MATH 565: Monte Carlo Methods in Finance

Instructions:

a. *Show all your work to justify your answers.*

b. *Group work can be done in groups of no more than THREE students.*

c. *Individual work must be done on your own.*

d. *What you submit should represent your own work. If you use other sources, cite them.*

e. *You may use sample code from the class without citation.*

f. *Please include your group members' names in your homework and submit it to Blackboard using the following method* Math565 + Group# + HW2 (e.g. Math565Group1HW2.pdf)

g. *MATLAB code must be submitted to Blackboard using the following method.* Math565 + Group# + HW2 (e.g. Math565Group1HW2.m)

Assignment 2 Due 11:59 pm Tuesday, October 4, 2022 (Group Work)

1. For the situation in Assignment 1 problem 1, construct a confidence interval for the probability that the time to wait of a bus is greater than 8 minutes using 10^6 Monte Carlo samples and `binomialCI`.

2. Consider the following three dimensional integral

$$\mu = \int_{[-1,1]^3} \cos\left(\sqrt{x_1^2 + x_2^2} + x_3\right) \, d\boldsymbol{x}.$$

The following are six IID $U[0,1]$ random numbers:

$$0.1135, 0.9745, 0.7287, 0.3515, 0.7076, 0.7996$$

Use these to form $\hat{\mu}_n$, a Mont Carlo estimate of μ. Granted, n cannot be very large given only six IID $U[0,1]$ random numbers, how large can you n be?

3. Computer Problem: Consider the following integral, which arises in the pricing of an Asian arithmetic mean call option:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max\left(\frac{1}{2}[S_1(\boldsymbol{z}) + S_2(\boldsymbol{z})] - 100, 0\right) \frac{\exp(-\boldsymbol{z}^T\Sigma^{-1}\boldsymbol{z}/2)}{\sqrt{4\pi^2 \det(\Sigma)}} d\boldsymbol{z}, \qquad \Sigma = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1 \end{pmatrix},$$

$$S_1(\boldsymbol{z}) = 100\exp(-0.0225 + 0.3z_1), \qquad S_2(\boldsymbol{z}) = 100\exp(-0.045 + 0.3z_2).$$

Approximate the integral using `meanMC_g` with an error tolerance of 0.02.

4. Y has a triangular probability density function (PDF) of

$$\rho(y) = \begin{cases} \frac{y-10}{100}, & 10 \le y \le 20, \\ \frac{30-y}{100}, & 20 < y \le 30, \\ 0, & \text{otherwise.} \end{cases}$$

Verify that ρ is a PDF. Use the inverse transform method to generate Y using the following six IID $U[0,1]$ random numbers:

$$0.1135, 0.9745, 0.7287, 0.3515, 0.7076, 0.7996$$

5. Computer Problem: Consider a distribution with density function as

$$\rho(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 \le x \le 1,$$

where $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \Gamma(\cdot)$ is the gamma function. Derive an acceptance-rejection method for generating random variables with above pdf with $\alpha = 2, \beta = 2$ from $U[0,1]$ random variables. Generate 1000 such random numbers.