MATH 565: Monte Carlo Methods in Finance

Instructions:

a. *Show all your work to justify your answers.*

b. *Group work can be done in groups of no more than THREE students.*

c. *Individual work must be done on your own.*

d. *What you submit should represent your own work. If you use other sources, cite them.*

e. *You may use sample code from the class without citation.*

f. *Please include your group members' names in your homework and submit it to Blackboard using the following method* Math565 + Group# + HW4 (e.g. Math565Group1HW4.pdf)

g. *MATLAB code must be submitted to Blackboard using the following method.* Math565 + Group# + HW4 (e.g. Math565Group1HW4.m)

Assignment 4 due Thursday, November 10, 2022 (Group Work)

1. For these next problems use the GAIL software and consider a stock with an initial price of \$30, an interest rate of 1%, and a volatility of 40%, being monitored weekly for 6 weeks.

   a) What is the price of an Asian arithmetic mean call option to the nearest \$0.1 if the strike price is \$30?

   b) What savings in number of paths and time do you find, if any, if you use a European call option as a control variate?

   c) What savings in number of paths and time do you find, if any, if you use antithetic variates?

2. Let $f : [0, 2] \to \mathbb{R}$ be some function whose integral you wish to compute with respect to a probability density function, ρ, i.e.,

$$\mu = \int_0^2 f(x)\rho(x)dx,$$

   where $\rho = \frac{1}{4}(x + 1)$. Suppose that it is difficult to generate random variables with PDF ρ, but easy to generate random variables with PDF $\tilde{\rho} = \frac{1}{2}$.

   a) Derive an acceptance-rejection method for generating variables X_i IID $\sim \rho$ from $\widetilde{X}_i$ IID $\sim \tilde{\rho}$. If you have $\widetilde{X_1} = 1.5765$ and $U_1 = 0.6929 \sim \mathcal{U}[0, 1]$, should you accept $\widetilde{X_1}$ to be X_1?

   b) Suppose $f(x) = x^2$, construct an estimate of μ using importance sampling with $\widetilde{X_1}, \ldots, \widetilde{X_4}$ IID $\sim \tilde{\rho}$. You can generate $\widetilde{X_i}$ by the following IID uniform random numbers $U_i \sim U[0, 1]$:

i	1	2	3	4
U_i	0.4914	0.2845	0.0733	0.7632

3. We want to estimate $\mu = \mathbb{E}(Y)$, where $Y = f(X)$, $f(x) = 2x^2$, and $X \sim U[-1, 2]$.

   a) Given four IID $\mathcal{U}[0, 1]$ random numbers as in 2b) Use simple Monte Carlo method with small sample size to estimate μ.

   b) We consider to use stratified sampling to estimate μ. Please use the above random numbers to construct a stratified sampling estimator with $K = 4, M = 1$ to estimate μ.

   c) We consider to use antithetic variates to estimate μ.