Bayesian AB Testing

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Agenda

p-value in H0, H1 framework

"if your p-value is 0.05, that means that 5% of the time you would see a test statistic at least as extreme as the one you found if the null hypothesis was true"

1. p-value is used in thousands of research papers
2. p-value is extremely popular for its easy interpretation
3. easy to calculate confidence intervals

p-value in H0, H1 framework

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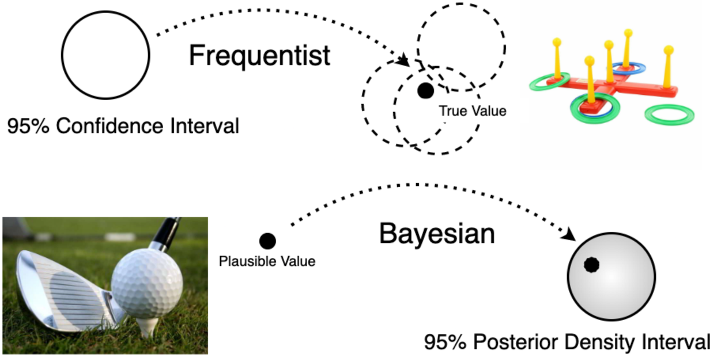
Are you sure?

Do you understand the nature of the p-value?

Disclaimer: I do not advocate against p-values, just know your tools.

Interpreting p-values

Greatest insights into p-values:



Suggested Reading

Explanation of P-values by Joe Felsenstein

Hypothesis Testing in H0, H1 framework

You should know what is hypothesis testing, t-test, p-values.

- 1 sample mean test $t = \frac{Z}{s} = \frac{\bar{X} - \mu}{\widehat{\sigma}/\sqrt{n}}$
- 2 sample mean test $t = \frac{\bar{X}_1 - \bar{X}_2}{s_p\sqrt{\frac{2}{n}}}, \quad s_p = \sqrt{\frac{s_{X_1}^2 + s_{X_2}^2}{2}}, \ldots$
- 2 sample not equal variances, now equal sample sizes test
  $\ldots, s_p = \sqrt{\frac{(n_1 - 1)\, s_{X_1}^2 + (n_2 - 1)\, s_{X_2}^2}{n_1 + n_2 - 2}}$

Too Complicated

The less assumptions we have, the more complicated is math and implementation

Bayesian Tools

1. Highest Density Interval
2. Region of Practical Equivalence
3. Bayes Factor
4. Custom

Highest Density Interval

HDI The most popular way to interpret the posterior

1. Represents a range of most probable values
2. Easy to interpret and calculate
3. Easy to visualize

Example

- With 95% probability effect size in range [A, B]
- Range [A, B] represents 95% of most probable effect sizes

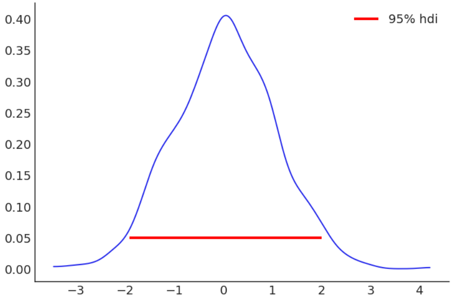


Figure: Highest Density Interval

Region of Practical Equivalence

RoPE is a common way to say if a parameter estimate is "significant". The use case:

1. You do not care if the effect size is less than 0.1
2. Plot the region overlapping with the posterior
3. Decide

Example

The effect size "E" is out of the region of practical equivalence so we treat it as a significant one

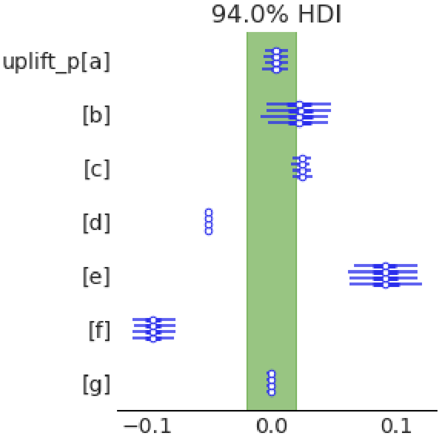


Figure: Rope Plot

Bayes Factor

IMO the most complicated to explain statistic.

1. Similar to the Frequentist p-value
2. Harder to interpret and explain to people
3. Checks H0 vs H1 for x_0

Definition

Bayes Factor is defined as the ratio of the likelihood of one particular hypothesis to the likelihood of another hypothesis

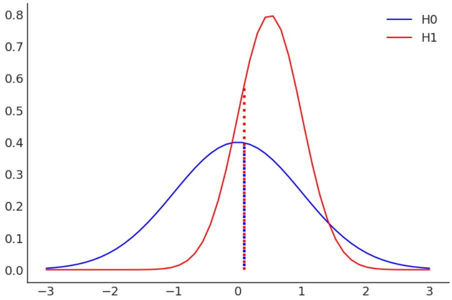


Figure: $\mathrm{BF} = \frac{\mathrm{pdf}_{H1}(x_0)}{\mathrm{pdf}_{H0}(x_0)}$

Custom Queries

You can do much more!

1. $P(A < 0)$
2. $P(A > B)$
3. $P(\max(A) > \max(B))$
4. $P(A = \arg\max(A, B, C, D))$
5. $P(\text{profit}(X, \Theta) > \$100)$
6. Quantiles - $Q_{0.05}(\text{profit}(X, \Theta))$

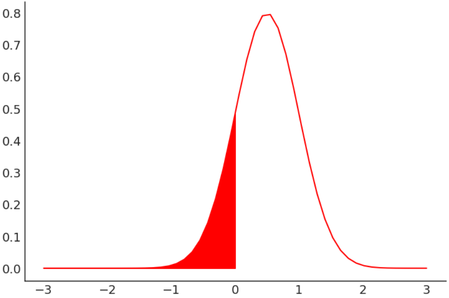


Figure: $P(A < 0)$

Takeouts

Bayesians have a Swiss Knife for Hypothesis Checking

1. Numerous ways to interpret results
2. Not a Yes/No answer
3. Uncertainty is obviously represented
4. Flexibility in analysis
5. Easy to implement
6. Easy to interpret

Figure: Bayesian Hypothesis Testing

Types of Problems

Bayesian AB testing is widely applicable

1. Discrete Observations (views and clicks)
2. Continuous Observations (read time, spent amount)
3. With Context Predictors (CUPED[1])
4. With Hierarchies (Regions)

A/B

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The hard part

Most of the above methods are still in development

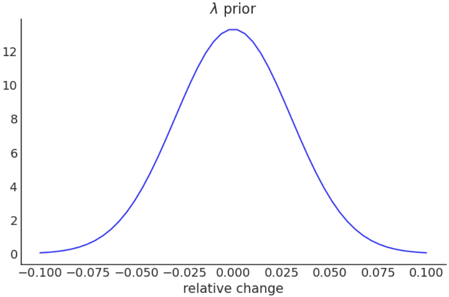
A/B

Approaching Priors

Uplift λ

Relative change to the baseline

When you start the experiment, don't you know anything about the set of possible outcomes?



Setting priors for Uplift

You are in the preparation to run an experiment B vs holdout A. You might be interested in increasing the mean of statistics (average bill)

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Relative or Absolute change?

Make it clear if the change is relative or absolute!

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- Do you expect after changes in B you have a 100% increase? Very sure No

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- Do you expect after changes in B you have a 100% increase? Very sure No
- Do you expect after changes in B you have a 10% increase? Unlikely

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- Do you expect after changes in B you have a 100% increase? Very sure No
- Do you expect after changes in B you have a 10% increase? Unlikely
- Do you expect after changes in B you have a 3% increase? Maybe

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- Do you expect after changes in B you have a 3% increase? Maybe
- Do you expect after changes in B you have a 3% decrease? Maybe

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- Do you expect after changes in B you have a 3% increase? Maybe
- Do you expect after changes in B you have a 3% decrease? Maybe
- Do you expect after changes in B you have an X% decrease? Your answer

Relative or Absolute change?

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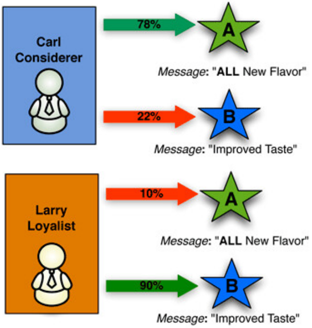
Binomial Model Example

- The example is binary Yes/No choice
- Observations follow the Bernoulli likelihood

$$x_i^A \sim \text{Bernoulli}(p_A)$$
$$x_i^B \sim \text{Bernoulli}(p_B)$$

Do we have additional information?



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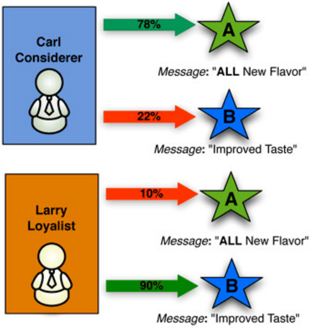
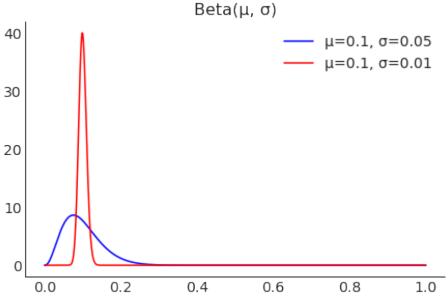
Do we have additional information?

- Historical $\bar{p}$
- Expected improvement $\pm\bar{\sigma}\%$ (e.g. $\pm 0.01\%$)



Adding Additional Information

We can parametrize Beta distribution in a special way



$$G \in \{A, B\}$$
$$x_i^G \sim \text{Bernoulli}(p_G)$$
$$p_G \sim \text{Beta}(\alpha_G, \beta_G)\ s.t.$$
$$\mathbb{E}p_G = \bar{p},$$
$$\text{Var}\ p_G = \bar{\sigma}^2$$

Adding Additional Information

We can parametrize Beta distribution in a special way

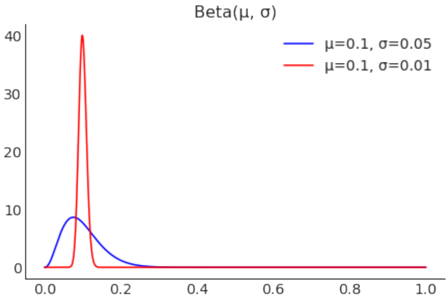
$$X \sim \text{Beta}(\alpha, \beta)$$

$$\mu = \frac{\alpha}{\alpha + \beta}$$

$$\sigma = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$X \sim \text{Beta}(\mu, \sigma) \Rightarrow$$

$$\Rightarrow \begin{cases} \alpha & = \mu\kappa \\ \beta & = (1-\mu)\kappa \\ \text{where} & \kappa = \frac{\mu(1-\mu)}{\sigma^2} - 1 \end{cases}$$



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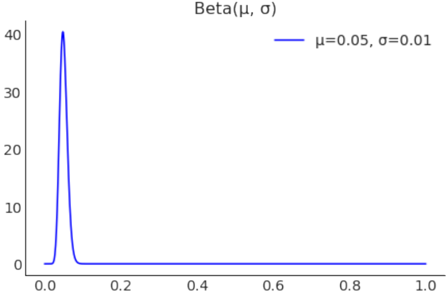
Prior Specification

Case Study

Our historical levels of conversion are about 5% (and fixed). We expect about 1% **absolute** change ($\bar{\sigma}$) after implementing the solution. Or, similarly, 20% **relative** change ($\bar{\delta}$).



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\begin{aligned}
\bar{p} &= 0.05 \\
\bar{\sigma} &= 0.01 = \bar{\delta} \cdot 0.05 \\
G &\in \{A, B\} \\
p_G &\sim \text{Beta}(\mu = \bar{p}, \sigma = \bar{\sigma})
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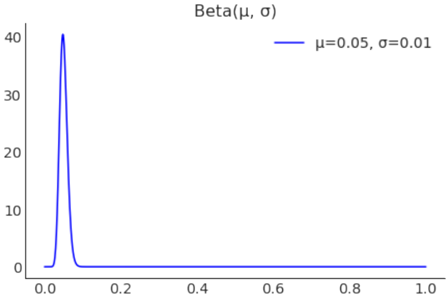

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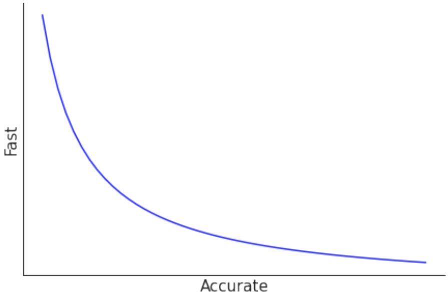
Takeout

Special Beta parametrization leads to more interpretable priors

Key questions be for you start

- How much time can be allocated for the test?
    - How accurate is the decision then?
- How accurate should be the decision?
    - How much time will be allocated for the test?



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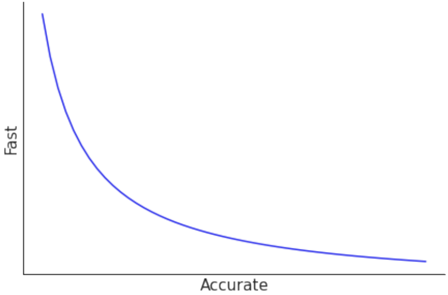
Impossibility

You can't be fast in data collection and accurate at the same time



Parameter Recovery Study

Parameter recovery is a simulated experiment to know your model better.

1. Generate data from a model configuration
2. Pretend you do not know the true values
3. Run inference for your model
4. Compare estimated parameters and ground truth ones

Given the results

- How well can you infer the model state?
- How does data size affects the results?
- Are there unidentifiable parameters?

Suggested Reading

Chapter 4 in Bayesian Workflow

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Suggested Reading

Chapter 4 in Bayesian Workflow

Parameter Recovery for AB testing

Given:

- Effect is significant if $|p - \bar{p}| > \bar{\sigma}$

Recall the model

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i &\in 1 \ldots N \\
x_i &\sim \text{Bernoulli}(p) \\
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- $N = 0,\ N = 1000,\ N = 100000$?

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- $N = 0$, $N = 1000$, $N = 100000$?
- What metric to use to evaluate detect effectiveness?

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Key observation

Effect detection is a classification problem. E.g. negative, neutral, positive effects. We can use ROC-AUC for multiclass

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AB Testing as classification

Some definitions of our classification setup

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Posterior $p(p \mid X_{1:N})$

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3. Predictions (probabilities using the posterior):
   - $P(\text{p is negative} \mid X_{1:N})$
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Posterior $p(p \mid X_{1:N})$

Run the simulation study

1. for $\hat{p} \in \dots$, for $N \in \dots$ get $p(p \mid X_{1:N})$
2. for $N \in \dots$ calculate ROC-AUC

ROC-AUC in Action

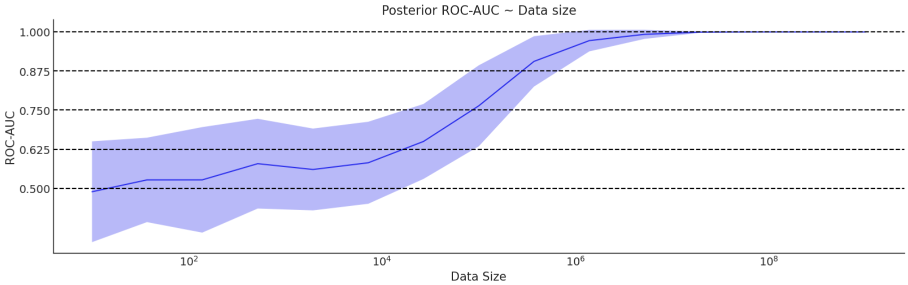

Figure: ROC-AUC increases as you get more data

Time is constraint:

1. Discuss maximum affordable time
2. Consult the plot for the expected ROC-AUC in decision

ROC-AUC is constraint:

1. Discuss minimum required ROC-AUC
2. Consult the plot for the expected data size

After the Inference

Situation: you've run the test for the aforehand specified duration. Key questions:

1. Which alternative to choose?
2. What is the comparison criterion?
3. Is the criterion connected to the real life?

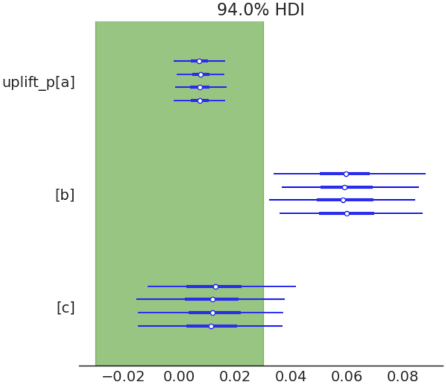

Figure: Example ROPE plot

After the Inference

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Key questions:

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A better metric

A good metric is the one that is connected to expected profit.

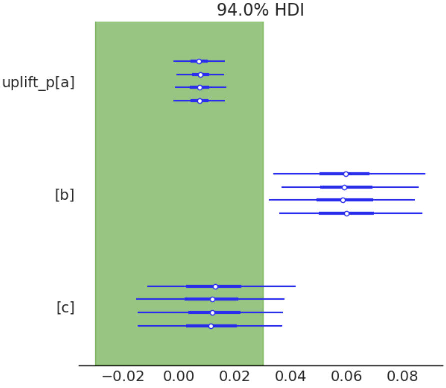

Figure: Example ROPE plot

Interpreting the Posterior

How can we calculate a better mectic?

It could look like this:

Interpreting the Posterior

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- Connect the conversion rate p_A or p_B to the company size audience

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$$\text{Monetization}_A = \\ \text{(Per User Value)} \times \text{(Num Users)} \times \Delta p_A - \text{(Implementation Cost)}$$

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- Use "Customer Value" as a proxy for money effect

It could look like this:

$$\text{Monetization}_A =$$
$$(\text{Per User Value}) \times (\text{Num Users}) \times \Delta p_A - (\text{Implementation Cost})$$

Use the posterior

We can calculate $p(\text{Monetization}_A \mid X_A)$ out of $p(p_A \mid X_A)$

Monetization Posterior

$$(\text{Per User Value}) \times (\text{Num Users}) \times \Delta p_A - (\text{Implementation Cost})$$

- Implementation cost might differ
- Per User Value might have scenarios
  - Positive
  - Negative
  - Average
- You connect the experiment with business
- Compare outcomes with uncertainty

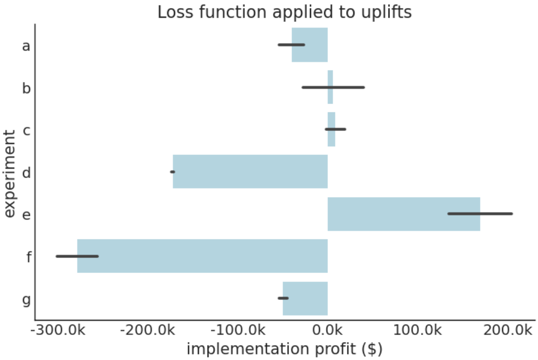

Figure: $p(\text{Monetization}_G \mid X_G)$

Takeouts

Real Life AB testing is full of challenges. Bayesian tools are still considered novel.

1. Framing the statistical test
    - Setting priors
    - Setting likelihood
2. Decision making before the test
    - Parameter recovery study
3. Bayesian decision making
    - Loss functions
    - Scenario testing

References I

R. Kohavi, A. Deng, Y. Xu, and T. Walker.
Improving the sensitivity of online controlled experiments by utilizing pre-experiment data.
02 2013.