POLYTECHNIC UNIVERSITY OF CATALONIA

Mini Project 1

Position Analysis of wing mechanisms and quadrotor formations

GEOMETRIC FUNDAMENTALS FOR ROBOT DESIGN

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In the mini project presented here, we are asked to put in practice all we have learned during the course, regarding the position analysis methods. As a first exercise it will be simulated the Smartbird's wing mechanism, which is constructible and can be analyzed using the Geaogebra software. The second exercise consists in the position analysis of a quadrotor localization problem, an equivalent 3-R$\underline{\text{P}}$R structure, which is no constructible, hence CUIK is a useful tool in this case.

1 Simulation of Smartbird's wing

Figure 1 shows the mechanical structure of Smartbird's wing. Every small circle represents a revolute joint and vertexs A and B are attached to the body (ground).

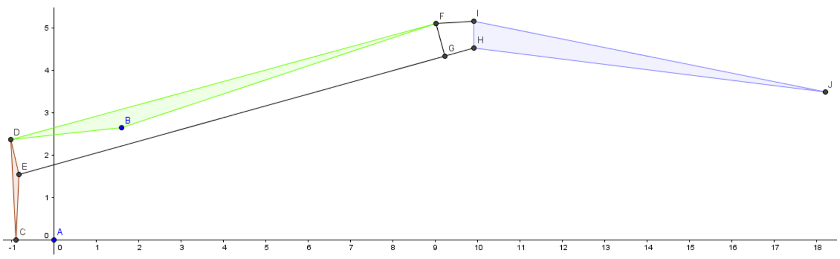

Figure 1: Mechanical structure of Smartbird's wing

First Step

To start with, it will be proved that the mechanism of the Figure 1 is constructive, describing its connections and bilaterations operations.

1. Known points A and B are set to their locations: $A = (0, 0)$ and $B = (1.596, 2.638)$
2. An angle θ is created in order to place the point C with respect to the horizontal line. With that angle and the distance $\overline{AC}$ the point C is placed.
3. Knowing the distances $\overline{CD}$ and $\overline{BD}$ point B and C are bilaterated to find D
4. Bilateratinf C and D with the proper distances, the point E is placed.
5. With the same procedure, points B and D and $\overline{BF}$ and $\overline{DF}$ point F is placed.
6. Drawing a circle over F with disctance $\overline{FG}$ and finding the one of its tangents passing through E, point G is found as the tangent point.
7. Computing the intersction between the previous tangent and a circle centered in E with radious $\overline{EH}$ point H is placed.
8. Bilaterating F and H with the proper lengths, I is placed.
9. Bilaterating H and I with the proper lengths, J is placed.

Second Step

Next, it is built the smartbird's wing mechanism in the Geogebra software. At the end, it is obtained the resultant structure shown in the Figure 1. A *ggb* file showing the result is attached.

Third Step

As the exercise ask, it is activated the motion of the link AC by using the angle variable θ. Next, it is added the trace of the vertex F, H and J. The results of this operation can be seen in the Figure 2.

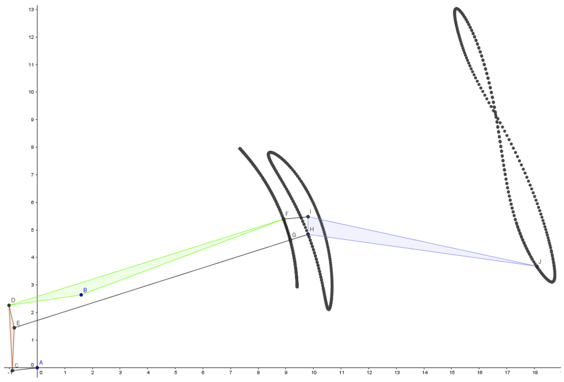

Figure 2: Trace of vertex F, H and J

2 Quadrotor localization

Problem formulation

The problem can be formulated by using distance constraints of the form $(x_i - x_j)^2 + (y_i - y_j)^2 = d_{ij}^2$. This constraints should be applied to all the measured and known distances. We need to take into account the angle θ. Furthermore, we know that point 1 is located at the origin and the position of sensor 2 is in the x axis.

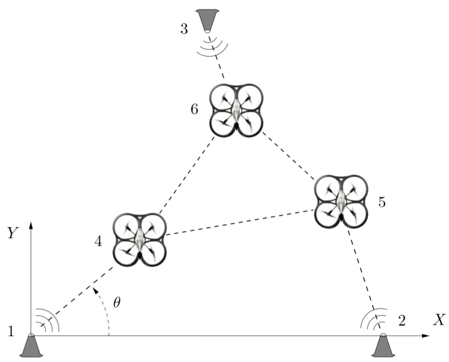

Figure 3: Problem description

Given the previous information we have:

$$x_1 = 0 \tag{1}$$
$$y_1 = 0 \tag{2}$$
$$y_2 = 0 \tag{3}$$
$$x_4 = \cos(\theta) d_{14} \tag{4}$$
$$y_4 = \sin(\theta) d_{14} \tag{5}$$
$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = d_{12}^2 \tag{6}$$
$$(x_1 - x_3)^2 + (y_1 - y_3)^2 = d_{13}^2 \tag{7}$$
$$(x_3 - x_2)^2 + (y_3 - y_2)^2 = d_{32}^2 \tag{8}$$
$$(x_4 - x_5)^2 + (y_4 - y_5)^2 = d_{45}^2 \tag{9}$$
$$(x_4 - x_6)^2 + (y_4 - y_6)^2 = d_{46}^2 \tag{10}$$
$$(x_6 - x_5)^2 + (y_6 - y_5)^2 = d_{65}^2 \tag{11}$$
$$(x_2 - x_5)^2 + (y_2 - y_5)^2 = d_{25}^2 \tag{12}$$
$$(x_3 - x_6)^2 + (y_3 - y_6)^2 = d_{36}^2 \tag{13}$$

As CuikSuite does not deal with sinusoidal functions we can replaced the equations related to point 4 coordinates by:

$$x_4 = c_1 d_{14} \tag{14}$$
$$y_4 = s_1 d_{14} \tag{15}$$
$$c_1^2 + s_1^2 = 0 \tag{16}$$

The points 1, 2, 3 are fixed and known, and so are their coordinates. Hence, equations 1, 2, 3, 6, 7 and 8 can be considered as the definition of the framework. We still consider the angle θ unknown due to its uncertainty (it's value, although bounded, is unknown). Thus, we can consider the system of equations defining the problem as follows:

$$(x_4 - x_5)^2 + (y_4 - y_5)^2 = d_{45}^2 \tag{17}$$
$$(x_4 - x_6)^2 + (y_4 - y_6)^2 = d_{46}^2 \tag{18}$$
$$(x_6 - x_5)^2 + (y_6 - y_5)^2 = d_{65}^2 \tag{19}$$
$$(x_2 - x_5)^2 + (y_2 - y_5)^2 = d_{25}^2 \tag{20}$$
$$(x_3 - x_6)^2 + (y_3 - y_6)^2 = d_{36}^2 \tag{21}$$
$$x_4 = c_1 d_{14} \tag{22}$$
$$y_4 = s_1 d_{14} \tag{23}$$
$$c_1^2 + s_1^2 = 1 \tag{24}$$

Being x_4, y_4, x_5, y_5, x_6, y_6, s_1 and c_1 unknowns and x_1, y_1, x_2, y_2, x_3 and y_3 are constants.

Forcing angular constraint

We need to ensure that $\theta \in [\theta_{min}, \theta_{max}]$. As the angle is not directly a variable of our system but its sinus and cosinus, we can constrain its cosinus because

$$\cos(\alpha) \in [\cos(\beta), 1] \tag{25}$$

is equivalent to

$$\alpha \in [-\beta, \beta] \tag{26}$$

So, given a constraint in the form of 26 we can transform it to the form 25. It should be noticed that this procedure is only valid when $\theta_{min} = -\theta_{max}$, but not in a general case when the angle range is not centered in 0.

Analysis of the system

We consider the system in 17-24, so we have 8 equations. The number of variables, as stated in first section is also 8.

Box bounding

In order to reduce the computational cost of the CUIK software, the problem can be bounded by reducing the dimension of the initial search box. Observing the Figure 3 it can be seen that several circles are limiting how farther the quadrotors formation can be. Namely, if it is considered the distances d_{14} and d_{45}, the point 5 can be as further as a circle of radius $d_{14} + d_{45}$. Hence, extending this conclusion to all the sides, it can be described the following limits. The distances between quadrotors are considered equal (d_q).

$$-d_{14} < x_4 < d_{14} \tag{27}$$

$$-d_{14} < y_4 < d_{14} \tag{28}$$

$$-d_{14} - d_q < x_5 < d_{14} + d_q \tag{29}$$

$$-d_{14} - d_q < y_5 < d_{14} + d_q \tag{30}$$

$$-d_{14} - d_q < x_6 < d_{14} + d_q \tag{31}$$

$$-d_{14} - d_q < y_6 < d_{14} + d_q \tag{32}$$

Furthermore, since the cosine and sinus of the angel ϕ are considered as variables, they have to be limited between -1 to 1, or to another number depending the specifications of the problem.

$$-1 < \cos(\phi) < 1 \tag{33}$$

$$-1 < \sin(\phi) < 1 \tag{34}$$

CUIK solver

Now that the problem is well defined, it can be translated to the scribes needed for CUIK. First of all, the problems specifies a $\sigma = 0.05$ that will be introduced in the

*.param file, where all the CUIK solver parameters are defined. Next, it is created a new file *.cuik, where all the equations and constraints will be paced (see annex to check the final file). The statement propose three different configurations (Table 1) to be tested and evaluated. In further sections are shown the results.

Situation	d_s	d_q	d_{14}	d_{25}	$d_{3,5}$	θ_{min}	θ_{min}
A	3	1	2	2	2	-3ž	3ž
B	3	1	2	2	2	0ž	60ž
C	3	3	4	4	4	0ž	360ž

Table 1: Studying Cases

Quadrotor configurations

In this section, The different solutions for every case are shown in the Figures 4, 5 and 6.

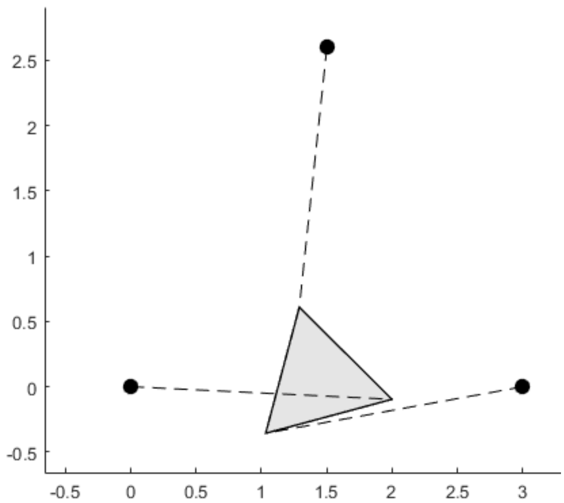

Figure 4: Unique solution of the system in case A

For the first case, CUIK only give one solution (Figure 4). This is produced due to the dimensions given to the different parameters, such as the distance between sensors (d_s), the distance between quadrotors (d_q) and the length imposed from quadrotor to sensor (d_{14},d_{36} and d_{25}).

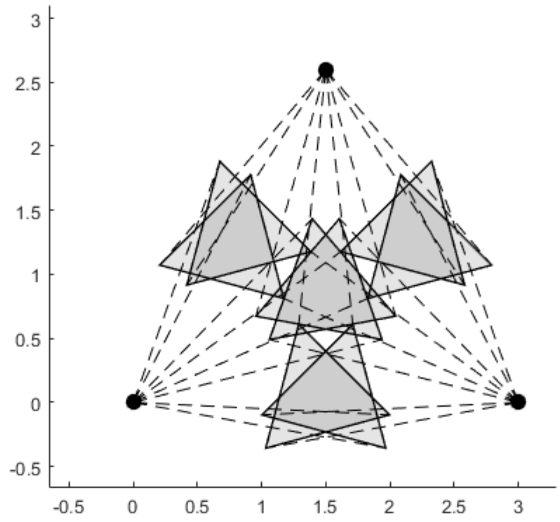

Figure 5: Solutions of the system in case B

In the case B, the configuration between parameters allow the system adopt 8 solutions (Figure 5). Hence, the solution is not unique.

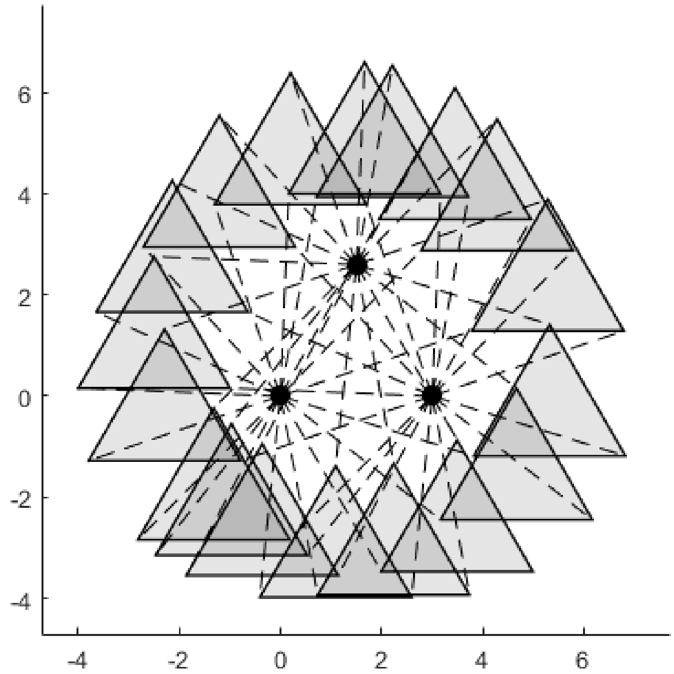

Figure 6: Some of the solutions of case C. Only some reduced number of solutions are plotted in order to obtain a clear image.

The last configuration (case C) have considerable amount of solutions. In Figure 6 are draw several of them, however, a circle can be appreciated as the path followed by the possible configurations.

Observing variables y_4 and x_5

Case A

In this case, a single solution is found, so it is clear that there is only a single combination of x_5 and y_4 (a single point) that corresponds to a solution of the system.

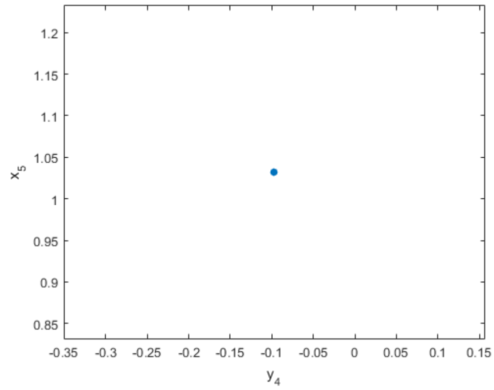

Figure 7: Variables x_5 and y_4 of the solutions of case A. Single point.

Case B

In this case, as the constraints are relaxed (allowing $\theta \in (0, 360)$) we observe many isolated points corresponding to a rotated/mirrored versions of the previous one. Still, we have a finite set of solution points.

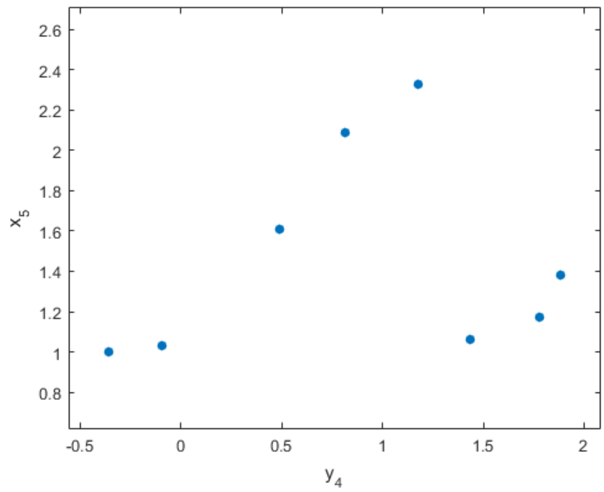

Figure 8: Variables x_5 and y_4 of the solutions of case B. Finite set of points.

Case C

Due to the particular geometry of this case (the triangle of sensors and quadrotors are the same and all sensor-quadrotor distances are also the same) the lines joining each quadrotor to its corresponding sensor are aligned. This gives rise to an infinite solution set. Intuitively as both quadrotor and sensor shapes are the same, regarding the closed curve, we can consider the system as composed by two points separated a certain distance. Thus, fixing one of them (for instance the sensor equivalent point) and moving the other we have a circular trajectory) as seen in figure 9.

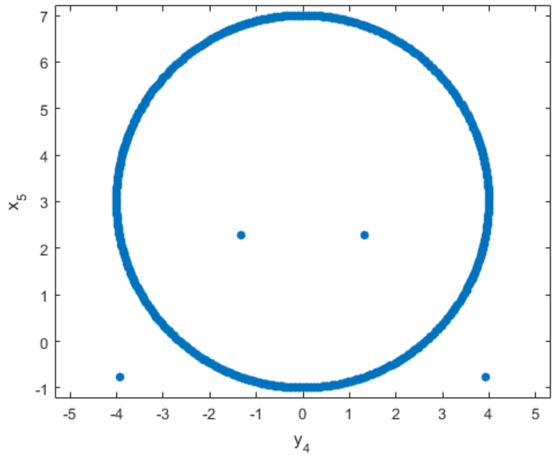

Figure 9: Variables x_5 and y_4 of the solutions of case C

The isolated points correspond to configurations where sensor-quadrotor lines are not parallel and hence no movement can be done without changing the sensor-quadrotor distances. Figure 10 shows the four isolated points in figure 9

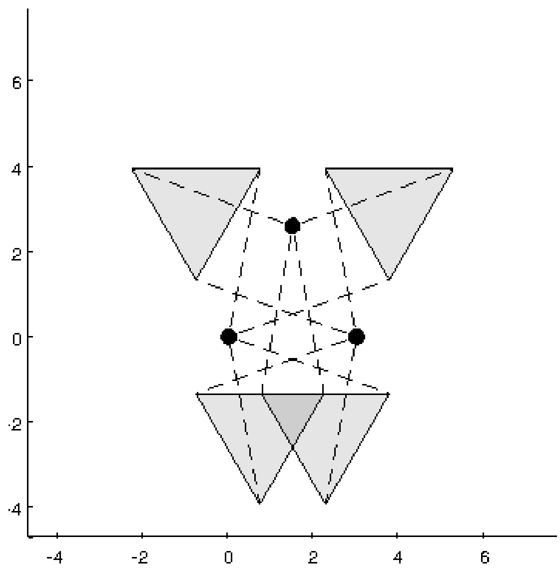

Figure 10: Isolated configurations of case C

3 Annex 1

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[CONSTANTS]

% Situation A
d14:=2.0
d25:=2.0
d36:=2.0
ds:=3.0
dq:=1.0
phimin:=-3.0*pi/180
phimax:=3.0*pi/180
X1:=0
Y1:=0
X2:=ds
Y2:=0
X3:=ds/2
Y3:=sin(pi/3)*ds

[SYSTEM VARS]

x4:[-d14,d14]
y4:[-d14,d14]
x5:[-d14-dq,d14+dq]
y5:[-d14-dq,d14+dq]
x6:[-d14-dq,d14+dq]
y6:[-d14-dq,d14+dq]
s1:[-1,1]
c1:[cos(phimin),1]
%c1:[-1,1]

[SYSTEM EQS]

% Distance equations
x4^2+x5^2-2*x5*x4+y4^2+y5^2-2*y5*y4 = dq^2;
X2^2+x5^2-2*x5*X2+Y2^2+y5^2-2*y5*Y2 = d25^2;
X3^2+x6^2-2*x6*X3+Y3^2+y6^2-2*y6*Y3 = d36^2;
x4^2+x6^2-2*x6*x4+y4^2+y6^2-2*y6*y4 = dq^2;
x6^2+x5^2-2*x5*x6+y6^2+y5^2-2*y5*y6 = dq^2;
x4 = d14*c1;
y4 = d14*s1;

% Circle equations
s1^2 + c1^2 = 1;
```