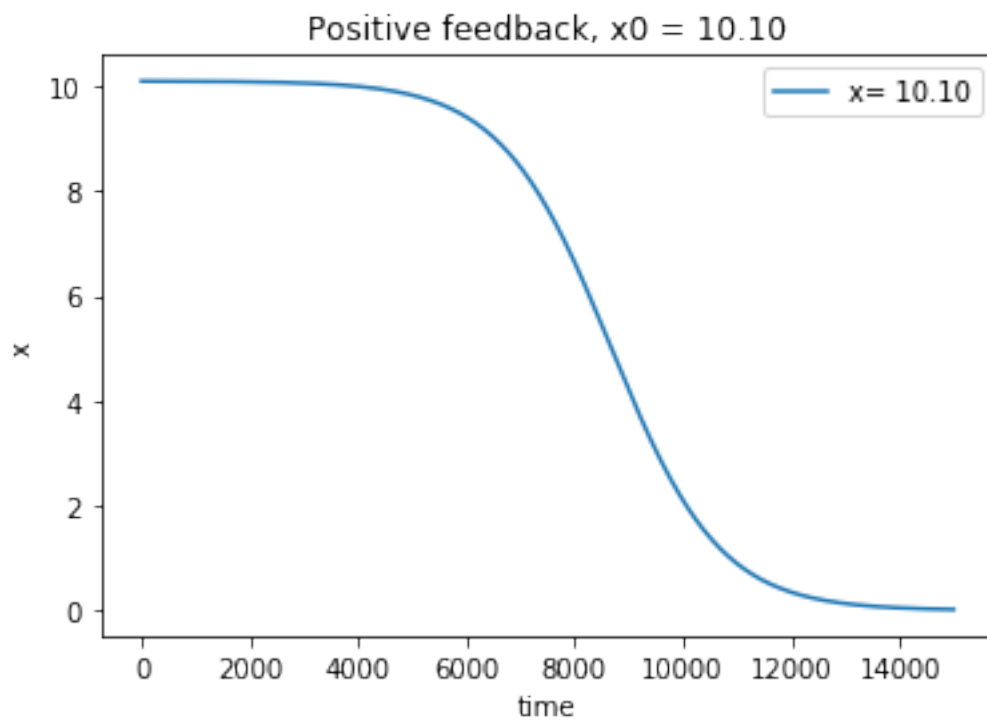
The plots show two equilibrium points: one around 0 nM and 989.898 nM approximately. Note that, when the starting concentration is larger than the second equilibrium point, it will decrease, and if it is smaller (but larger than 10 approx.) it will increase, which is expected from an attractor point.

The threshold value lies between 10 and 11 approximately, as showed above. We can try to close up a little bit more in that interval:

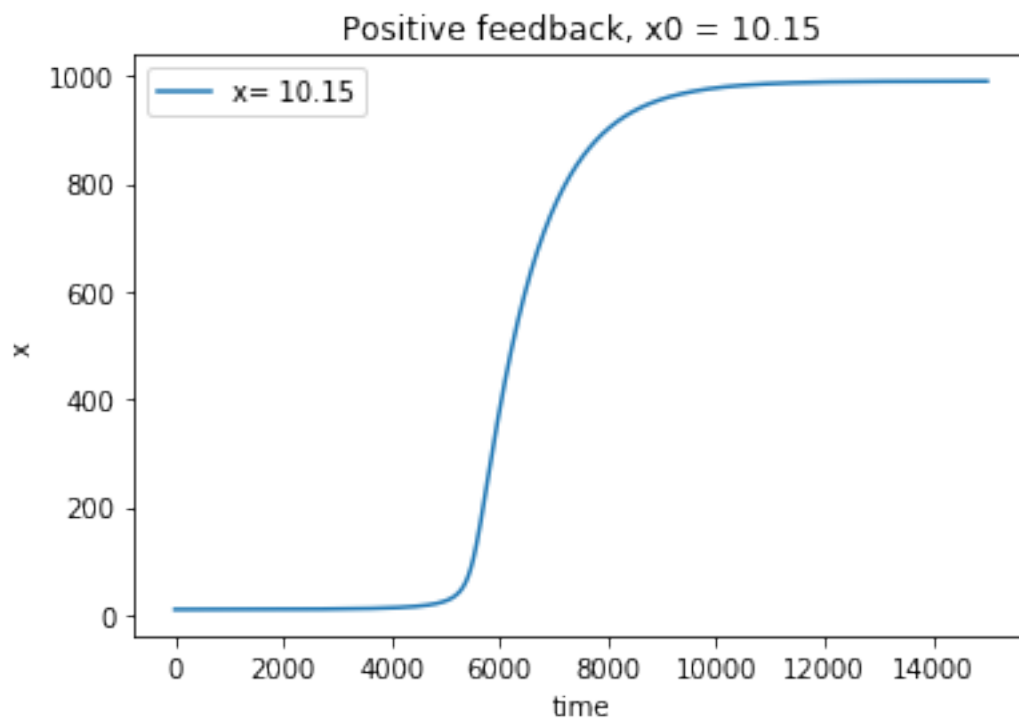
```
[6]: x0=[10, 10.1, 10.15, 10.2,]
for x in x0:
    y0 = odeint(dx_dt, x, t)
    plt.plot(t, y0)
    plt.xlabel('time')
    plt.ylabel('x')
    plt.legend(["x= %.2f" %x])
    if x == 10.2:
        print("\n **Now look what happens**")
    plt.title('Positive feedback, x0 = %.2f'%x)
    plt.show()
    len_y0=len(y0)
    print("The function tends to the value: ", round(float(y0[len_y0-1]),4))
```



The function tends to the value: 0.0003

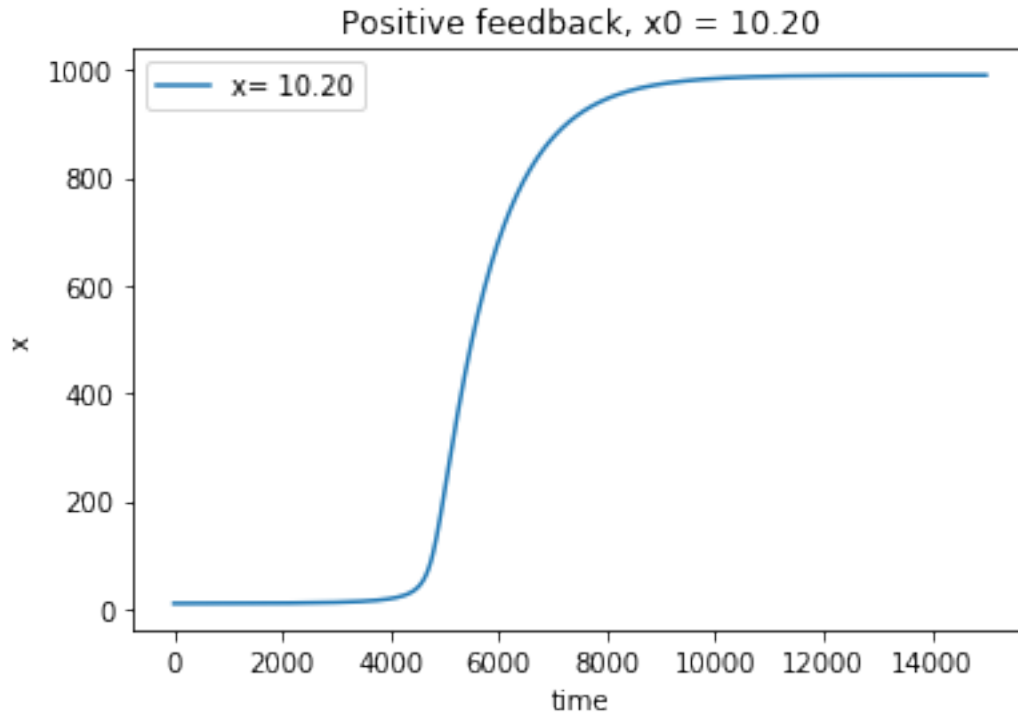


The function tends to the value: 0.0179



The function tends to the value: 989.8047

**\*\*Now look what happens\*\***



The function tends to the value: 989.8521

the threshold lies between 10.1 and 10.2.

### 0.0.3 Exercise 2

Consider the following activator-repressor model discussed in class (slide titled “From oscillations to pulses”):

Simulate this model for the following parameter values:  $a_2=0.025$  nM/s,  $b_1=15$  nM/s,  $b_2=0.8$  nM/s,  $d_1=d_2=5 \cdot 10^{-5}$  s $^{-1}$ ,  $g=2.5 \cdot 10^{-7}$  nM $^{-1}$ s $^{-1}$ ,  $K_1=3000$  nM,  $K_2=750$  nM, and  $n=m=2$ . Vary  $a_1$  as shown in the slides. Reproduce the dynamics obtained there.

The setup is similar as before, but more complex:

```
[7]: a2 = 0.025
     b1 = 15
     b2 = 0.8
     d1 = d2 = 5*10**(-5)
     g = 2.5*10**(-7)
     K1 = 3000
     K2 = 750
     n = m = 2
     y = 0.001
     x = 3
```



```

# Note that now we need to return dx/dt and dy/dt
def dx_dy(y, t,a1):
    """
    Activator-repressor model from class. Ex2
    """
    x, y = y[0], y[1]

    dxdt = (a1+((b1*x**n)/(K1**n + x**n)))-g*x*y-d1*x
    dydt = (a2+((b2*x**m)/(K2**m + x**m)))-d2*y

    return [dxdt, dydt]

```

Set the initial state (y0 and time stamps).

```

[8]: y0 = [1, 2]
     t = np.linspace(0, 1000000,1000)

```

And plot

```

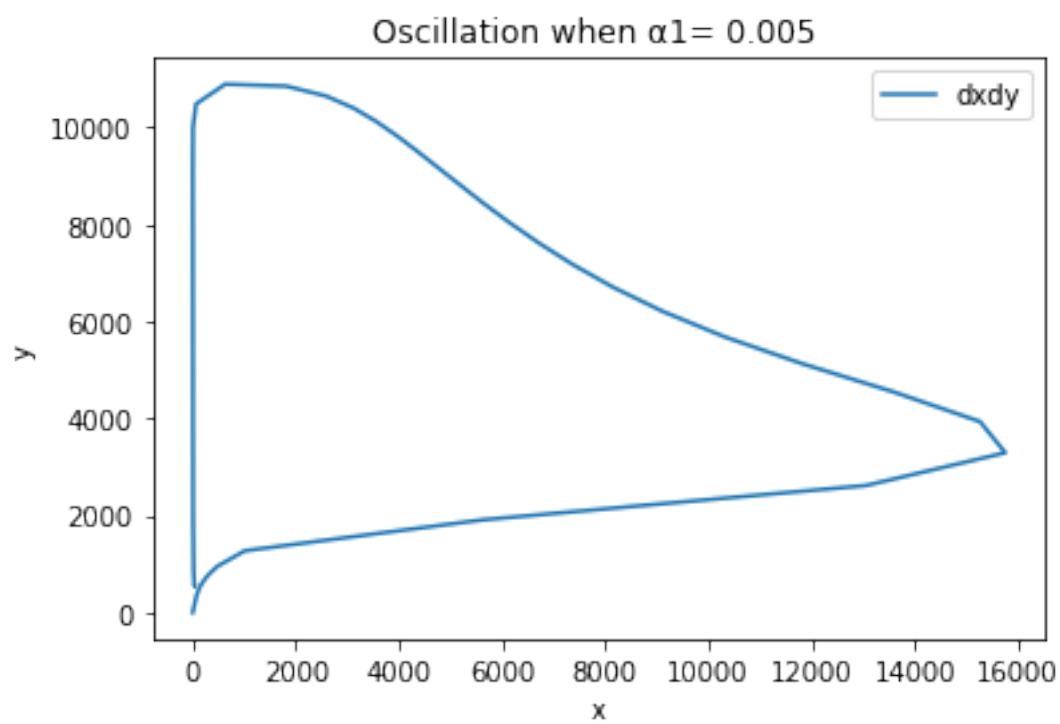
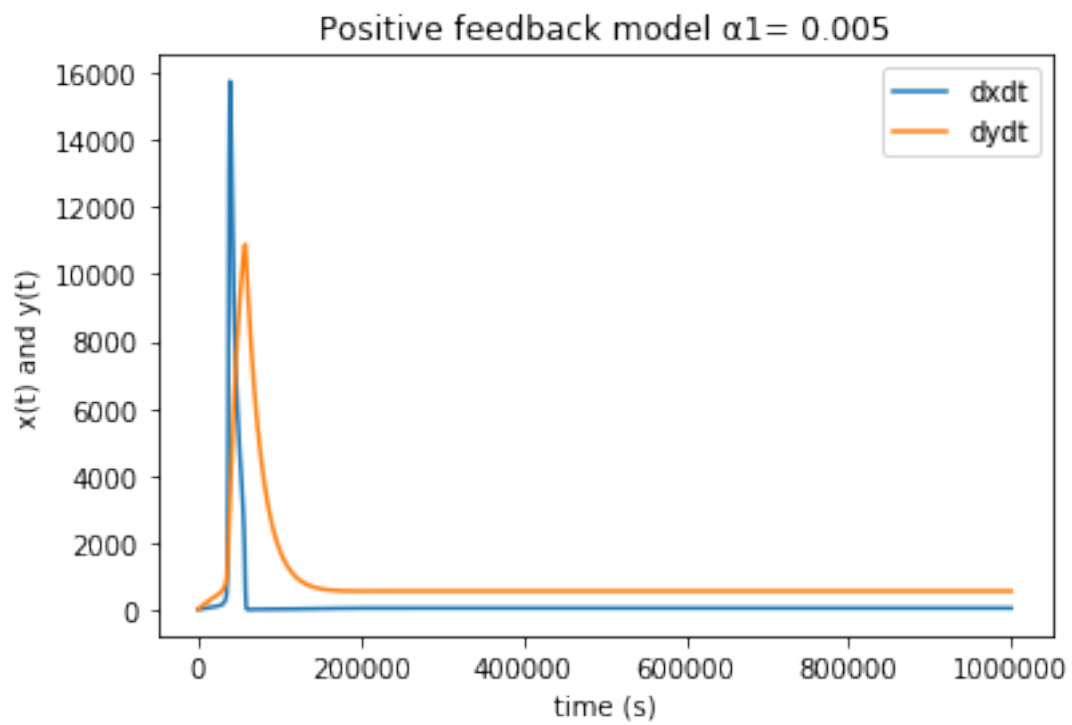
[9]: a1=[0.005,0.007,0.01]

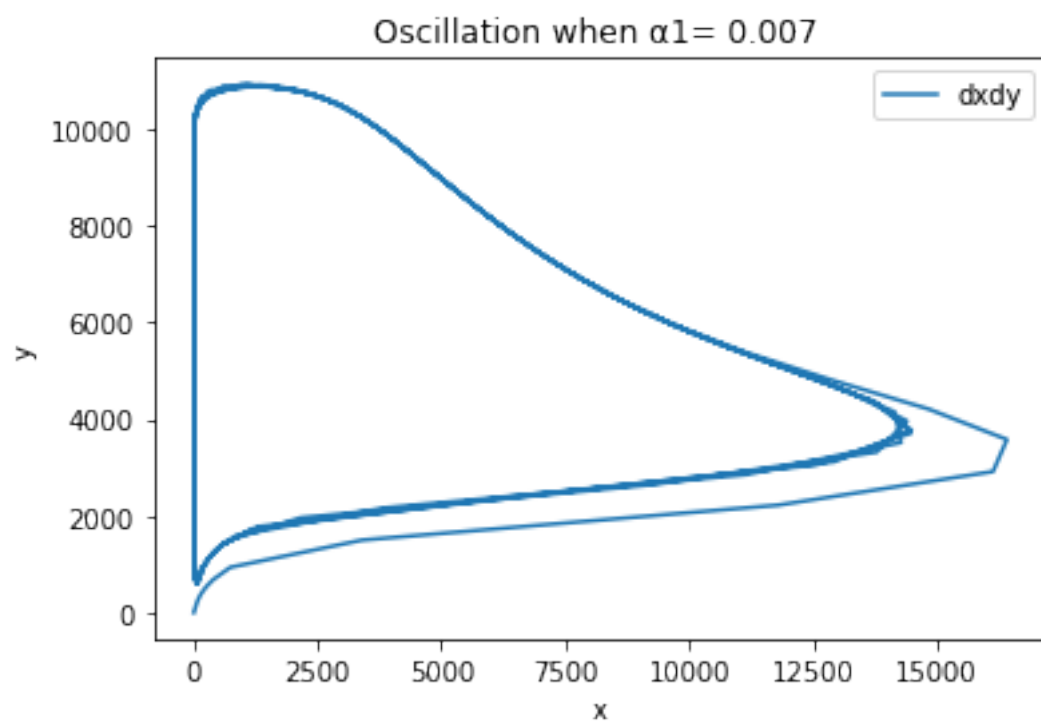
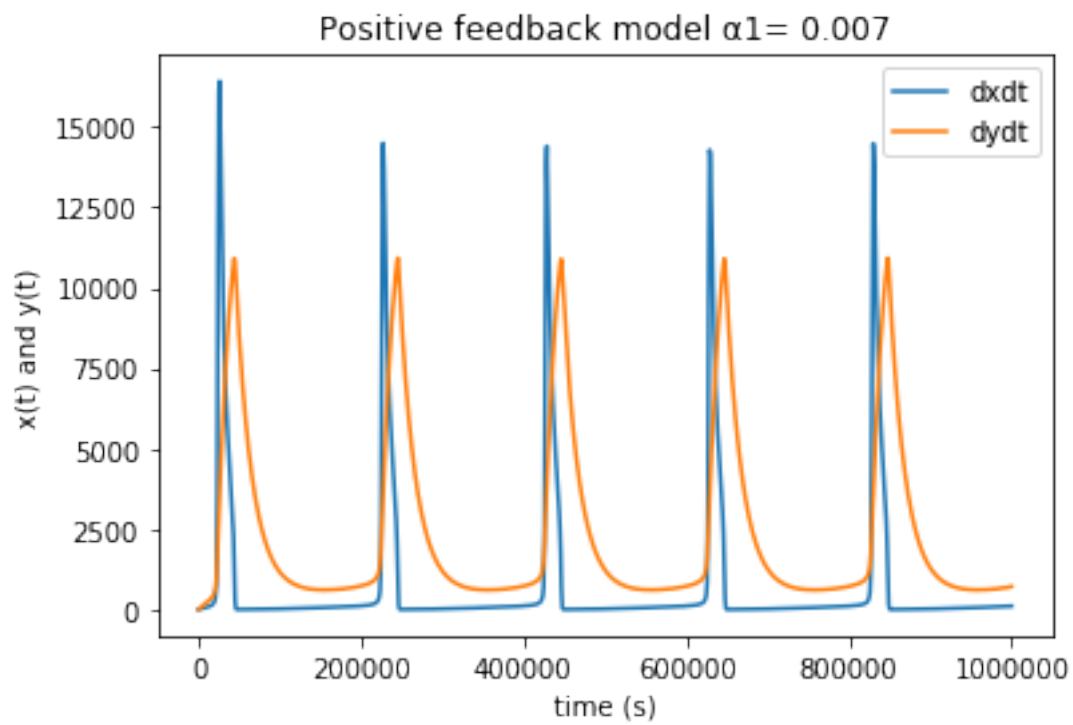
for a in a1:

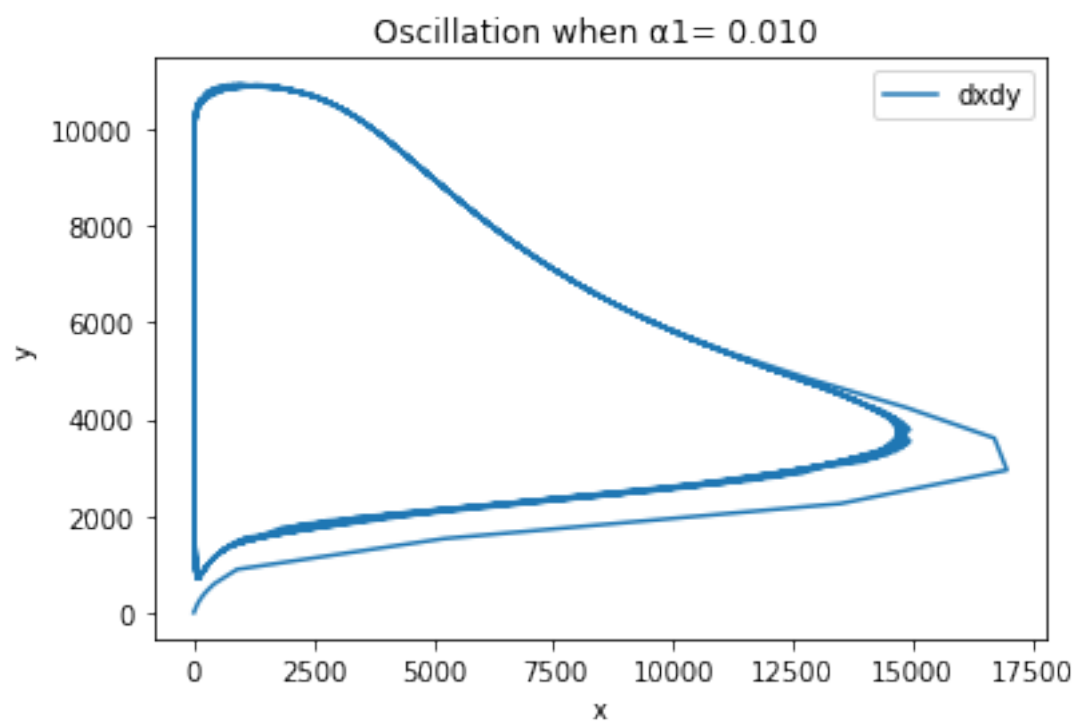
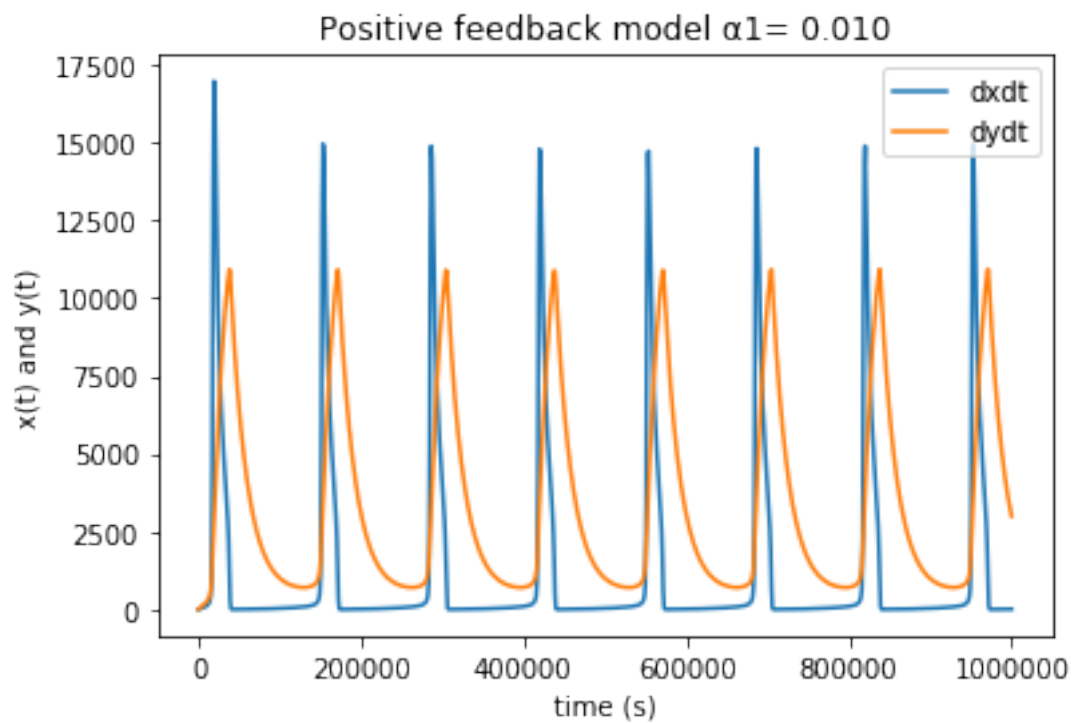
    y1 = odeint(dx_dy, y0, t, args=(a,))
    dx= y1[:,0]
    dy=y1[:,1]
    plt.plot(t, dx)
    plt.plot(t, dy)
    plt.xlabel('time (s)')
    plt.ylabel('x(t) and y(t)')
    plt.legend(["dxdt","dydt"])
    plt.title('Positive feedback model 1= %.3f'%a)
    plt.show()

    #oscillation
    plt.plot(dx,dy)
    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend(["dx dy"])
    plt.title('Oscillation when 1= %.3f '%a)
    plt.show()

```







See how when  $a_1$  is 0.005, the system does not oscillate, the phase diagram starts and ends in one single turn, but it begins to oscillate with the next two values of  $a_1$ , see how the line in the phase plots is passing through the same circular path over and over. The plots with time in the x axis look consistent with this.

### 0.0.4 Exercise 3.

The following model describes genetic competence in *B. subtilis*:

Simulate this model for the parameter values given in the table below:

Next, vary the parameters  $k$ ,  $s$  and  $s$  (one at a time) and study the response of the system in the different situations. Reproduce the different dynamical regimes studied in class.

```
[21]: a_k = 0.0875
a_s = 0.0004
bk = 7.5
dk = ds = 1*10**(-4)
kk = 5000
ks = 833
yk = ys = 0.001
Tk = 25000
Ts = 20
n = 2
p = 5

def dK_dS(y, t, a_k, a_s, bs):
    """
    Genetic competence of B Subtilis. Ex3
    """
    K, S = y[0], y[1]

    dKdt = (a_k + ((bk*(K**n))/(kk**n + K**n))) - ((yk*K)/(1+(K/Tk)+(S/Ts))) - (dk*K)
    dSdt = (a_s + (bs/(1 + ((K/ks)**p)))) - ((ys*S)/(1+(K/Tk)+(S/Ts))) - (ds*S)

    return [dKdt, dSdt]
```

Setting up:

```
[22]: y0 = [1, 0.1]
t = np.linspace(0, 1000000, 10000)
```

And now solve it:

```
[25]: y1 = odeint(dK_dS, y0, t, args=(a_k, a_s, bs))
dK= y1[:,0]
dS=y1[:,1]
```

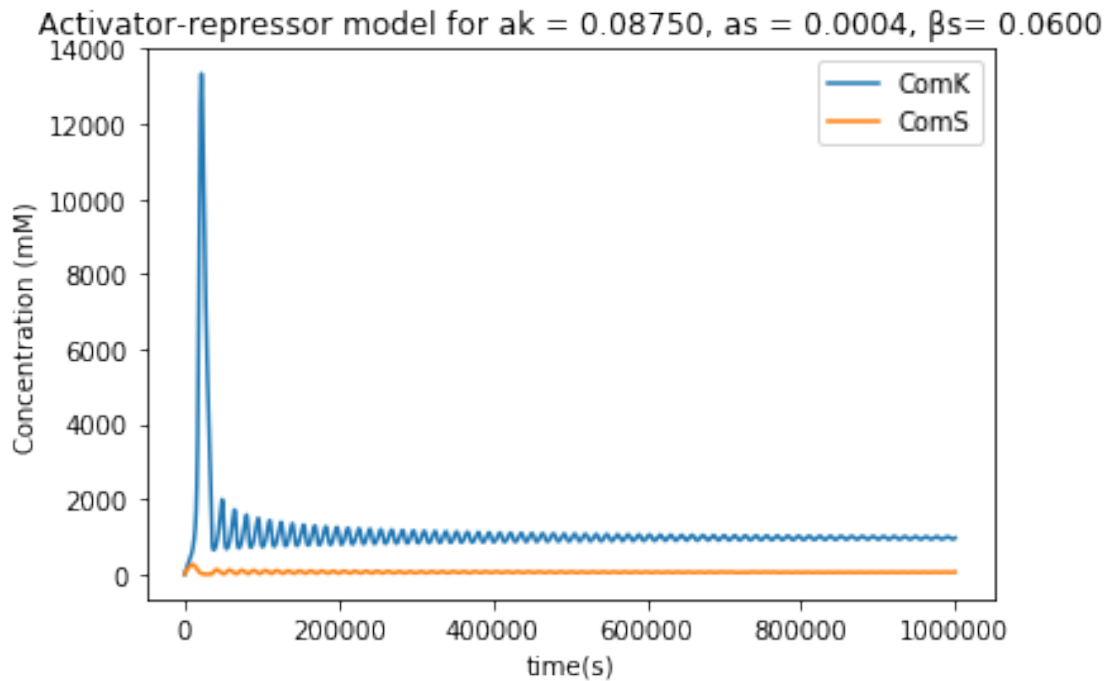
```

plt.plot(t, dK)
plt.plot(t, dS)
plt.xlabel('time(s)')
plt.ylabel('Concentration (mM)')
plt.legend(["ComK", "ComS"])
plt.title('Activator-repressor model for ak = %.5f, as = %.4f, s= %.4f'%(a_k,
→a_s, bs))
plt.show()
len_yS=len(dS)
len_yK=len(dK)
print("[ComS] tends to the value: ", round(float(dS[len_yS-1]),3))
print("[ComK] tends to the value: ", round(float(dK[len_yK-1]),3))

plt.plot(dK,dS)
plt.xlabel('comK')
plt.ylabel('comS')
plt.title('Oscillation when k = 0.00875, s = 0.0004, s=0.06')

plt.show()

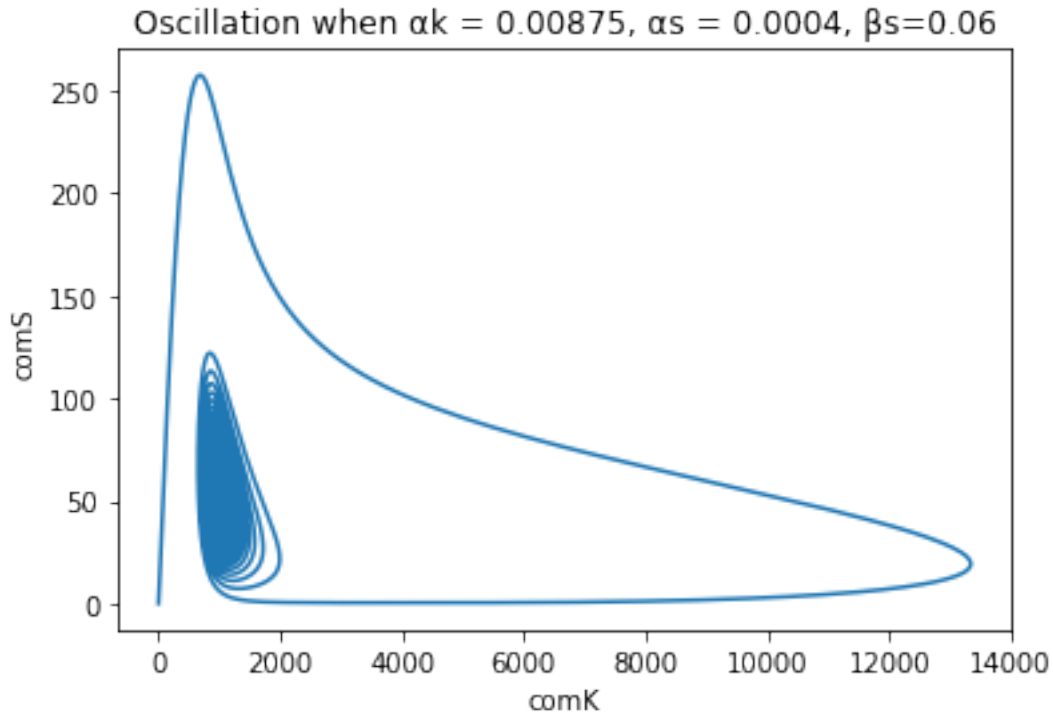
```



```

[ComS] tends to the value: 57.738
[ComK] tends to the value: 966.31

```



### 0.0.5 Changing the basal expression rate of ComK ( k ) value.

When  $\alpha_k = 0.000875, 0.0875, 0.5, 0.875, 2$  and  $10$

```
[20]: a_k = [0.000875,0.0875,0.5,0.875,2,50]

for ak in a_k:
    a_s = 0.004
    bs = 0.06

    y1 = odeint(dK_dS, y0, t, args=(ak, a_s,bs))
    dK = y1[:, 0]
    dS = y1[:, 1]
    plt.plot(t, dK)
    plt.plot(t, dS)
    plt.xlabel('time(s)')
    plt.ylabel('Concentration (mM)')
    plt.legend(["ComK","ComS"])
    plt.title('Activator-repressor model for k = %.4f and s = %.4f'%(ak, a_s))
    plt.show()
    len_yS=len(dS)
    len_yK=len(dK)
    print("When k = %.4f" %ak)
    print("[ComS] tends to the value: ", round(float(dS[len_yS-1]),3))
```

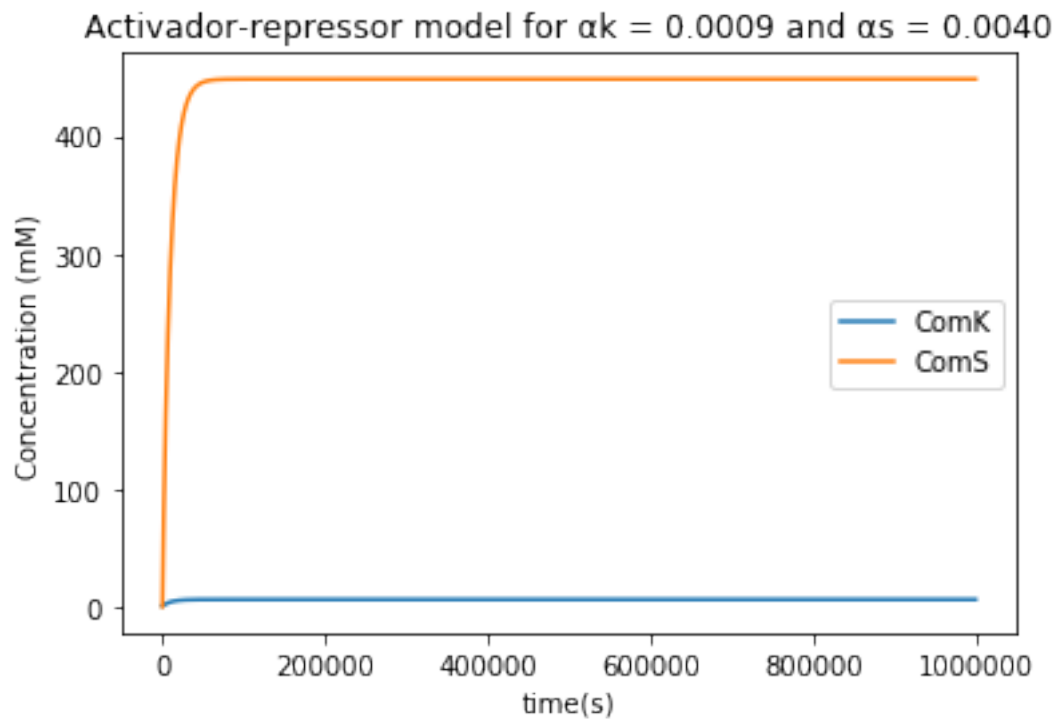
```

print("[ComK] tends to the value: ", round(float(dK[len_yK-1]),3))

plt.plot(dK, dS)
plt.xlabel('ComK')
plt.ylabel('ComS')
plt.title(' ComK vs ComS when k = %.4f and s = %.4f' % (ak, a_s))

plt.show()

```

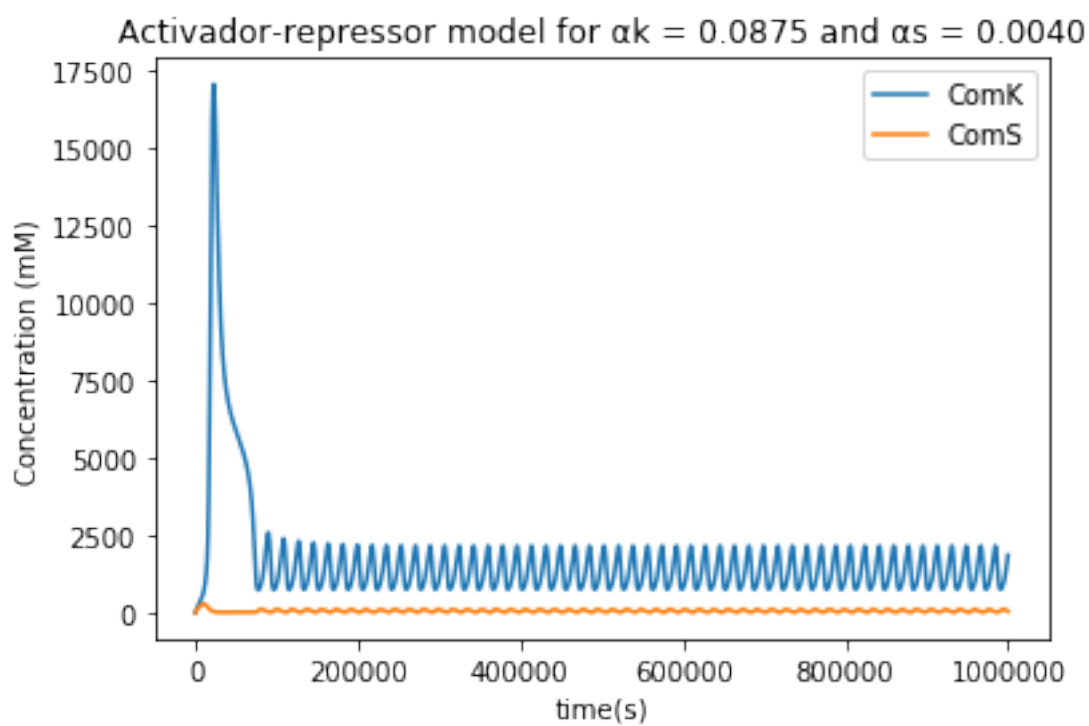
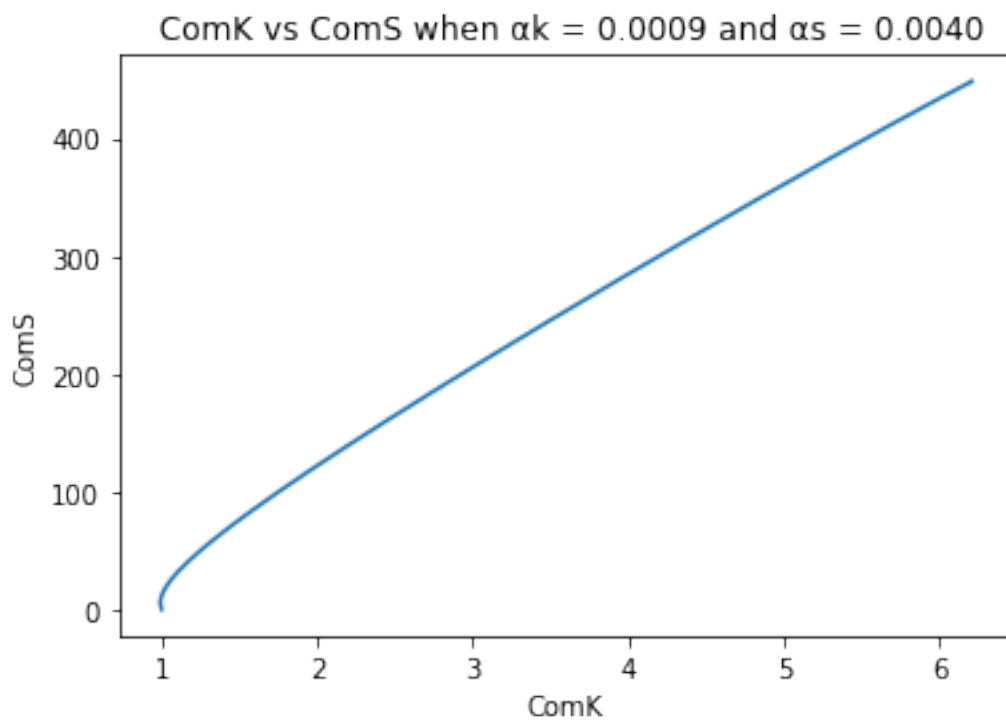


When  $k = 0.0009$

[ComS] tends to the value: 448.539

[ComK] tends to the value: 6.214

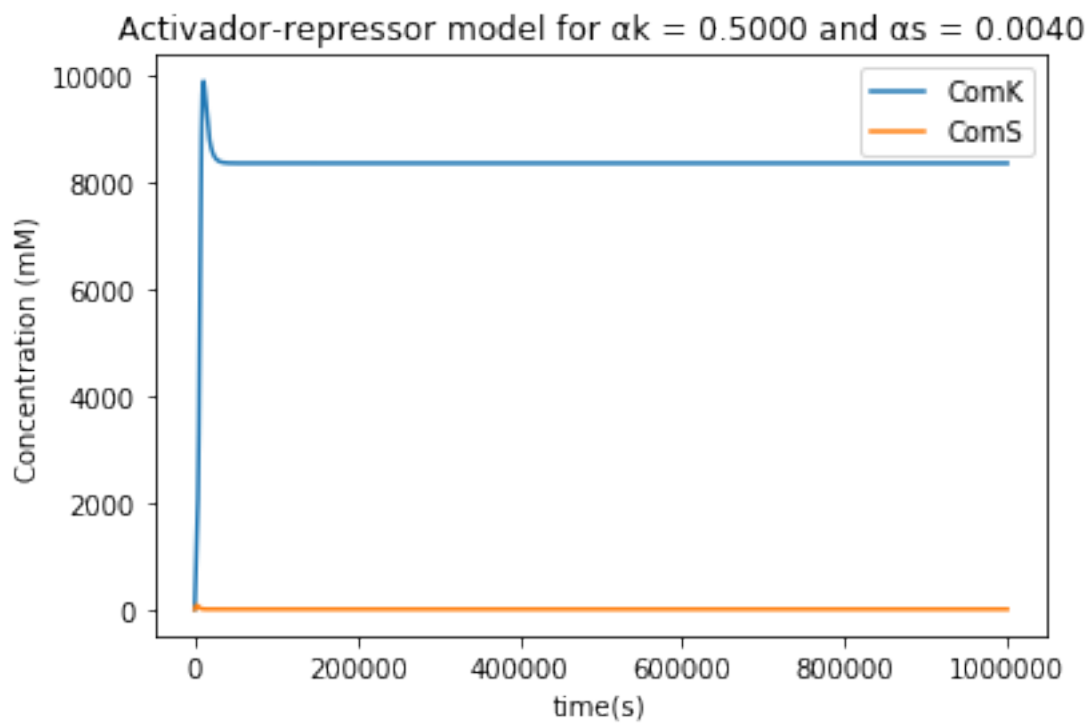
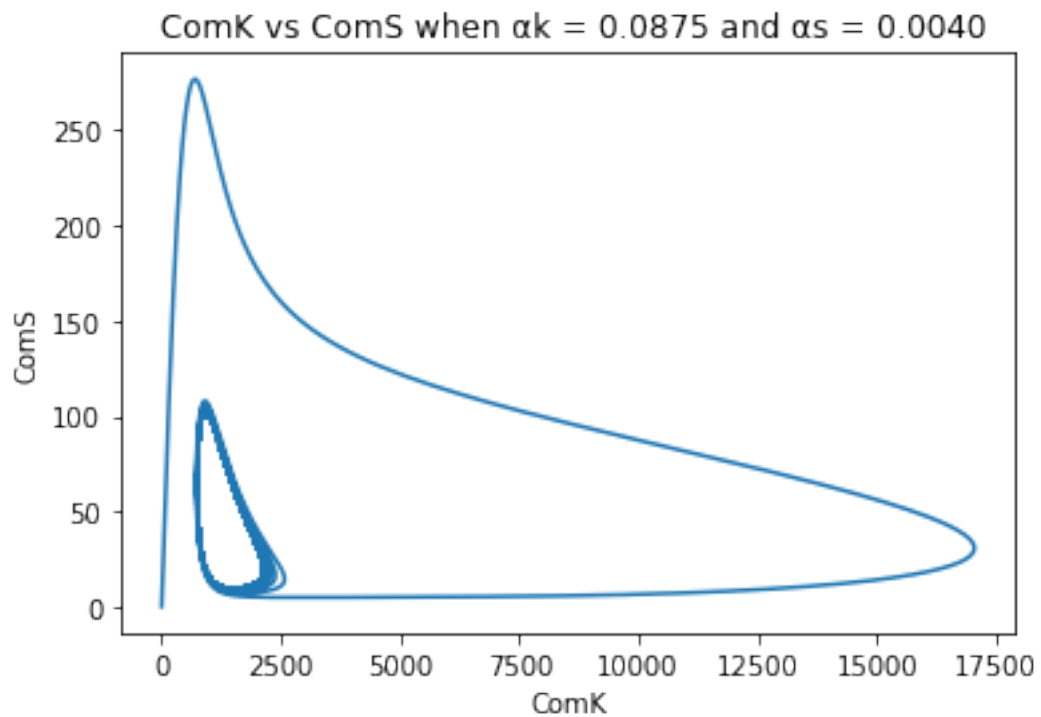




When  $k = 0.0875$

[ComS] tends to the value: 40.804

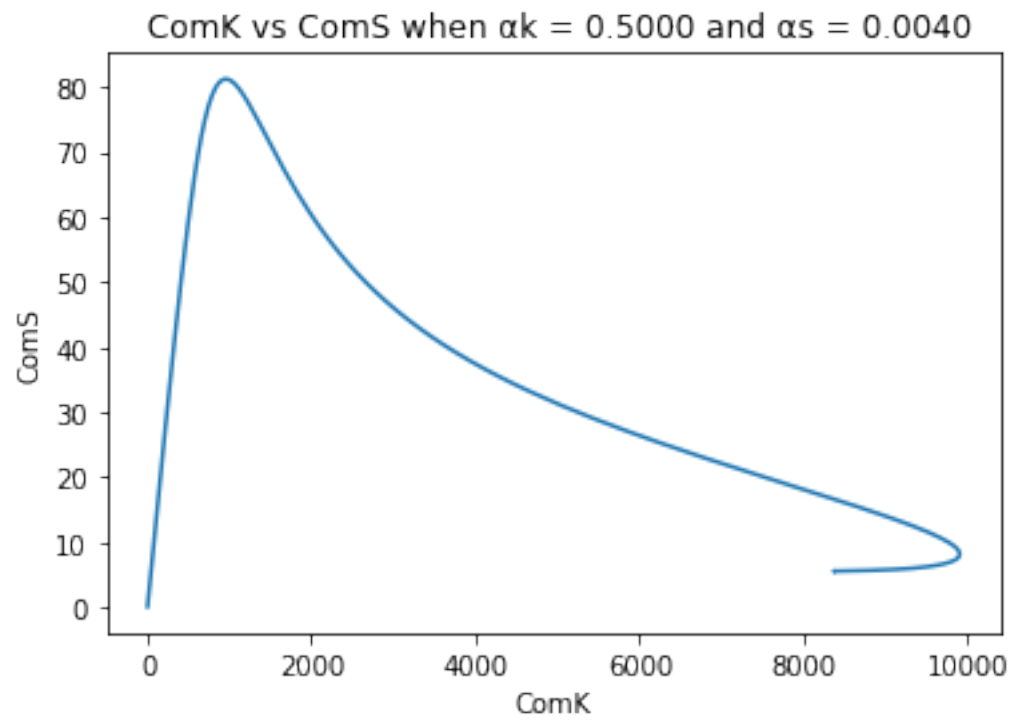
[ComK] tends to the value: 1848.559

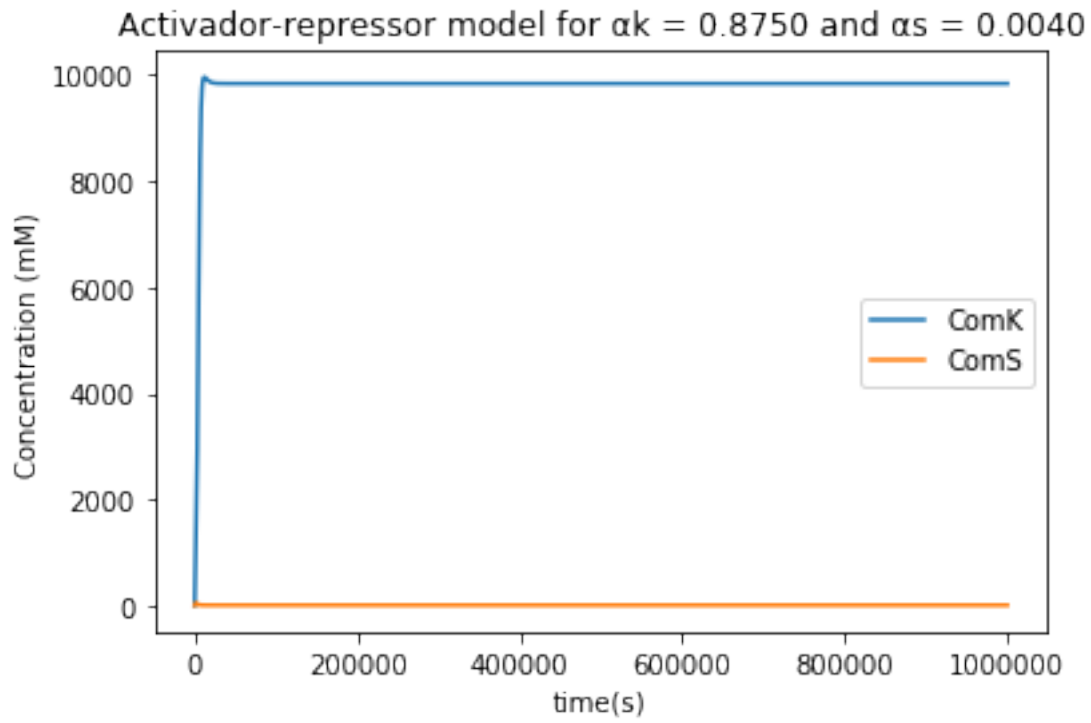


When  $k = 0.5000$

[ComS] tends to the value: 5.555

[ComK] tends to the value: 8370.096

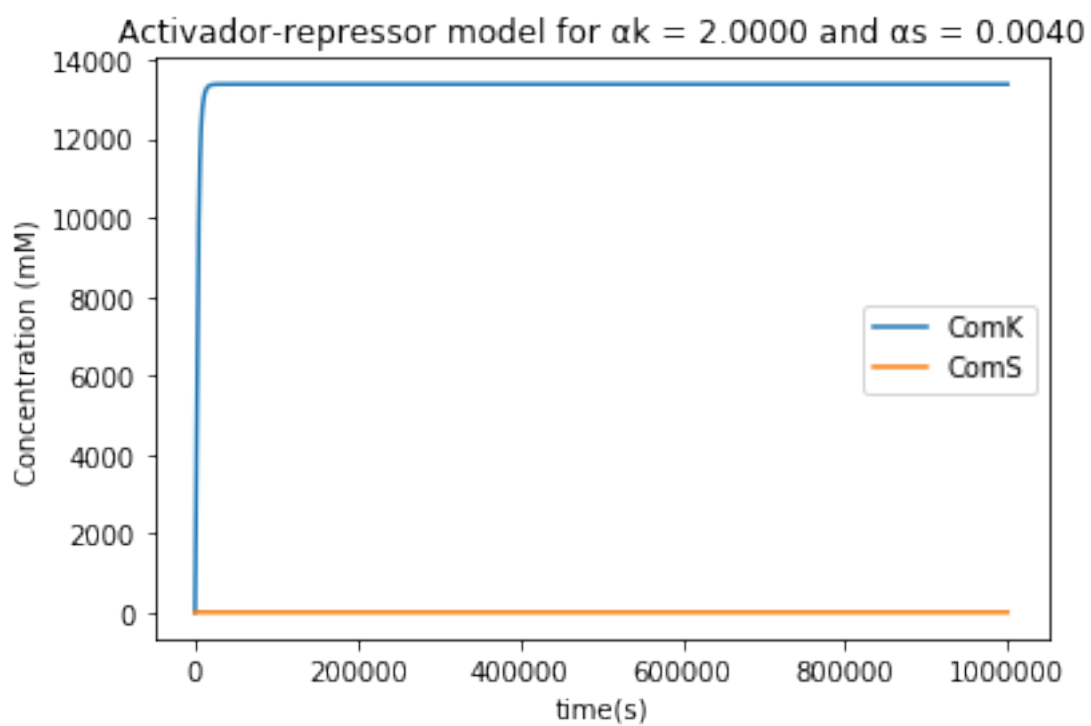
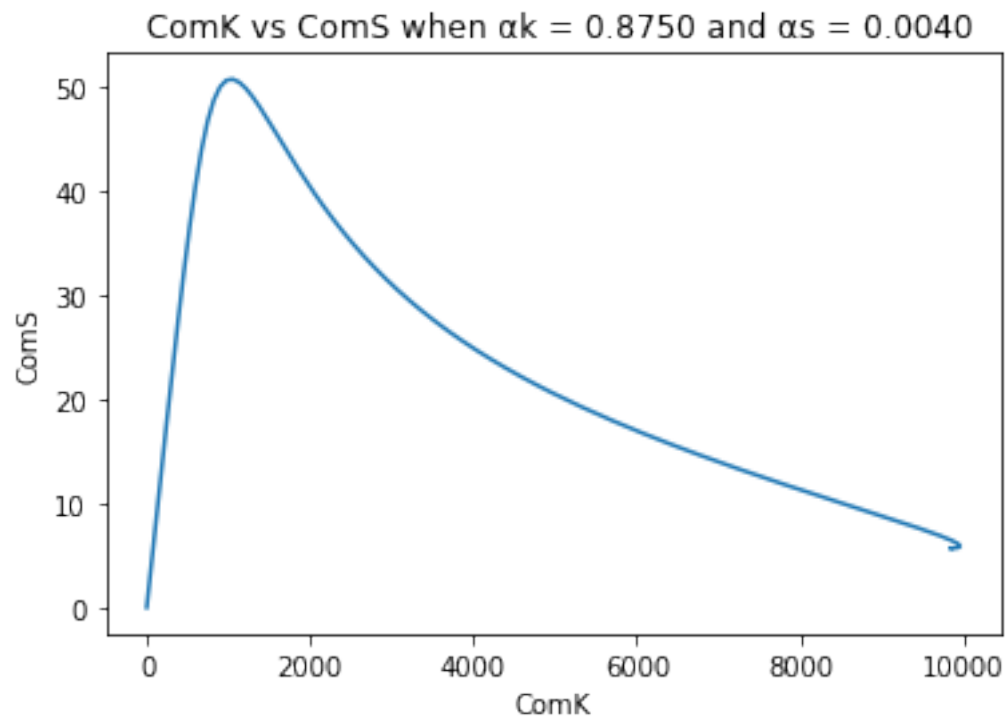




When  $k = 0.8750$

[ComS] tends to the value: 5.758

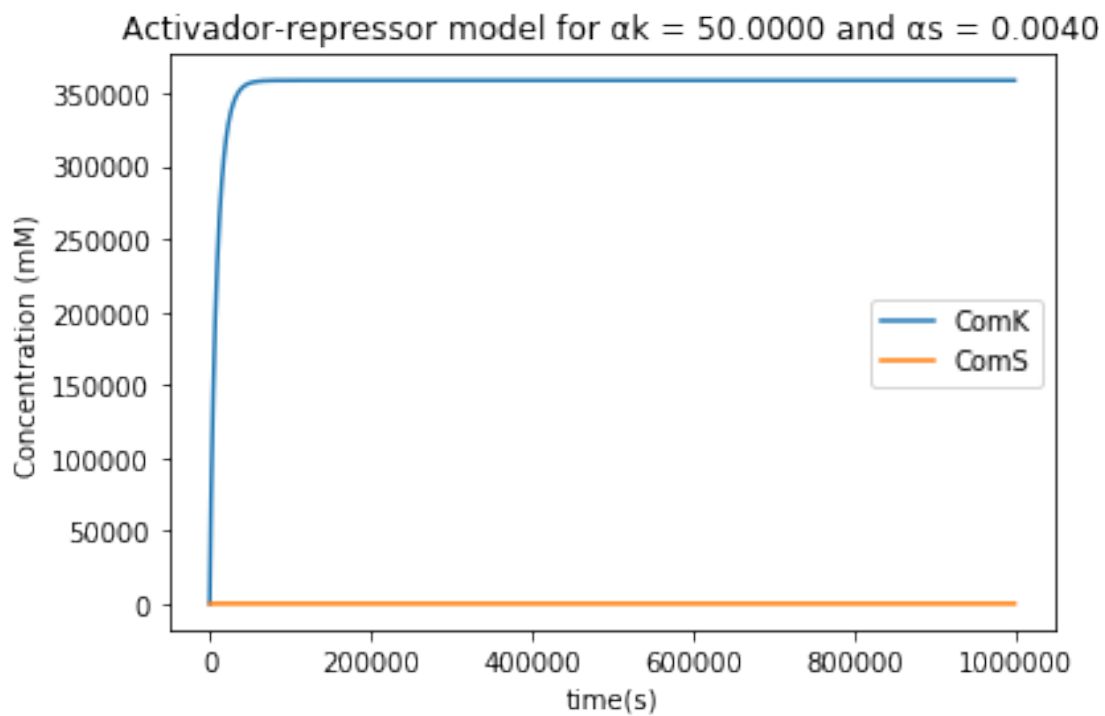
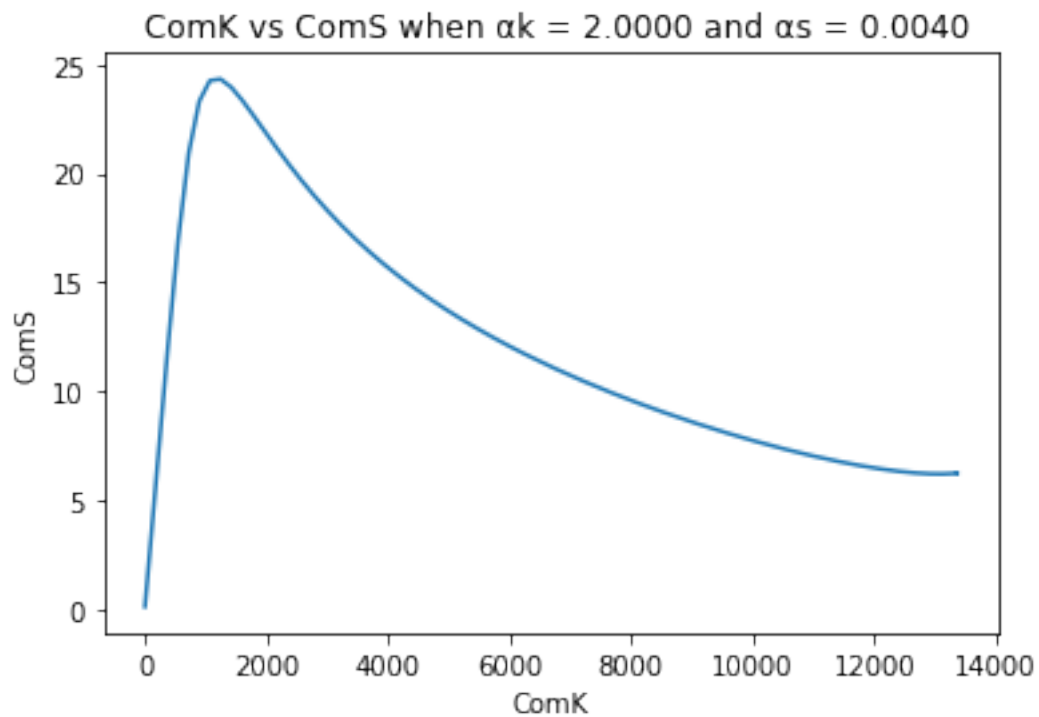
[ComK] tends to the value: 9840.131



When  $k = 2.0000$

[ComS] tends to the value: 6.236

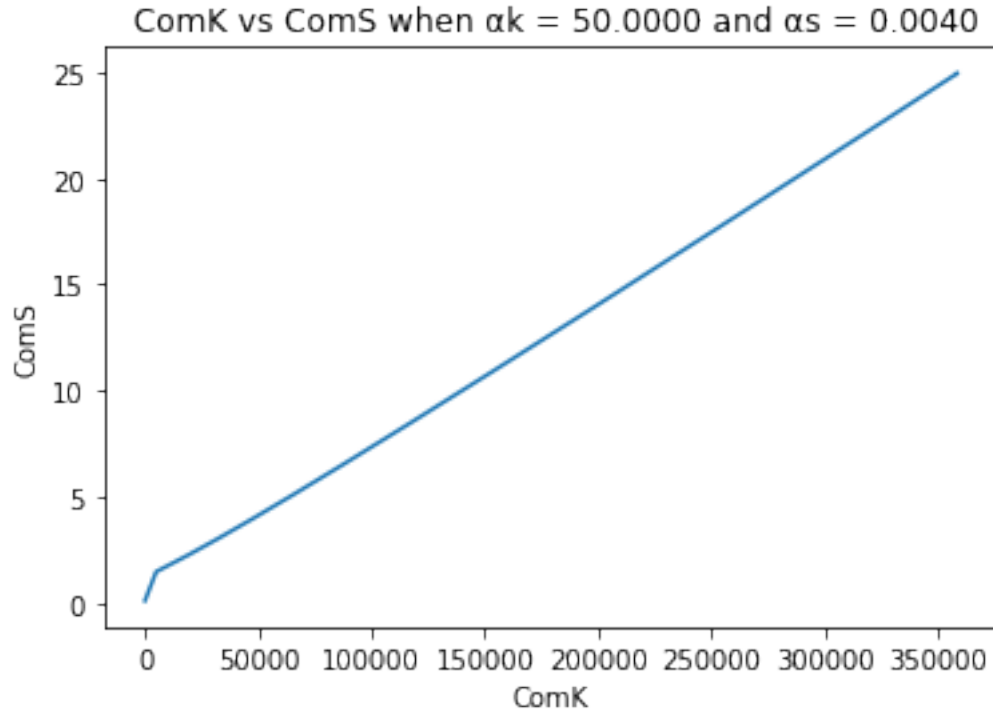
[ComK] tends to the value: 13376.666



When  $k = 50.0000$

[ComS] tends to the value: 24.963

[ComK] tends to the value: 358839.829



In this system we have the combination of positive and negative feedback, ComK and ComS respectively.

Observing the plot above, we can notice that when the basal expression rate of ComK gets lower the concentration of ComS does not change whereas the concentration of ComK gets lower as well. For instance, when:  $k = 0.00875$  and  $k = 0.0009$  the concentration of ComS is 413.235 mM. This may be the reason that there are not oscillation behavior of the system since the ComS is suppressing the expression of ComK in a negative feedback. That stable fixed point corresponds to the vegetative state.

However when the basal expression rate of ComK gets higher values ( $k = 0.0875$  to  $0.875$ ) we can notice that the concentration of ComS decrease significantly (from 57.728 to 0.448) while the concentration on ComK gets much higher (from 966.518 to 5548.956 mM). These may be because the ComS is not expressed enough to inhibit the expression of ComK. Also, we can see that within these values the system is unstable. The system exhibits limit cycle oscillations between a mid-ComK and a high-ComK level. In this case oscillation of the system is clearer to see. In contrast, with lower values of  $k$  there are not oscillation, we can see that the curve tends to the 400 mM and there not a recirculation of the reaction. But when  $k$  get higher values than 0.5 the oscillation disappear and the system reach a higher concentration of ComK but it stays in a stable state

without concentration changes in ComS and ComK.

### 0.0.6 Changing the basal expression rate of ComS ( s ) value

```
[14]: # solve the ODE problem
a_s1 = [0.004,0.05,0.85,1,2,5,10]

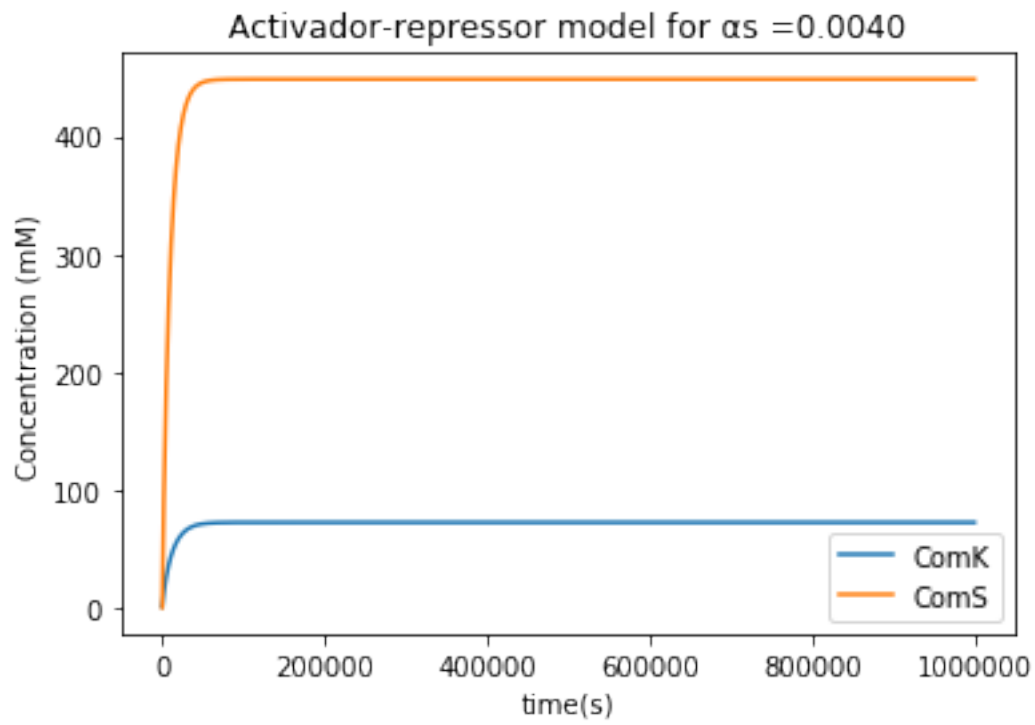
for a_s in a_s1:
    a_k = 0.00875
    bs = 0.06

    y1 = odeint(dK_dS, y0, t, args=(a_k, a_s,bs))
    dK = y1[:, 0]
    dS = y1[:, 1]
    plt.plot(t, dK)
    plt.plot(t, dS)
    plt.xlabel('time(s)')
    plt.ylabel('Concentration (mM)')
    plt.legend(["ComK","ComS"])
    plt.title('Activador-repressor model for s =%.4f'%a_s)
    plt.show()
    len_yS=len(dS)
    len_yK=len(dK)
    print("When s = %.4f" %a_s)
    print("[ComS] tends to the value: ", round(float(dS[len_yS-1]),3))
    print("[ComK] tends to the value: ", round(float(dK[len_yK-1]),3))

    plt.plot(dK, dS)
    plt.xlabel('ComK')
    plt.ylabel('ComS')
    plt.title('ComK vs ComS s =%.4f' % a_s)

    plt.show()
```

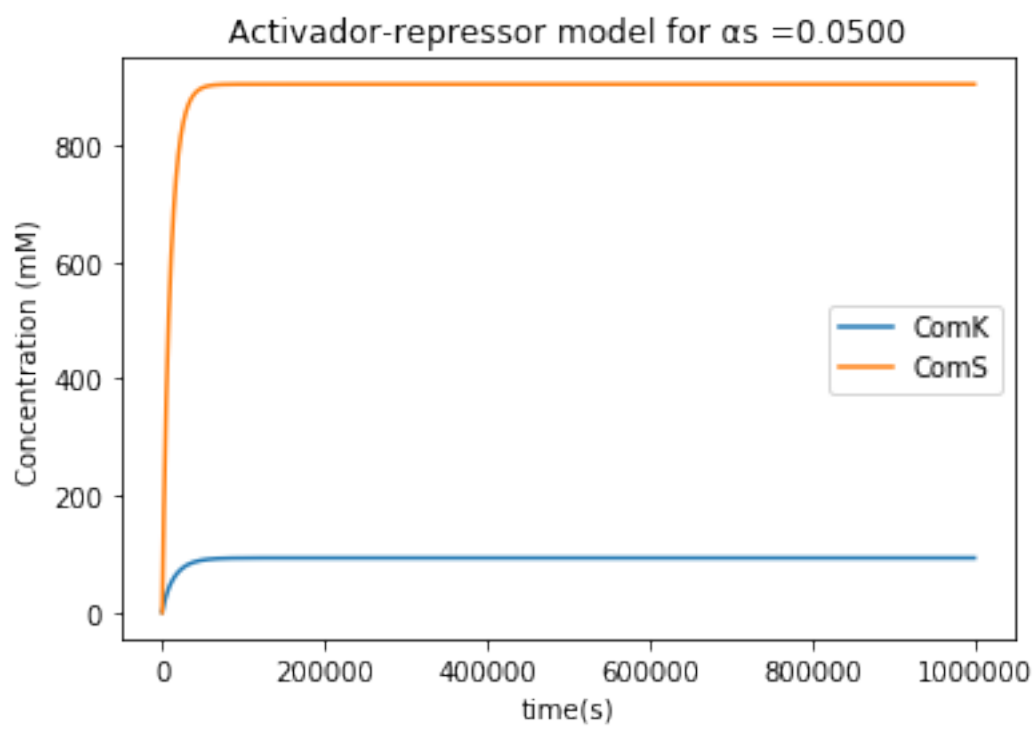
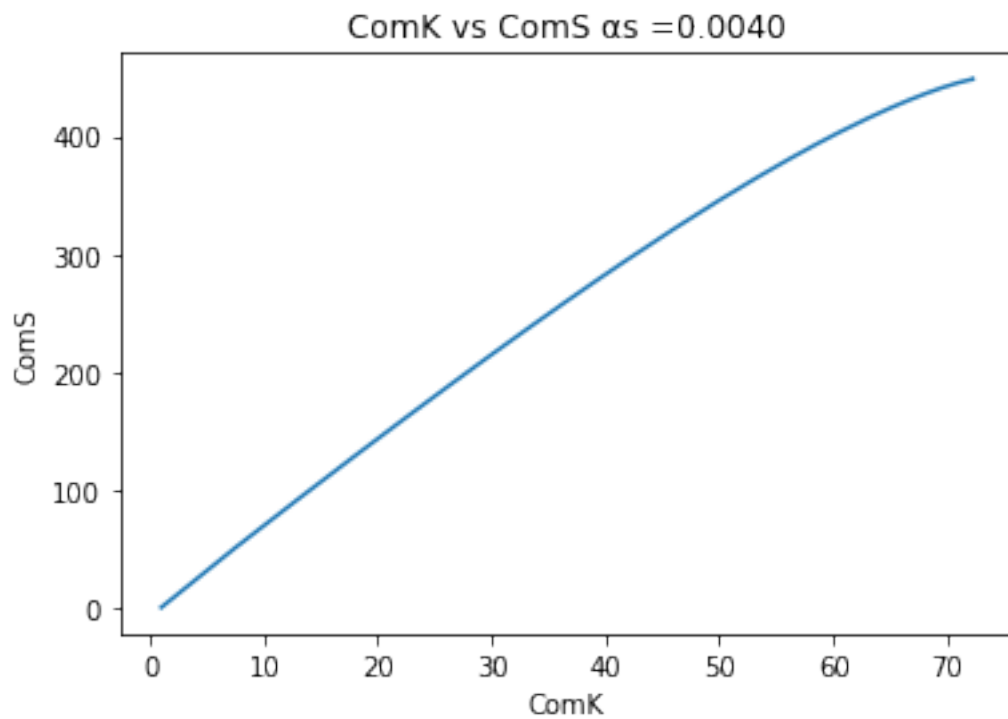




When  $s = 0.0040$

[ComS] tends to the value: 448.558

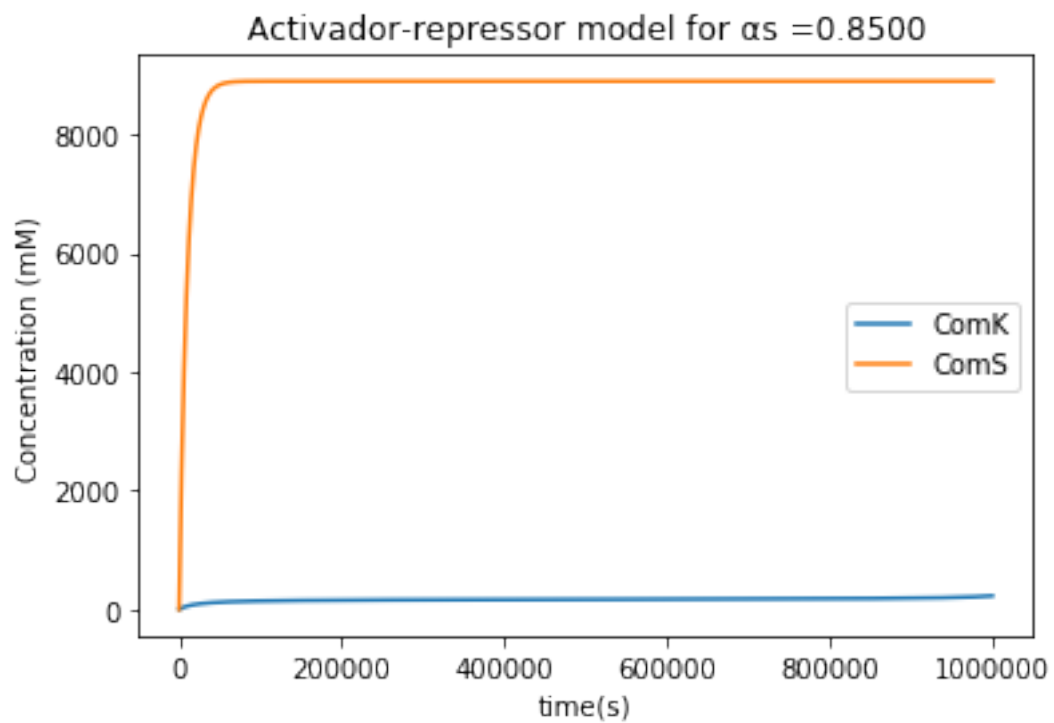
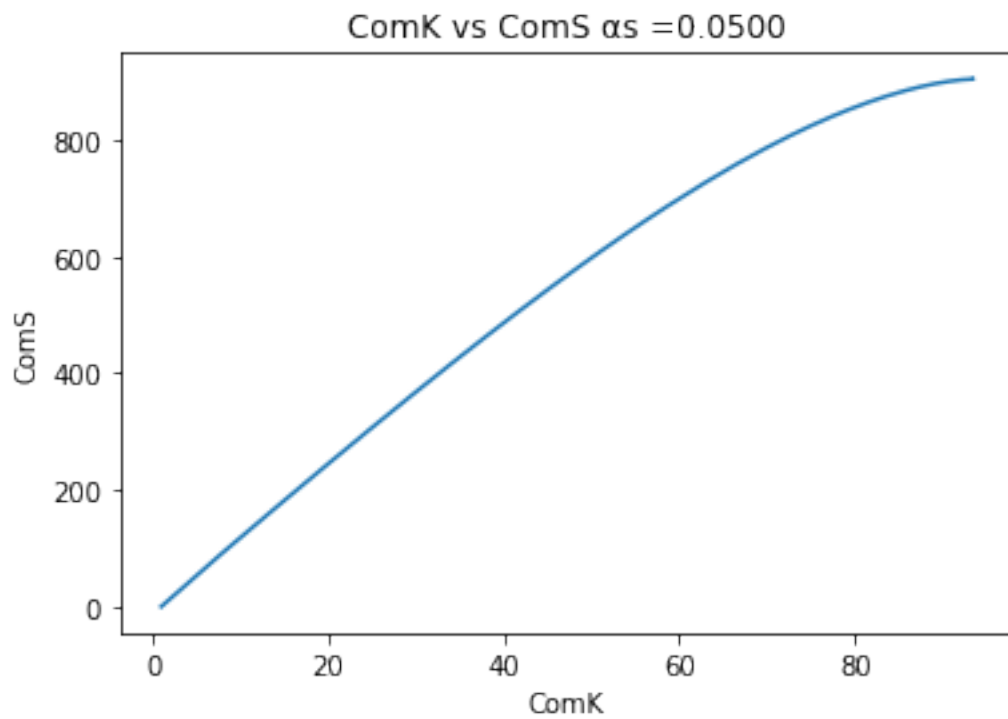
[ComK] tends to the value: 72.322



When  $s = 0.0500$

[ComS] tends to the value: 904.333

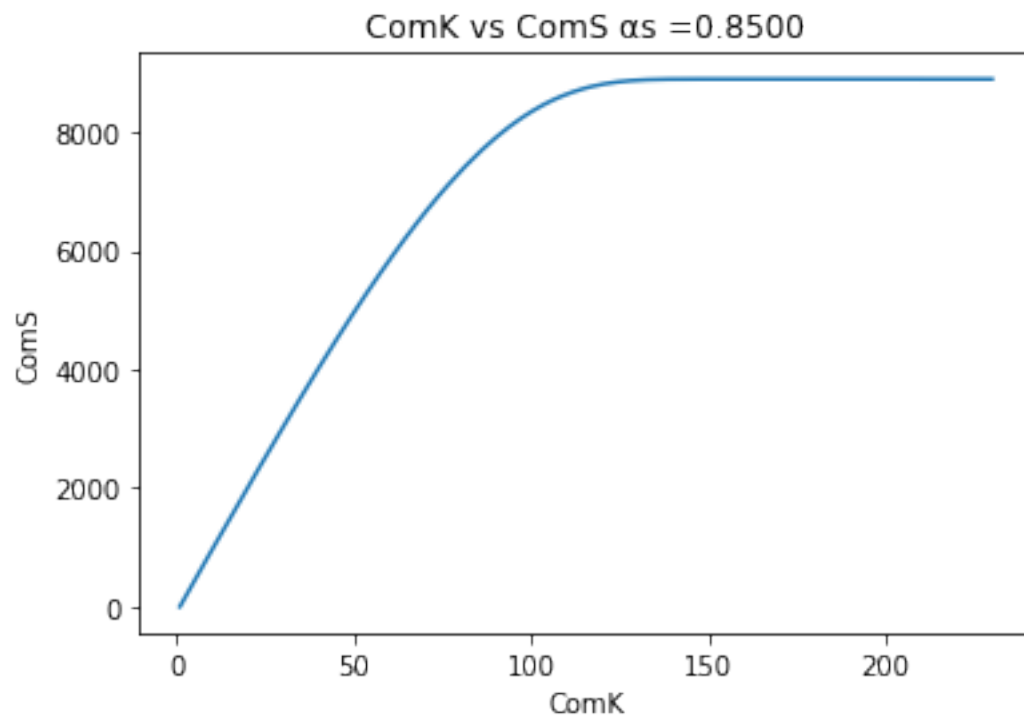
[ComK] tends to the value: 93.482

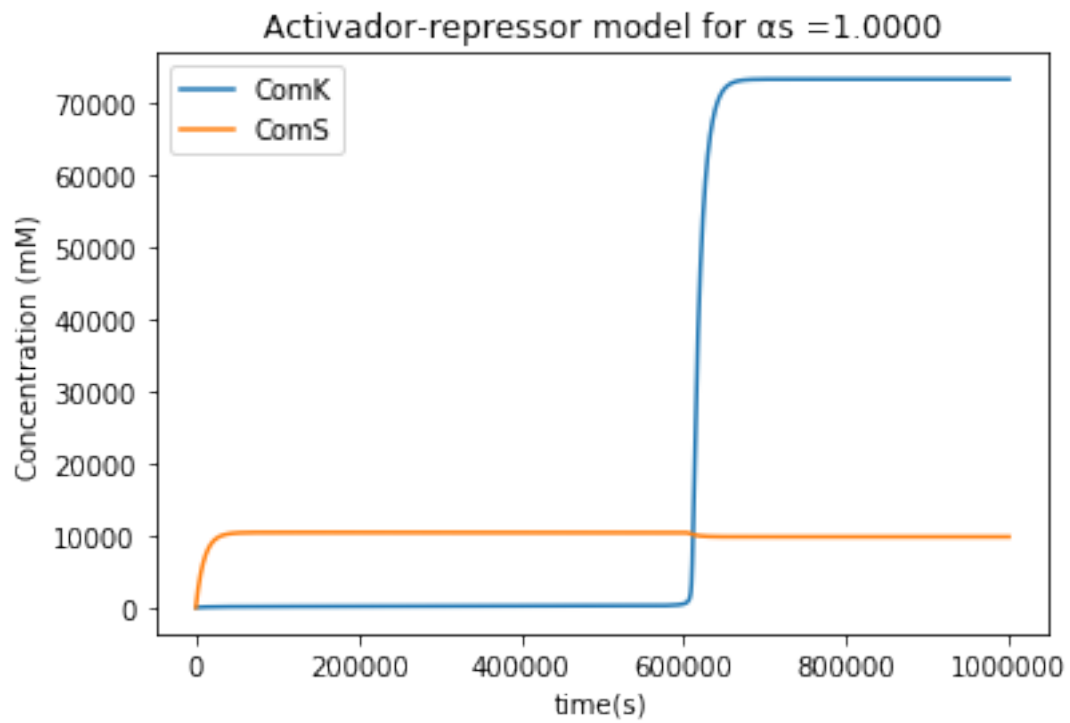


When  $s = 0.8500$

[ComS] tends to the value: 8899.643

[ComK] tends to the value: 229.975

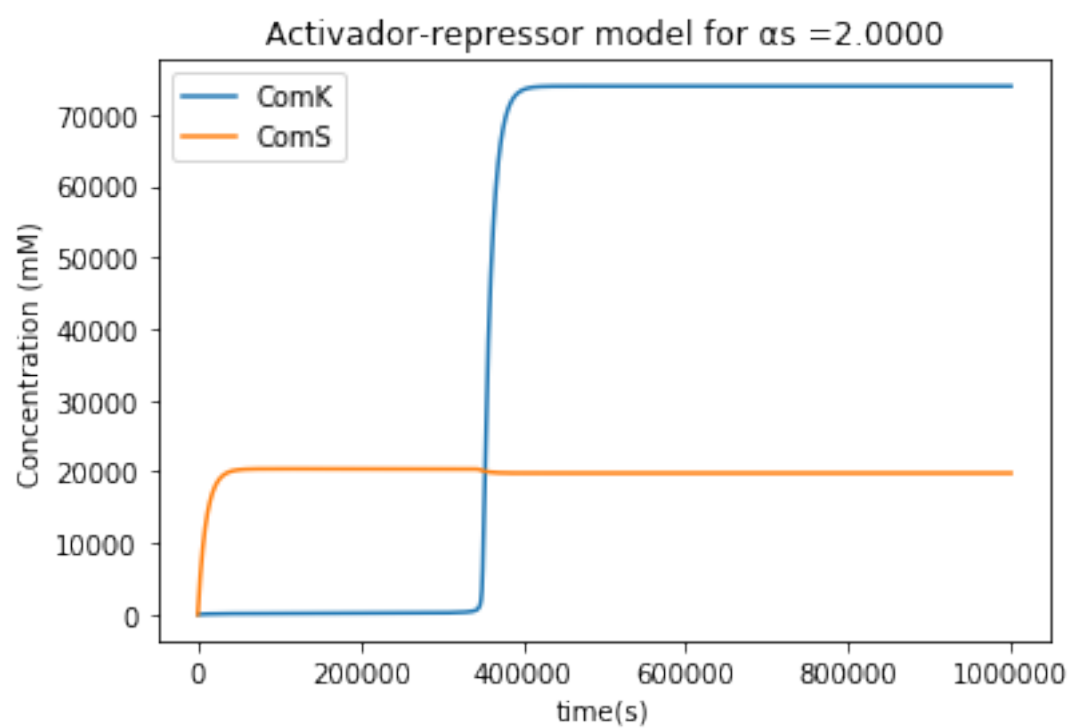
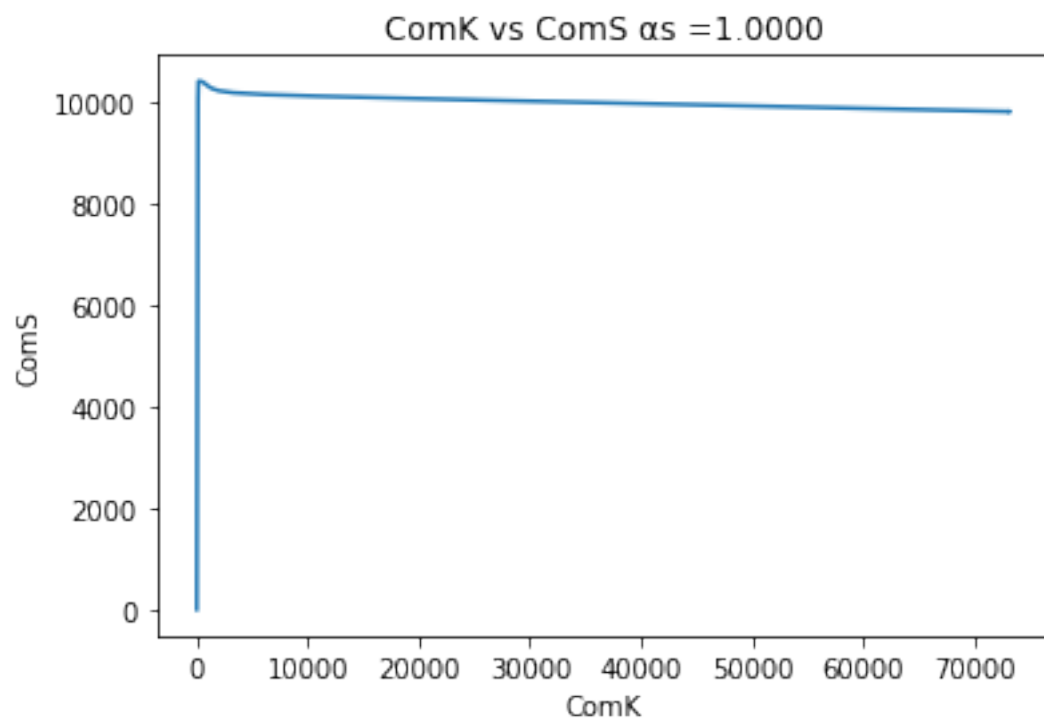




When  $s = 1.0000$

[ComS] tends to the value: 9801.591

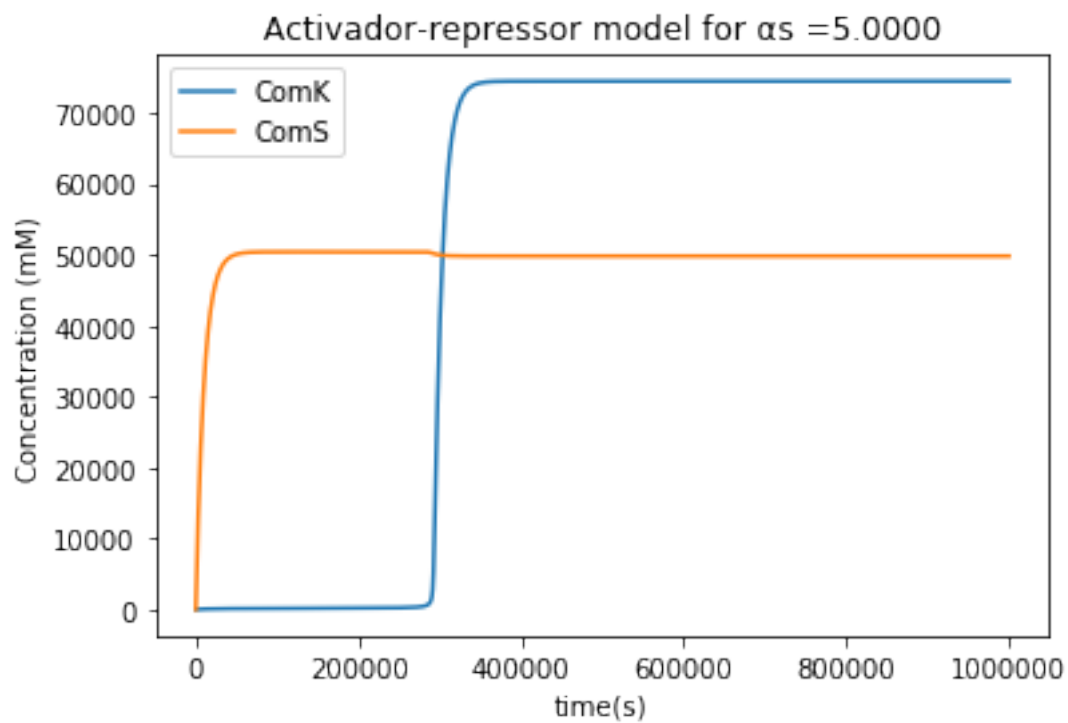
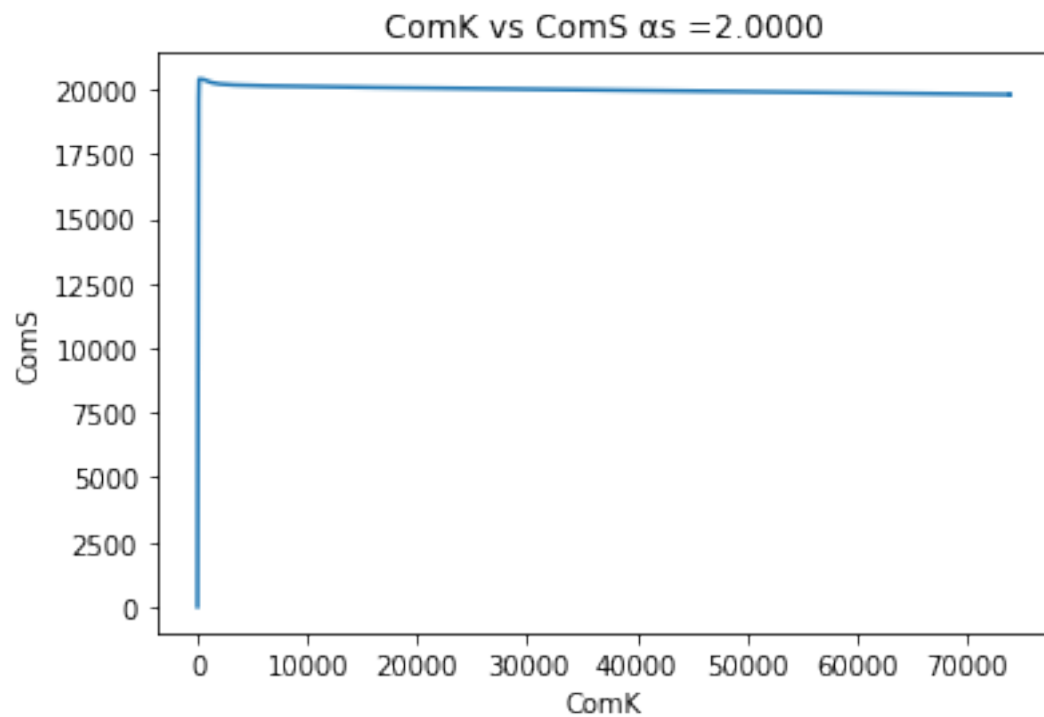
[ComK] tends to the value: 73256.832



When  $s = 2.0000$

[ComS] tends to the value: 19800.797

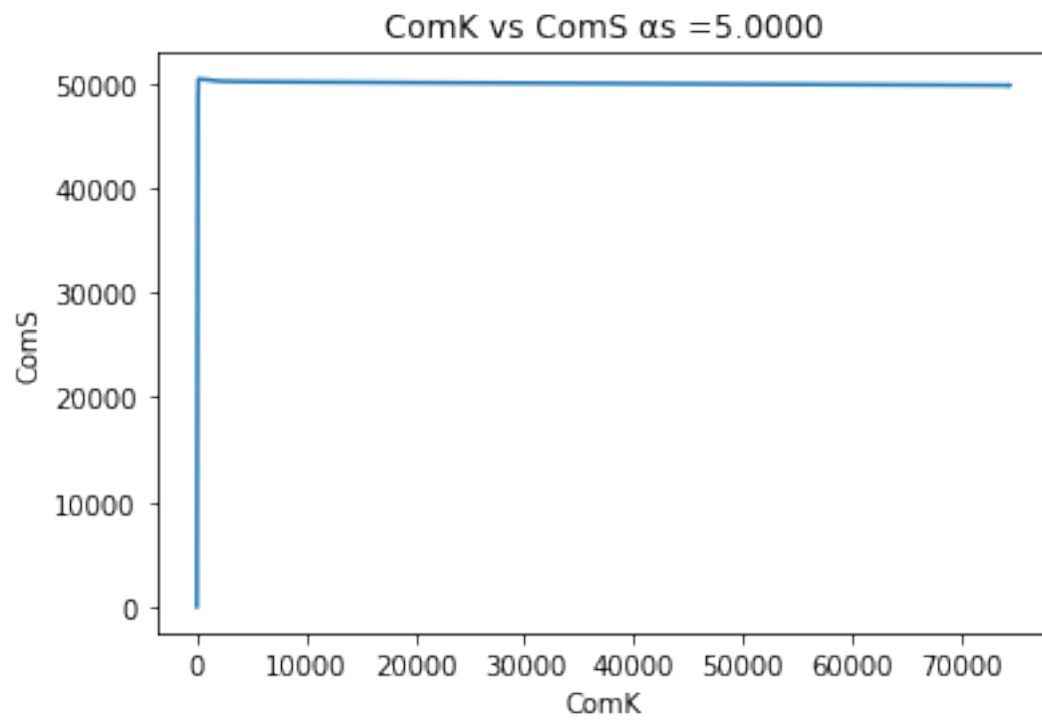
[ComK] tends to the value: 74002.184



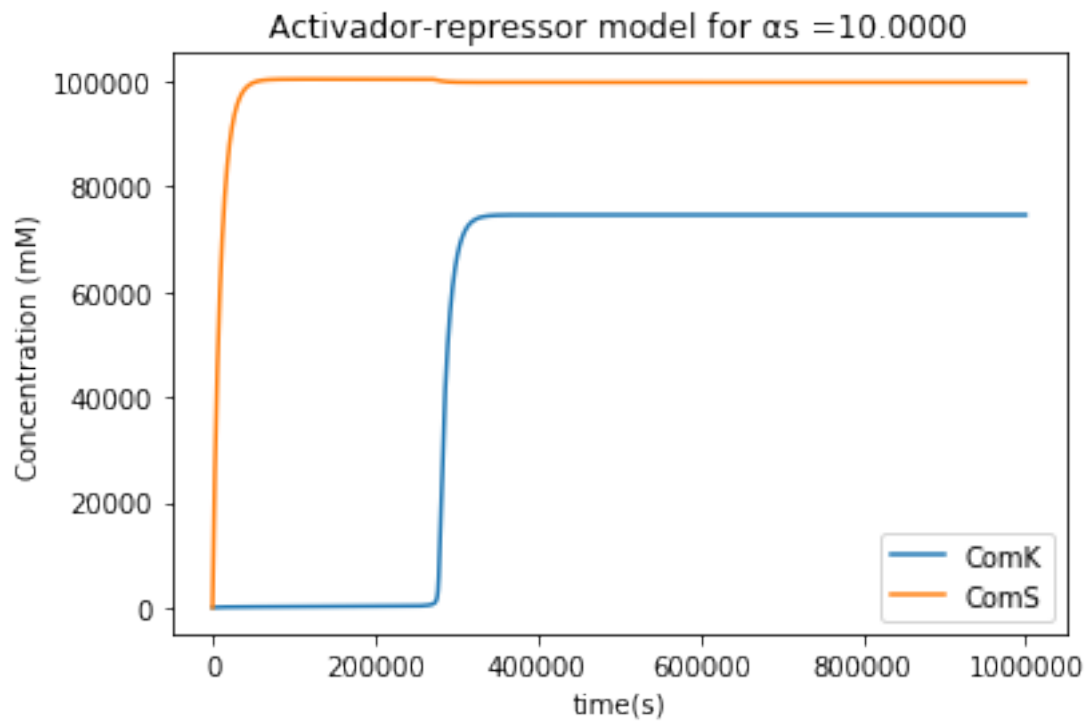
When  $s = 5.0000$

[ComS] tends to the value: 49800.319

[ComK] tends to the value: 74452.236



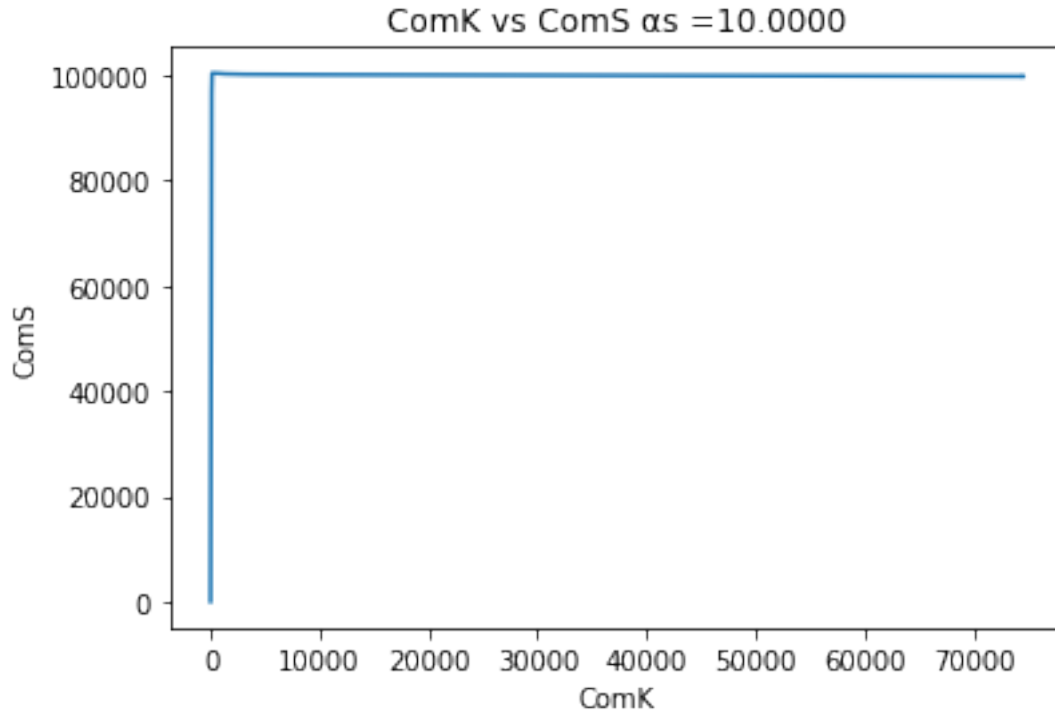




When  $s = 10.0000$

[ComS] tends to the value: 99800.16

[ComK] tends to the value: 74602.729



Observing the behaviour of the system when we change the value of the basal expression rate of ComS, we can notice that raising the value from 0.0004 to 0.8 the concentration of comS increase significantly faster than the concentration of ComK. Once these concentration get its maximum value they stay in a stable state. In other words, they do not change along the time. So, it makes sense that there are not oscillation in the system. However, once the  $s$  is higher than 0.85 the behaviour changes. Now we can see that the concentration of ComK increase sharply from 0 to 74500 nm at a certain moment of time, whereas the concentration of ComS remains the same all the time. Although, we can also notice that the higher is the  $s$  the higher concentration values comes gets while the max concentration of ComK doesn't change, but the time in which comK reach its maximum concentration gets smaller. In addition, we can notice that the change of the  $s$  does not produce any oscillation state or unstable state.

#### 0.0.7 Changing unrepressed expression rate of ComS ( $s$ ) variable:

```
[15]: # solve the ODE problem
bs1 = [0.006,0.2, 0.68,0.8,0.92,2,10]

for bs in bs1:
    a_s = 0.0004
    a_k = 0.00875

    y1 = odeint(dK_dS, y0, t, args=(a_k, a_s,bs))
    dK = y1[:, 0]
    dS = y1[:, 1]
```

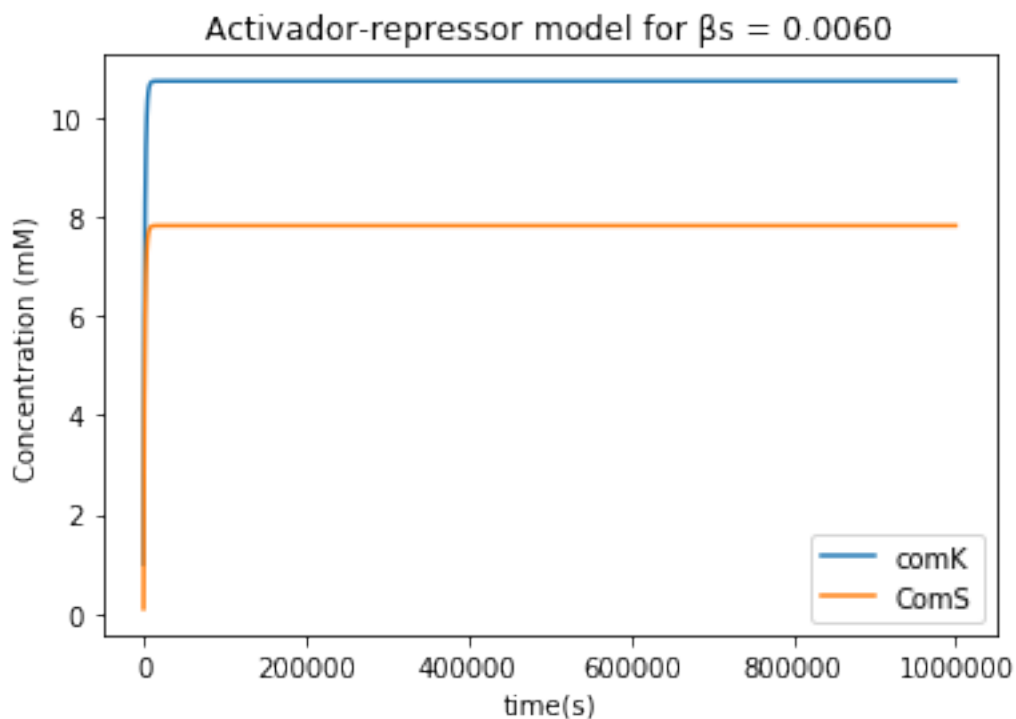
```

plt.plot(t, dK)
plt.plot(t, dS)
plt.xlabel('time(s)')
plt.ylabel('Concentration (mM)')
plt.legend(["comK", "ComS"])
plt.title('Activador-repressor model for s = %.4f'%bs)
plt.show()
len_yS=len(dS)
len_yK=len(dK)
print("When s = %.4f" %bs)
print("[ComS] tents to the value: ", round(float(dS[len_yS-1]),3))
print("[ComK] tents to the value: ", round(float(dK[len_yK-1]),3))

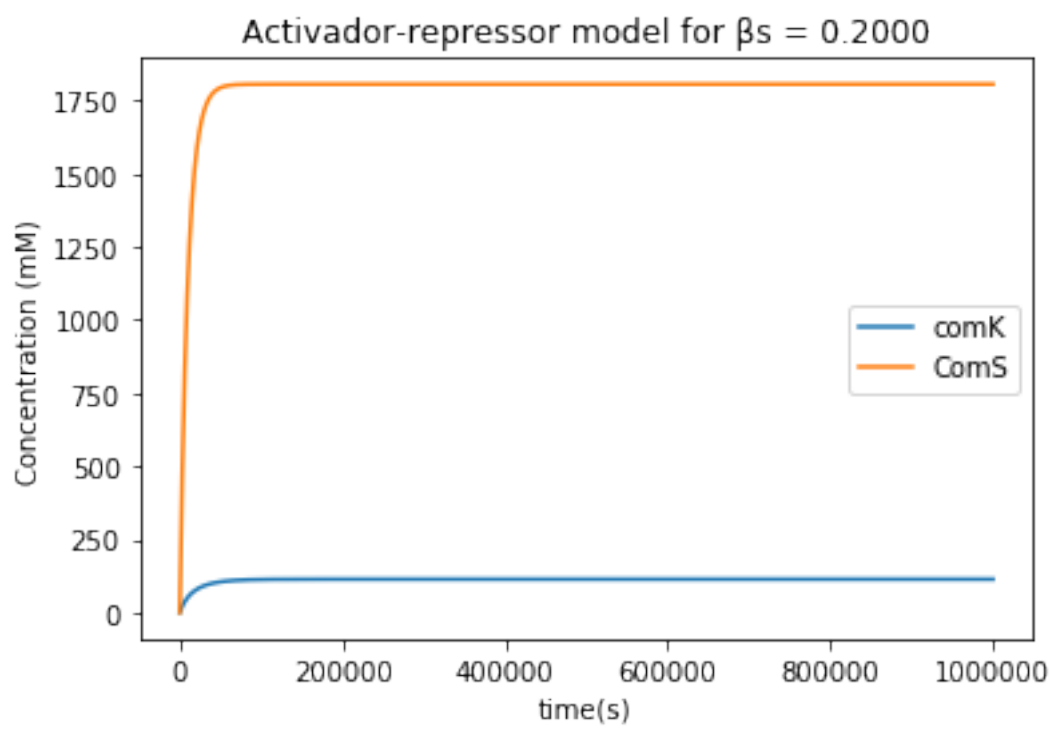
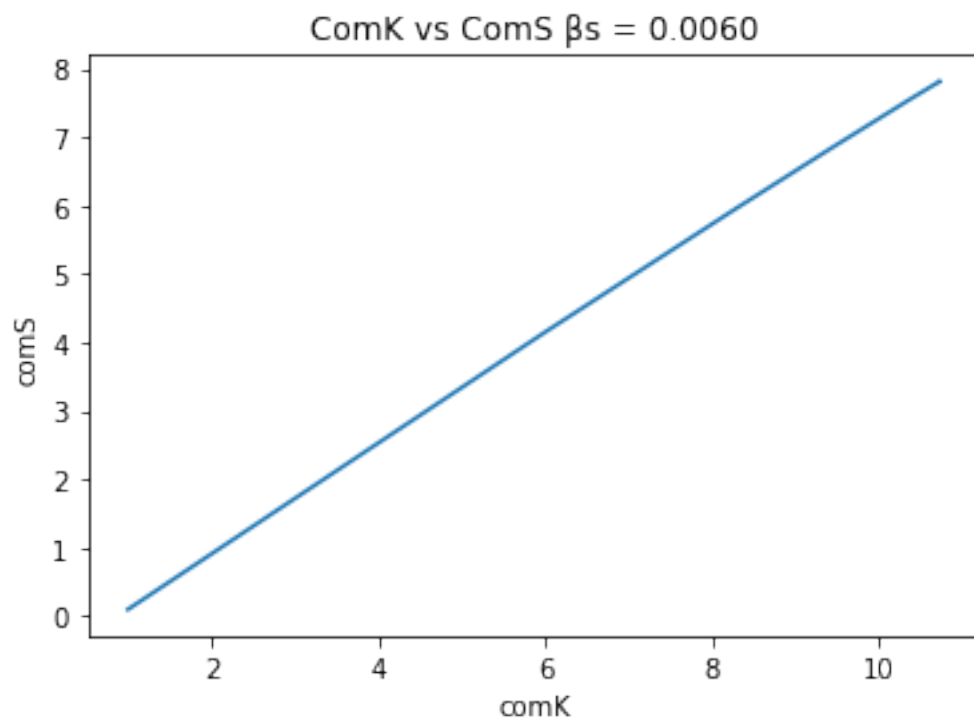
plt.plot(dK, dS)
plt.xlabel('comK')
plt.ylabel('comS')
plt.title('ComK vs ComS s = %.4f' % bs)

plt.show()

```



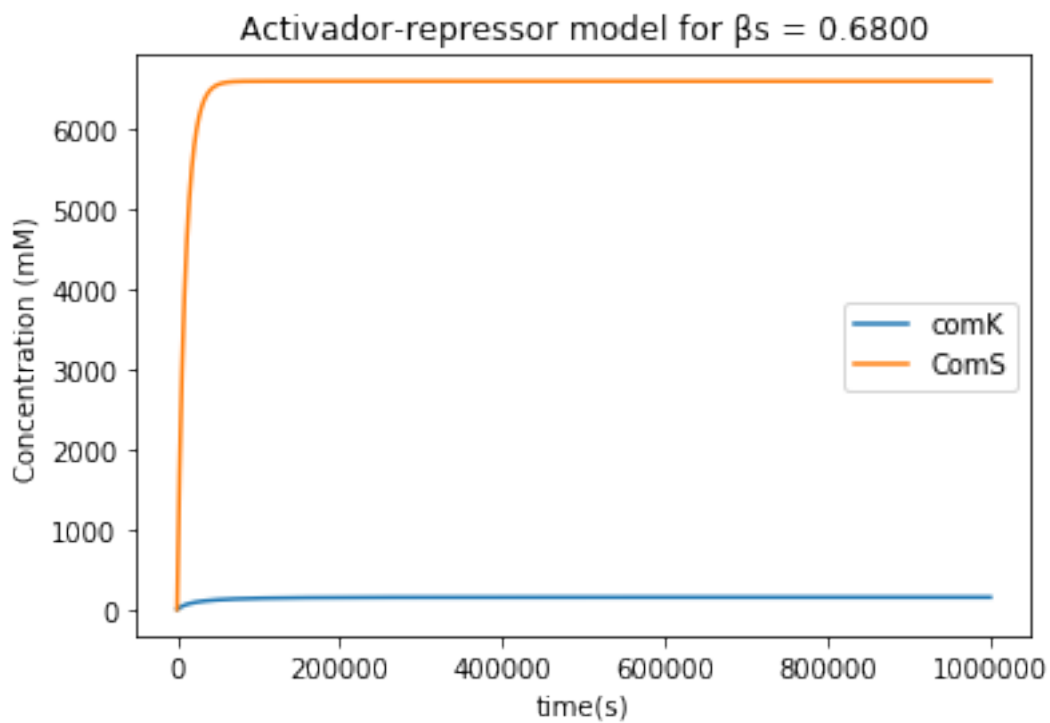
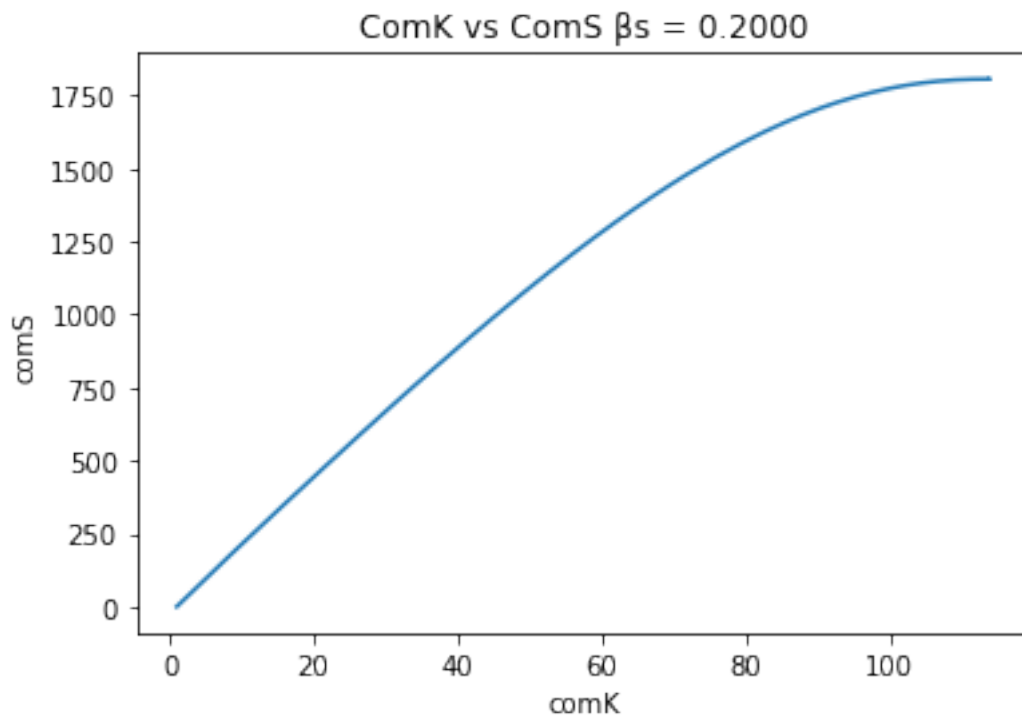
When  $s = 0.0060$   
 [ComS] tents to the value: 7.817  
 [ComK] tents to the value: 10.729



When  $s = 0.2000$

[ComS] tends to the value: 1806.104

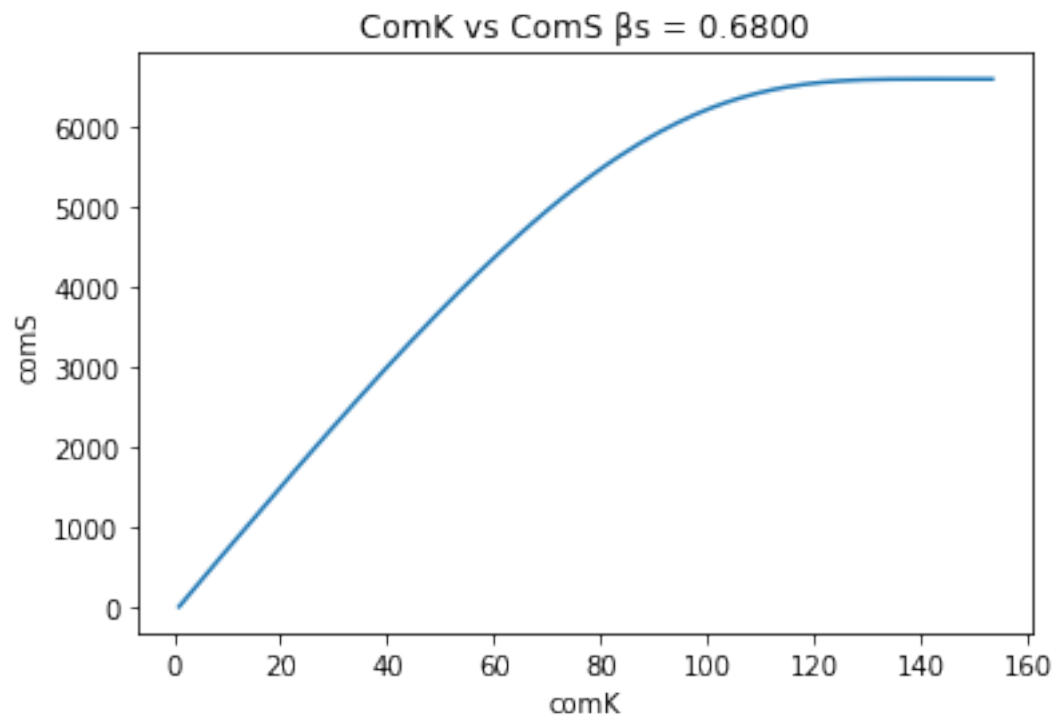
[ComK] tends to the value: 113.96

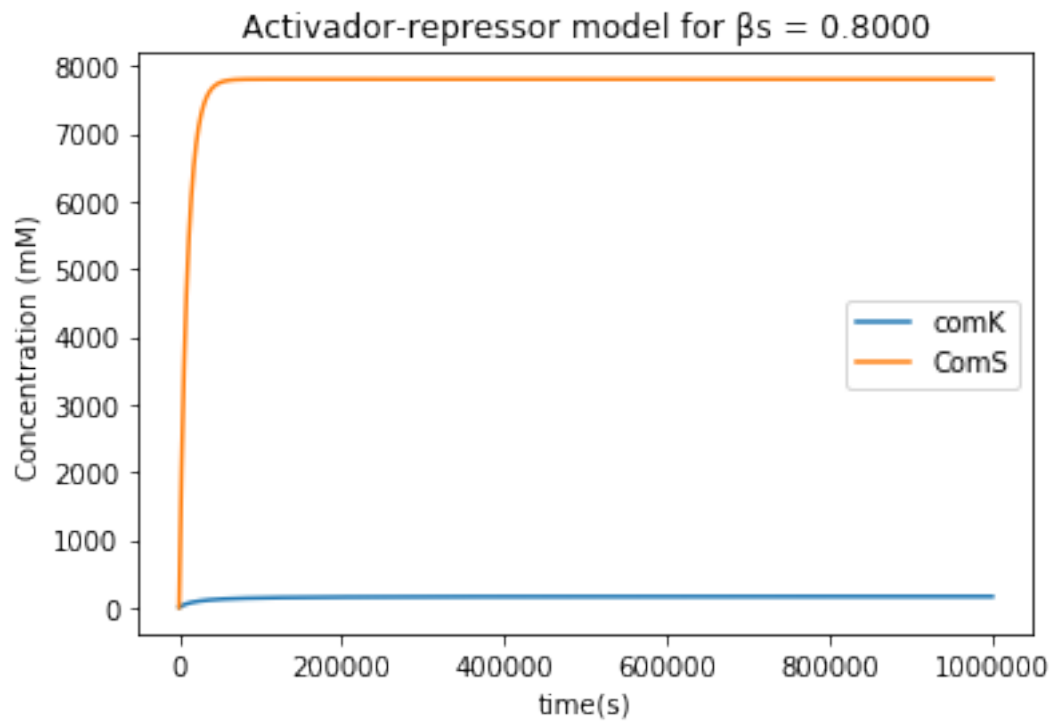


When  $s = 0.6800$

[ComS] tends to the value: 6603.169

[ComK] tends to the value: 153.365

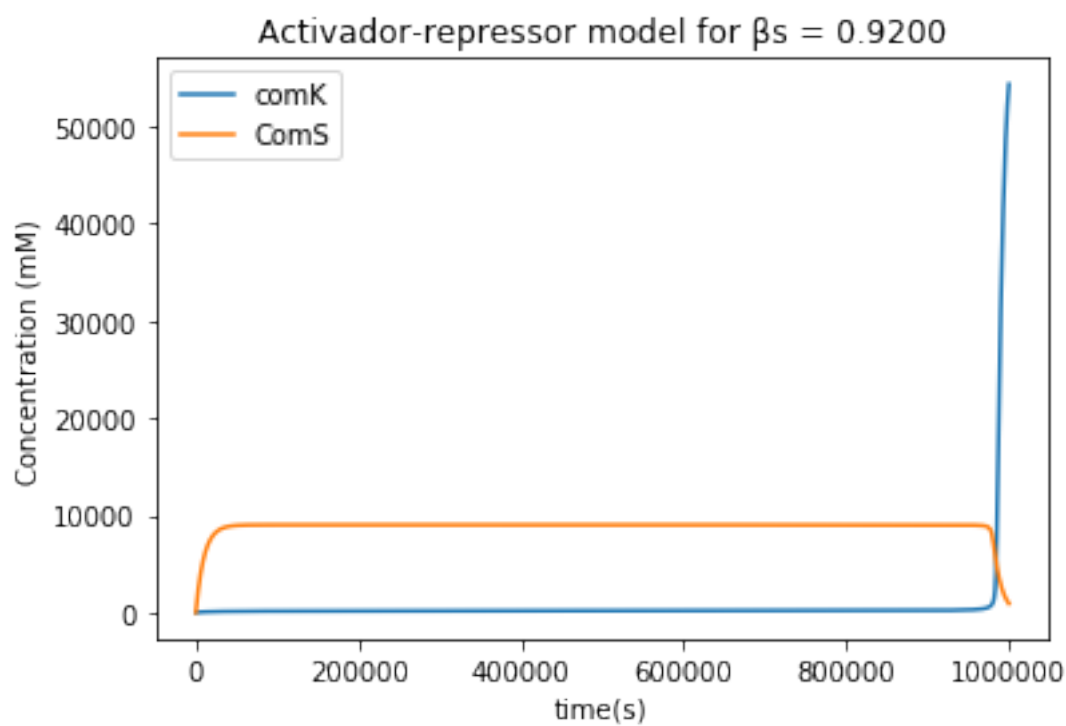
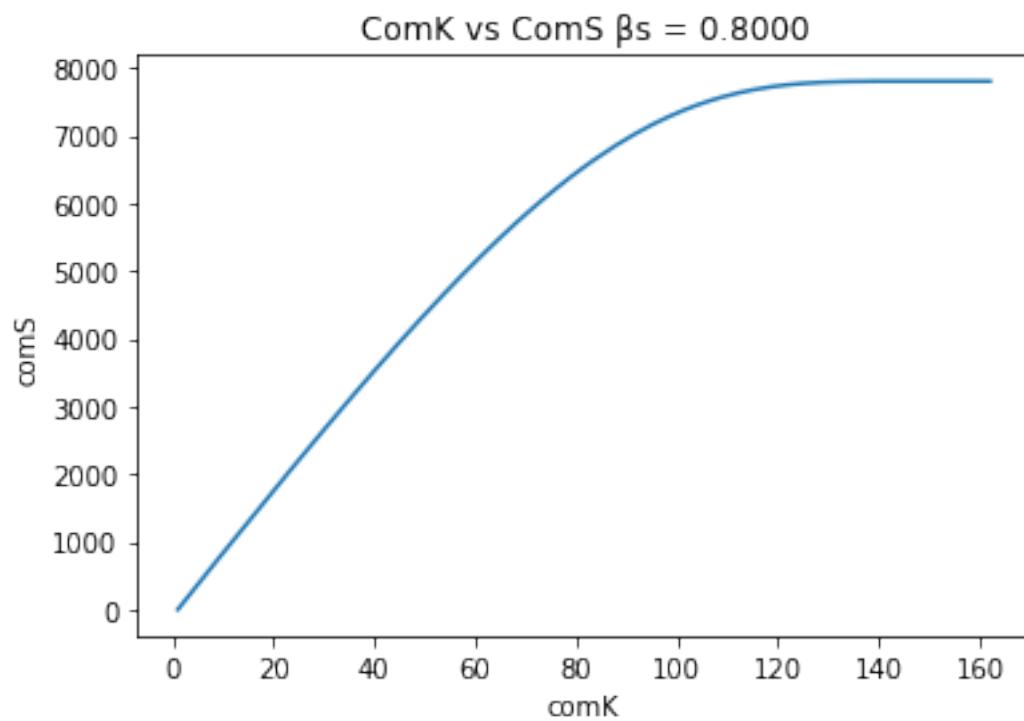




When  $s = 0.8000$

[ComS] tends to the value: 7802.285

[ComK] tends to the value: 162.067

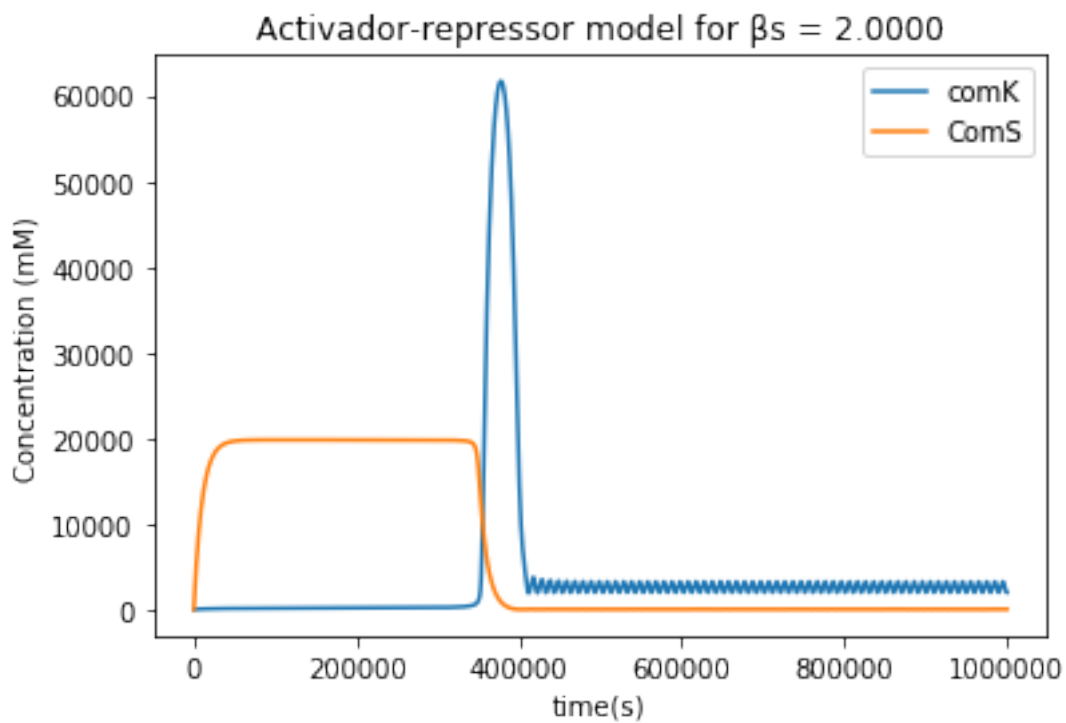
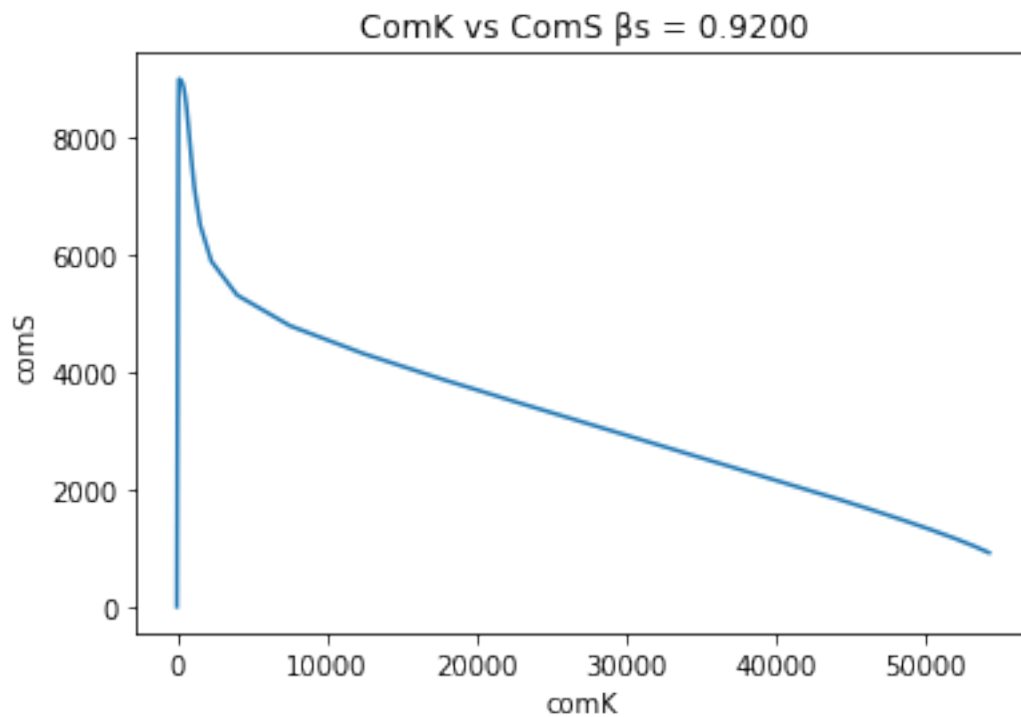




When  $s = 0.9200$

[ComS] tends to the value: 920.625

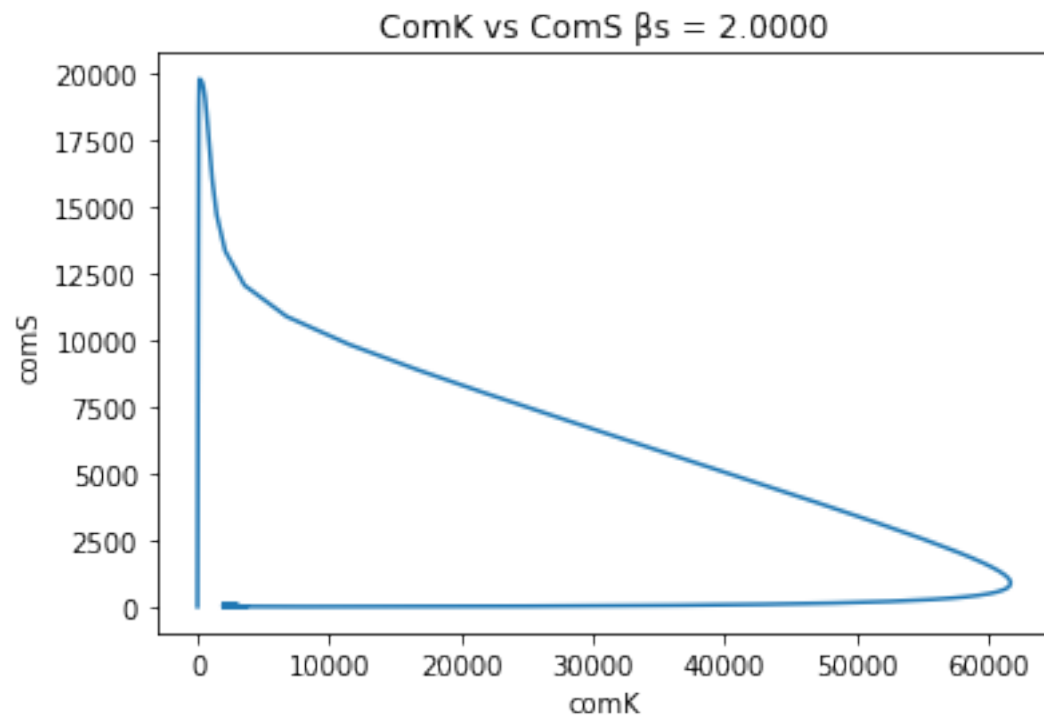
[ComK] tends to the value: 54289.4

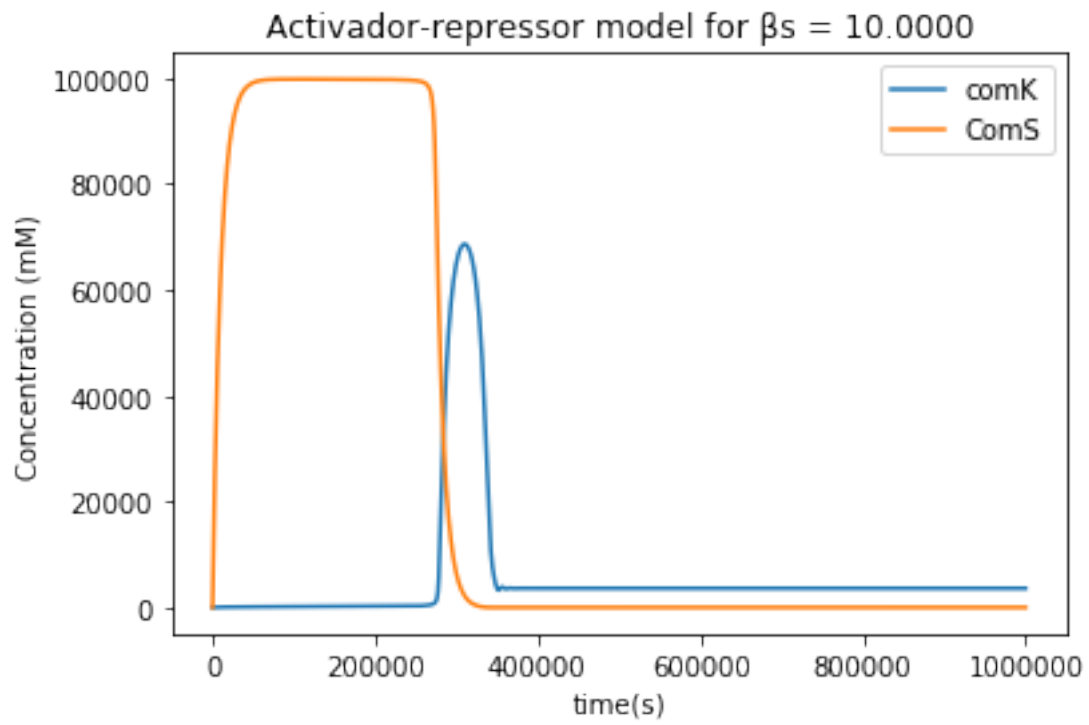


When  $s = 2.0000$

[ComS] tends to the value: 30.337

[ComK] tends to the value: 2004.644

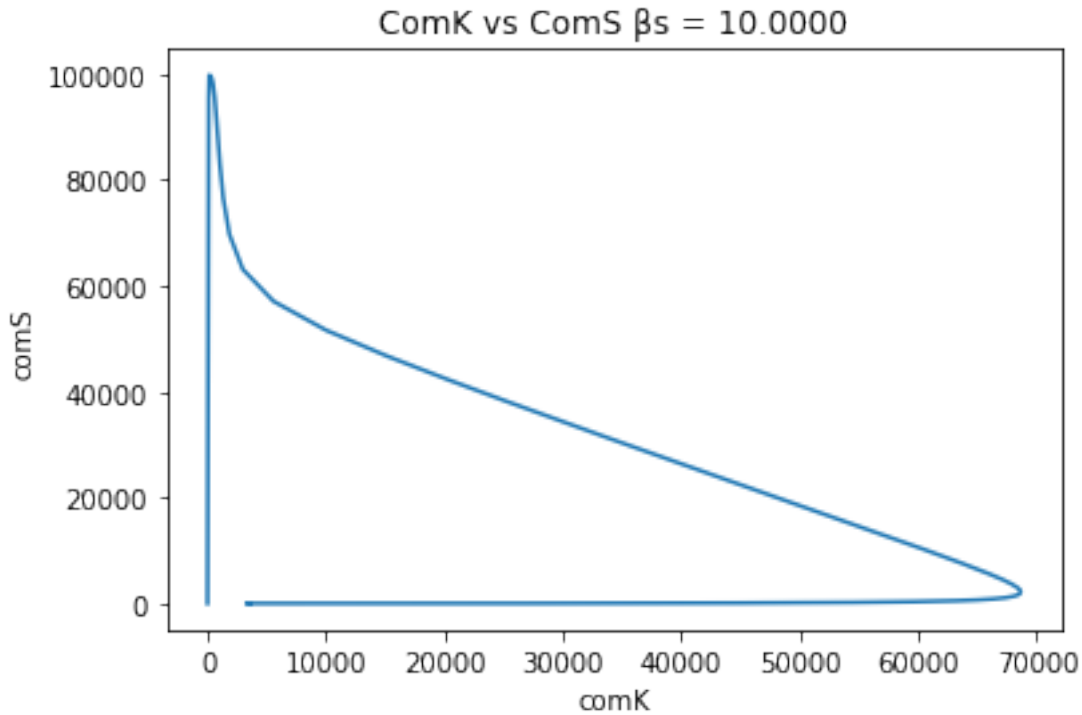




When  $s = 10.0000$

[ComS] tends to the value: 9.649

[ComK] tends to the value: 3614.644



In the first plot we can see that when we low the  $s$  value the concentration of ComS and ComK decrease significantly, but the first one does it faster than the other one. As we can see that the concentration of ComK get higher values. Also, we can see that the there is not oscillation state, the plot of ComK vs ComS is just a straight line. We can notice that when  $s$  increase its value from 0.06 to 0.92 the concentration of ComS increase extremely fast and the concentration of ComK increase slowly. Within these values the system reaches a steady state without changes and no-oscillation.

However, when  $s$  gets values higher than 0.92 we can start seeing some changes. The concentration of ComK stays lower than ComS but at a certain moment the concentration of ComK increase sharply and as soon as this happens the concentration of ComS drops to the minim. Also, in the following cases we can easily see that this sharp concentration raise happens earlier when  $s$  gets higher values. In addition, we can notice that when  $s$  get higher values such as 2 and 10, the concentration on ComS tends to be higher than ComK from the first period of time and suddenly ComK raises its concentration to its maximum and ComS drops its concentration. Also, it is seen how the concentration of ComK becomes unstable which it can been seen in the oscillation plot how the system is unstable. We can also say that the pick which is seen in the “ComK vs ComS” plot when  $s = 2$  and  $s = 10$  is due to this both changes of concentration.

[ ]: