

Assignment No 3- CS 538, S2023

Date Due: March 10th, 2023

1. In the Simplex approach, show the following: (here recall that $\hat{c}_j = c_j - \sum_{i \in B} c_i d_{ij}$).
 - (a) Let y be a solution to the constraints $Ay = b$, not necessarily a basic feasible solution. Let $\hat{c}(B)$ be the relative cost vector defined w.r.t. a basis B . Show that $\hat{c}^T y = g - f$ where f is the cost of the current solution associated with B and g is the cost of y .
 - (b) Suppose the Simplex is at basis B and selects a basis B' by choosing a column j s.t. $\hat{c}_j < 0$. Is the cost of the solution guaranteed to decrease? Under which condition would it not.
2. Investigate the following and report your findings:
 - (a) How does the Simplex method find the first Basic Feasible solution
 - (b) Can the Simplex Method cycle, i.e. repeat a basis? under what conditions.
 - (c) What is the complexity of the Simplex method.
3. In the system $Ax = b, x \geq 0$ solved via Simplex, can two different basis in A result in the same vertex. If so, give an example. If not prove your statement.
4. Can a column vector in A that has just left the basis B when Simplex moves to B' return to the basis at the very next step?
5. A network problem is formulated for a directed graph $G = (V, E)$ using the node-arc incidence matrix which represent the matrix A in the Simplex method. Show that a set of $|V| - 1$ columns is linearly independent if and only if the corresponding arcs, considered as undirected edges in the undirected version of G , is a tree. Interpret the pivot step of the simplex method in the light of this fact.