Assignment No 1- CS 538, S2023

Date Due: Feb 5th, 2023

1. Consider a mobile company that wants to connect wireless clients to base stations. Further suppose that each base station has a capacity of C clients. A client can connect to a base station only if the base station set is less than or equal to distance r away. Given a collection of base stations, wireless clients and the geographic location of base stations and wireless clients, design a method to determine an assignment of clients to base stations such that the number of clients serviced is maximized.

2. There are n doctors that need to be scheduled for duty in a hospital during holiday periods. There are m vacation periods (Thanksgiving, Christmas etc.) each of period p_i. However there must be at least one doctor on call at the hospital every day during the vacation period. Each doctor D_i specifies the set of vacation days W_i that he can work. How will you determine a schedule such that each doctor is not working more than c days in any vacation period.

3. Describe the proof of Hall's theorem discussed in class for a bipartite graph $G = (U, V, E)$. In particular, in the induction step show that if there is a subset $S \subseteq U$ with neighbor set $N(S)$, where $|N(S)| = |S|$ then S and T satisfy the hypothesis of the the theorem, i.e. for every subset in S(or T) the neighborhood set is of size greater than or equal to S (or T).

4. Show that $y(\theta) = \theta x + (1-\theta)y, 0 \leq \theta \leq 1$ represents all the points on the line segment defined by two endpoints x and y. This is true in $\mathcal{R}^d$ for any dimension d.

5. Show that the following sets are convex:

   (i) Lines and Line segments in $\mathcal{R}^d$.

   (ii) Ellipsoids in $\mathcal{R}^d$.

   (iii) Halfspaces defined by the hyperplane $a^T x = b$.

6. Let $C \subseteq R^n$ be a convex set, with $x_1 \ldots x_k \in C$. Show that $\theta_1 x_1 + \ldots + \theta_k x_k \in C$ where $\theta_1, \ldots \theta_k \in R$ satisfy $\theta_i \geq 0, \theta_1 + \cdots + \theta_k = 1$.

7. Show that if C is a convex set then $C' = T(C)$ is a convex set, where $T(C) = Ax + b, x \in C$.