MATH 565: Monte Carlo Methods in Finance

Instructions:

a. *Show all your work to justify your answers.*

b. *Group work can be done in groups of no more than THREE students.*

c. *Individual work must be done on your own.*

d. *What you submit should represent your own work. If you use other sources, cite them.*

e. *You may use sample code from the class without citation.*

f. *Please include your group members' names in your homework and submit it to Blackboard using the following method* Math565 + Group# + HW3 (e.g. Math565Group1HW3.pdf)

g. *MATLAB code must be submitted to Blackboard using the following method.* Math565 + Group# + HW3 (e.g. Math565Group1HW3.m)

Assignment 3 Due 11:59 pm Tuesday, October 25, 2022 (Group Work)

1. Consider the following pairs of options. Explain which one has a higher price and why? Except where indicated, assume that all parameters are the same for both options.

Option A	Option B
European put option with strike price \$100	European put option with strike price \$90
European put option with an initial stock of price \$100	European put option with an initial stock of price \$90
European put option where the initial stock price is \$50, and the strike price is \$40	A barrier down-and-out put option with a barrier of \$30.

2. (a) Consider the following IID $\mathcal{N}(0,1)$ random variables X_j that come from a pseudo-random number generator:

j	1	2	3	4
X_j	-1.3011	-0.9032	-0.0339	1.3695

Use these random numbers to construct the values of a Brownian motion, B_1, at weekly times for four weeks. Specifically, find $B_1(t_j)$ for $t_j = j/52$ years with $j = 0, 1, 2, 3, 4$.

(b) Consider a stock that is modeled by a geometric mean Brownian motion with initial price S(0) = \$45, interest rate 1%, and volatility 60%, for four weeks. Use the Brownian motion B_1 from (a) to construct one stock price path $S_1(t_j), j = 0, 1, 2, 3, 4$.

(c) Another path, S2, given below:

j	0	1	2	3	4
t_j	0	1/52	1/26	3/52	1/13
$S_2(t_j)$	45.00	47.23	48.99	52.04	50.69

For S_1 and S_2, what are the discounted payoffs of a lookback call option that expires four weeks from now? Are these payoffs less than, greater than, or equal to the discounted payoffs of a European call option with a strike price of \$45? Is the price of the lookback call option less than, greater than, or equal to the price of the European call option?

(d) Using the stock path generated in b), what is the discounted payoff of an American put option with a strike price of \$45 that expires four weeks from the initial time? Assume that the exercise boundary is given as follows:

j	0	1	2	3	4
t	0	1/52	1/26	3/52	1/13
$b(t)$	40.00	41.00	42.00	43.00	45.00

3. (Computer problem) Consider the problem of pricing *lookback* options for a stock modeled by a geometric Brownian motion with an initial price of \$100, a volatility of 40%, and zero interest rate. Let the expiry time be 24 weeks in the future (consider 52 weeks a year), and let the monitoring frequency be weekly.

   a) Use the GAIL software to find the price of both the put and call options to the nearest \$0.1.

   b) Does the put or the call have a higher price? What is a possible intuitive explanation?