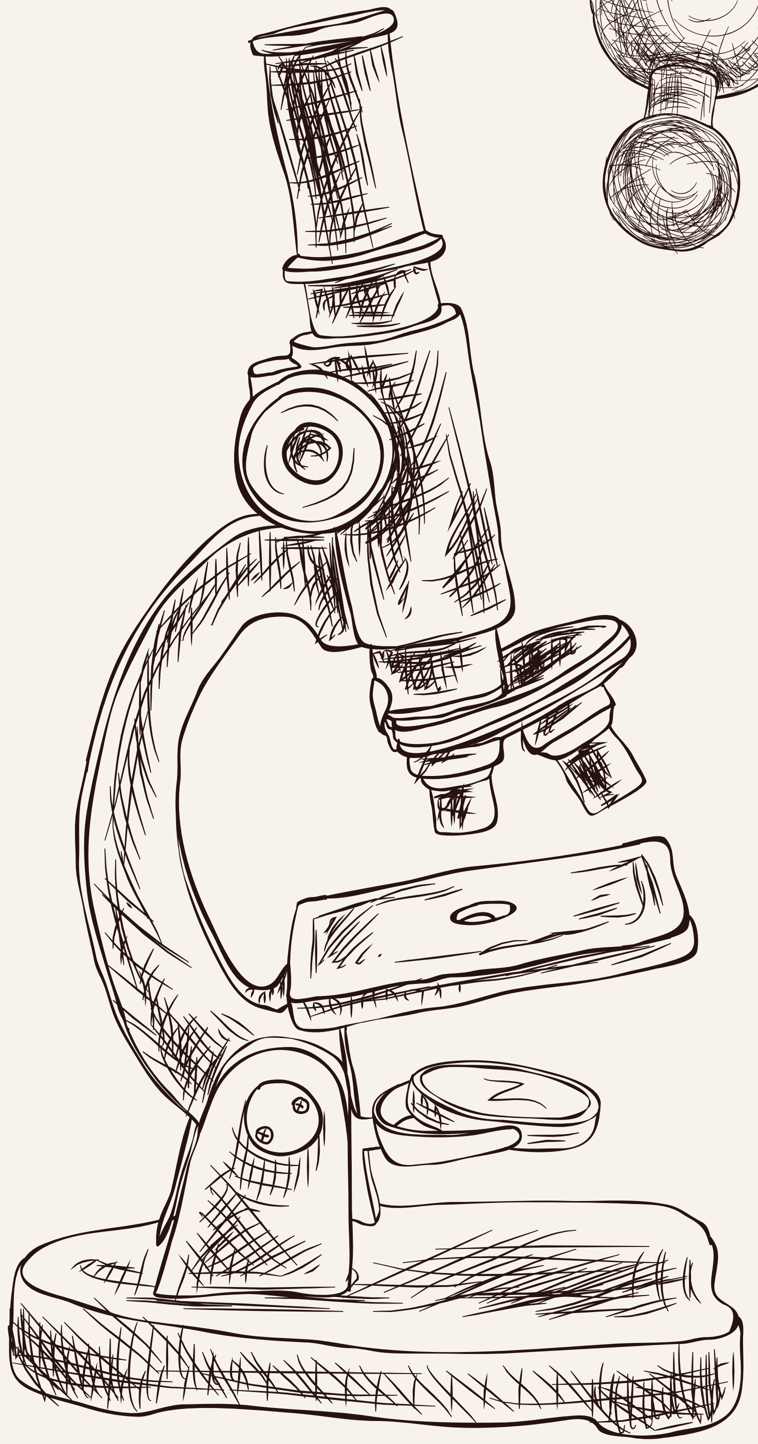
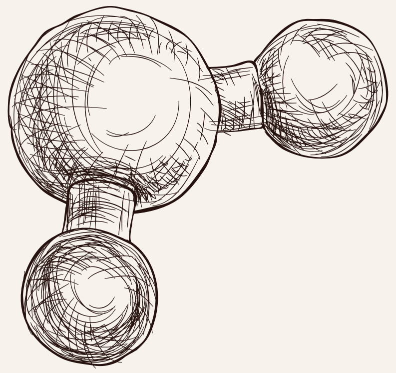
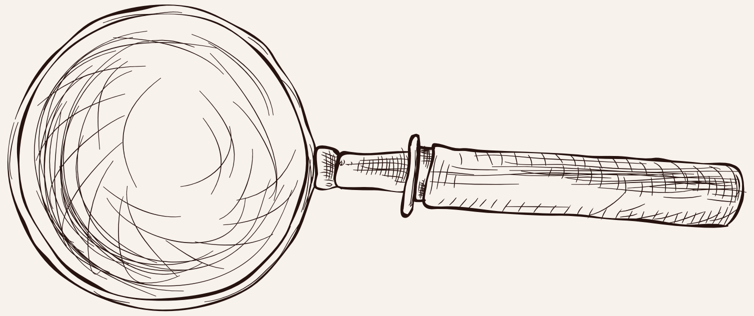
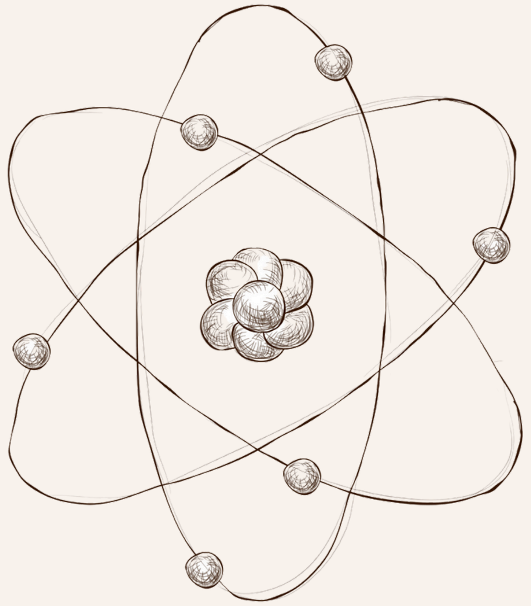


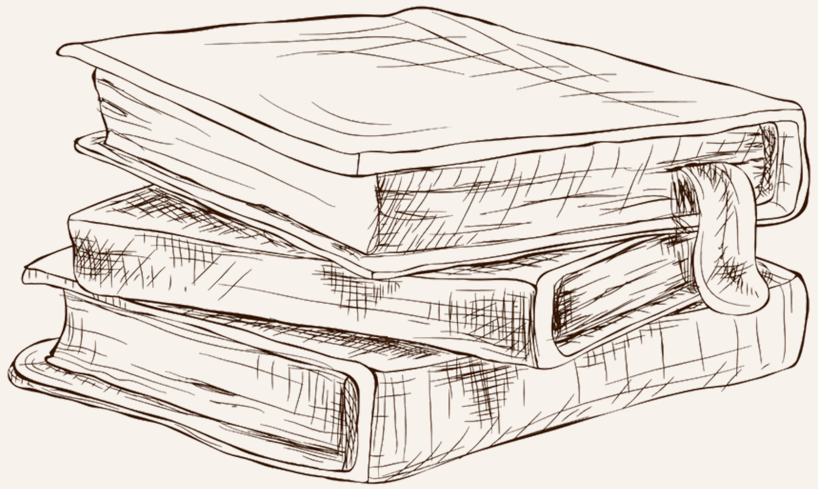






SISTEMAS LINEARES

ANA CAROLINA, CESAR HIPOLITO, EDUARDO BITTAR, LORENA ANDRADE, MATHEUS BRITO

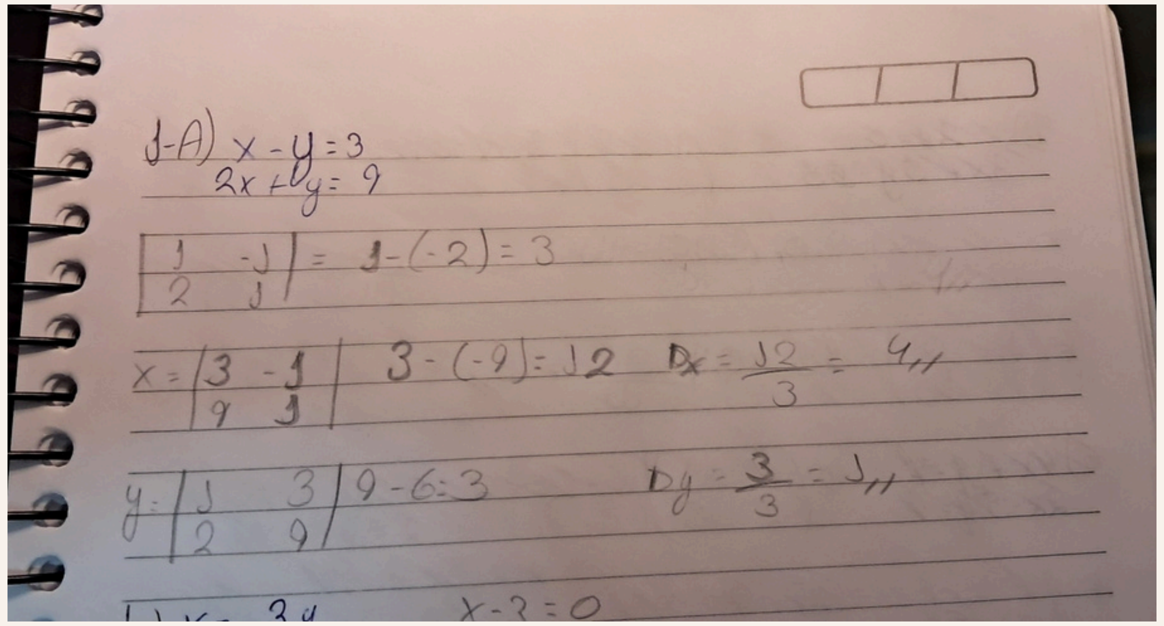


METODO DE CRAMER

Método de Cramer é um método utilizado para encontrar o conjunto solução de um sistema de equação linear possível determinado. Essa regra utiliza o determinante das matrizes associadas ao sistema para encontrar as soluções do sistema. O método é aplicado principalmente em sistemas lineares que possuem 3 incógnitas e 3 equações, mas pode ser empregado também em sistemas lineares 2 por 2.

1) METODO DE CRAMER

a)

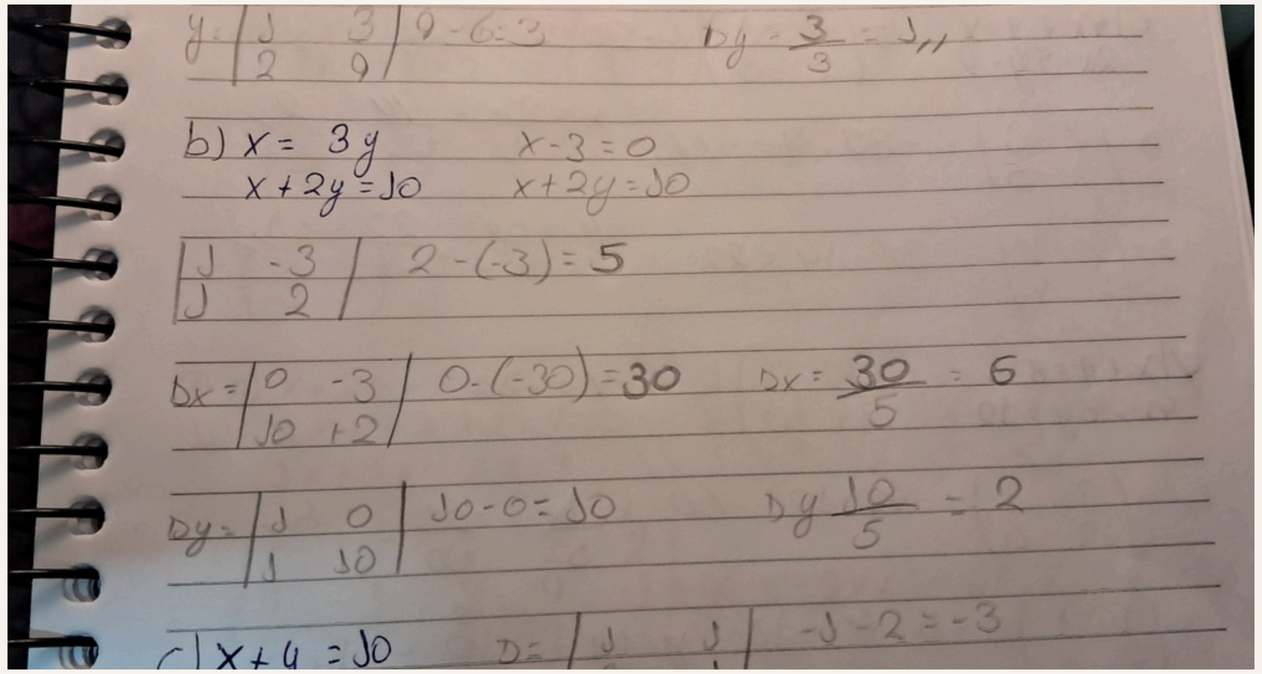

1-A) $x - y = 3$
$2x + y = 9$

$\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1-(-2) = 3$

$x = \begin{vmatrix} 3 & -1 \\ 9 & 1 \end{vmatrix}$ $3-(-9) = 12$ $Dx = \frac{12}{3} = 4$

$y: \begin{vmatrix} 1 & 3 \\ 2 & 9 \end{vmatrix}$ $9-6=3$ $Dy = \frac{3}{3} = 1$

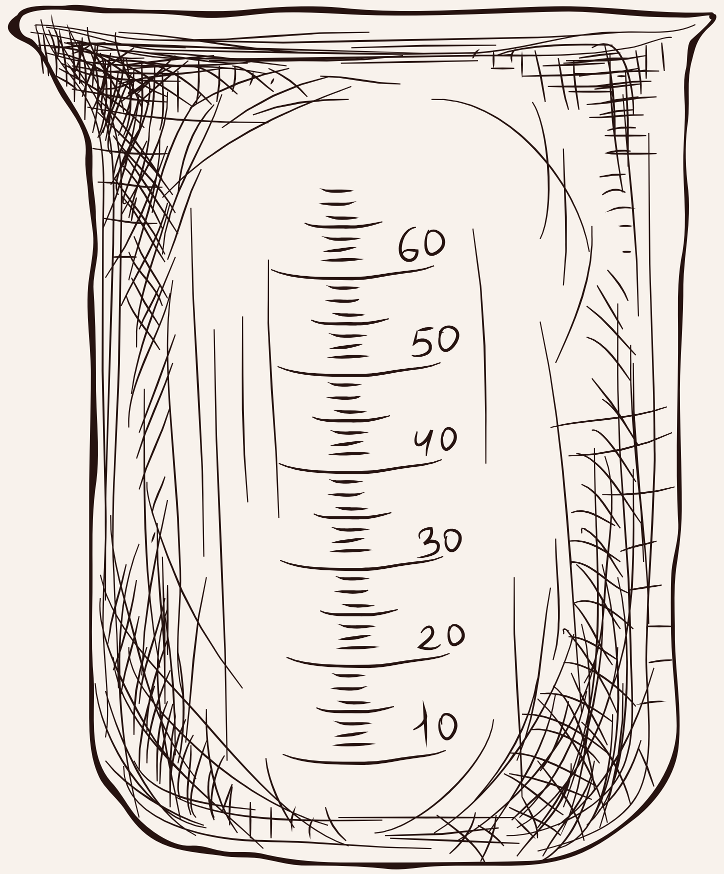
b)

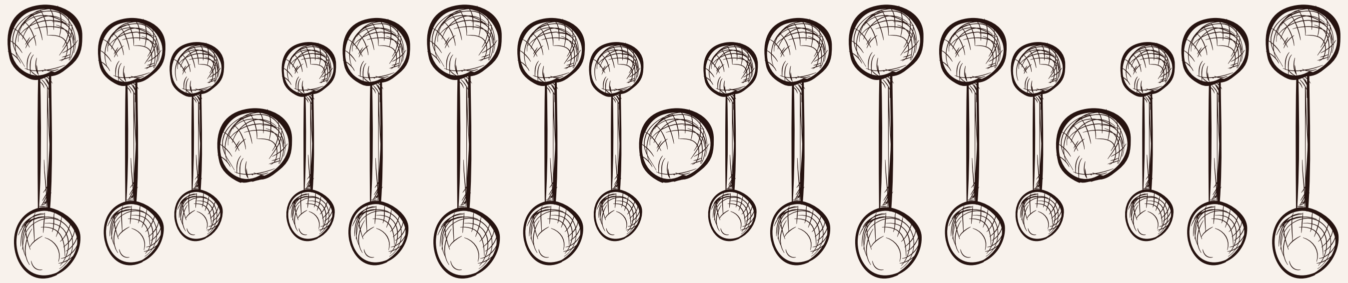

b) $x = 3y$ $\quad x-3=0$
$x+2y=10$ $\quad x+2y=10$

$\begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix}$ $2-(-3) = 5$

$Dx = \begin{vmatrix} 0 & -3 \\ 10 & 2 \end{vmatrix}$ $0-(-30) = 30$ $Dx = \frac{30}{5} = 6$

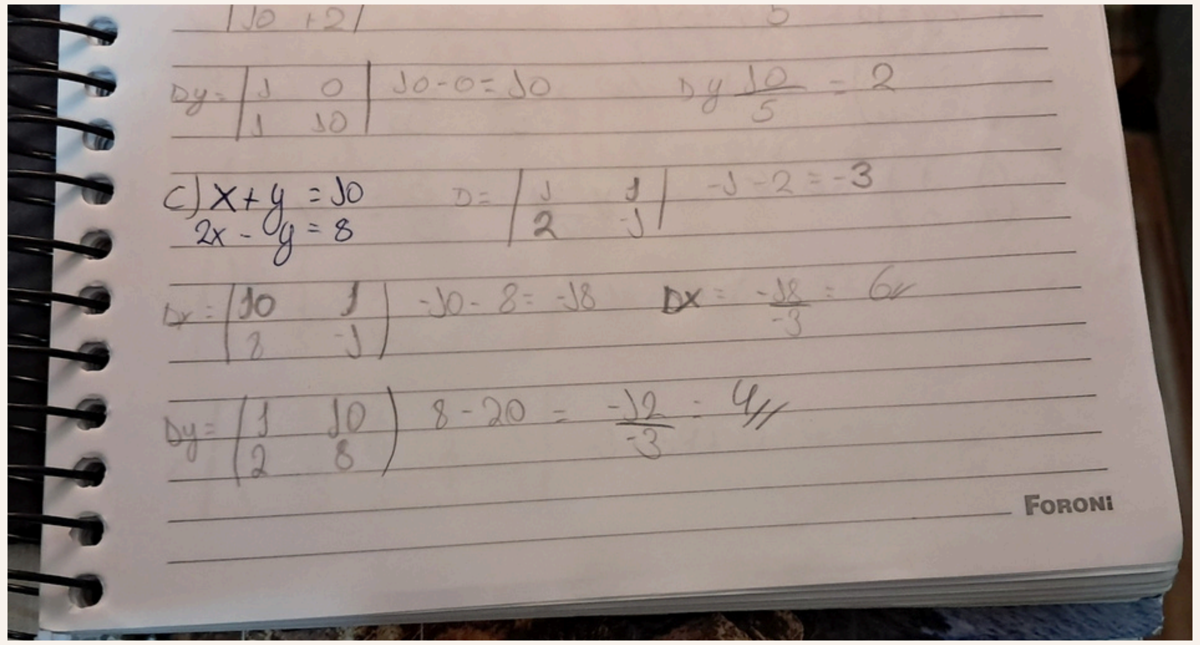
$Dy = \begin{vmatrix} 1 & 0 \\ 1 & 10 \end{vmatrix}$ $10-0=10$ $Dy \frac{10}{5} = 2$



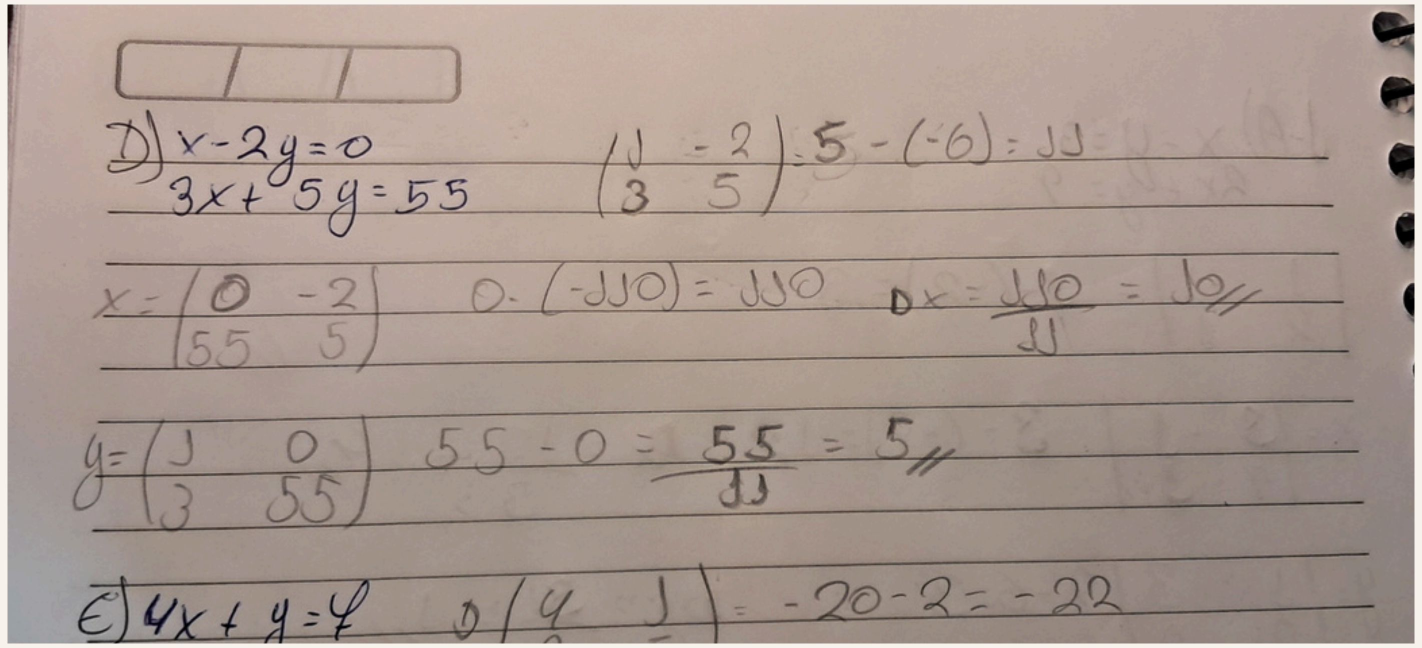


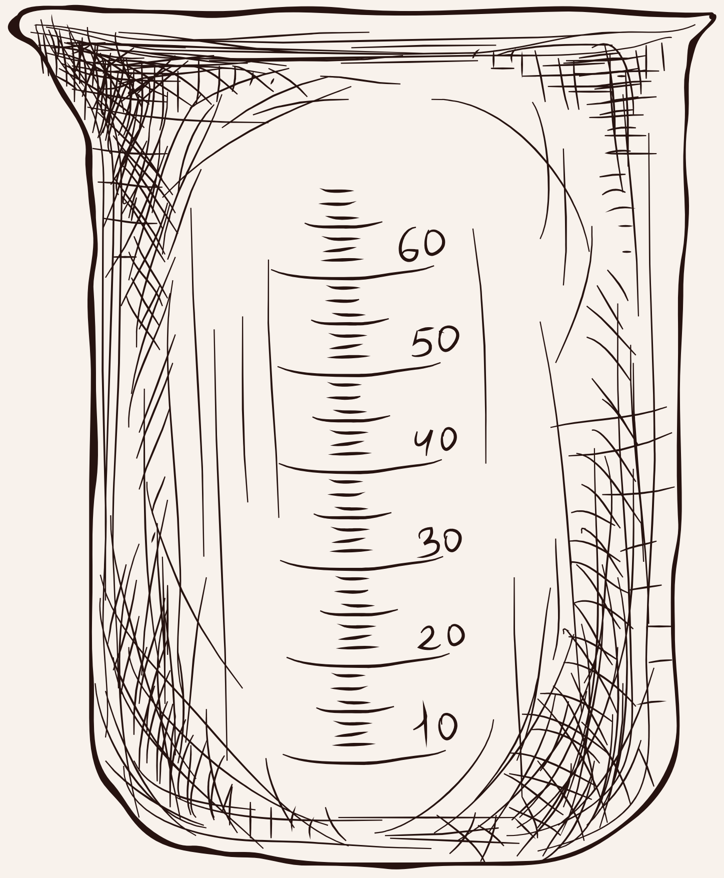
1) METODO DE CRAMER

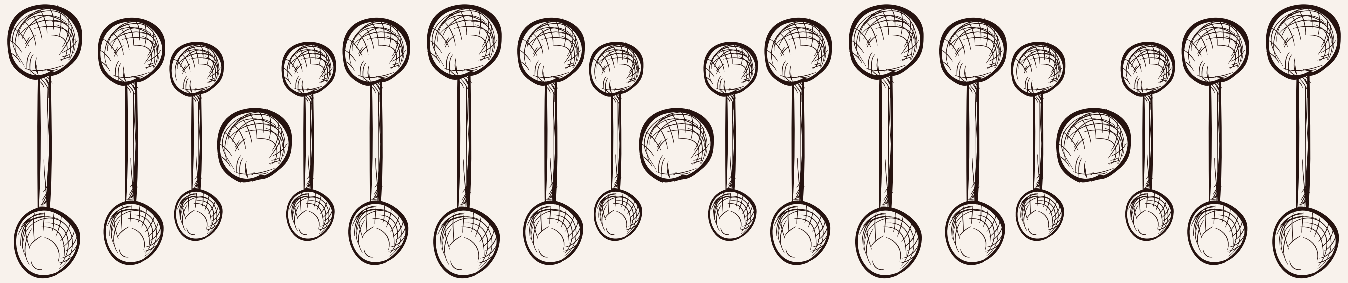
c)



d)

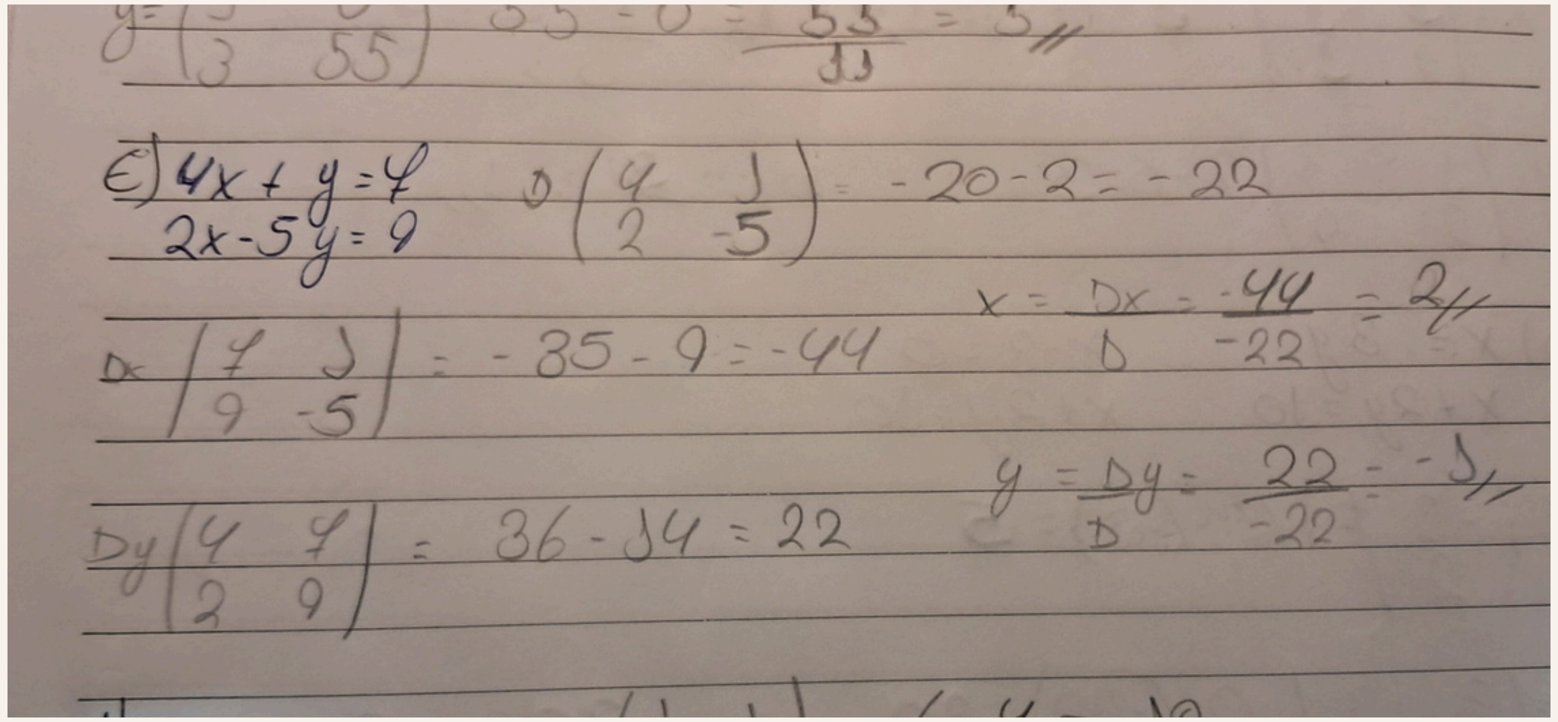






1) METODO DE CRAMER

e)



$y = \begin{pmatrix} 3 & 0 \\ 3 & 55 \end{pmatrix} \; 33 - 0 = \frac{33}{11} = 3$

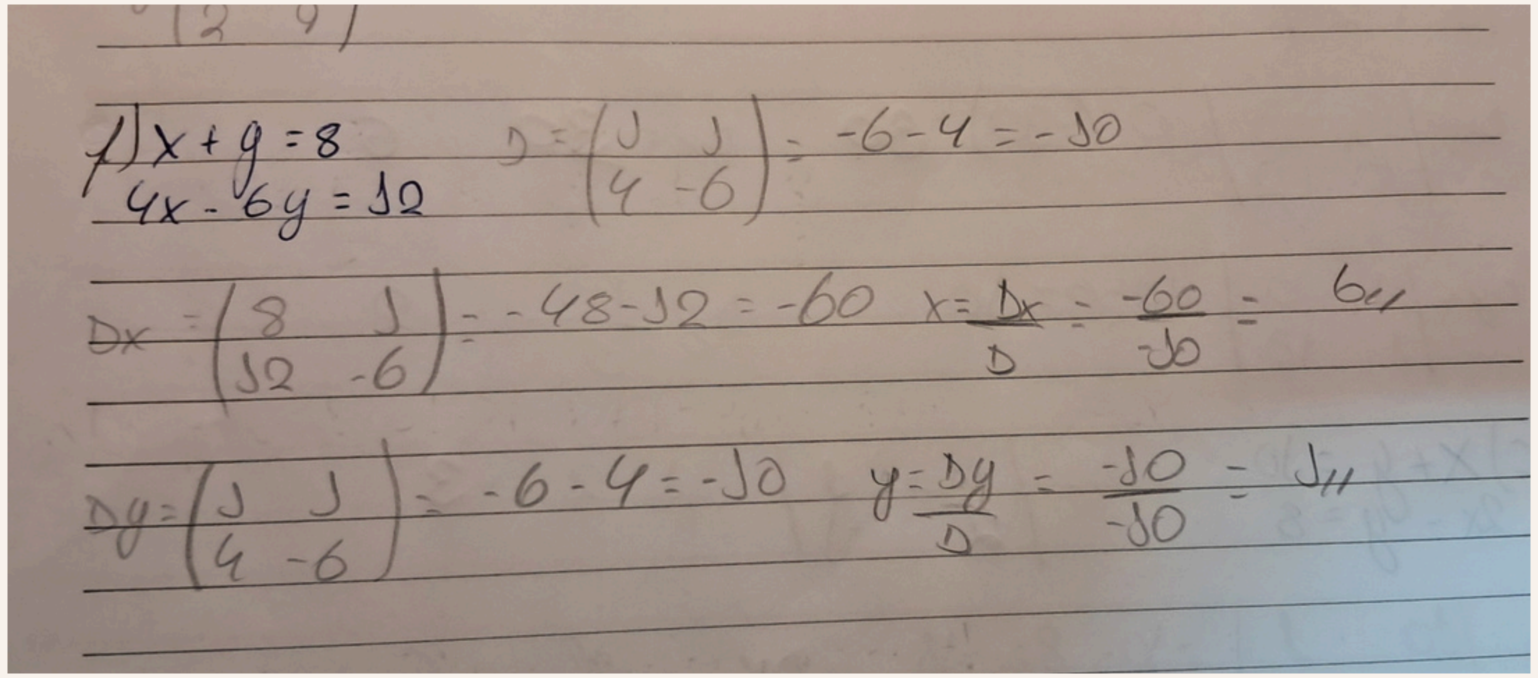
e) $4x + y = 7$
$2x - 5y = 9$

$D \begin{pmatrix} 4 & 1 \\ 2 & -5 \end{pmatrix} = -20 - 2 = -22$

$D_x \begin{vmatrix} 7 & 1 \\ 9 & -5 \end{vmatrix} = -35 - 9 = -44 \qquad x = \frac{D_x}{D} = \frac{-44}{-22} = 2$

$D_y \begin{pmatrix} 4 & 7 \\ 2 & 9 \end{pmatrix} = 36 - 14 = 22 \qquad y = \frac{D_y}{D} = \frac{22}{-22} = -1$

f)

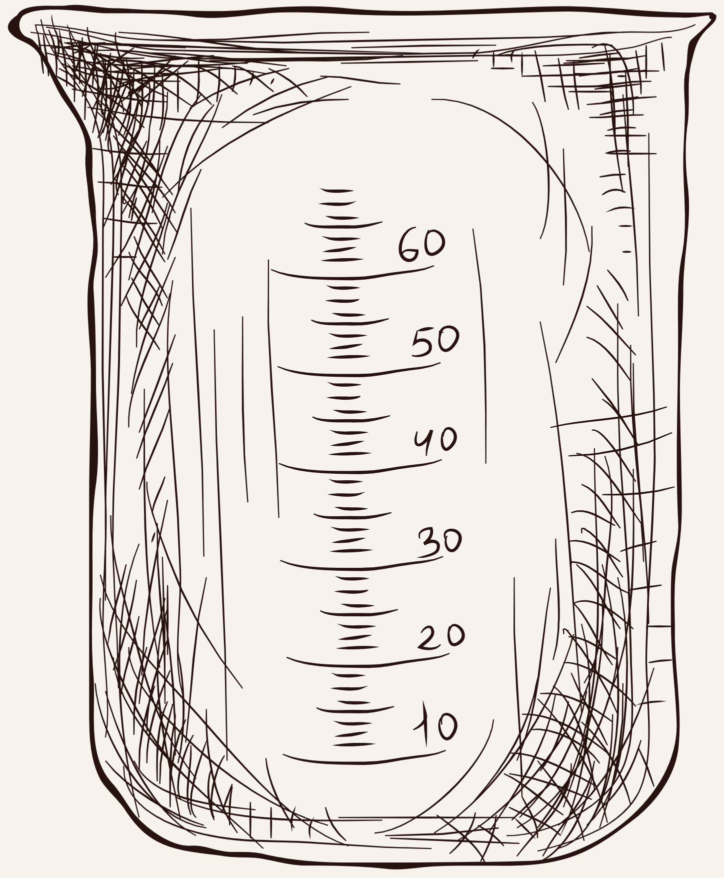


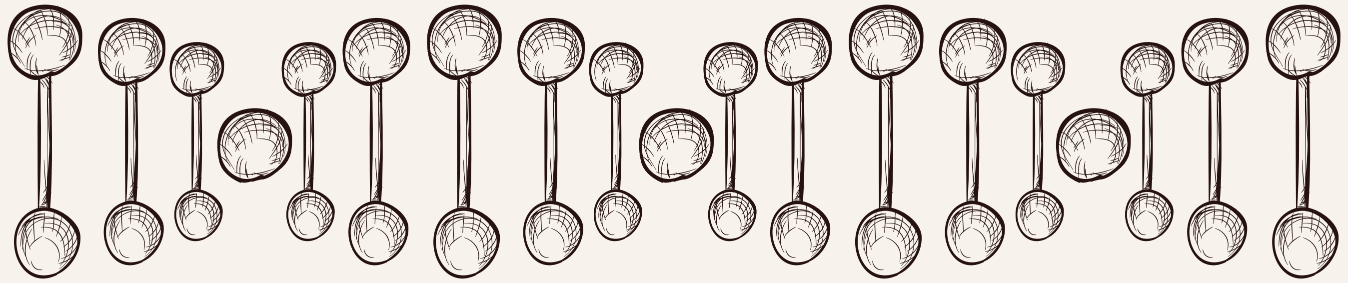
f) $x + y = 8$
$4x - 6y = 12$

$D = \begin{pmatrix} 1 & 1 \\ 4 & -6 \end{pmatrix} = -6 - 4 = -10$

$D_x = \begin{pmatrix} 8 & 1 \\ 12 & -6 \end{pmatrix} = -48 - 12 = -60 \qquad x = \frac{D_x}{D} = \frac{-60}{-10} = 6$

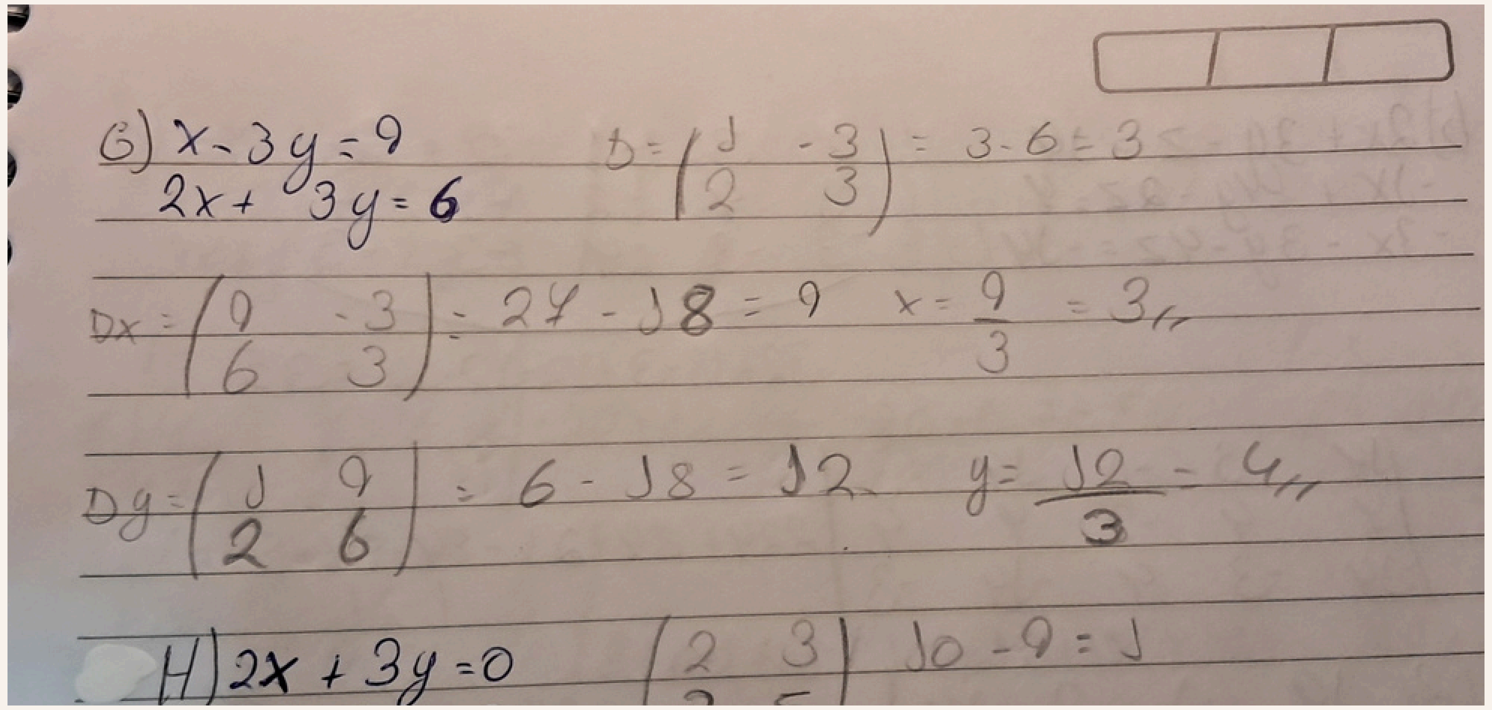
$D_y = \begin{pmatrix} 1 & 1 \\ 4 & -6 \end{pmatrix} = -6 - 4 = -10 \qquad y = \frac{D_y}{D} = \frac{-10}{-10} = 1$





1) METODO DE CRAMER

g)



G) $x - 3y = 9$
$2x + 3y = 6$

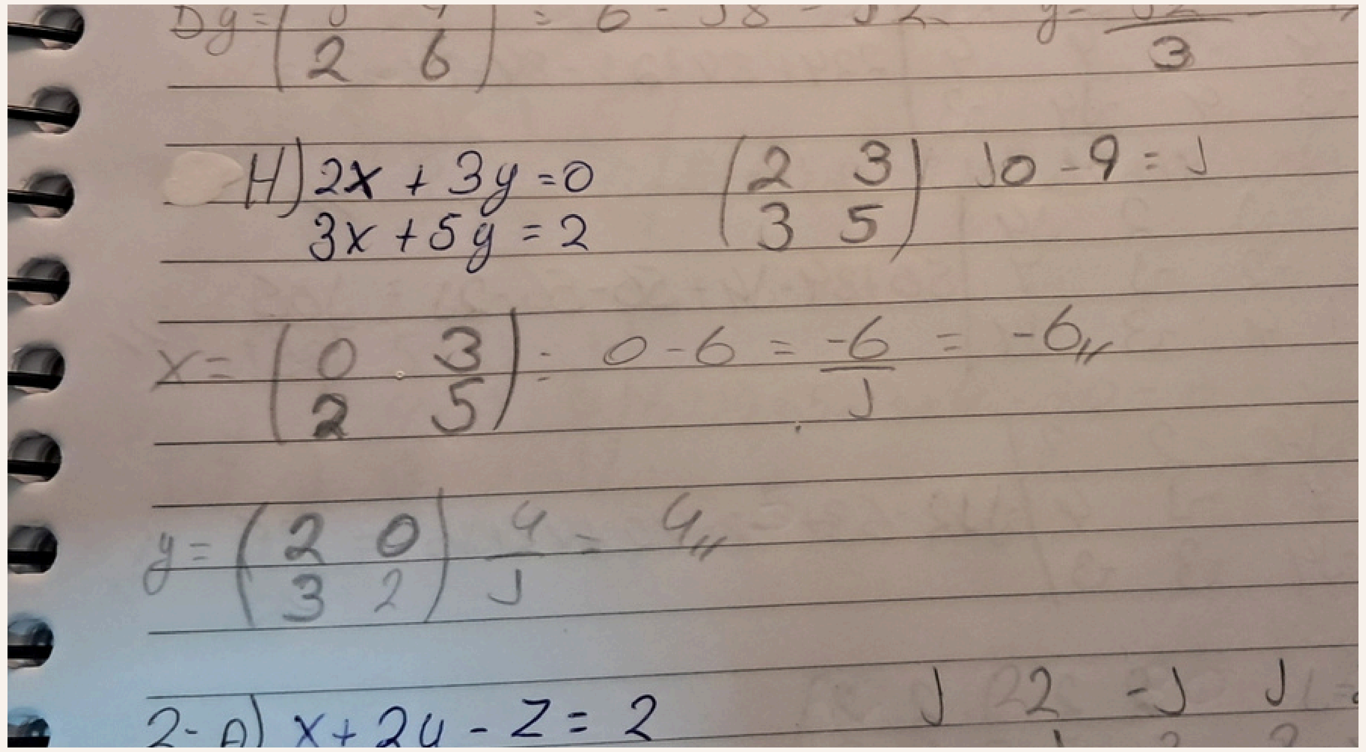
$D = \begin{pmatrix} 1 & -3 \\ 2 & 3 \end{pmatrix} = 3 - 6 = 3$

$Dx = \begin{pmatrix} 9 & -3 \\ 6 & 3 \end{pmatrix} = 27 - 18 = 9 \quad x = \frac{9}{3} = 3$

$Dy = \begin{pmatrix} 1 & 9 \\ 2 & 6 \end{pmatrix} = 6 - 18 = 12 \quad y = \frac{12}{3} = 4$

H) $2x + 3y = 0$ $\quad \begin{pmatrix} 2 & 3 \end{pmatrix}$ $10 - 9 = 1$

h)

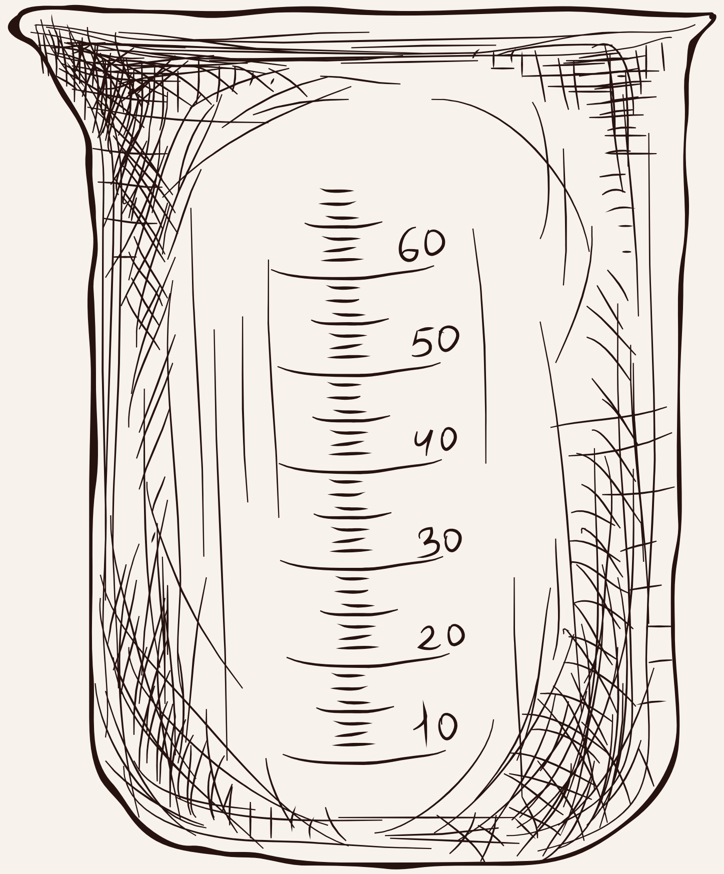


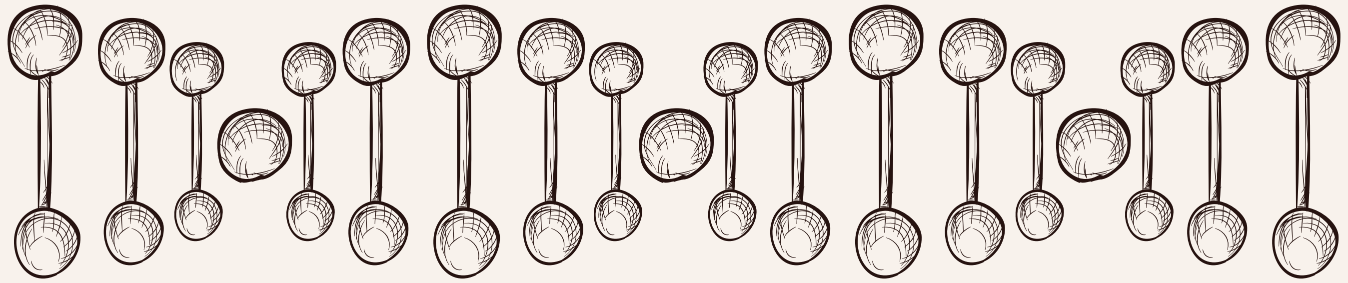
H) $2x + 3y = 0$
$3x + 5y = 2$ $\quad \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$ $10 - 9 = 1$

$x = \begin{pmatrix} 0 & 3 \\ 2 & 5 \end{pmatrix} = 0 - 6 = \frac{-6}{1} = -6$

$y = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix} \frac{4}{1} = 4$

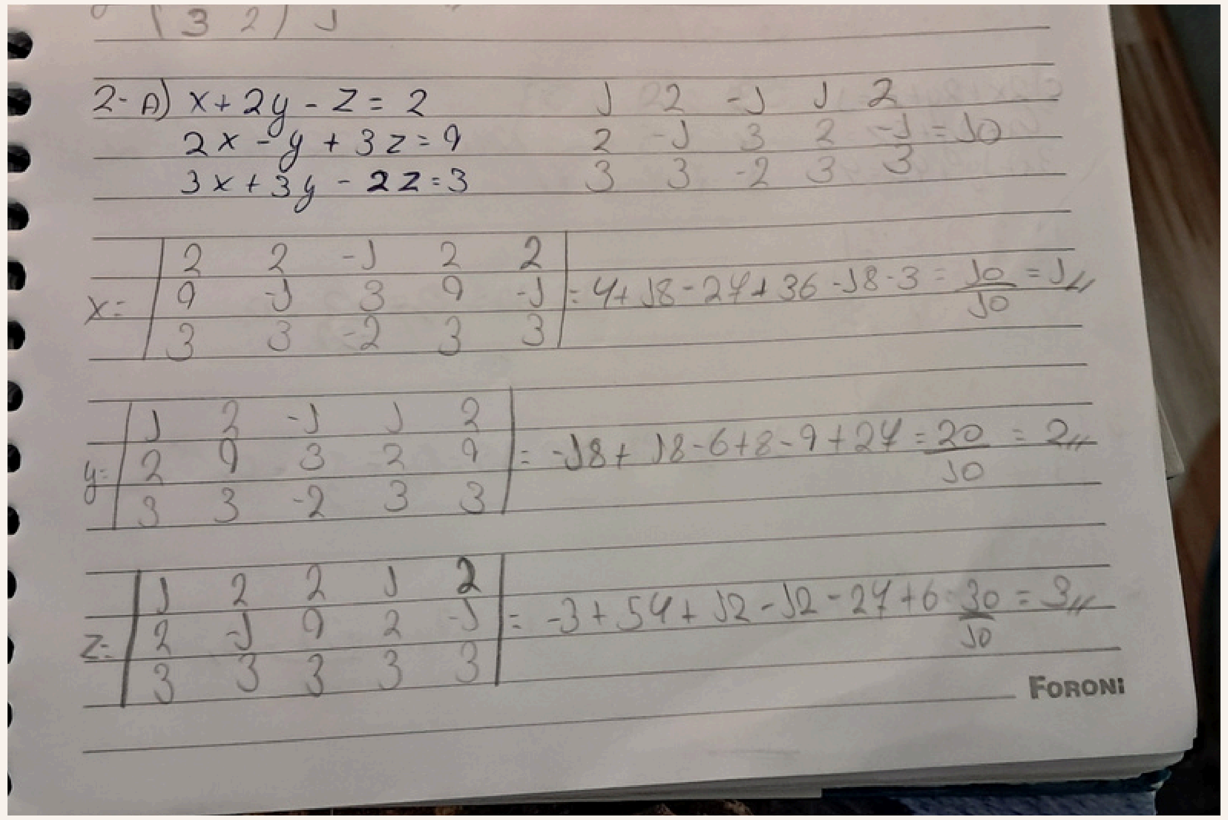
2-A) $x + 2y - z = 2$ $\quad 1 \quad 2 \quad -1 \quad 1$





2) METODO DE CRAMER

a)



2- A) $x + 2y - z = 2$
$2x - y + 3z = 9$
$3x + 3y - 2z = 3$

$$\begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} = 10$$

$$x: \begin{vmatrix} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} = 4 + 18 - 24 + 36 - 18 - 3 = \frac{10}{10} = 1$$

$$y: \begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{vmatrix} = -18 + 18 - 6 + 8 - 9 + 24 = \frac{20}{10} = 2$$

$$z: \begin{vmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{vmatrix} = -3 + 54 + 12 - 12 - 24 + 6 = \frac{30}{10} = 3$$

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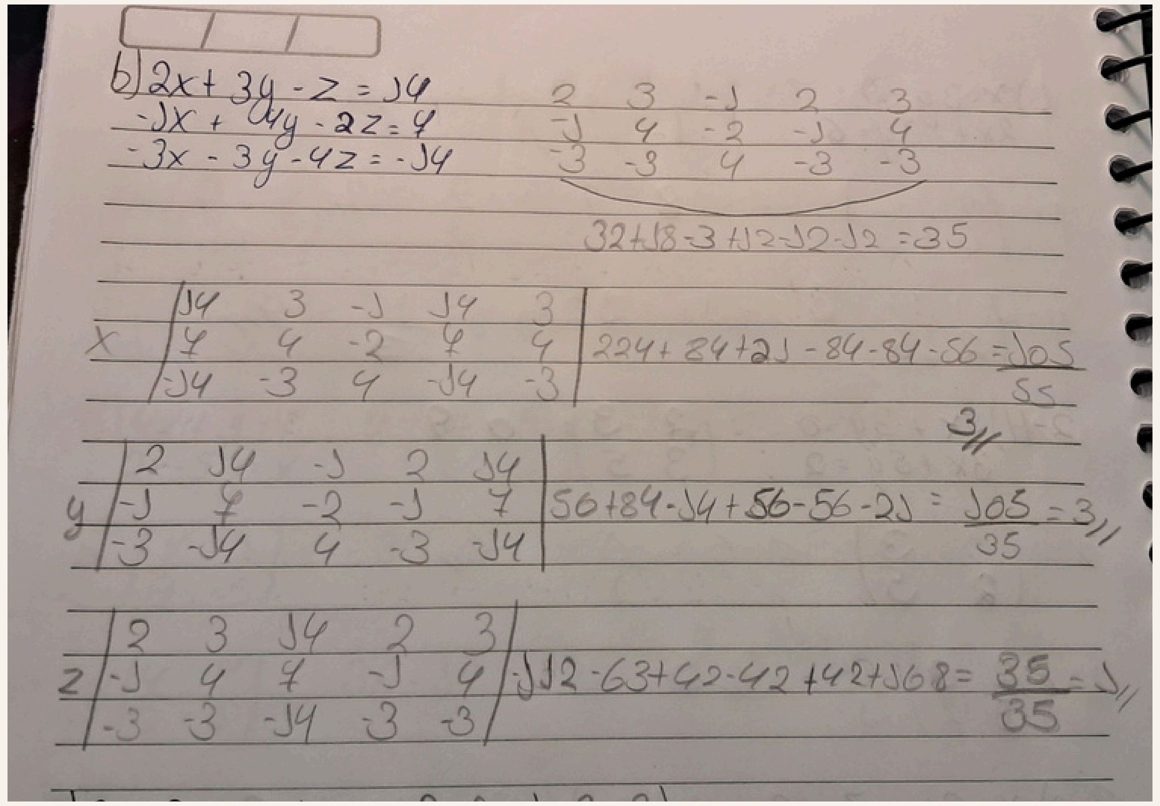
b)



b) $2x + 3y - z = 14$
$-1x + 4y - 2z = 4$
$-3x - 3y - 4z = -14$

$$\underbrace{\begin{vmatrix} 2 & 3 & -1 & 2 & 3 \\ -1 & 4 & -2 & -1 & 4 \\ -3 & -3 & 4 & -3 & -3 \end{vmatrix}}_{32+18-3+12-12-12=35}$$

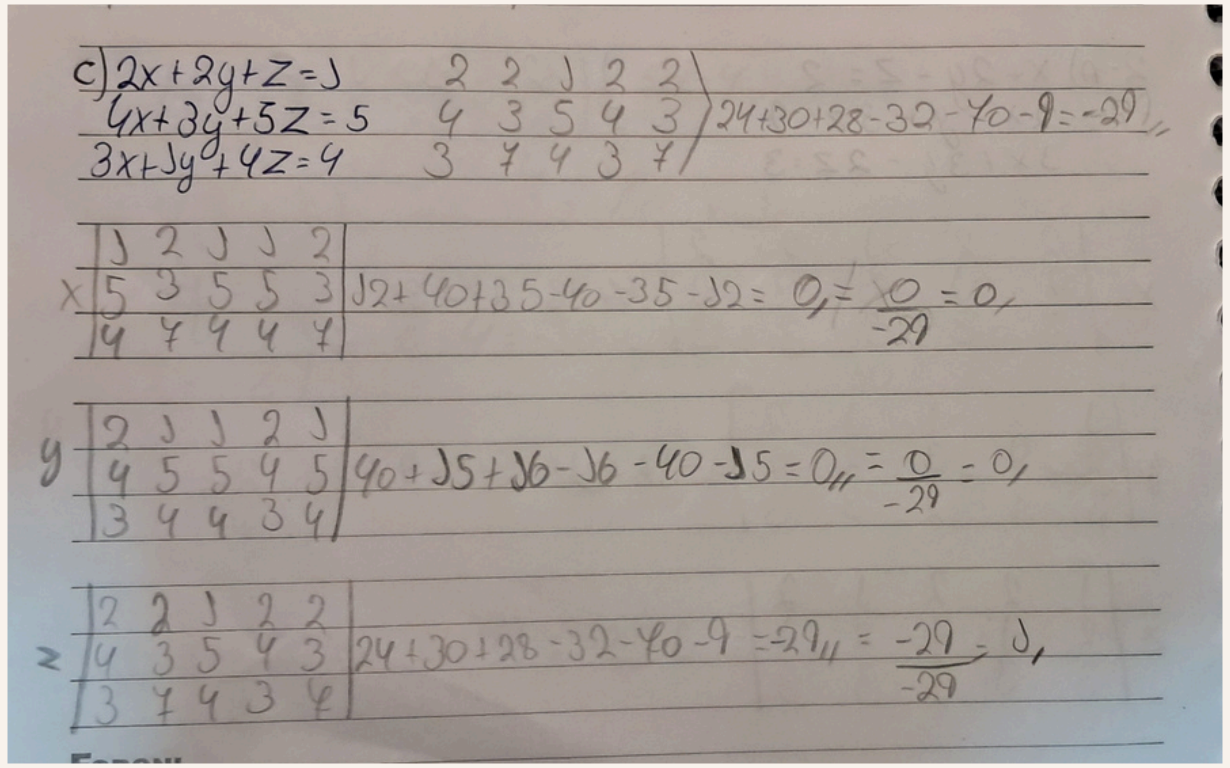
$$x \begin{vmatrix} 14 & 3 & -1 & 14 & 3 \\ 4 & 4 & -2 & 4 & 4 \\ -14 & -3 & 4 & -14 & -3 \end{vmatrix} 224 + 84 + 21 - 84 - 84 - 56 = \frac{105}{35} = 3$$

$$y \begin{vmatrix} 2 & 14 & -1 & 2 & 14 \\ -1 & 4 & -2 & -1 & 4 \\ -3 & -14 & 4 & -3 & -14 \end{vmatrix} 56 + 84 - 14 + 56 - 56 - 21 = \frac{105}{35} = 3$$

$$z \begin{vmatrix} 2 & 3 & 14 & 2 & 3 \\ -1 & 4 & 4 & -1 & 4 \\ -3 & -3 & -14 & -3 & -3 \end{vmatrix} 112 - 63 + 42 - 42 + 42 + 168 = \frac{35}{35} = 1$$

2) METODO DE CRAMER

c)



c) $2x+2y+z=1$
$4x+3y+5z=5$
$3x+1y+4z=4$

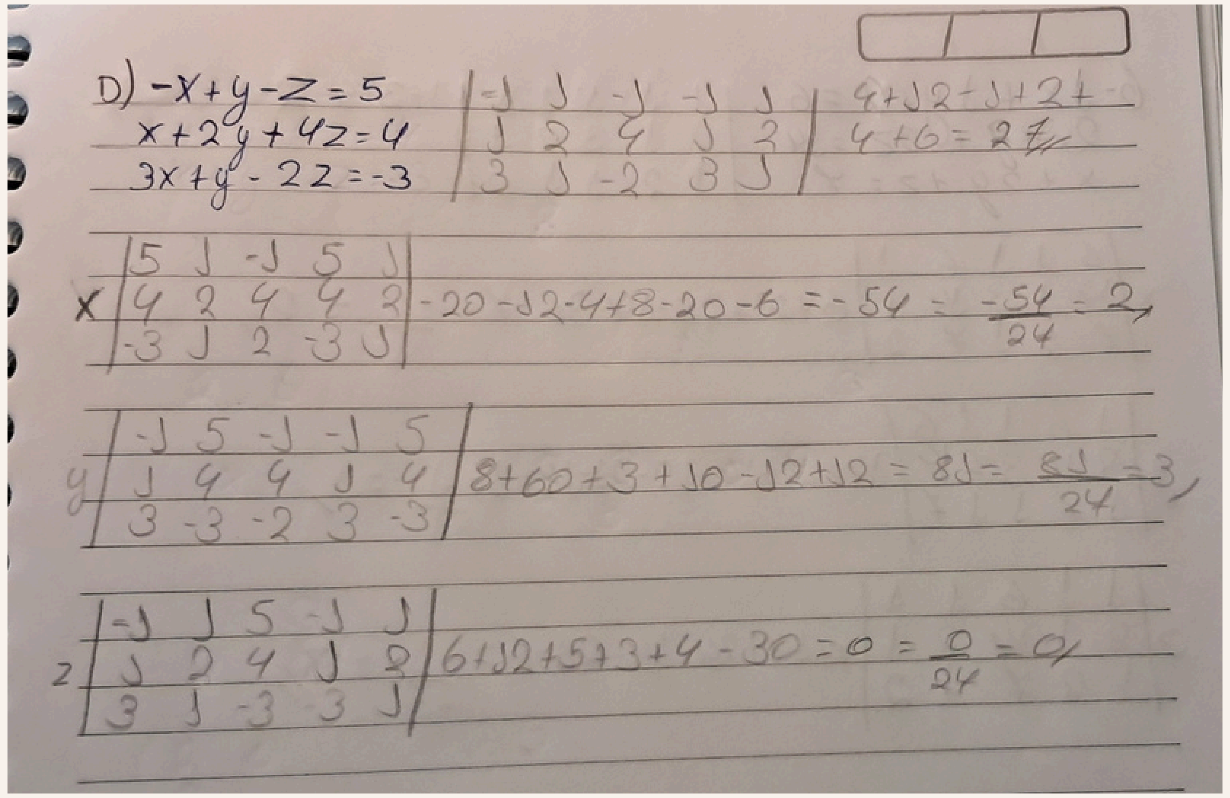
$\begin{matrix} 2 & 2 & 1 & 2 & 2 \\ 4 & 3 & 5 & 4 & 3 \\ 3 & 7 & 4 & 3 & 7 \end{matrix}$ $24+30+28-32-70-9=-29$

$x \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 5 & 3 & 5 & 5 & 3 \\ 4 & 7 & 4 & 4 & 7 \end{vmatrix}$ $12+40+35-40-35-12=0 = \frac{0}{-29}=0$

$y \begin{vmatrix} 2 & 1 & 1 & 2 & 1 \\ 4 & 5 & 5 & 4 & 5 \\ 3 & 4 & 4 & 3 & 4 \end{vmatrix}$ $40+15+16-16-40-15=0 = \frac{0}{-29}=0$

$z \begin{vmatrix} 2 & 2 & 1 & 2 & 2 \\ 4 & 3 & 5 & 4 & 3 \\ 3 & 7 & 4 & 3 & 4 \end{vmatrix}$ $24+30+28-32-70-9=-29 = \frac{-29}{-29}=1$

d)



D) $-x+y-z=5$
$x+2y+4z=4$
$3x+y-2z=-3$

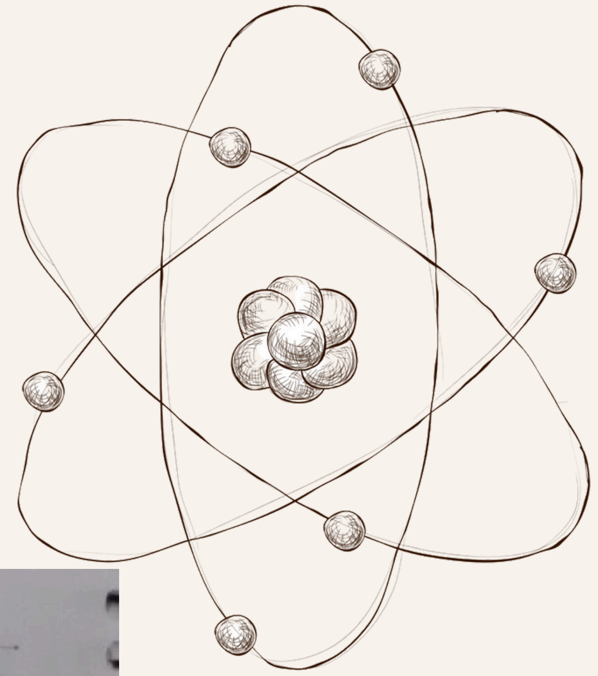
$\begin{vmatrix} -1 & 1 & -1 & -1 & 1 \\ 1 & 2 & 4 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \end{vmatrix}$ $4+12-1+2+4+6=27$

$x \begin{vmatrix} 5 & 1 & -1 & 5 & 1 \\ 4 & 2 & 4 & 4 & 2 \\ -3 & 1 & 2 & -3 & 1 \end{vmatrix}$ $-20-12-4+8-20-6=-54 = \frac{-54}{24}=2$

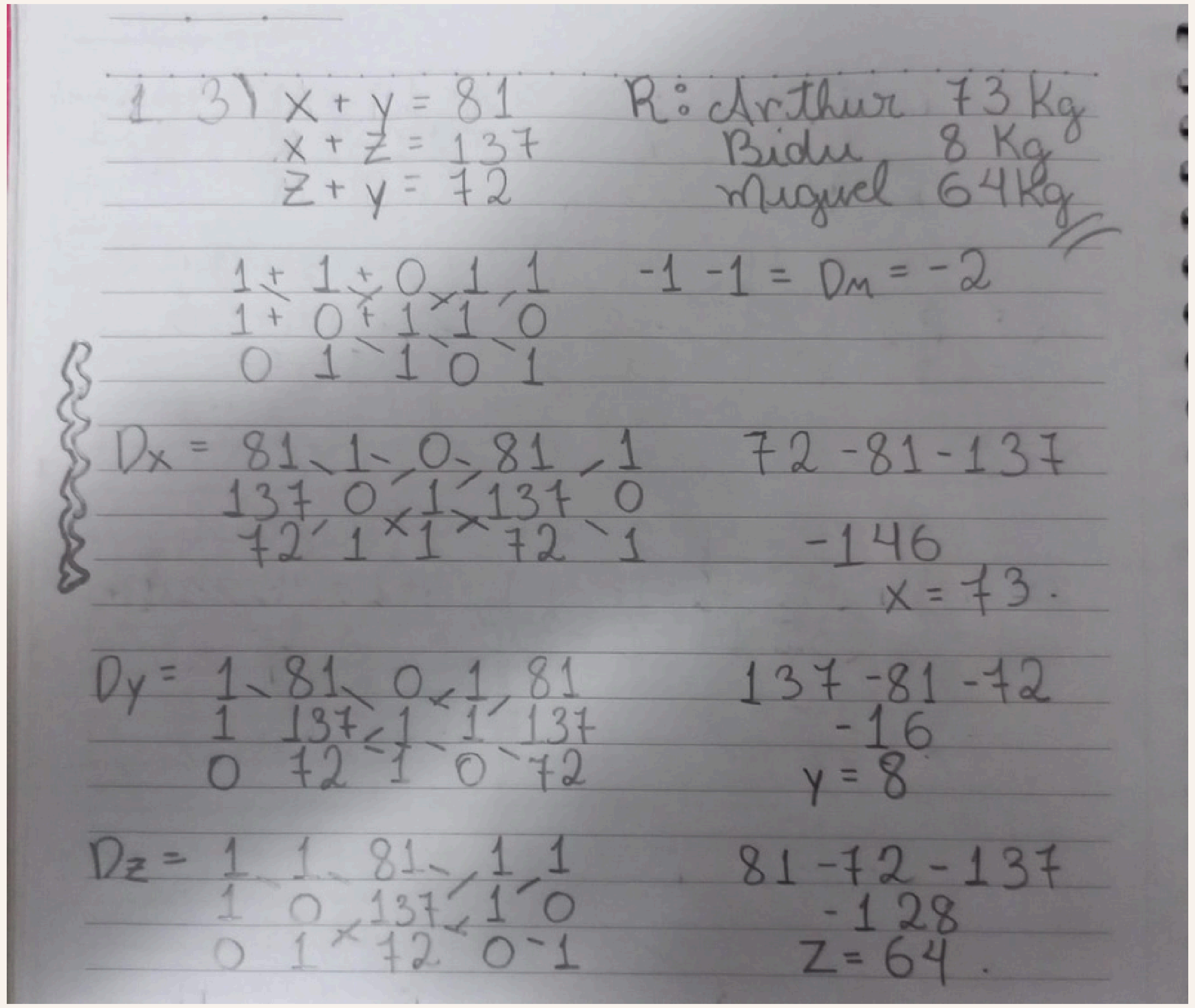
$y \begin{vmatrix} -1 & 5 & -1 & -1 & 5 \\ 1 & 4 & 4 & 1 & 4 \\ 3 & -3 & -2 & 3 & -3 \end{vmatrix}$ $8+60+3+10-12+12=81 = \frac{81}{24}=3$

$z \begin{vmatrix} -1 & 1 & 5 & -1 & 1 \\ 1 & 2 & 4 & 1 & 2 \\ 3 & 1 & -3 & 3 & 1 \end{vmatrix}$ $6+12+5+3+4-30=0 = \frac{0}{24}=0$

3) Arthur e seu irmão Miguel foram com seu cachorro Bidu à farmácia de seu avô. Lá encontraram uma velha balança com defeito, que só indicava corretamente pesos superiores a 60 kg. Assim, eles se pesaram dois a dois e obtiveram as seguintes marcas:



- Arthur e Bidu pesam juntos 81kg
- Arthur e Miguel pesam juntos 137Kg
- Miguel e Bidu pesam juntos 72Kg



1 3) $x + y = 81$
$x + z = 137$
$z + y = 72$

R: Arthur 73 Kg
Bidu 8 Kg
Miguel 64 Kg

$$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{matrix}$$

$-1 -1 = D_M = -2$

$$D_x = \begin{matrix} 81 & 1 & 0 & 81 & 1 \\ 137 & 0 & 1 & 137 & 0 \\ 72 & 1 & 1 & 72 & 1 \end{matrix}$$

$72 - 81 - 137$
-146
$x = 73$.

$$D_y = \begin{matrix} 1 & 81 & 0 & 1 & 81 \\ 1 & 137 & 1 & 1 & 137 \\ 0 & 72 & 1 & 0 & 72 \end{matrix}$$

$137 - 81 - 72$
-16
$y = 8$

$$D_z = \begin{matrix} 1 & 1 & 81 & 1 & 1 \\ 1 & 0 & 137 & 1 & 0 \\ 0 & 1 & 72 & 0 & 1 \end{matrix}$$

$81 - 72 - 137$
-128
$z = 64$.

4) Resolva o sistema linear a seguir

$$\begin{cases} 2x + y - z = 1 \\ x - y + z = 2 \\ x + 3y - 2z = 3 \end{cases}$$

4)

$2 + 1 - 1 = 1$
$1 - 1 + 1 = 2$
$1 + 3 - 2 = 3$

$$D_M = \begin{matrix} 2 & 1 & -1 & 2 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 3 & -2 & 1 & 3 \end{matrix} \qquad 4 + \cancel{1} - 3 + 2 - 6 - \cancel{1} \quad -3$$

$$D_x = \begin{matrix} 1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 1 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{matrix} \qquad +\cancel{2} + \cancel{3} - 6 + 4 - \cancel{3} - 3 \quad \frac{-3}{-3}$$

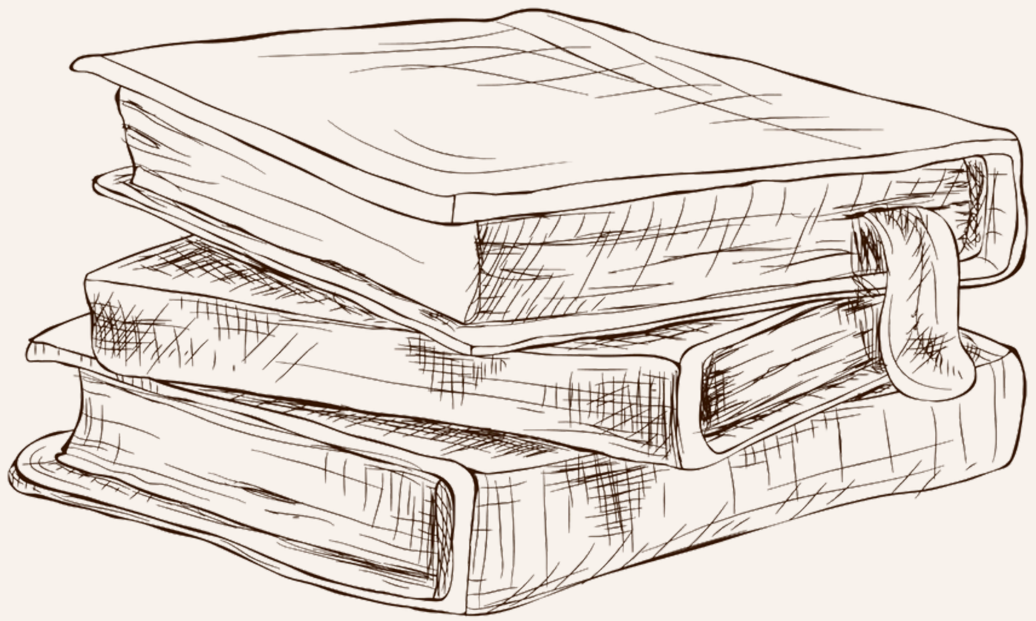
$x = 1$

$$D_y = \begin{matrix} 2 & 1 & -1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 1 & 3 & -2 & 1 & 3 \end{matrix} \qquad -8 + 1 - 3 + 2 - 6 + 2 \quad -17 + 5 \quad \frac{-12}{-3}$$

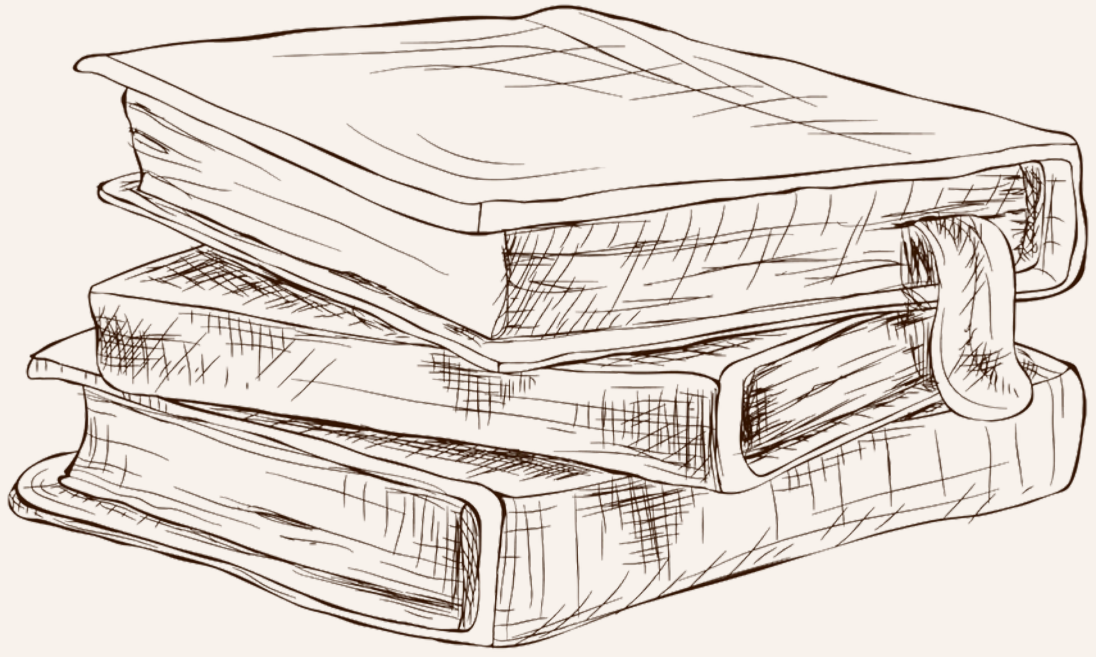
$y = 4$

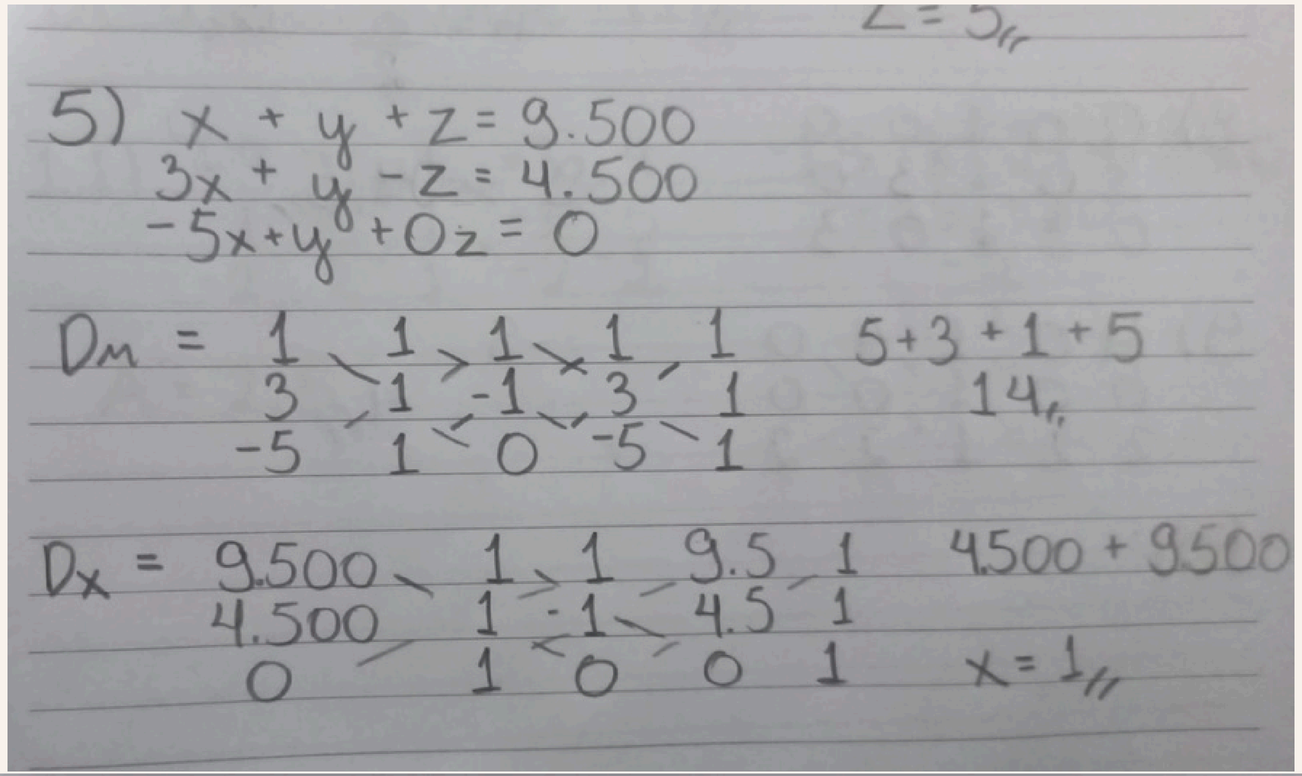
$$D_z = \begin{matrix} 2 & 1 & 1 & 2 & 1 \\ 1 & -1 & 2 & 1 & -1 \\ 1 & 3 & 3 & 1 & 3 \end{matrix} \qquad -6 + 2 + 3 - 3 - 12 + 1 \quad \frac{-15}{-3}$$

$z = 5$



5) Calcule os valores dos pesos x,y e z para os quais as balanças estão equilibradas





$z = 5$

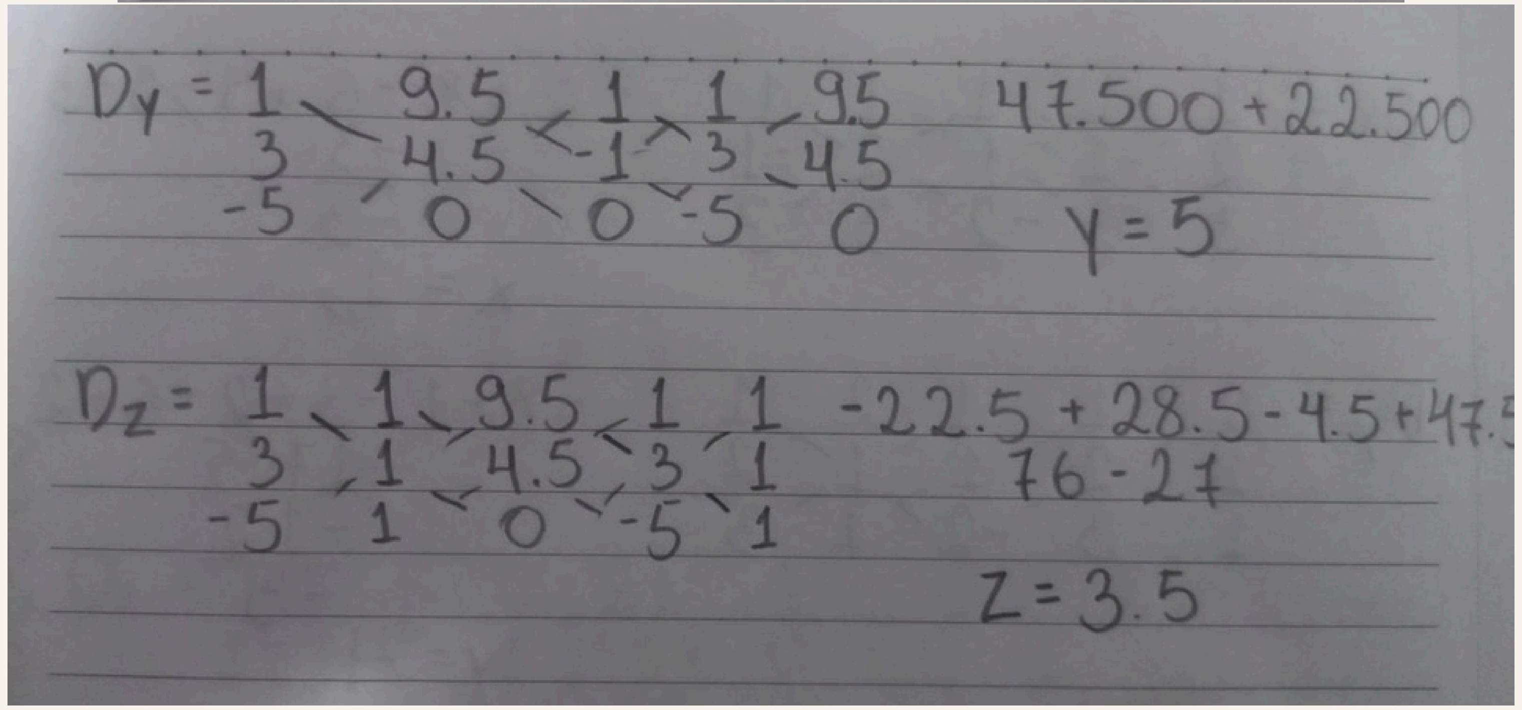
5) $x + y + z = 9.500$

$3x + y - z = 4.500$

$-5x + y + 0z = 0$

$$D_m = \begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & 3 & 1 \\ -5 & 1 & 0 & -5 & 1 \end{array} \qquad \begin{array}{c} 5+3+1+5 \\ 14 \end{array}$$

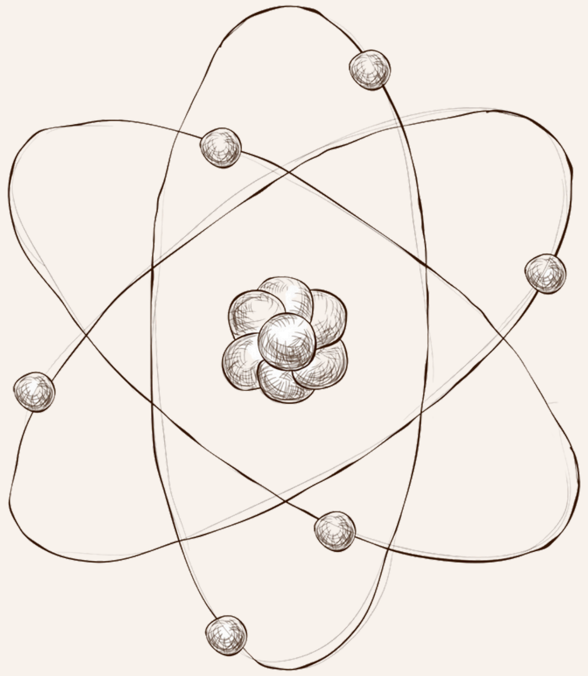
$$D_x = \begin{array}{ccccc} 9.500 & 1 & 1 & 9.5 & 1 \\ 4.500 & 1 & -1 & 4.5 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{array} \qquad \begin{array}{c} 4500 + 9500 \\ \\ x = 1 \end{array}$$



$$D_y = \begin{array}{ccccc} 1 & 9.5 & 1 & 1 & 9.5 \\ 3 & 4.5 & -1 & 3 & 4.5 \\ -5 & 0 & 0 & -5 & 0 \end{array} \qquad \begin{array}{c} 47.500 + 22.500 \\ \\ y = 5 \end{array}$$

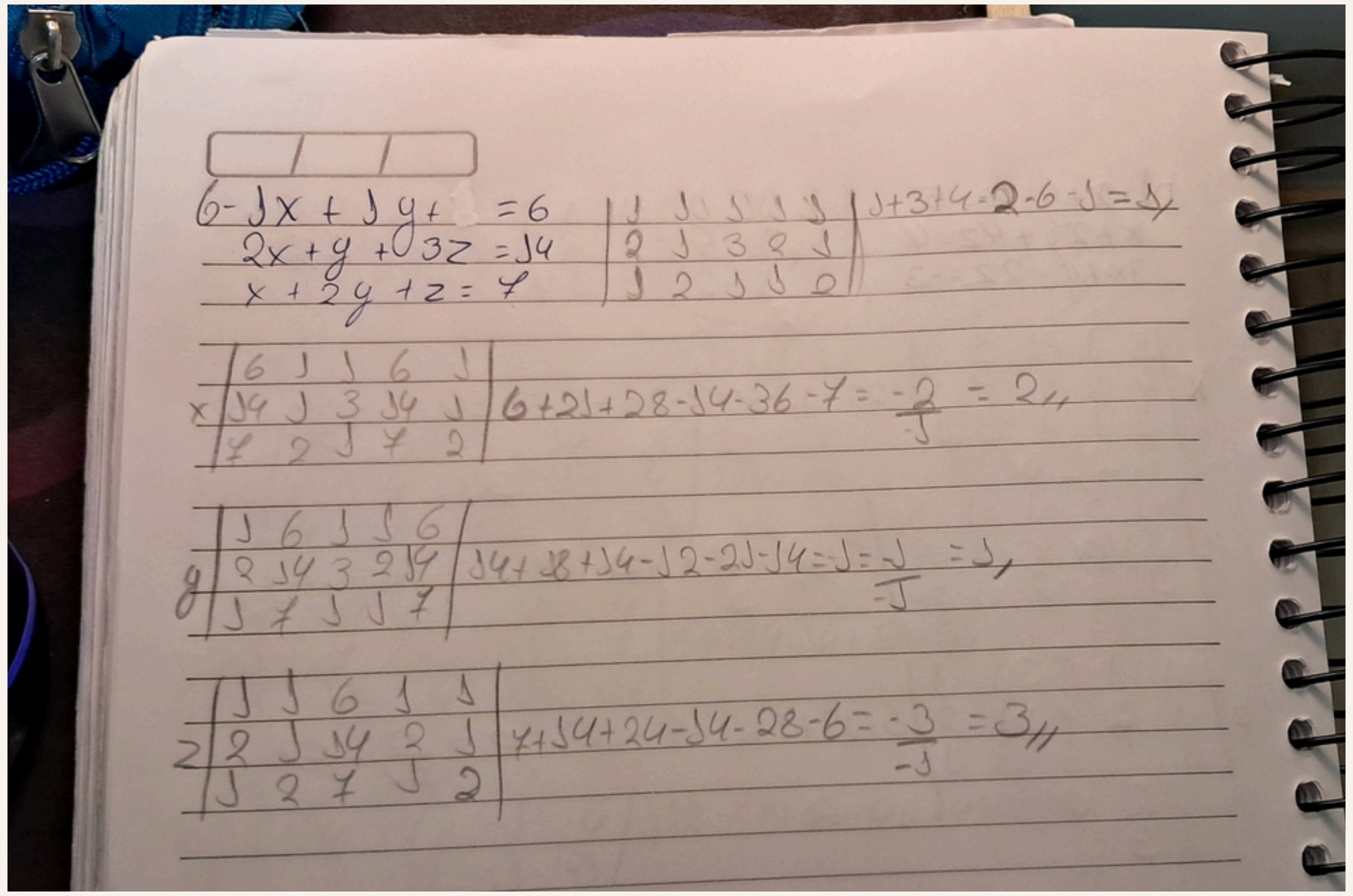
$$D_z = \begin{array}{ccccc} 1 & 1 & 9.5 & 1 & 1 \\ 3 & 1 & 4.5 & 3 & 1 \\ -5 & 1 & 0 & -5 & 1 \end{array} \qquad \begin{array}{c} -22.5 + 28.5 - 4.5 + 47.5 \\ 76 - 27 \\ \\ z = 3.5 \end{array}$$

6) Antônio Carlos gosta muito de desafios e resolveu fazer uma brincadeira com seus fregueses. Em vez de colocar os preços de cada uma de suas frutas (maçã, limão e melão), colocou as tabuletas:

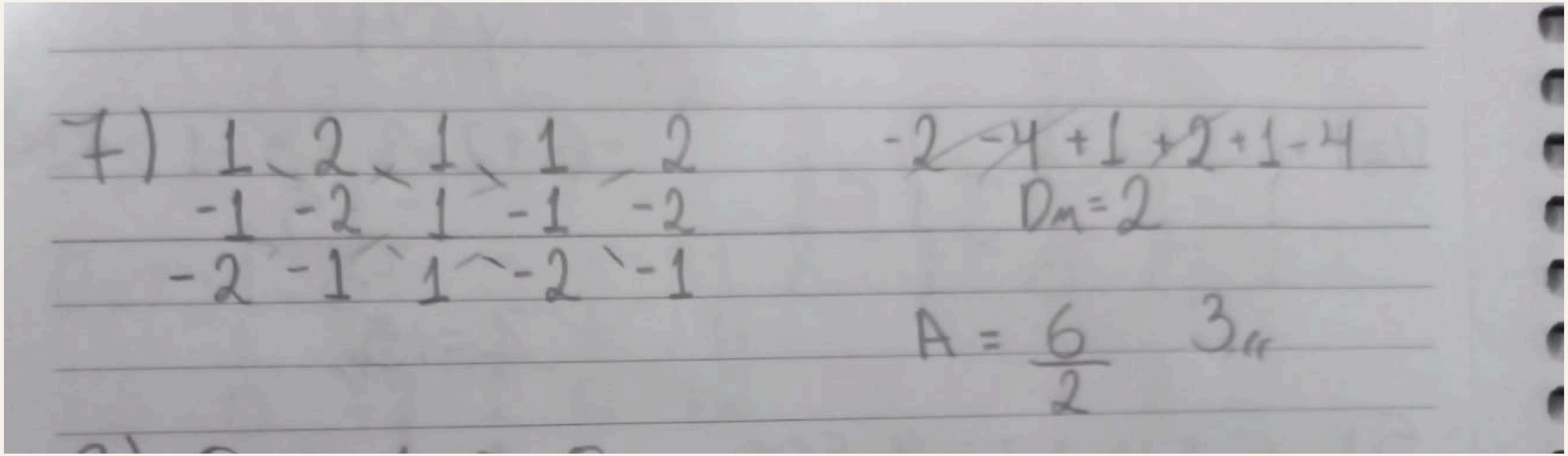


Quanto custa a duzia dos limões?

A duzia do limão custa R$12,00



7) A area do triangulo de vértices A(1;2), B(-1;-2) e C(-2;-1) é:



8) A area do triangulo de vértices A(0;0), B(3;0 e C(0,3) é:

2

8)
$$\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 0 & 3 \end{array}$$

9

4,5

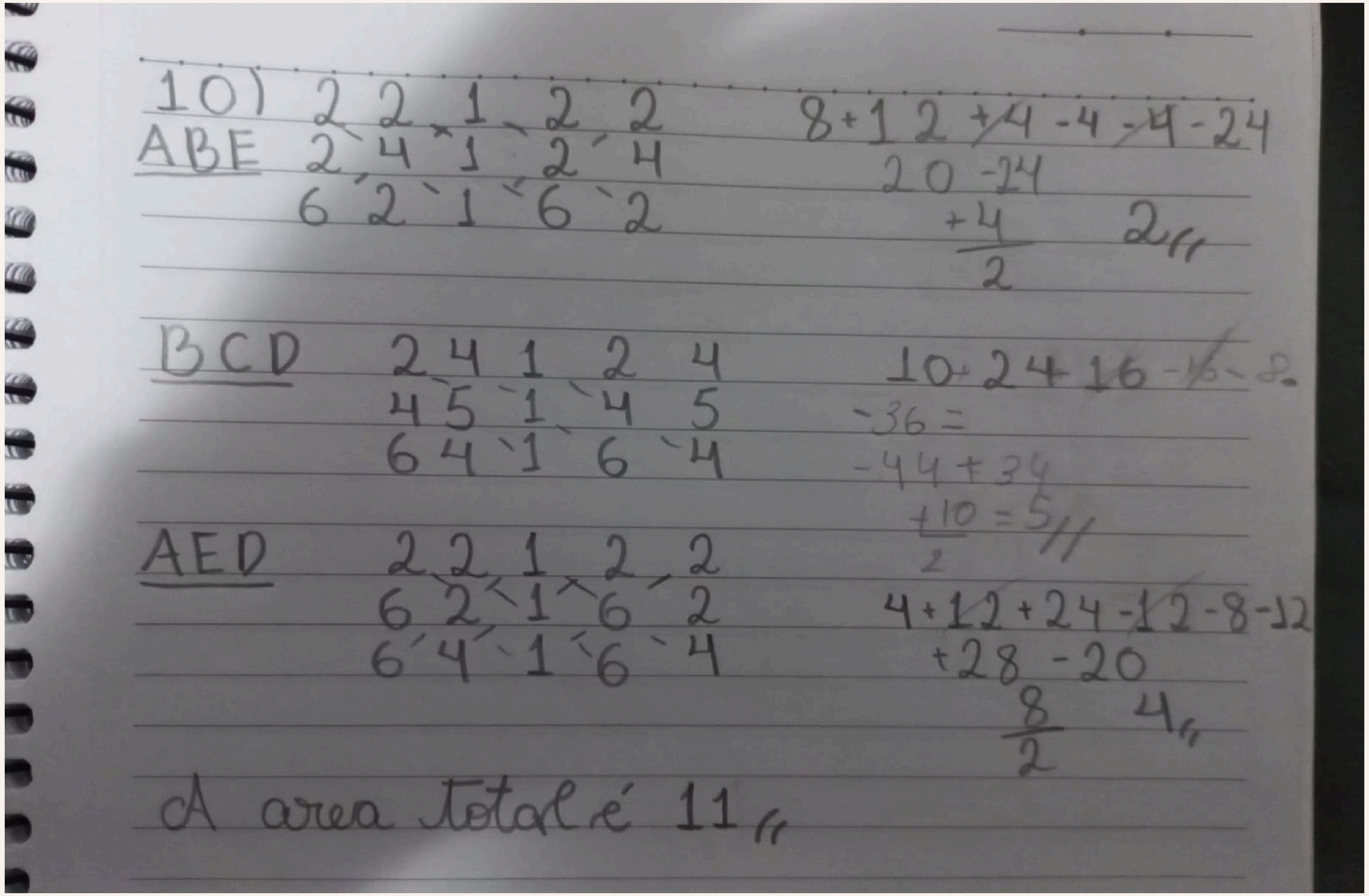
9) A area do triangulo de vértices A(4;0), B(0;0) e C(2;2) é:

9) $\begin{matrix} 4 & 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 2 & 1 & 2 & 2 \end{matrix}$

-8

4

10) Calcule a área da figura de vértices A=(2,2), B=(2,4), C=(4,5), D=(6,4), E=(6,2)



11) Calcule a área da figura de vértices A=(-3,4), B=(-1,5), C=(-2,2)

11)

$$\begin{array}{ccccc} -3 & 4 & 1 & -3 & 4 \\ -1 & 5 & 1 & -1 & 5 \\ -2 & 2 & 1 & -2 & 2 \end{array}$$

$$-15 - 8 - 2 + 4 + 6 + 10$$

$$-25 + 20$$

$$\frac{-5}{2}$$

A = 2,5