TESTE 3 - MAT 13S - LUISA DE SOUZA FERREIRA - 102026

1 - $\langle p(x), q(x) \rangle = p(0)\,q(0) + p(1)\,q(1)$

$p(x) = -x + 2$ e $q(x) = x - 1$

$\langle p(x), q(x) \rangle = 2.(-1) + \cancel{(1).(0)}^{\,0} = -2$ //

$\langle q(x), q(x) \rangle = (-1)(-1) + (0).(0) = 1$ //

$\dfrac{\langle p(x), q(x) \rangle}{\|q(x)\|^2} = \dfrac{-2}{1} = -2$ //

letra C

2- 2 pontos que contem a reta r:

$B = (2, -1, 1)$ e $C = (4, -3, 1)$

3 pontos $A = (1, 0, -1)$, $B = (2, -1, 1)$ e $C = (4, -3, 1)$

$\overrightarrow{AB} = B - A = (1, -1, 2)$

$\overrightarrow{CB} = B - C = (-2, 2, 0)$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & -1 & 2 & 1 & -1 \\ -2 & 2 & 0 & -2 & 2 \end{array}\right| = 0i - 4j + 2k - 2k + 4i + 0j$$

$= 4i - 4j + 0$

$n = (4, -4, 0)$

$n = \frac{1}{4}(1, -1, 0)$

-1

$\rightarrow x - y - 1 = 0$

$A = (1, 0, -1) = 1 - 0 - 1 = 0 \rightarrow 0 = 0$ ✓ confere

letra C

3- $(0,2,-1) \neq \alpha(1,-2,2)$ NÃO SÃO PARALELAS

Seja $P=(1,0,-1)$ um ponto da reta r e $Q=(0,0,-2)$ um ponto da reta s.

$\vec{PQ} = Q-P = (-1,0,-1)$

$\{(0,2,-1), (1,-2,2), (1,-2,2)\}$ é LD ou LI?

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -2 & 2 \\ 1 & -2 & 2 \end{bmatrix} \begin{matrix} 0 & 2 \\ 1 & -2 \\ 1 & -2 \end{matrix} = 0+4+2-2+0-4$$

$= 0 \rightarrow$ concorrentes

letra D

$\langle (0,2,-1), (1,-2,2) \rangle = 0$

$0-4-2=-6 \neq 0 \rightarrow$ não perpendiculares

4- I. $\langle u,v \rangle = 0$

$\langle u, u+v \rangle = 0 \rightarrow \langle u,u \rangle + \cancel{\langle u,v \rangle}^{=0} = \langle u,u \rangle \neq 0$, então V

II- F! $u \parallel w$ e não $u \perp w$

III- $\langle w,u \rangle = 0$ e $\langle w,v \rangle = 0$

$\langle w, v+u \rangle = \langle w,v \rangle + \langle w,u \rangle = 0+0=0$, logo V

letra B

Digitalizado com CamScanner

5- $\{ \overset{v_1}{(1,1,0)}, \overset{v_2}{(1,0,1)}, \overset{v_3}{(0,1,1)} \}$

$u_1 = v_1$

$u_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{\|u_1\|^2} u_1$

$u_2 = v_2 - \frac{(1)(1)+(1)(0)+(0)(1)}{2} u_1 = v_2 - \frac{1}{2} u_1 = \left(\frac{1}{2}, 1, -\frac{1}{2}\right)$

letra A

6- $(0,0)$, $(1,2)$, $(-1,-1)$

$0 = a_0$

$2 = a_0 + a_1$ $\rightarrow$ $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$

$-1 = a_0 - a_1$

$A^t . A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}_{2\times3} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}_{3\times2} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$Z = (A^t A^{-1}) . A^t\ b$

$Z = \left(\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}_{2\times3} \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}_{3\times1}$

$y = \frac{2}{6} + \frac{9x}{6}$

$Z = \frac{1}{6} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times1}$

$A + B$

$\frac{2}{6} + \frac{9}{6} = \frac{11}{6}$

$Z = \frac{1}{6} \begin{bmatrix} 2 \\ 9 \end{bmatrix} =$

letra B

7- $\det(A - \lambda I) = 0$

$\begin{vmatrix} a+1 & b \\ c & d+1 \end{vmatrix} = 0$

$(a+1)(d+1) + cd = 0$

$\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = 0$

$(a-1)(d-1) + cd = 0$

$(A - \lambda I)v = 0$

$\begin{bmatrix} a+1 & b \\ c & d+1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$ $\rightarrow a+1=0$ $\rightarrow c=0$

$(A - \lambda I)v = 0$

$\begin{bmatrix} a-1 & b \\ c & a-1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 0 \rightarrow -2a+2+b=0$

$-\cancel{2c}+d-1=0$

$a = 1$ //

letra D

8- $\det(A-\lambda I)=0$ → $\det(A-2I)$

$B = A^3 - 2A^2 = A^2(A-2I)$ $\lambda = 2$ com multiplicidade geométrica $= 2$

$\lambda_1 + \lambda_2 = 2+2 = 4$ //

letra C

9- $ax+by+c=0$ $\begin{bmatrix} x_i \\ y_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\langle (x_1, y_1), (x,y) \rangle = 0$

$\cdot\ x_1 . x + y_1 . y = 0$ x_i

x_1, y_1 é um ponto da reta!

$T = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$ não possui autovalor, logo

letra B

10 - $\det(A - \lambda I) = 0$

$$\begin{vmatrix} a+1 & b \\ b & c+1 \end{vmatrix} = 0$$

$(a+1)(c+1) - b^2 = 0$

$ac + a + c + ac - a^2c^2$

$-a^2c^2 + 2ac + a + c = 0$

$$\begin{cases} a + c = 0 \rightarrow a = -c \\ 4a - 4c = 0 \end{cases}$$

$b = a \cdot c$

$b = (a)(-a) = -a < 0$

letra B

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} a-4 & b \\ b & c-4 \end{vmatrix} = 0$$

$(a-4)(c-4) - b^2 = 0$

$ac - 4a - 4c + ac - a^2c^2$

$-a^2c^2 + 2ac - 4c - 4a = 0$

$(ax^2 - 2(ac)xy + cy^2) = 0$

$(x^2 - y^2 = 0 \rightarrow$ UMA HIPÉRBOLE

$ax^2 + 2bxy + cy^2 = 0$

$b = a \cdot c$

$b < 0$