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① Distribuição Binomial

$P(X=k) = \binom{n}{k} \cdot p^{k} \cdot q^{(n-k)}$

↳ fracasso
↳ sucesso

(a)
Solução:
$p \to$ ter defeito $= 0{,}12$
$q \to$ não ter defeito $= 0{,}88$
$n \to$ espaço amostral $= 40$
$k \to$ qtde. analisada

$\Rightarrow p = P(X=3) + P(X=4) + P(X=5)$

- $P(X=3) = \binom{40}{3} \cdot (0{,}12)^3 \cdot (0{,}88)^{40-3} = 0{,}1507$
- $P(X=4) = \binom{40}{4} \cdot (0{,}12)^4 \cdot (0{,}88)^{40-4} = 0{,}1901$
- $P(X=5) = \binom{40}{5} \cdot (0{,}12)^5 \cdot (0{,}88)^{40-5} = 0{,}1866$

$\boxed{p = 0{,}5274}$

(b)
$p = 1 - [P(X=1) + P(X=0)]$
- $P(X=1) = \binom{40}{1} \cdot (0{,}12)^1 \cdot (0{,}88)^{40-1} = 0{,}0328$
- $P(X=0) = \binom{40}{0} \cdot (0{,}12)^0 \cdot (0{,}88)^{40-0} = 0{,}0060$

$\boxed{p = 0{,}9612}$

(c) $p = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
- $P(X=2) = \binom{40}{2} \cdot (0{,}12)^2 \cdot (0{,}88)^{38} = 0{,}0872$

$\boxed{p = 0{,}2767}$

(d) $p = P(X=0) + P(X=1)$

$\boxed{p = 0{,}0388}$

(e) $p = P(X \geq 1) = 1 - P(X=0)$

$\boxed{p = 0{,}9940}$

② (i) Distribuição Binomial

$p \to$ ocorrer falha $= 0{,}6$
$q \to$ não ocorrer falha $= 0{,}4$
$n \to 3$
$X \to k$

$p = \binom{3}{k} \cdot (0{,}6)^k \cdot (0{,}4)^{3-k}$

k	X	p	p
0	0	$1 \cdot (0{,}6)^0 \cdot (0{,}4)^3$	0,0640
1	1	$3 \cdot (0{,}6)^1 \cdot (0{,}4)^2$	0,2880
2	2	$3 \cdot (0{,}6)^2 \cdot (0{,}4)^1$	0,4320
3	3	$1 \cdot (0{,}6)^3 \cdot (0{,}4)^0$	0,2160

(ii) Esperança matemática

$E(X=k) = \sum_{i=1}^{\infty} x_i \cdot p(x_i)$

$\Rightarrow E(X=k) = 0 \cdot 0{,}0640 + 1 \cdot 0{,}2880 + 2 \cdot 0{,}4320 + 3 \cdot 0{,}216$

$\Rightarrow \boxed{E(X=k) = 1{,}8}$

③ Distribuição de Poisson

$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

$\lambda \to$ nº esperado de sucessos ou taxa de ocorrência, $\lambda > 0$
$x \to$ nº de sucessos.
$e \to$ nº de Euler.

Solução:

(a) $P(4) = \frac{e^{-3} \cdot 3^4}{4!} = 0{,}1680$

(b) $P(0) = \frac{e^{-3} \cdot 3^0}{0!} = 0{,}0498$

(c) $\lambda = \frac{3}{1\,dia} \times 7\,dias = 21$ chamados.

$P(20) = \frac{e^{-21} \cdot 21^{20}}{20!} = 0{,}0867$

4

a) $n \to$ nº de ações que caíram de preço

$n = \frac{75}{100} \cdot 15 = 11,25$

b) $\sigma = \sqrt{n \cdot p \cdot q}$ Desvio Padrão da Distrib. Binomial

$\Rightarrow \sigma = \sqrt{15 \cdot 0,75 \cdot 0,25}$

$\boxed{\sigma = 1,6770}$

c)
$p' = P(X=15) = \binom{15}{15} \cdot (0,75)^{15} \cdot (0,25)^{0}$

$\Rightarrow \boxed{p' = 0,0133}$

d) $p' = P(X=10) = \binom{15}{10} \cdot (0,75)^{10} \cdot (0,25)^{5}$

$\boxed{p' = 0,1651}$

e)
$p' = P(X \geq 13) = P(X=13) + P(X=14) + P(X=15)$

- $P(X=13) = \binom{15}{13} \cdot (0,75)^{13} \cdot (0,25)^{2} = 0,1559$
- $P(X=14) = \binom{15}{14} (0.75)^{14} (0,25)^{1} = 0,0668$
- $P(X=15) = \binom{15}{15} \cdot (0,75)^{15} \cdot (0,25)^{0} = 0,0134$

$\boxed{p' = 0,2361}$

5 Distribuição normal

$Z(x) = \frac{x - \mu}{\sigma}$

$\mu \to$ média
$\sigma \to$ desvio padrão

Solução:

a) $\mu = 850$
$\sigma = 40$

$Z(x=700) = \frac{700-850}{40} = -3,75$
$Z(x=1000) = \frac{1000-850}{40} = 3,75$

$f(z=+3,75) = 0,99991$
$f(z=-3,75) = 0,00009$

$p' = 0,99991 - 0,00009$

$\boxed{p' = 0,99982}$

b) $Z(x=800) = \frac{800-850}{40} = \frac{-5}{4} = -1,25$

$\Rightarrow p' = 1 - f(z=-1,25) = 1 - 0,1056$

$\boxed{p' = 0,8944}$

c) $Z(x=750) = \frac{750-850}{40} = \frac{-5}{2} = -2,5$

$p' = f(z=-2,5) = 0,00621$

$\boxed{p' = 0,00621}$

(d) Em anexo.

6

a) $p(X=4) = \binom{6}{4} \cdot (0,5)^{4} \cdot (0,5)^{2}$

$\boxed{p = 0,2343}$

b) $p' = p(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$p' = 1 - \left(\binom{6}{0} \cdot (0,5)^{0} \cdot (0,5)^{6} + \binom{6}{1} \cdot (0,5)^{1} \cdot (0,5)^{5} \right)$

$p' = 1 - (0,0156 + 0,0937)$

$\boxed{p' = 0,8907}$

c) $p' = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$ $0,0156 + 0,0937 + 0,2343 + 0,3125$

$\Rightarrow p' = 0,0156 + 0,0937 + 0,2343 + 0,3125$

- $P(X=3) = \binom{6}{3} \cdot (0,5)^{3} \cdot (0,5)^{3} = 0,3125$
- $P(X=2) = \binom{6}{2} \cdot (0,5)^{2} \cdot (0,5)^{6} = 0,2343$

$\boxed{p' = 0,6561}$

7

a) $p' = P(X=4) = \binom{30}{4} \cdot (0,2)^{4} \cdot (0,8)^{26} \Rightarrow \boxed{p' = 0,1325}$

b) $p' = P(X \leq 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

- $P(X=0) = \binom{30}{0} \cdot (0,2)^{0} \cdot (0,8)^{30} = 0,0012$
- $P(X=1) = \binom{30}{1} \cdot (0,2)^{1} \cdot (0,8)^{29} = 0,0093$
- $P(X=2) = \binom{30}{2} \cdot (0,2)^{2} \cdot (0,8)^{28} = 0,0336$

$\boxed{p' = 0,9559}$

⑧ $\begin{cases} \mu = 4000 \\ \sigma = 200 \end{cases}$ $z(x) = \dfrac{x-\mu}{\sigma}$

a) $\begin{cases} z(x=3600) = \dfrac{3600-4000}{200} = -2 \\ z(x=4250) = \dfrac{4250-4000}{200} = 1,25 \end{cases}$

$\begin{cases} f(z=-2) = 0,0227 \\ f(z=1,25) = 0,8943 \end{cases}$ $p' = f(1,25) - f(-2)$

$p' = \boxed{0,8716}$

b) $z(x=3400) = \dfrac{34\cancel{0}\cancel{0} - 40\cancel{0}\cancel{0}}{2\cancel{0}\cancel{0}} = -3$

$f(z=-3) = 0,0013$

$p' = \boxed{0,0013}$

c) $z(x=4636) = \dfrac{4636-4000}{200} = 3,18$

$p' = 1 - f(z=3,18) \Rightarrow p' = \boxed{0,0008}$

d) Em anexo

⑨ a) $f(z=?) = 0,04 \Rightarrow z = -1,75$

$\Rightarrow -1,75 = \dfrac{x-1000}{100} \Rightarrow x = \boxed{825\ell}$

b) Em anexo.

⑩ $p(x \geq 4) = P(X=4) + P(X=5) + P(X=6)$

$\Rightarrow p(x \geq 4) = 0,01536 + 0,001536 + 0,000064$

$\boxed{p = 0,01696}$

⑪ $p(x=3) = \binom{100}{3} \cdot (0,001)^3 \cdot (0,999)^{997}$

$p = \boxed{0,61}$

a) $p(X=12) = \binom{30}{12} \cdot (0,5)^{30}$

$p = \boxed{0,08055}$

$p' = p(x>20) = \sum_{i=21}^{30} P(x=i)$

$\Rightarrow \boxed{p' = 0,0214}$

⑬ $\begin{cases} \mu = 5 \\ \sigma = 0,9 \end{cases} \Rightarrow Z = \dfrac{x-\mu}{\sigma} \Rightarrow x = 0,9$

(i) $P(Z) = 0,15 \Rightarrow z = -1,04 \Rightarrow x \leq 4,1$ kg (peq

(ii) $P(Z) = 0,15 + 0,5 \Rightarrow z = 0,39 \Rightarrow 4,1 < x < 5,4$ (m

(iii) $P(Z) = 0,15 + 0,5 + 0,2 \Rightarrow z = 1,04 \Rightarrow 5,4 < x < 5,9$ (gr

$\Rightarrow x > 5,9$ (extra)

⑭ a) $z(x=990) = \dfrac{990-1000}{10} = -1$

$f(z=-1) = 0,15866$

$p' = \boxed{0,15866}$

b) $x - \mu \leq 2\sigma \Rightarrow x \leq 2\sigma + \mu$

$\Rightarrow x \leq 1020.$

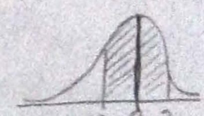
• $z(x=1020) = \dfrac{1020-1000}{10} = 2$

• $f(z=2) = 0,97725$

ou

• $z(x=8) = \dfrac{980-1000}{10} = -2$

• $f(z=-2) = 0,02275$



$\Rightarrow p = f(z=2) - f(z=-2)$

$p = 0,9545,$

o que representa 95% dos garrafos.

c)

$p(x>1002) = 1 - f\left(z = \dfrac{1002-1000}{10}\right) = 1 - f(z=0,5)$

$\Rightarrow P(x>1002) = 1 - 0,69146 = 0,30854$

$p' = P(X \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

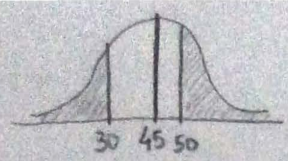
$\Rightarrow p' = \binom{4}{0} \cdot (0,30854)^0 \cdot (0,69146)^4 + \binom{4}{1} \cdot (0,30854)^1 \cdot (0,69146)^3$

$+ \binom{4}{2} \cdot (0,30854)^2 \cdot (0,69146)^2 + \binom{4}{3} \cdot (0,3085)^3 \cdot (0,69146)^1$

$+ \binom{4}{4} \cdot (0,30854)^4 \cdot (0,69146)^0$

$\boxed{p' = 0,3139}$

15 $\begin{cases} \mu = 45 \\ \sigma = 20 \\ E = 50 \end{cases}$



i) $z(50) = \dfrac{50-45}{20} = 0{,}25$

$z(30) = \dfrac{30-45}{20} = -0{,}75$

ii) $p' = 1 - f(z=0{,}25) + f(z=-0{,}75)$

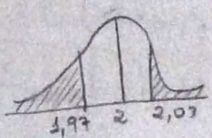
$p' = 1 - 0{,}59871 + 0{,}22663$

$\boxed{p' = 0{,}62792}$ qtde: $p' \times 50$ $\boxed{\text{qtde} = 31{,}396}$

16 $\begin{cases} \mu = 2 \\ \sigma = 0{,}01 \end{cases}$ $|x - \mu| > 0{,}03$

$1{,}97 < x < 2{,}03$

$\begin{cases} z(x=1{,}97) = -3 \\ z(x=2{,}03) = 3 \end{cases}$



$p = 1 - f(z=+3) + f(z=-3)$

$p = 1 - 0{,}99865 + 0{,}00135$

$p = 0{,}0027 \Rightarrow \boxed{p = 27\%}$

17 $\begin{cases} \mu = 8 \\ \sigma = 1{,}8 \\ p = 0{,}05 \\ x = ? \end{cases}$ $p(z) = 0{,}05$

$\Rightarrow z = -1{,}64$

$z(x) = -1{,}64$

$\Rightarrow -1{,}64 = \dfrac{x-8}{1{,}8}$

$\Rightarrow x = 8 - 2{,}952$

$\boxed{x = 5{,}048}$

18 $E(x) = \dfrac{1}{\lambda} \Rightarrow 5 = \dfrac{1}{\lambda} \Rightarrow \lambda = 0{,}2$

a) $P(x>5) = e^{-0{,}2 \cdot 5} = 0{,}3679$

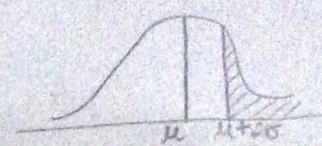
b) $P(x \leq 4) = 1 - e^{-0{,}2 \cdot 4} = 0{,}5507$

c) $P(3 < x < 8) = P(x>3) - P(x>8)$

$P(3 < x < 8) = e^{-0{,}2}(e^{3} - e^{8})$

$\boxed{P(3 < x < 8) = 0{,}3469}$

19 $\begin{cases} \mu = 129 \\ \sigma = 14 \end{cases}$



a) $z = \dfrac{2\sigma}{\sigma}$ $\nearrow z_1 = 2 \Rightarrow P_1 = 0{,}97725$; $\searrow z_2 = -2 \Rightarrow P_2 = 0{,}02275$

$p = 1 - 0{,}97725$

$\boxed{p = 0{,}2275}$

b) $z(100) = \dfrac{100-129}{14} = -2{,}07$

$f(z=-2{,}07) = 0{,}01923$

$\boxed{p = 0{,}01923}$

c) $p(z) = 0{,}95 \Rightarrow z = 1{,}65$

$1{,}65 = \dfrac{x-129}{14}$

$\Rightarrow x = 129 + 1{,}65 \cdot 14$

$\Rightarrow \boxed{x = 152{,}1}$

d) $z(199) = \dfrac{199-129}{14} = 5$

$f(z=5) < 0{,}00002 < 0{,}002\%$

A quantidade de cirurgias que são realizadas neste tempo é muito pequena.

20 $\mu = 600$ $z = \dfrac{312-600}{100} = -2{,}88$

$\sigma = 100$

$x = 312$

$n = 10\,000$

$\boxed{P = 0{,}00199}$

$\text{Qtde} = n \cdot p \Rightarrow \boxed{\text{qtde} = 19{,}9 \simeq 20}$