Project;Preparatory School for Stat.Phys

Statistical mechanics for natural flocks of birds

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Flock of Starling Birds

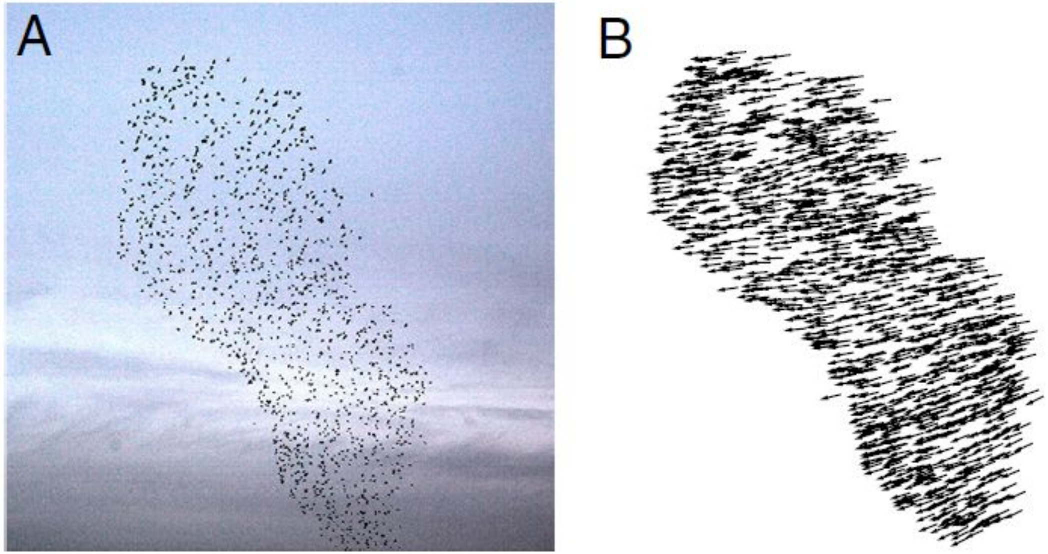


Fig. 1. The raw data. (*A*) One snapshot from flocking event 28 – 10, $N = 1,246$ birds (see *SI Appendix*, Table S1). (*B*) Instantaneous vector velocities of all the individuals in this snapshot, normalized as $\vec{s}_i = \vec{v}_i/|\vec{v}_i|$.

What we can measure from the data?

- We define the normalized velocity variables, $\vec{s}_i = \vec{v}_i / |\vec{v}_i|$

which are stationary states, so they have the jointly probability distribution, $P(\{\vec{s}_i\})$

- We want the probability that fit the data but we want it without make any unnecessary hypothesis on the model.
- We can’t obtain the the prob. distribution directly from the data. Instead, we can obtain is the correlation matrix of the normalized velocity, so we want to measure:

$$\langle \vec{s}_i \cdot \vec{s}_j \rangle_P = \langle \vec{s}_i \cdot \vec{s}_j \rangle_{\text{exp}},$$

- there exists a variety of distributions that can be fit by the data, but the minimum structure comes from Maximum Entropy Method .

  We just construct the model with the maximum information available!

Maximum Entropy Method

To obtain the probability consistent with the data, we need to extremize the Shannon Entropy. In general, the Lagrangian Multiplier of Shannon Entropy:

$$\mathcal{S}\left[P;\{\lambda_\nu\}\right] = S\left[P\right] - \sum_{\mu=0}^{K} \lambda_\mu \left[\langle f_\mu(\mathbf{x})\rangle_P - \langle f_\mu(\mathbf{x})\rangle_{\text{exp}}\right]$$

If we maximize with respect to the probability:

$$0 \;=\; \frac{\partial \mathcal{S}\left[P;\{\lambda_\nu\}\right]}{\partial P(\mathbf{x})} \longrightarrow P(\mathbf{x}) \;=\; \frac{1}{Z(\{\lambda_\nu\})} \exp\left[-\sum_{\mu=1}^{K} \lambda_\mu f_\mu(\mathbf{x})\right]$$

And if we optimize with respect to the multiplier:

$$0 \;=\; \frac{\partial \mathcal{S}\left[P;\{\lambda_\nu\}\right]}{\partial \lambda_\mu} \longrightarrow \Rightarrow \langle f_\mu(\mathbf{x})\rangle_{\text{exp}} \;=\; \frac{1}{Z(\{\lambda_\nu\})} \sum_{\mathbf{x}} f_\mu(\mathbf{x}) \exp\left[-\sum_{\mu=1}^{K} \lambda_\mu f_\mu(\mathbf{x})\right]$$

In our model, the variables are the correlation, which means we need a multiplier with two indices, the J. At last,

$$P(\{\vec{s}_i\}) = \frac{1}{Z(\{J_{ij}\})} \exp\left[\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \vec{s}_i \cdot \vec{s}_j\right]$$

The model

This model is the Heisenberg model of spins, which is good because we know how to deal with the Heisenberg model.

However, we need to make some simplifications due to the change of the bird's change of neighbours over the time. In this way, it does not make sense to talk about correlation matrix of labeled individuals.

So, we need to suppose that both strength of interaction and the number of neighbours are fixed, from which follows the probability distribution:

$$P(\{\vec{s}_i\}) = \frac{1}{Z(\{J_{ij}\})} \exp\left[\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\vec{s}_i \cdot \vec{s}_j\right] \longrightarrow P(\{\vec{s}_i\}) = \frac{1}{Z(J, n_c)} \exp\left[\frac{J}{2}\sum_{i=1}^{N}\sum_{j \in n_c^i} \vec{s}_i \cdot \vec{s}_j\right]$$

Now we need to extract from the data both values of J, n_c

Another way to do this is to make the maximum entropy method with respect to the average of correlation:

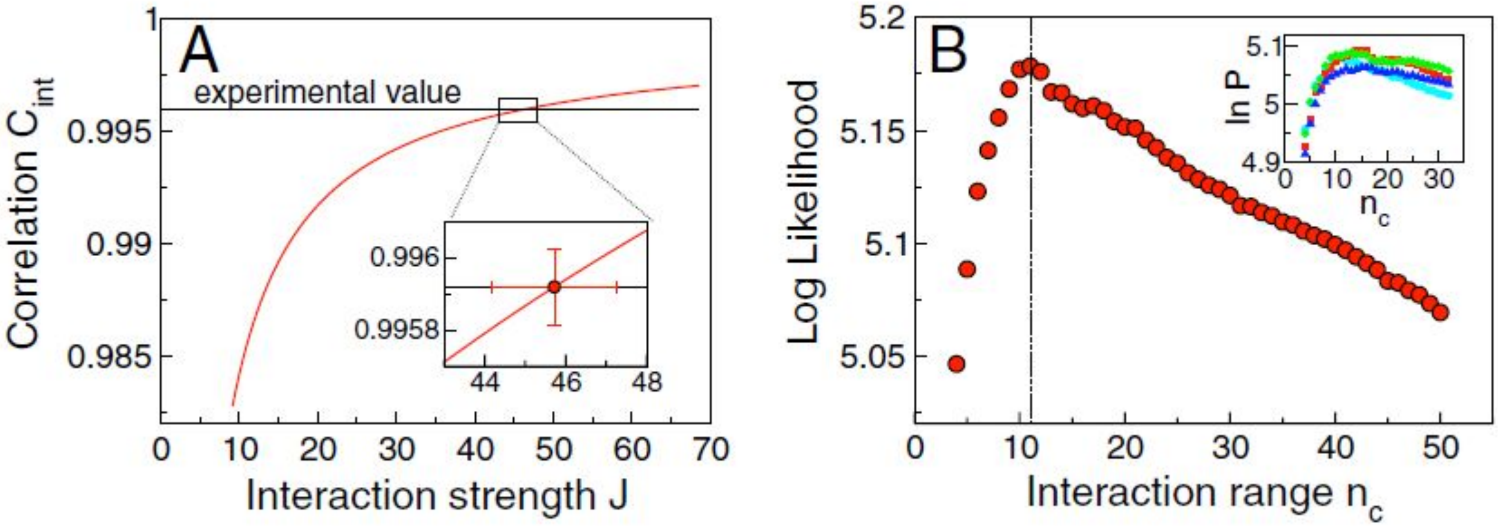
$$C_{\text{int}} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{n_c}\sum_{j \in n_c^i} \langle \vec{s}_i \cdot \vec{s}_j \rangle$$

Methods

- The data were obtained in the field using stereometric photography and computer vision techniques;
- The samples contain 21 distinct flocking events with the amount of 122 to 4268 individuals;
- The linear extensions among the starlings were between 9.10m and 85.70m and they use triangulation algorithms to measure it.

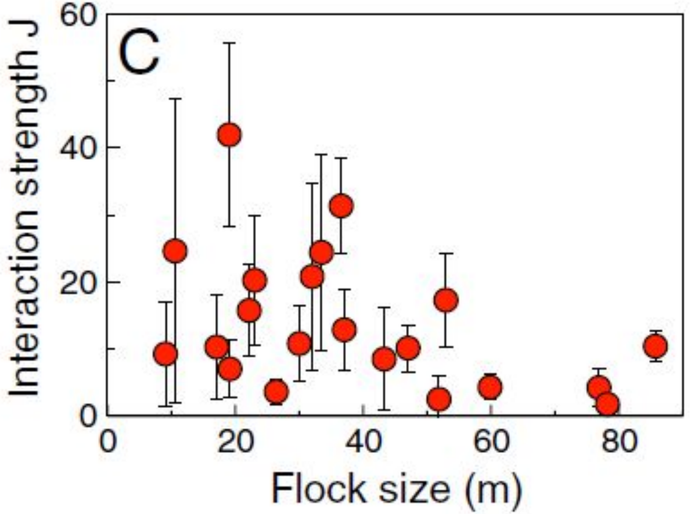
Results

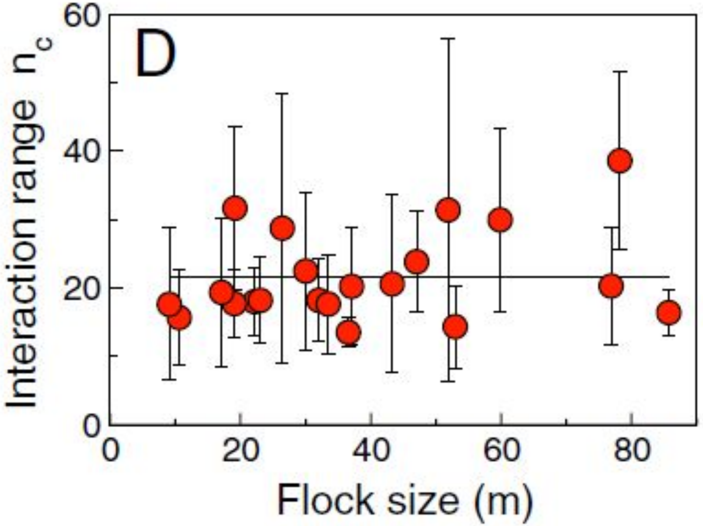
Predicted vs. Experimental Correlation and Maximum Entropy



- Find the Nc that maximizes the Shannon Entropy;
- Find the correlation value that matches with the found Nc;
- Find the correspondent value of J for the correlation;

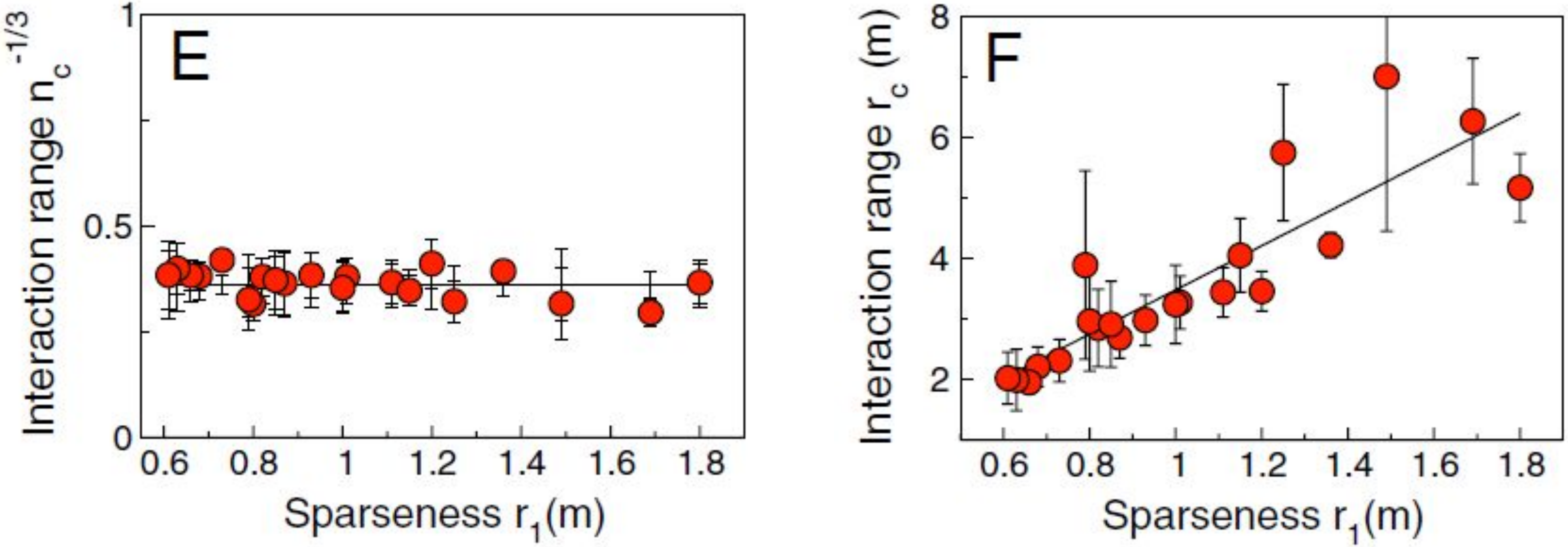
Interaction Strength J and Interaction Range Nc for all flocks





- Find an average of Nc for all flocks;
- Nc = 21.2 +- 1.7;

Metric Interaction Range vs. Topological Range



- We find the signature of interactions with a fixed number of topological neighbours;

Model Predictions

From a single input: C_{int} $\longrightarrow$ J and n_c $\longrightarrow$ Fix all the parameters.

Solve the system with the Maximum Entropy Model with:

- Fixed flight direction as a boundary condition.
- Spin wave approximation.

Calculate any observable from the model.

NOT AS A FIT, BUT AS A PREDICTION!

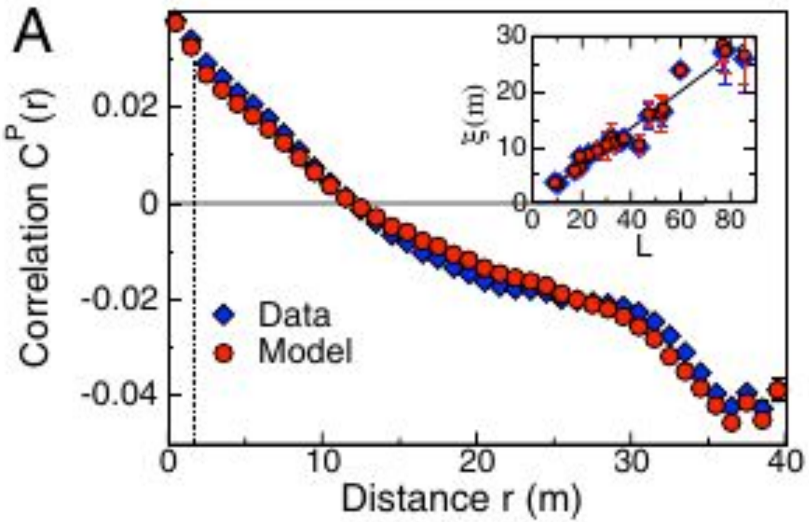
Correlation (L and P) as a function of the distance:

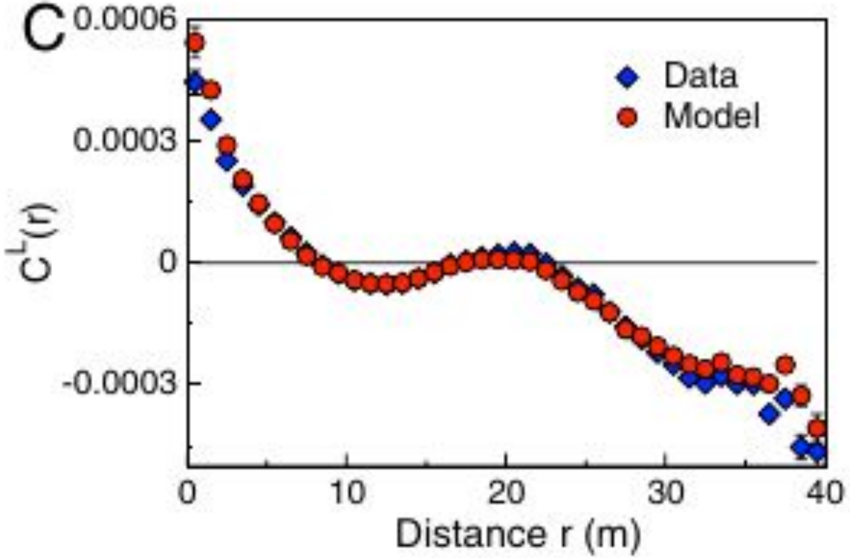
$$\vec{S} = (1/N)\sum_i \vec{s}_i = S\vec{n}$$

$$\vec{s}_i = s_i^L \vec{n} + \vec{\pi}_i$$

$$C^P(r) = \langle \vec{\pi}_i \cdot \vec{\pi}_j \rangle$$

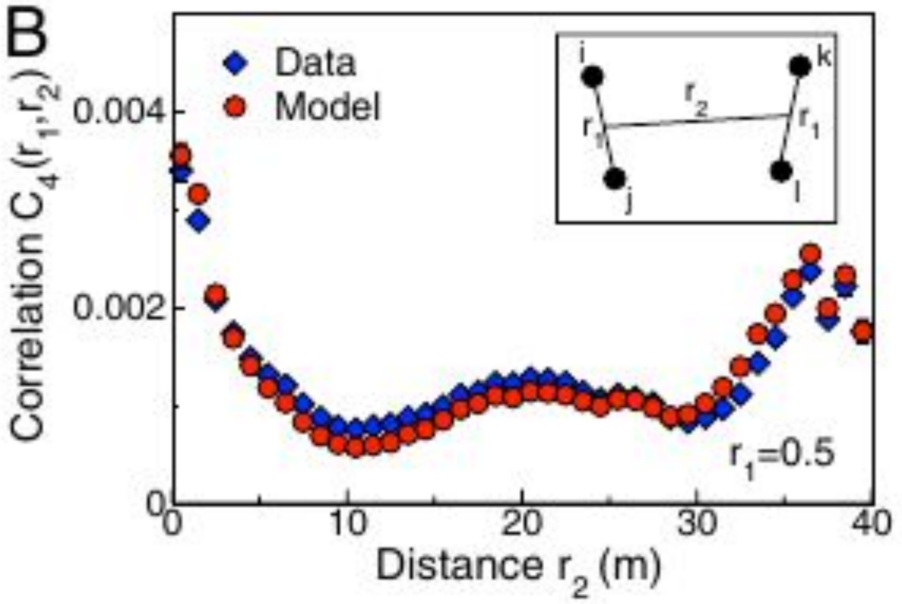
$$C^L(r) = \langle s_i^L s_j^L \rangle - S^2$$





Cavagna A, et al. (2010) Scale-free correlations in starling flocks.Proc Nati Acad Sci USA 107:11865-11870

Four point correlation function:

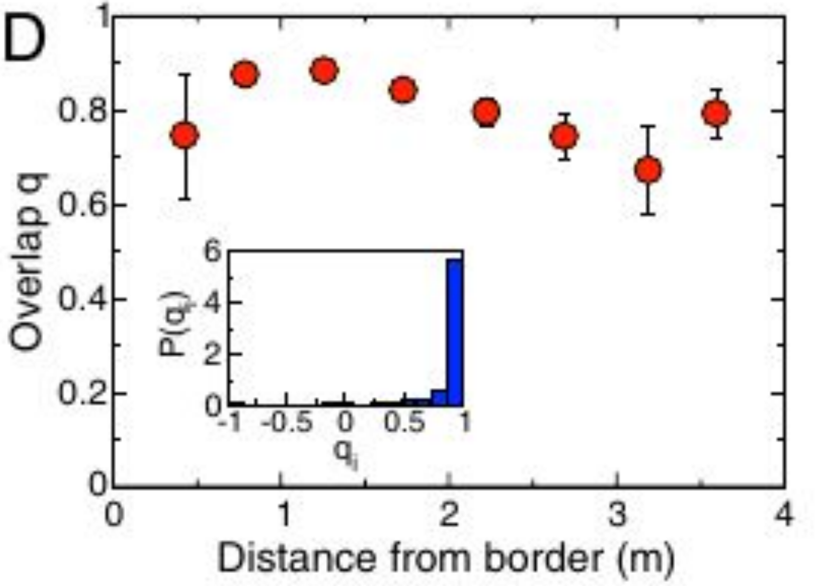


The model has no input about higher order correlation.

Surprisingly reproduce very well the data.

Overlap:

$$q_i = \langle \vec{\pi}_{\mathrm{i}} \rangle \cdot \vec{\pi}_{\mathrm{i}}^{\mathrm{exp}} / (|\langle \vec{\pi}_{\mathrm{i}} \rangle||\vec{\pi}_{\mathrm{i}}^{\mathrm{exp}}|)$$



Measures alignment between data velocities and model velocities.

Analogy with Heinsenberg model for spins

the mechanistic interpretation

Maximum entropy model

$$P(\{\vec{s}_i\}) = \frac{1}{Z(\{J_{ij}\})} \exp \underbrace{\left[\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \vec{s}_i \cdot \vec{s}_j \right]}_{\text{- Hamiltonian}}$$

$\vec{s}_i =$ velocity directions

Heinsenberg model for spins

$$H(\{\vec{s}_i\}) = -(1/2) \sum_{i,j} J_{ij} \vec{s}_i \cdot \vec{s}_j$$

$\vec{s}_i =$ spins

Hamiltonian $\Rightarrow$ Dynamics

Langevin dynamics

$$\frac{d\vec{s}_i}{dt} = -\frac{\partial H}{\partial \vec{s}_i} + \vec{\eta}_i(t) = \underbrace{\sum_{j=1}^{N} J_{ij} \vec{s}_j}_{\text{social forces}} + \underbrace{\vec{\eta}_i(t)}_{\text{noise}}$$

Testing the mechanistic interpretation. Can we take it seriously?

Simulation

- Self propelled particles
- Social forces

Compare

Microscopic parameters

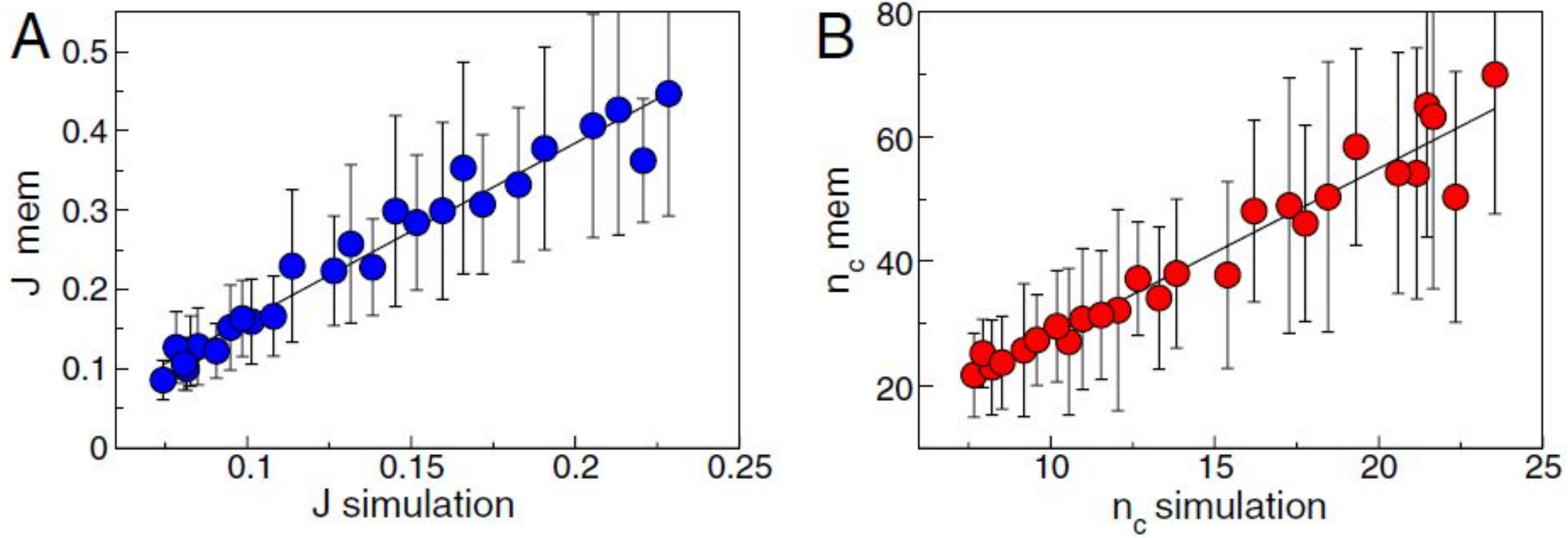
J_{sim} n_{sim}

- simulation inputs

Maximum entropy model

J_{mem} n_{mem}

- just like withe the real data



- proportional values, not equal $n_c^{\mathrm{mem}} \sim 3 \times n_c^{\mathrm{sim}}$

Calibration

Observed in real flocks (mem) $n_c = 21.2$

True interactions $n_c = 7.8$

Thank you !

simulation model

$$\vec{v}_i(t+1) = v_0 \Theta \left[\alpha \sum_{j \in n_c^i} \vec{v}_j(t) + \beta \sum_{j \in n_c^i} \vec{f}_{ij} + n_c \vec{\eta}_i \right]$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t),$$

$$\vec{f}_{ij}(r_{ij} < r_b) = -\infty \vec{e}_{ij},$$

$$\vec{f}_{ij}(r_b < r_{ij} < r_a) = \frac{1}{4} \cdot \frac{r_{ij} - r_e}{r_a - r_e} \vec{e}_{ij},$$

$$\vec{f}_{ij}(r_a < r_{ij} < r_0) = \vec{e}_{ij}.$$

Additional Material:

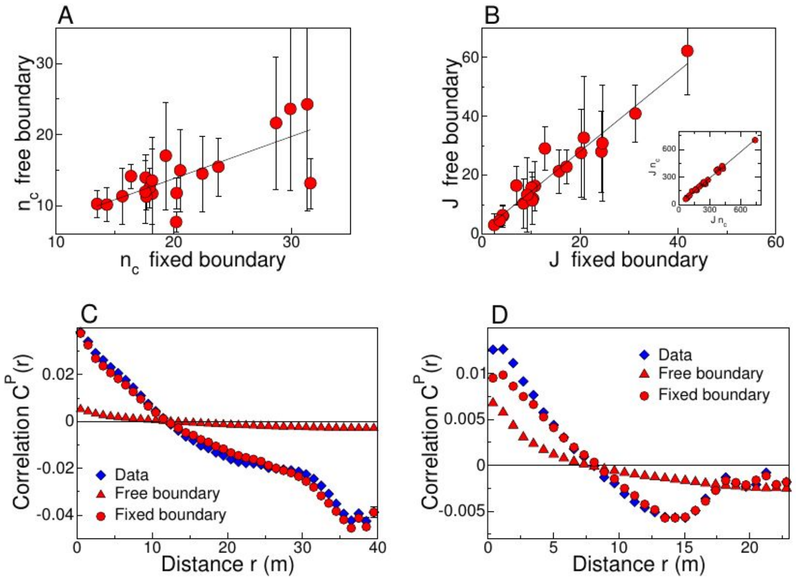
Experimental data:

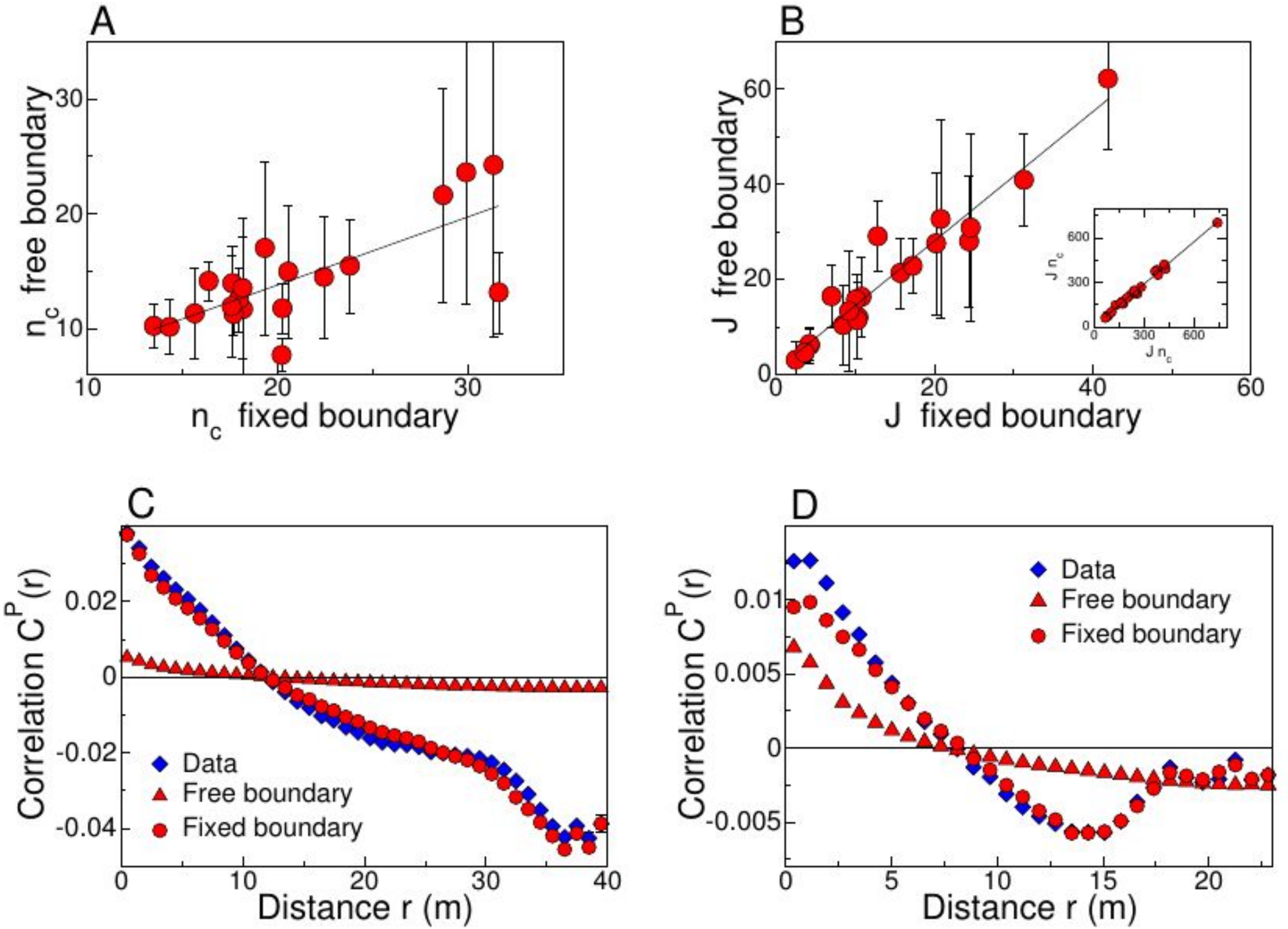
Event[a]	N	S	v_0 (m/s)	L (m)
17–06	552	0.935	9.4	51.8
21–06	717	0.973	11.8	32.1
25–08	1571	0.962	12.1	59.8
25–10	1047	0.991	12.5	33.5
25–11	1176	0.959	10.2	43.3
28–10	1246	0.982	11.1	36.5
29–03	440	0.963	10.4	37.1
31–01	2126	0.844	6.8	76.8
32–06	809	0.981	9.8	22.2
42–03	431	0.979	10.4	29.9
49–05	797	0.995	13.9	19.2
54–08	4268	0.966	19.1	78.7
57–03	3242	0.978	14.1	85.7
58–06	442	0.984	10.1	23.1
58–07	554	0.977	10.5	19.1
63–05	890	0.978	9.9	52.9
69–09	239	0.985	11.8	17.1
69–10	1129	0.987	11.9	47.3
69–19	803	0.975	13.8	26.4
72–02	122	0.992	13.2	10.6
77–07	186	0.978	9.3	9.1

4 Bird correlation:

$$C_4(r_1;r_2) = \frac{\sum_{\mathrm{ijkl}} \langle (\vec{\pi}_{\mathrm{i}} \cdot \vec{\pi}_{\mathrm{j}})(\vec{\pi}_{\mathrm{i}} \cdot \vec{\pi}_{\mathrm{j}}) \rangle \Delta_{\mathrm{ijkl}}}{\sum_{\mathrm{ijkl}} \Delta_{\mathrm{ijkl}}}, \qquad \text{(S61)}$$

$$\Delta_{\mathrm{ijkl}} = \delta(r_{\mathrm{ij}} - r_1)\delta(r_{\mathrm{kl}} - r_1)\delta(r_{\mathrm{ij-kl}} - r_2)$$





A
B
C
D
n_c free boundary
n_c fixed boundary
J free boundary
J fixed boundary
J n_c
Correlation $C^P(r)$
Distance r (m)
Data
Free boundary
Fixed boundary