Customer Lifetime

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Agenda

What is it?

The model rescribes customer relations, they can be the following

	Non-Contractual Unobserved quit	**Contractual** Observed quit
Continuous non-scheduled	e-commerce grossery purchase	credit cards SIM cards
Discrete scheduled	event tickets weekly magazine	netflix gym membership

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Hard to differentiate between customers who have indefinitely churned and those who will return in the future.

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BG-NBD model can help

The Intuition of the BG-NBD Model

BG-NBD is Beta-Gamma Negative Binomial Distribution

- focuses on predicting the number of transactions

It is a part of the LTV model

$$\text{LTV} = \text{number of transactions} \times \text{value of transaction}$$

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- value of transaction is usually an easy thing to get
- we mostly care about number of transactions

Example

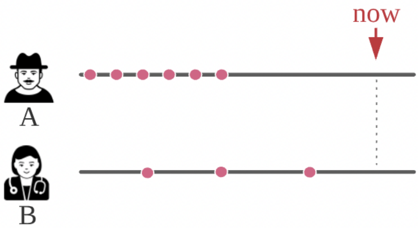
- You are a bakery owner
- You have purchase records of your customers (with id)
- You want to plan next year revenue

Practical Aspects

In addition to standard assumptions:

- Once quit, users never return
- Unobserved frequency is constant in time

There is couple more:



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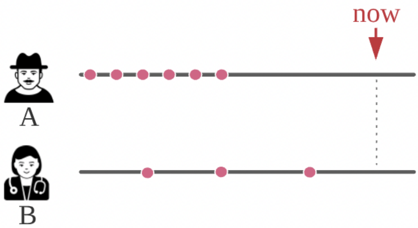

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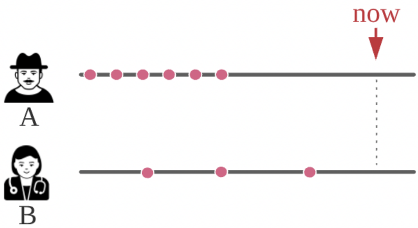

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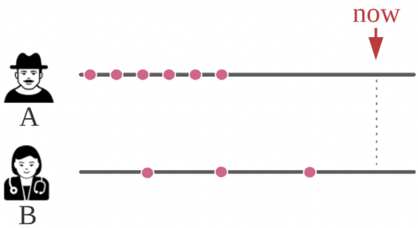


Interpretation

So we treat each user separately which is more realistic.
The retention follows the same idea.

How priors help

There are number of assumptions we may want to bring in:

- We're sure that most of our customers make purchases at a rate of 4 purchases a week
- Our users are "addicted" to the product and have low quit probability, below 30% in a year

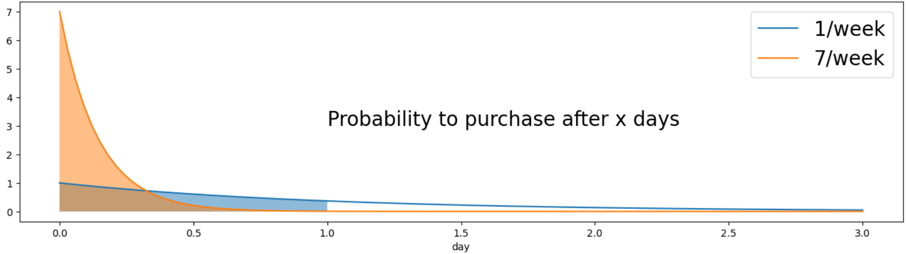
Uncertainty

Our assumptions may be not very certain and we'll reflect them in the priors

The Poisson Process

Let's break complicated likelihood in parts.

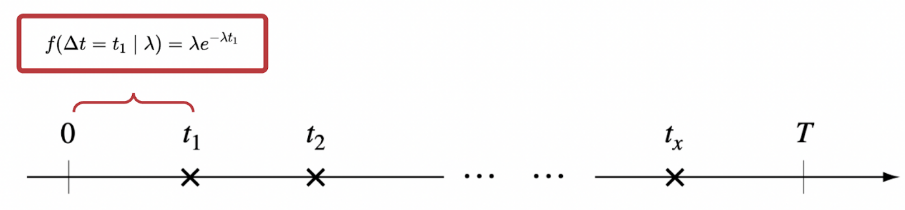
- Users follow Poisson Process to make (discrete) purchases
- They make purchases at rate $\lambda = 1$ and 7 per week.
- With this assumption, we can model the time-to-next-purchase Δ_t as an exponential distribution Exponential(λ)



We are certain the Orange customer will make a purchase next day

Time to purchase

Every gap can be measured with Exponential(λ)



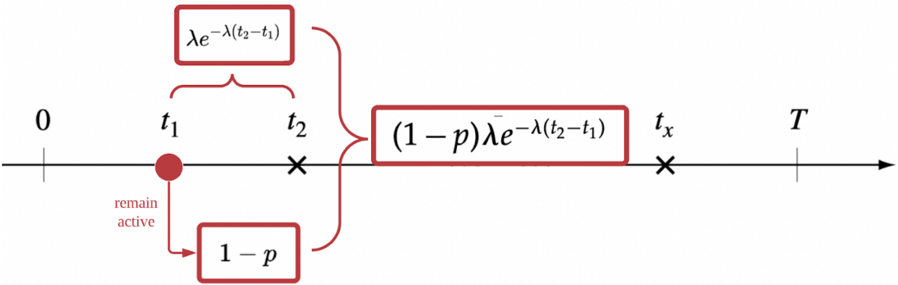
$$P(t_1, \ldots, t_x \mid \lambda) = \prod_{i=1}^{i=x} \lambda e^{-\lambda(t_i - t_{i-1})} = \lambda^x e^{-\lambda t_x} = P(x, t_x \mid \lambda)$$

Insight

You do not need a sequence $t_1, \ldots, t_x$, you only need $t_x =$ "Age of the customer at last purchase **x**" and $\mathbf{x} =$ "Number of purchases"

Deactivation Probability

Every step follows a change to deactivate

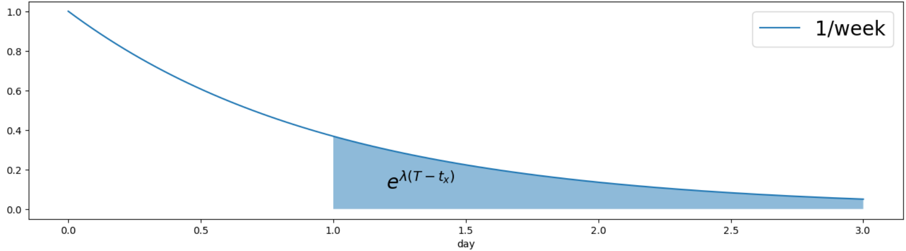


In the previous formula we add $(x-1)$ cases of non deactivation.

$$P(x, t_x \mid \lambda, p) = (1-p)^{x-1}\lambda^x e^{-\lambda t_x}$$

Deactivation after t_x

1. The customer did not deactivate after t_x, this probability is $(1-p)e^{-\lambda(T-t_x)}$
2. Customer deactivated with probability p and never returns



Remainder probability is

$$P(T \mid \lambda, t_x, p) = p + (1-p)e^{-\lambda(T-t_x)}$$

Total Probability

Step by step we figure out the final formula to use

$$P(x, t_x, T \mid \lambda, p) = \underbrace{\left(p + (1-p)e^{-\lambda(T-t_x)}\right)}_{P(T \mid \lambda, p, t_x)} \times \underbrace{(1-p)^{x-1}\lambda^x e^{-\lambda t_x}}_{P(x, t_x \mid \lambda, p)}$$

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$$\begin{aligned} P(x, t_x, T \mid \lambda, p) &= p(1-p)^{x-1}\lambda^x e^{-\lambda t_x} \\ &+ (1-p)e^{-\lambda(T-t_x)}(1-p)^{x-1}\lambda^x e^{-\lambda t_x} \end{aligned}$$

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$$
\begin{aligned}
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P(x, t_x, T \mid \lambda, p) &= p(1-p)^{x-1}\lambda^x e^{-\lambda t_x} \\
&\quad + (1-p)e^{-\lambda(T-t_x)}(1-p)^{x-1}\lambda^x e^{-\lambda t_x} \\
P(x, t_x, T \mid \lambda, p) &= p(1-p)^{x-1}\lambda^x e^{-\lambda t_x} + (1-p)^x\lambda^x e^{-\lambda T}
\end{aligned}
$$

Important

This formula is only applicable to customers made at least one purchase, those have $x > 0$ and t_x. We need a formula for all customers including $x = 0$ and absence of t_x

Total Probability for all customers

Assumption is that all users after they enroll are active by default.
Given a user without any purchases, we have

$$P(T \mid \lambda, x = 0) = e^{-\lambda T}$$

Combining it with the previous formula we have

$$P(x, t_x, T \mid \lambda, p) = \begin{cases} p(1-p)^{x-1}\lambda^x e^{-\lambda t_x} + (1-p)^x \lambda^x e^{-\lambda T} & x > 0 \\ e^{-\lambda T} & x = 0 \end{cases}$$

Or, equivalently

$$P(x, t_x, T \mid \lambda, p) = \delta_{x>0}\left[p(1-p)^{x-1}\lambda^x e^{-\lambda t_x}\right] + (1-p)^x \lambda^x e^{-\lambda T}$$

Transferring to PyMC

There is no such a distribution in PyMC, so you need to implement one. This is what is done with `pm.DensityDist`

```
def logp_x_tx_T(value, p, lam):
    # value.shape = (n_obs, 3)
    x, tx, T = value.T[0], value.T[1], value.T[2]
    delta_x = at.where(x>0, 1, 0)
    A1 = x*at.log(1-p) + x*at.log(lam) - lam*T
    A2 = (at.log(p) + (x-1)*at.log(1-p) + x*at.log(lam) - lam*tx)
    A3 = at.log(at.exp(A1) + delta_x * at.exp(A2))
    return A3
```

```
with pm.Model() as model:
    ...
    pm.DensityDist(
        "obs", p, lam,
        logp=logp_x_tx_T,
        observed=data # (n_obs, 3)
    )
```

References I

A. Meraldo.
Bayesian customer lifetime values modeling using pymc3, 2022.

A. Meraldo.
Customer lifetime value estimation via probabilistic modeling, 2022.