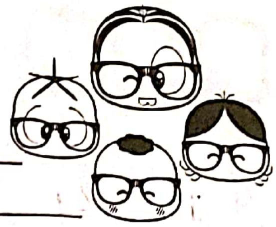


TESTE 1 - MAT 135

1- $2A - AA^T \leftrightarrow 2A \cdot I_n - A \cdot A^T \leftrightarrow A(2I_n - A^T)$

$A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \quad A^T = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \quad 2I_n = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$A \, (2I_n - A^T)$$

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \right)$$

C

$$\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0-1 & 2+2 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$$

2. $D = \begin{array}{c} \begin{array}{cccc} & & C_2 & \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & -2 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \end{array}$

Por Laplace:

$\det(D) = (-2)(-1)^{2+3} \begin{vmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} = (-2)(-1)(1) = 2$

$\begin{vmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 1 & -1 \\ 0 & -2 \end{matrix}$ $\det = -1+0+2-0-0-0$

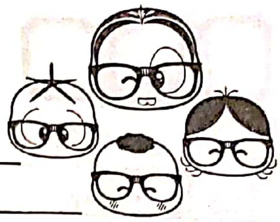
$\det = 1$

D

3. $A \xrightarrow{L_1 \leftarrow 2L_1} X \xrightarrow{L_2 \leftrightarrow L_1} Y \xrightarrow{L_2 \leftarrow L_2 - 2L_1} B$ $\det(B) = -2$

$\det(A) = 1$ multiplica pela constante $\det(X) = 2$ muda sinal $\det(Y) = -2$ não muda

C



4- $\begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix} \underset{L_1 \leftarrow 2L_1}{\sim} \begin{bmatrix} -2 & 4 \\ 1 & 1 \end{bmatrix} \underset{L_2 \leftrightarrow L_1}{\sim} \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \sim$

$\underset{L_2 \leftarrow L_2 - 2L_1}{\sim} \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$ D

5- $A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj} A$

$A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{matrix} -1 & 0 \\ 0 & -1 \\ 2 & 0 \end{matrix}$ $\det(A) = 3+0+0-2+0+0 = 1$ //

$\bar{A} \rightarrow a_{11} = \Delta_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} = (1)(-3) = -3$

$a_{12} = \Delta_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = (-1)(0) = 0$

$a_{13} = \Delta_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = (1)(2) = 2$

$a_{21} = \Delta_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -1 \\ 0 & 3 \end{vmatrix} = (-1)(0) = 0$

$a_{22} = \Delta_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -1 \\ 2 & 3 \end{vmatrix} = (1)(-1) = -1$

$a_{23} = \Delta_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} = (-1)(0) = 0$

$a_{31} = \Delta_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = (1)(-1) = -1$

$a_{32} = \Delta_{32} = (-1)^{3+2} \begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix} = (-1)(0) = 0$

$a_{33} = \Delta_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = (1)(1) = 1$

$$\bar{A} = \begin{bmatrix} -3 & 0 & 2 \\ 0 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad adj(A) = \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{detA} adjA$$

$$A^{-1} = \frac{1}{1} \cdot adjA$$

SEGUNDA LINHA

(B)

$$A^{-1} = adjA = \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \rightarrow 0 + (-1) + 0 = -1$$

6 - I - V $(0z = 0)$

II - V $(4z = 0 \rightarrow z = 0/4 \rightarrow z = 0)$

III - X se $a \neq 0$, então S só tem uma solução

$G\ y = 2$

(A)

7 - I - NÃO! FALSO, isso só seria correto se o sistema fosse homogêneo (ou seja, $b = 0$) F

II - SIM! Aqui sim o sistema é homogêneo. V

III - $Au = b$ $\qquad A(3v) = 0$

$u = A^{-1}b$ $\qquad 3v = A^{-1} \cdot 0 = 0$

$A(u + 3v) = b$

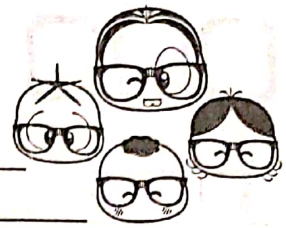
$A(A^{-1} \cdot b + 0) = b$ $\qquad$ logo, a afirmação é verdadeira

$A \cdot A^{-1} \cdot b = b$

$I_n \cdot b = b$

$b = b$ V

(B)



$1 + y - y = 1 \rightarrow 1 = 1$

8- $\begin{cases} x - y = 1 \\ x + z = 2 \\ y + z = 1 \end{cases}$

$x + z = 2 \rightarrow x + 1 - y = 2$

$x = 1 + y$

$z = 1 - y$

$S = \{(1+y, y, 1-y) / y \in \mathbb{R}\}$

$S = \{(1+a, a, 1-a) / a \in \mathbb{R}\}$

B

9- $\begin{cases} x + 5y + 10z = 100 \\ x + y + z = 15 \end{cases}$

$z = 10$ > 100 (inválido, porque passa de 100 reais)

$z = 9$ < 100, $x + y = 6$ $(x = 5, y = 1)$ [uma solução pro sistema)

$z = 5$ < 100, $\begin{cases} x + y = 10 \\ x + 5y = 50 \end{cases}$ $(x = 0, y = 10)$ [uma solução)

$z = 4$ < 100, $\begin{cases} x + y = 11 \rightarrow x = 11 - y \\ x + 5y = 60 \end{cases}$

$11 - y + 5y = 60$

$4y = 49$

$y = 49/4$ → A solução precisa ser inteira

Logo, $5 \leq z \leq 9$

D

10- $A = L.U$

$$A = \begin{bmatrix} -1 & 1 \\ 4 & 1 \end{bmatrix} \underset{\underset{m}{L_1 \leftarrow -1L_1}}{\sim} \begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix} \underset{\underset{m_1}{L_2 \leftarrow -4L_1 + L_2}}{\sim} \begin{bmatrix} 1 & -1 \\ 0 & -3 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 \\ m.m_1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (-1)(-4) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$l_{21} = 4$,

D