Assignment No 2- CS 538, S2023

Date Due: Feb 19th, 2023

1. Design the dual of the following Linear Problem:

$$\max c^T x$$

such that

$$a_i^T x \leq b_i, \quad 1 \leq i \leq m'$$
$$a_j^T x = b_j, \quad m' + 1 \leq j \leq m$$
$$x_k \geq 0, 1 \leq k \leq r$$

where $x = (x_1, x_2 \dots x_n), x \in R^n$

2. Show that all linear programs can be expressed as

$$\min c^T x$$
$$Ax = b$$
$$x \geq 0$$

where $x = (x_1, x_2 \dots x_n), , x \in R^n$.

3. We consider an attacker on your flow network. Consider a bipartite network $N(E, d)$ where $E = \{e_1, e_2, \cdots, e_m\}$. We denote your (the designer's) budget by B_D. The flow design is represented by a vector $f \in [0,1]^m$ with the requirement $\sum_{i=1}^m f[i] = 1$, where $f[i]$ is the flow amount on edge e_i. For each edge e_i with flow amount $f[i]$, the designer's cost is $d[i]f[i]$. We say a design f is within budget if $\sum_{i=1}^m d[i]f[i] \leq B_D$. An attack is a vector $X \in [0,1]^m$ indicating the attack on edges, where $X[i]$ denotes the attack, or attack level, on edge e_i, the fraction of flow captured on e_i.

   The adversary's benefit is the sum of flow she captures over the bipartition and defined as $\sum_{j=1}^k f[j] \cdot X[j]$. Your goal is to minimize the attacker's benefit.

   Given the adversary's strategy $X[j]$ as fixed and provided, describe a linear program to optimize the designer's strategy and find its dual.

4. Set up a linear program for determining a shortest path in a weighted directed graph. Find the dual of this LP.

5. Complete the proof from Class: Compute the dual of linear program for the maximum flow problem using path structures instead of flow conservation at vertices. Use variables for flows on s-t paths, where a path is the sequence of edges from s to t. Explain how this can be used to prove the strong duality of the maximum flow problem.

6. A system $Ax \leq b, x \in R^n, A \in R^{m \times n}$ may be infeasible, i.e. has no solution. Show that this is true iff there exists a $y \in R^m$ such that $A^T y = 0, b^T y < 0$, and $y \geq 0$.