

SISTEMAS LINEARES

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METODO DE CRAMER

Método de Cramer é um método utilizado para encontrar o conjunto solução de um sistema de equação linear possível determinado. Essa regra utiliza o determinante das matrizes associadas ao sistema para encontrar as soluções do sistema. O método é aplicado principalmente em sistemas lineares que possuem 3 incógnitas e 3 equações, mas pode ser empregado também em sistemas lineares 2 por 2.



1) METODO DE CRAMER

a)

Handwritten notes for solving a system of linear equations:

$\begin{array}{l} J-A) x-y=3 \\ 2x+y=9 \end{array}$

$\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 - (-2) = 3$

$x = \frac{1}{3} \begin{vmatrix} 3 & -1 \\ 9 & 1 \end{vmatrix} = \frac{3 - (-9)}{3} = \frac{12}{3} = 4$

$y = \frac{1}{3} \begin{vmatrix} 1 & 3 \\ 2 & 9 \end{vmatrix} = \frac{9 - 6}{3} = 1$

$D = 3$

b)

Handwritten notes for solving a system of linear equations:

$\begin{array}{l} y \\ 2 \\ 9 \end{array} \begin{array}{l} 3 \\ 9 \end{array} \begin{array}{l} 9-6=3 \\ D_y = \frac{3}{3} = 1 \end{array}$

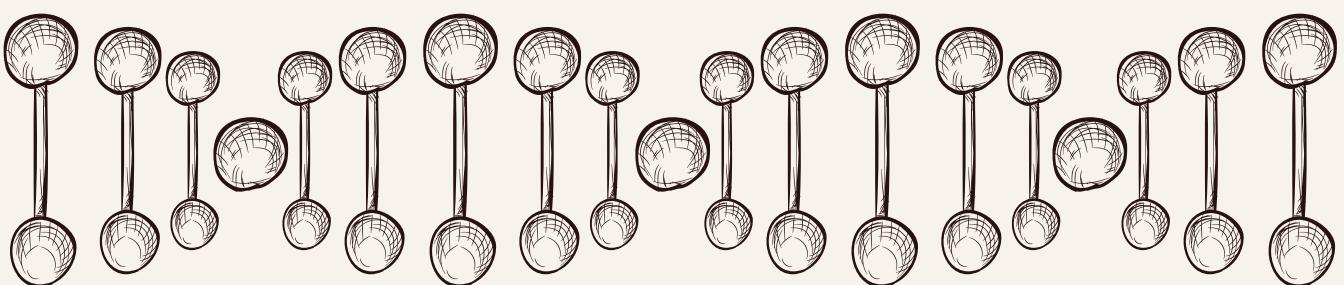
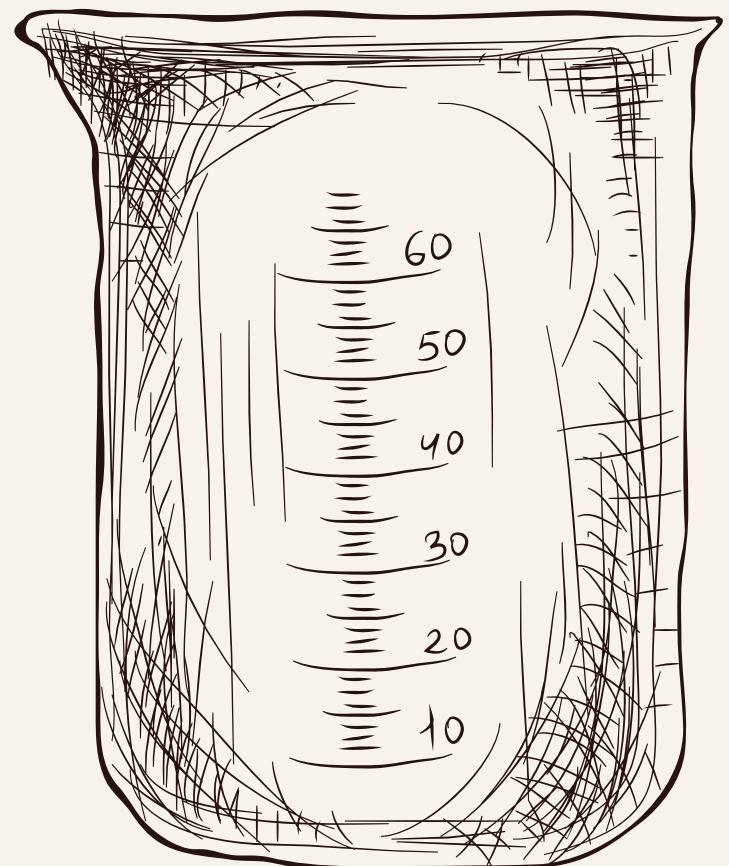
$\begin{array}{ll} b) x = 3y & x-3=0 \\ x+2y=10 & x+2y=10 \end{array}$

$\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix} = 2 - (-3) = 5$

$D_x = \frac{1}{5} \begin{vmatrix} 0 & -3 \\ 10 & 2 \end{vmatrix} = \frac{0 - (-30)}{5} = 6$

$D_y = \frac{1}{5} \begin{vmatrix} 3 & 0 \\ 1 & 10 \end{vmatrix} = \frac{30 - 0}{5} = 6$

$c) x+4=10 \quad D = \frac{1}{5} \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = \frac{-1 - 2}{5} = -3$



1) METODO DE CRAMER

c)

Handwritten notes for Cramer's method:

$\begin{array}{|cc|} \hline & 10 & 2 \\ \hline D_y = & \begin{array}{|cc|} 1 & 0 \\ 1 & 10 \end{array} & 10 - 0 = 10 & D_y \frac{10}{5} = 2 \\ \hline \end{array}$

$C) \begin{array}{l} x + y = 10 \\ 2x - y = 8 \end{array} \quad D = \begin{array}{|cc|} 1 & 1 \\ 2 & -1 \end{array} \quad -1 - 2 = -3$

$D_x = \begin{array}{|cc|} 10 & 1 \\ 8 & -1 \end{array} \quad 10 - 8 = 2 \quad D_x = \frac{2}{-3} = -\frac{2}{3}$

$D_y = \begin{array}{|cc|} 1 & 10 \\ 2 & 8 \end{array} \quad 8 - 20 = -12 \quad D_y = \frac{-12}{-3} = 4$

d)

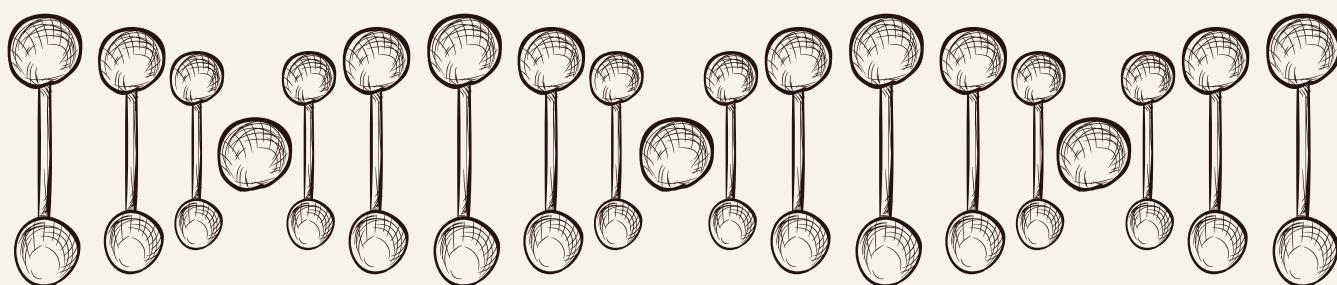
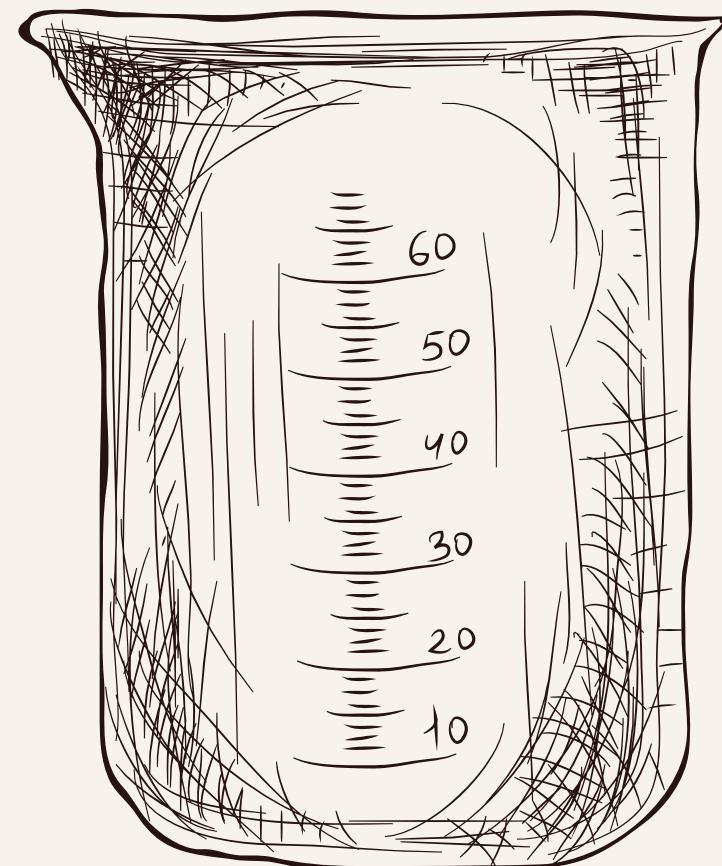
Handwritten notes for Cramer's method:

$\begin{array}{|cc|} \hline & & \\ \hline D) & x - 2y = 0 & (1 \quad -2) \cdot 5 - (-6) = 15 - (-6) = 21 = 1 \cdot x \\ 3x + 5y = 55 & (3 \quad 5) & \end{array}$

$x = \begin{array}{|cc|} 10 & -2 \\ 55 & 5 \end{array} \quad 0 - (-55) = 55 \quad D_x = \frac{55}{21} = \frac{55}{21}$

$y = \begin{array}{|cc|} 1 & 0 \\ 3 & 55 \end{array} \quad 55 - 0 = \frac{55}{21} = \frac{55}{21}$

E) $4x + y = 4 \quad 0 \begin{pmatrix} 4 & 1 \end{pmatrix} = -20 - 2 = -22$



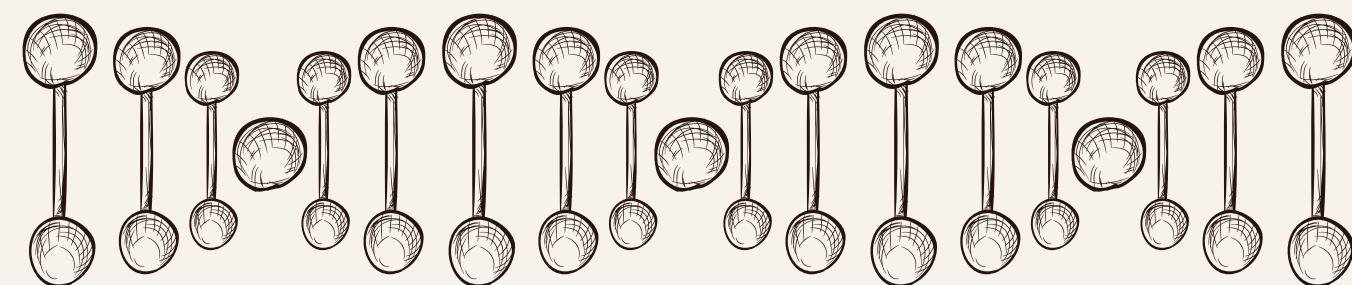
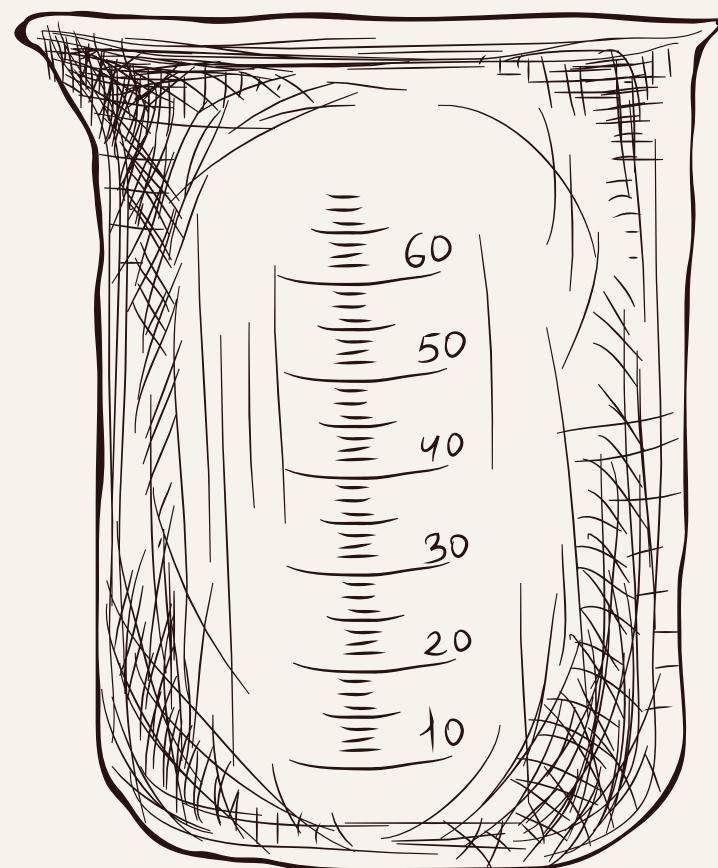
1) METODO DE CRAMER

e)

$$\begin{aligned} & D = \begin{vmatrix} 3 & 5 \\ 2 & -5 \end{vmatrix} = 3 \cdot (-5) - 2 \cdot 5 = -15 - 10 = -25 \\ & \begin{cases} 4x + y = 4 \\ 2x - 5y = 9 \end{cases} \quad D_x = \begin{vmatrix} 4 & 1 \\ 2 & -5 \end{vmatrix} = -20 - 2 = -22 \\ & D_y = \begin{vmatrix} 3 & 5 \\ 2 & 9 \end{vmatrix} = 3 \cdot 9 - 2 \cdot 5 = 27 - 10 = 17 \\ & x = \frac{D_x}{D} = \frac{-22}{-25} = \frac{22}{25} \\ & y = \frac{D_y}{D} = \frac{17}{-25} = -\frac{17}{25} \end{aligned}$$

f)

$$\begin{aligned} & \begin{cases} x + y = 8 \\ 4x - 6y = 52 \end{cases} \quad D = \begin{vmatrix} 1 & 1 \\ 4 & -6 \end{vmatrix} = -6 - 4 = -10 \\ & D_x = \begin{vmatrix} 8 & 1 \\ 52 & -6 \end{vmatrix} = -48 - 52 = -100 \quad x = \frac{D_x}{D} = \frac{-100}{-10} = 10 \\ & D_y = \begin{vmatrix} 1 & 1 \\ 4 & -6 \end{vmatrix} = -6 - 4 = -10 \quad y = \frac{D_y}{D} = \frac{-10}{-10} = 1 \end{aligned}$$



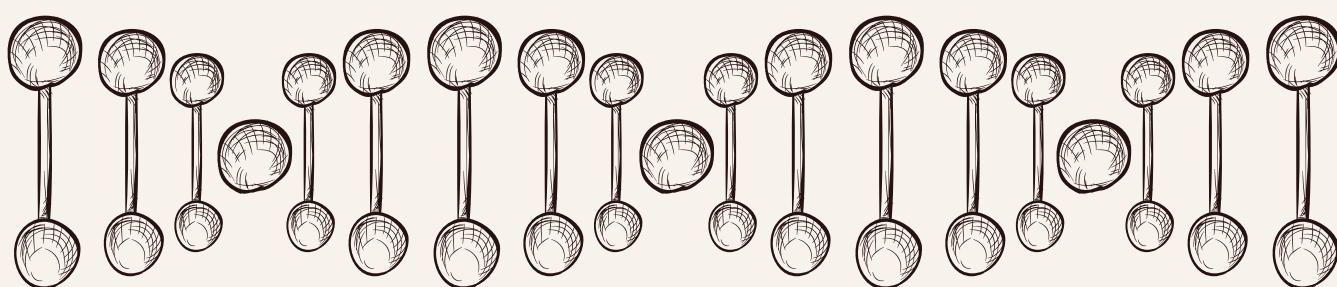
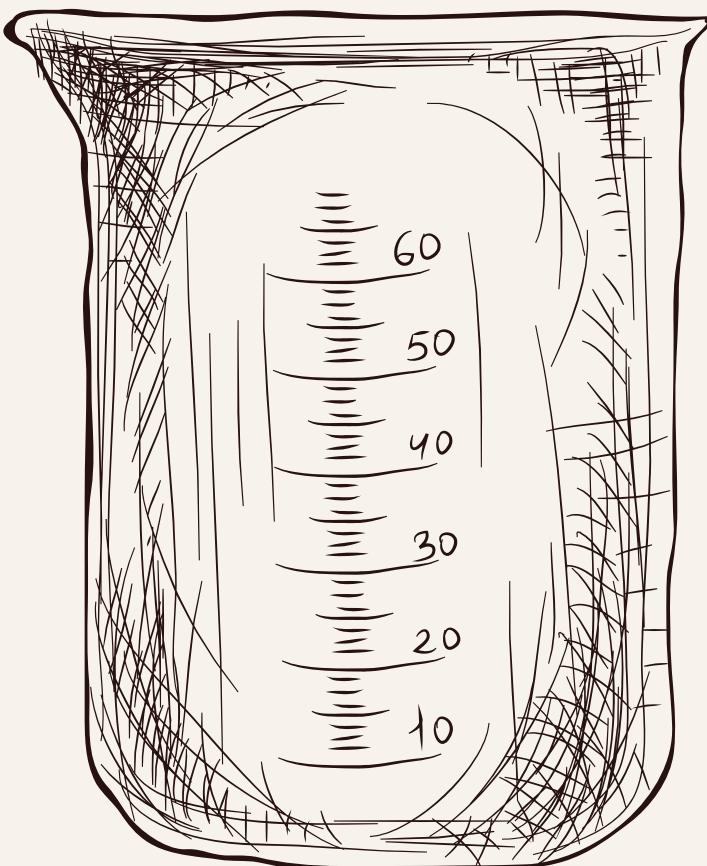
1) METODO DE CRAMER

g)

$$\begin{array}{l} \text{G) } \begin{array}{l} x - 3y = 9 \\ 2x + 3y = 6 \end{array} \quad D = \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = 3 - 6 = 3 \\ \text{D}_x = \begin{vmatrix} 9 & -3 \\ 6 & 3 \end{vmatrix} = 24 - 18 = 6 \quad x = \frac{6}{3} = 2, \\ \text{D}_y = \begin{vmatrix} 1 & 9 \\ 2 & 6 \end{vmatrix} = 6 - 18 = -12 \quad y = \frac{-12}{3} = 4, \\ \text{H) } 2x + 3y = 0 \quad \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix} = 10 - 9 = 1 \end{array}$$

h)

$$\begin{array}{l} \text{D}_y = \begin{vmatrix} 0 & 7 \\ 2 & 6 \end{vmatrix} = 0 - 14 = -14 \quad y = \frac{-14}{3} = -\frac{14}{3}, \\ \text{H) } \begin{array}{l} 2x + 3y = 0 \\ 3x + 5y = 2 \end{array} \quad \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \\ x = \begin{vmatrix} 0 & 3 \\ 2 & 5 \end{vmatrix} = 0 - 6 = \frac{-6}{1} = -6, \\ y = \begin{vmatrix} 2 & 0 \\ 3 & 2 \end{vmatrix} = \frac{4}{1} = 4, \\ \text{2-A) } x + 2y - z = 2 \quad \begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 1 \end{array}$$



2) METODO DE CRAMER

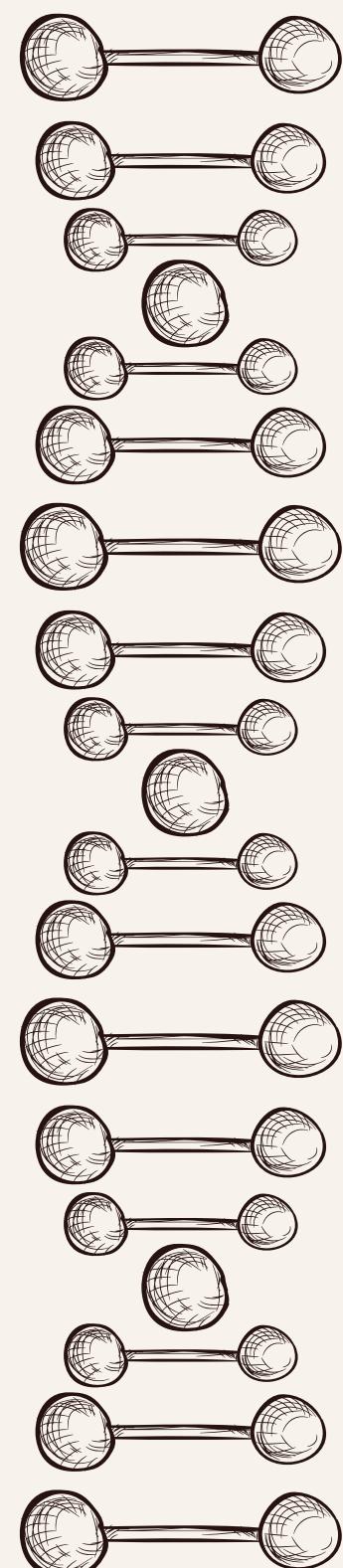
a)

$$\begin{aligned}
 & \text{2-A)} \quad \begin{array}{l} \boxed{3 \ 2 \ 1} \\ \boxed{2 \ x + 2y - z = 2} \\ \boxed{2x - y + 3z = 9} \\ \boxed{3x + 3y - 2z = 3} \end{array} \quad \begin{array}{c} \boxed{2 \ 2 \ -1 \ 1 \ 2} \\ \boxed{2 \ -1 \ 3 \ 3 \ -1} = 30 \\ \boxed{3 \ 3 \ -2 \ 3 \ 3} \end{array} \\
 & x = \frac{\begin{vmatrix} 3 & 2 & -1 & 2 & 2 \\ 0 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix}}{30} = \frac{4 + 18 - 24 + 36 - 18 \cdot 3}{30} = \frac{10}{30} = 1 \\
 & y = \frac{\begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 0 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix}}{30} = \frac{-18 + 18 - 6 + 8 - 9 + 24}{30} = \frac{20}{30} = \frac{2}{3} \\
 & z = \frac{\begin{vmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 0 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{vmatrix}}{30} = \frac{-3 + 54 + 12 - 12 - 24 + 6 \cdot 30}{30} = \frac{30}{30} = 1
 \end{aligned}$$

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b)

$$\begin{aligned}
 & \text{b)} \quad \begin{array}{l} \boxed{2 \ 3 \ 1} \\ \boxed{2x + 3y - z = 14} \\ \boxed{-3x + 4y - 2z = 4} \\ \boxed{-3x - 3y - 4z = -14} \end{array} \quad \begin{array}{c} \boxed{2 \ 3 \ -1 \ 2 \ 3} \\ \boxed{-3 \ 4 \ -2 \ -1 \ 4} \\ \boxed{-3 \ -3 \ 4 \ -3 \ -3} \end{array} \\
 & 32 + 18 - 3 + 12 - 12 - 12 = 35 \\
 & x = \frac{\begin{vmatrix} 14 & 3 & -1 & 14 & 3 \\ 4 & 4 & -2 & 4 & 4 \\ -14 & -3 & 4 & -14 & -3 \end{vmatrix}}{35} = \frac{224 + 84 + 21 - 84 - 84 - 86}{35} = \frac{105}{35} = 3 \\
 & y = \frac{\begin{vmatrix} 2 & 14 & -1 & 2 & 14 \\ -1 & 4 & -2 & -1 & 7 \\ -3 & -14 & 4 & -3 & -14 \end{vmatrix}}{35} = \frac{56 + 84 - 14 + 56 - 56 - 21}{35} = \frac{105}{35} = 3 \\
 & z = \frac{\begin{vmatrix} 2 & 3 & 14 & 2 & 3 \\ -3 & 4 & 4 & -1 & 4 \\ -13 & -3 & -14 & -3 & -3 \end{vmatrix}}{35} = \frac{112 - 63 + 42 - 42 + 42 + 168}{35} = \frac{35}{35} = 1
 \end{aligned}$$



2) METODO DE CRAMER

c)

$$\begin{array}{l}
 \text{c)} \left. \begin{array}{l} 2x+2y+z=5 \\ 4x+3y+5z=5 \\ 3x+5y+4z=4 \end{array} \right\} \begin{array}{l} 2 \ 2 \ 1 \ 2 \ 2 \\ 4 \ 3 \ 5 \ 4 \ 3 \\ 3 \ 7 \ 4 \ 3 \ 7 \end{array} \quad 24+30+28-32-40-9 = -29 \\
 \begin{array}{r} | \\ \times \\ | \end{array} \begin{array}{r} 1 \ 2 \ 1 \ 1 \ 2 \\ 5 \ 3 \ 5 \ 5 \ 3 \\ 1 \ 4 \ 7 \ 9 \ 4 \ 4 \end{array} \quad 12+40+35-40-35-12 = 0, = \frac{0}{-29} = 0, \\
 \text{d)} \left. \begin{array}{l} 2 \ 1 \ 1 \ 2 \ 1 \\ 4 \ 5 \ 5 \ 4 \ 5 \\ 3 \ 4 \ 4 \ 3 \ 4 \end{array} \right\} 40+35+36-36-40-35 = 0, = \frac{0}{-29} = 0, \\
 \text{e)} \left. \begin{array}{l} 2 \ 2 \ 1 \ 2 \ 2 \\ 4 \ 3 \ 5 \ 4 \ 3 \\ 3 \ 7 \ 4 \ 3 \ 4 \end{array} \right\} 24+30+28-32-40-9 = -29, = \frac{-29}{-29} = 1,
 \end{array}$$

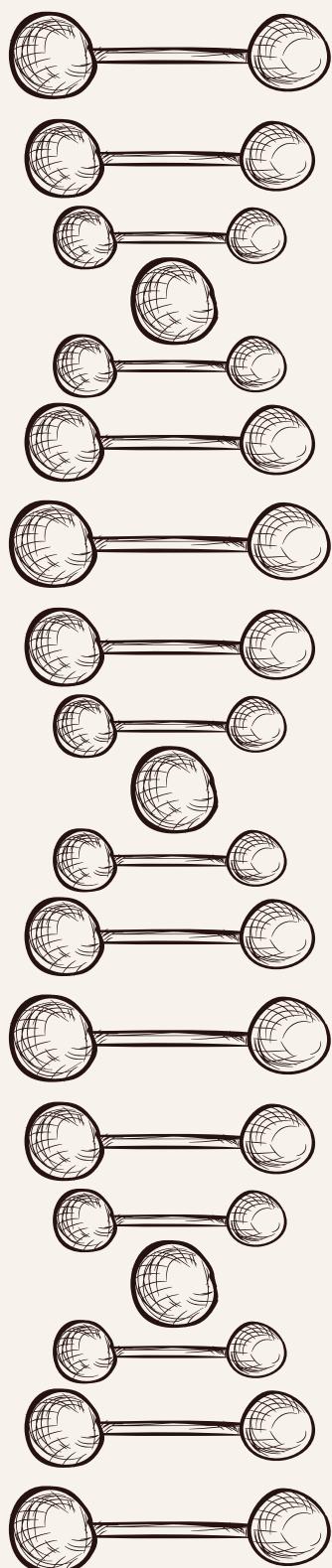
d)

$$D) \begin{array}{l} -x + y - z = 5 \\ x + 2y + 4z = 4 \\ 3x + y - 2z = -3 \end{array} \left| \begin{array}{cccc|c} 1 & 1 & -1 & 1 & 5 \\ 1 & 2 & 4 & 3 & 4 \\ 3 & 1 & -2 & 3 & -3 \end{array} \right. \quad \begin{array}{l} 4+12-3+2=11 \\ 4+6=10 \\ 4-6=-2 \end{array}$$

$$x \left| \begin{array}{cccc|c} 5 & 1 & -1 & 5 & 15 \\ 4 & 2 & 4 & 3 & 12 \\ -3 & 1 & 2 & -3 & 15 \end{array} \right. \quad -20-12-4+8-20-6 = -54 = \frac{-54}{24} = 2,$$

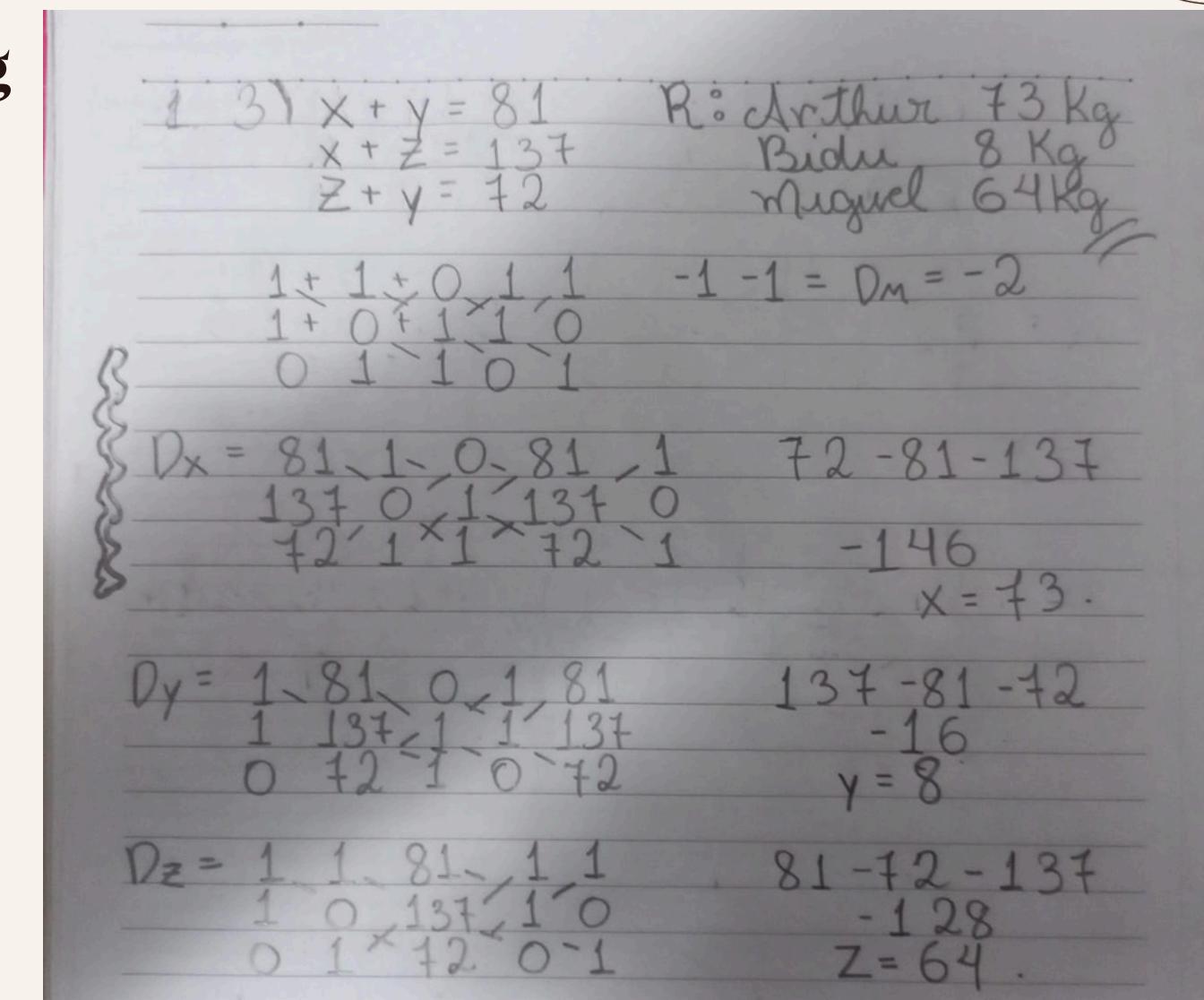
$$y \left| \begin{array}{cccc|c} -1 & 5 & -1 & -1 & 5 \\ 1 & 9 & 9 & 1 & 9 \\ 3 & -3 & -2 & 3 & -3 \end{array} \right. \quad 8+60+3+10-12+12 = 80 = \frac{80}{24} = 3,$$

$$2 \left| \begin{array}{cccc|c} -1 & 1 & 5 & -1 & 1 \\ 1 & 9 & 9 & 1 & 9 \\ 3 & 3 & -3 & 3 & 3 \end{array} \right. \quad 6+12+5+3+4-30 = 0 = \frac{0}{24} = 0$$



3) Arthur e seu irmão Miguel foram com seu cachorro Bidu à farmácia de seu avô. Lá encontraram uma velha balança com defeito, que só indicava corretamente pesos superiores a 60 kg. Assim, eles se pesaram dois a dois e obtiveram as seguintes marcas:

- Arthur e Bidu pesam juntos 81kg
 - Arthur e Miguel pesam juntos 137Kg
 - Miguel e Bidu pesam juntos 72Kg

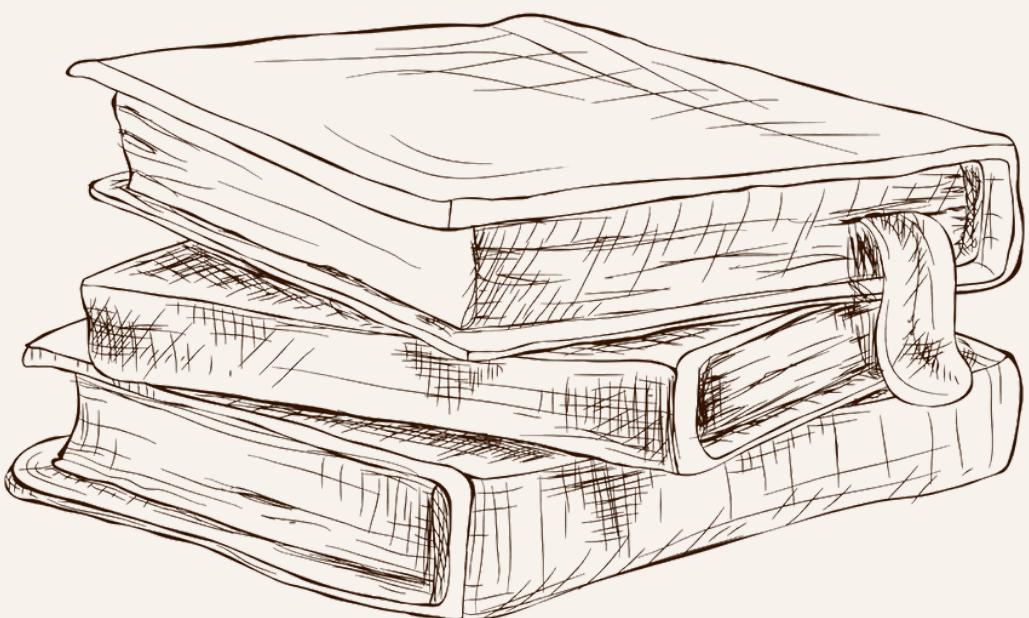


4) Resolva o sistema linear a seguir

4)

$$\begin{array}{r} 2 + 1 - 1 = 1 \\ 1 - 1 + 1 = 2 \\ 1 + 3 - 2 = 3 \end{array}$$

$$D_M = \begin{vmatrix} 2 & 1 & -1 & 2 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 3 & -2 & 1 & 3 \end{vmatrix} \quad 4+1-3+2-6-1$$



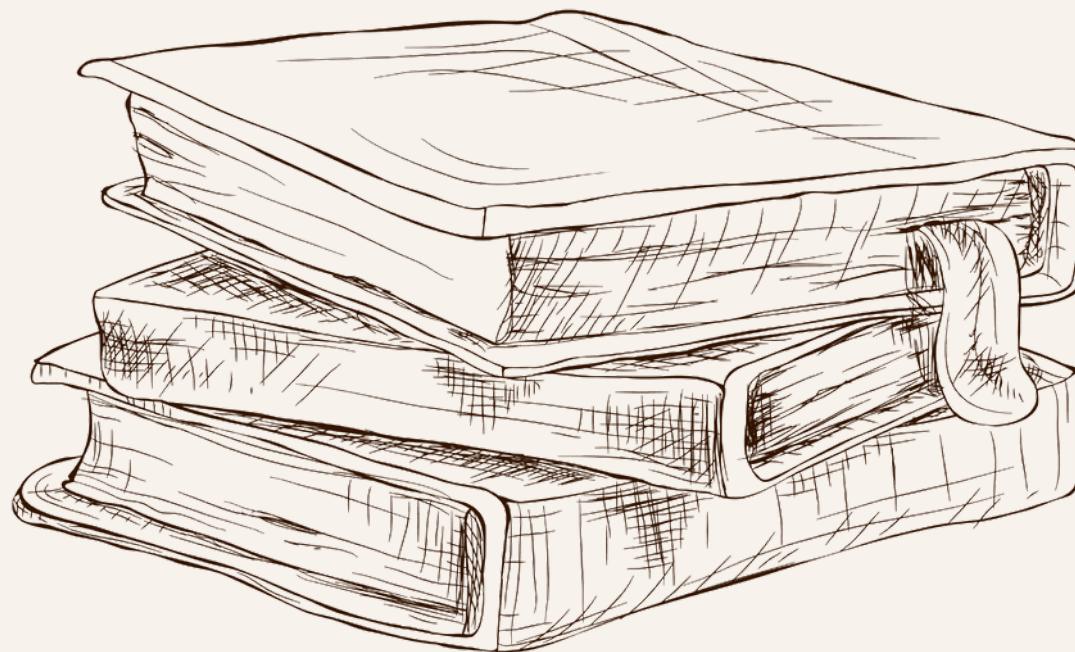
$$\begin{cases} 2x + y - z = 1 \\ x - y + z = 2 \\ x + 3y - 2z = 3 \end{cases}$$

$$D_x = \begin{vmatrix} 1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 1 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} + 2 + 3 - 6 + 4 - 3 - 3$$
$$x = 1$$

$$D_y = \begin{vmatrix} 2 & 1 & -1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 1 & 3 & -2 & 1 & 3 \end{vmatrix} - 8 + 1 - 3 + 2 - 6 + 2$$
$$y = 4$$

$$D_z = \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 1 & -1 & 2 & 1 & -1 \\ 1 & 3 & 3 & 1 & 3 \end{vmatrix} - 6 + 2 + 3 - 3 - 12 + 1$$
$$z = 5$$

5) Calcule os valores dos pesos x,y e z para os quais as balanças estão equilibradas



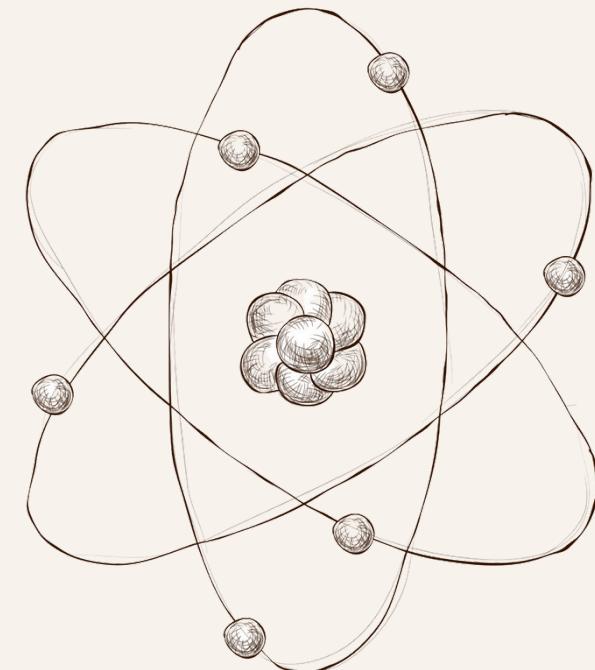
$$\begin{aligned} & \text{L = } 5, \\ & 5) \quad x + y + z = 9.500 \\ & \quad 3x + y - z = 4.500 \\ & \quad -5x + y + 0z = 0 \\ & D_m = \frac{1}{3} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow -1 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 3 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \quad \frac{5+3+1+5}{14}, \\ & D_x = \frac{9.500}{4.500} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow -1 \end{array} \begin{array}{l} \nearrow 9.5 \\ \searrow 4.5 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \quad \frac{4.500 + 9.500}{1} \quad x = 1, \end{aligned}$$

$$\begin{aligned} & D_y = \frac{1}{3} \begin{array}{l} \nearrow 9.5 \\ \searrow 4.5 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow -1 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 3 \end{array} \begin{array}{l} \nearrow 9.5 \\ \searrow 4.5 \end{array} \quad \frac{47.500 + 22.500}{0} \quad y = 5 \\ & D_z = \frac{1}{3} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \begin{array}{l} \nearrow 9.5 \\ \searrow 4.5 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 0 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow -5 \end{array} \begin{array}{l} \nearrow 1 \\ \searrow 1 \end{array} \quad \frac{-22.5 + 28.5 - 4.5 + 47.5}{76 - 21} \\ & \qquad \qquad \qquad z = 3.5 \end{aligned}$$

6) Antônio Carlos gosta muito de desafios e resolveu fazer uma brincadeira com seus fregueses. Em vez de colocar os preços de cada uma de suas frutas (maçã, limão e melão), colocou as tabuletas:

Quanto custa a duzia dos limões?

A duzia do limão custa R\$12,00



The notebook contains the following handwritten work:

$$\begin{array}{rcl} 6 - jx + jy + & = 6 \\ 2x + y + 3z = j4 \\ x + 2y + z = j4 \end{array}$$

$j+3+4-2-6-j=5$

$$\begin{array}{rcl} 6 & & 6 \\ j4 & & j3 \\ j2 & & j7 \end{array}$$

$6+21+28-j4-36-j = -j = 2$

$$\begin{array}{rcl} j & & j \\ j4 & & j2 \\ j7 & & j7 \end{array}$$

$j4+j8+j4-j2-2j-j4=j=j$

$$\begin{array}{rcl} j & & j \\ j4 & & j2 \\ j7 & & j2 \end{array}$$

$j4+j4+24-j4-28-6=-j=3$

7) A área do triângulo de vértices $A(1;2)$, $B(-1;-2)$ e $C(-2;-1)$ é:

$$\begin{aligned} 7) \quad & \begin{array}{r} 1 \ 2 \\ -1 \ -2 \\ \hline -2 \ -1 \end{array} \quad \begin{array}{r} 1 \ 1 \\ 1 \ -1 \\ \hline 1 \ -2 \end{array} \quad \begin{array}{r} 2 \\ -2 \\ \hline -1 \end{array} \\ & -2 -4 + 1 + 2 + 1 - 4 \\ & D_M = 2 \\ & A = \frac{6}{2} \quad 3 \pi \end{aligned}$$

8) A área do triângulo de vértices A(0;0), B(3;0) e C(0,3) é:

The image shows handwritten calculations on lined paper. At the top right, there is a small number '2'. Below it, the letter 'g' is written next to the number '4,5,'. To the left of these, there is a large calculation for the area of a triangle. The calculation is set up as a determinant:

$$\frac{1}{2} \begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 0 & 3 \end{vmatrix}$$

The calculation is worked out as follows:

$$\frac{1}{2} [0(0-3) - 0(1-0) + 1(0-0)] = \frac{1}{2} [0 - 0 + 0] = 0$$

Below this, there is a large '9'.

9) A área do triângulo de vértices A(4;0), B(0;0) e C(2;2) é:

9) $\frac{1}{2} \times 4 \times 2 = 4$

10) Calcule a área da figura de vértices A=(2,2), B=(2,4), C=(4,5), D=(6,4), E=(6,2)

The image shows handwritten calculations for three triangles on lined paper. The top row shows the vertices A(2,2), B(2,4), and E(6,2) with their differences: (2,2), (1,2), (2,2) and (8,12), (4,4), (-4,-4). The second row shows the vertices B(2,4), C(4,5), and D(6,4) with their differences: (2,4), (1,4), (2,4) and (10,24), (16,16), (-8,-8). The third row shows the vertices A(2,2), E(6,2), and D(6,4) with their differences: (2,2), (4,1), (2,2) and (4,12), (24,12), (-12,-12). Below these rows, the total area is given as 11.

10) $\begin{array}{r} 2 \ 2 \\ - 2 \ 4 \\ \hline 6 \ 2 \end{array}$ $\begin{array}{r} 1 \ 2 \\ - 2 \ 4 \\ \hline 6 \ 2 \end{array}$ $\begin{array}{r} 2 \ 2 \\ - 2 \ 4 \\ \hline 2 \ 4 \end{array}$

ABE $\begin{array}{r} 8 \ 12 \\ + 4 \ 4 \\ \hline 20 \ 24 \end{array}$ $\begin{array}{r} -4 \ -4 \\ - 4 \ -4 \\ \hline -16 \end{array}$ $\begin{array}{r} 24 \\ -16 \\ \hline 8 \end{array}$

BCD $\begin{array}{r} 2 \ 4 \ 1 \ 2 \ 4 \\ - 4 \ 5 \ 1 \ 4 \ 5 \\ \hline 6 \ 4 \ 1 \ 6 \ 4 \end{array}$ $\begin{array}{r} 10 \ 24 \ 16 \\ - 16 \ - 8 \\ \hline -8 \end{array}$

AED $\begin{array}{r} 2 \ 2 \ 1 \ 2 \ 2 \\ - 6 \ 2 \ 1 \ 6 \ 2 \\ \hline 6 \ 4 \ 1 \ 6 \ 4 \end{array}$ $\begin{array}{r} 4 \ 12 \ 24 \\ + 24 \ -12 \ -12 \\ \hline 28 \ -20 \end{array}$ $\begin{array}{r} -12 \\ -20 \\ \hline 8 \ 4 \end{array}$

A área total é 11,

11) Calcule a área da figura de vértices A=(-3,4), B=(-1,5), C=(-2,2)

11) $\begin{vmatrix} -3 & 4 & 1 \\ -1 & 5 & 1 \\ -2 & 2 & 1 \end{vmatrix}$ $\frac{-15 - 8 - 2 + 4 + 6 + 10}{2} = \frac{-25 + 20}{2} = \frac{-5}{2}$

$A = 2,5$