Star Battle

Resolução de Problema de Decisão usando Programação em Lógica com Restrições

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Resumo Este artigo complementa o segundo projecto da Unidade Curricular de Programação em Lógica, do Mestrado Integrado em Engenharia Informática e de Computação. O projecto consiste num programa, escrito em Prolog, capaz de resolver qualquer tabuleiro do jogo Star Battle, que é um problema de decisão.

Keywords: star battle, sicstus, prolog, feup

1 Introdução

asdasdad
asdads

2 Descrição do Problema

3 Abordagem

3.1 Variáveis de Decisão

3.2 Restrições

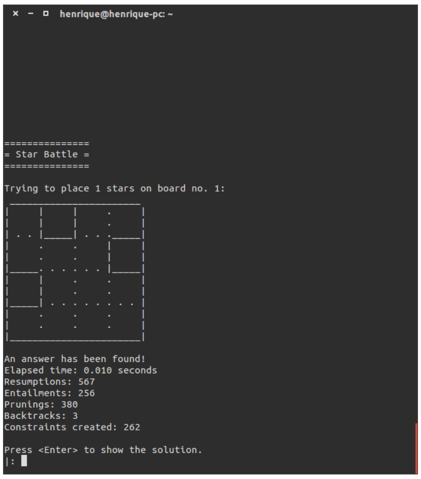
3.3 Função de Avaliação

4 Visualização da Solução

5 Resultados

6 Conclusões

Escrever conclusões aqui



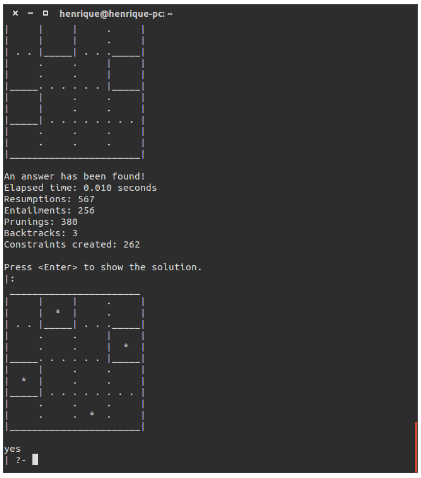
```
===============
= Star Battle =
===============

Trying to place 1 stars on board no. 1:
 _______________________
|     |     |     .     |
|     |     |     .     |
| . . |_____| . . ._____|
|     .     .     |     |
|     .     .     |     |
|_____. . . . . . |_____|
|     |     .     .     |
|     |     .     .     |
|_____| . . . . . . . . |
|     .     .     .     |
|     .     .     .     |
|_______________________|

An answer has been found!
Elapsed time: 0.010 seconds
Resumptions: 567
Entailments: 256
Prunings: 380
Backtracks: 3
Constraints created: 262

Press <Enter> to show the solution.
|:
```

Figura 1. caption.



```
|     |     |     .     |
|     |     |     .     |
| . . |_____| . . ._____|
|     .     .     |     |
|     .     .     |     |
|_____. . . . . . |_____|
|     |     .     .     |
|     |     .     .     |
|_____| . . . . . . . . |
|     .     .     .     |
|     .     .     .     |
|_______________________|

An answer has been found!
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Press <Enter> to show the solution.
|:
 _______________________
|     |     |     .     |
|     |  *  |     .     |
| . . |_____| . . ._____|
|     .     .     |     |
|     .     .     |  *  |
|_____. . . . . . |_____|
|     |     .     .     |
|  *  |     .     .     |
|_____| . . . . . . . . |
|     .     .     .     |
|     .     .  *  .     |
|_______________________|

yes
| ?-
```

Figura 2. caption.

Referências

1. Star Battle rules,
   http://logicmastersindia.com/lmitests/dl.asp?attachmentid=430
2. SICStus Prolog, https://sicstus.sics.se/
3. SWI-Prolog, http://www.swi-prolog.org/

Anexo

Código fonte

starBattle.pl

```
%=============================================%
%=                                           =%
%=           ..:: STAR BATTLE ::..           =%
%=                                           =%
%=        Type 'starBattle.' to start        =%
%=                                           =%
%=============================================%
%=                                           =%
%=             ..:: Authors ::..             =%
%=                                           =%
%=     Henrique Ferrolho && Joao Pereira     =%
%=                FEUP - 2014                =%
%=                                           =%
%=============================================%

%===============%
%= @@ includes =%
%===============%
:- use_module(library(clpfd)).
:- include('containers.pl').
:- include('printer.pl').
:- include('solver.pl').
:- include('starBattleTestBoards.pl').
:- include('utilities.pl').

%====================%
%= @@ game launcher =%
%====================%
starBattle:-
    clearConsole,
    write('To run the program type:'), nl,
    nl,
    write('\tstarBattle(NumBoard, NumStars).'),nl,
    nl,
    write('- NumBoard'), nl,
    write('number of the board you wish to test.'), nl,
    nl,
    write('- NumStars'), nl,
    write('number of stars you wish to place on each row, column and region.'), nl,
    nl.

starBattle(BoardNumber, NumStars):-
    clearConsole,
    write('==============='), nl,
    write('= Star Battle ='), nl,
    write('==============='), nl,
    nl,
    format('Trying to place ~d stars on board no. ~d:', [NumStars, BoardNumber]), nl,

    getBoard(BoardNumber, Board),
    printBoard(Board), !,

    solveBoard(Board, NumStars, Result), !,
    pressEnterToContinue,

    %getBoardSize(Board, BoardSize),
    %printResult(Result, BoardSize, NumStars),
    printResultBoard(Board, Result, NumStars), !.
```

solver.pl

```
solveBoard(Board, S, Result):-
    getBoardSize(Board, N),

    % a board NxN can not have more than N/2 stars
    S #=< (N - 1) // 2 + 1,

    ResultLength #= N * S,

    length(Result, ResultLength),
    length(ResultRegions, ResultLength),

    domain(Result, 1, N),
    domain(ResultRegions, 1, N),

    % 1st restriction
    validateNumOfOccurrencesForEachElem(Result, S, N),

    % 2nd restriction
    fetchResultRegions(Board, Result, N, S, ResultRegions),
    validateNumOfOccurrencesForEachElem(ResultRegions, S, N),

    % 3rd restriction
    noAdjacentStars(Result, S, N),

    statistics(walltime, _),
    labeling([bisect], Result),
    statistics(walltime, [_, ElapsedTime | _]),
    format('An answer has been found!~nElapsed time: ~3d seconds', ElapsedTime), nl,
    fd_statistics,
    nl.


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getBoardSize([Head|_], N):-
    length(Head, N).


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validateNumOfOccurrencesForEachElem(Elements, NumOfOccurrences, N):-
    validateNumOfOccurrencesForEachElem(Elements, NumOfOccurrences, N, 1).

validateNumOfOccurrencesForEachElem(Result, S, N, N):-
    exactly(N, Result, S).
validateNumOfOccurrencesForEachElem(Result, S, N, I):-
    exactly(I, Result, S),
    I1 #= I + 1,
    validateNumOfOccurrencesForEachElem(Result, S, N, I1).


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fetchResultRegions(Board, Result, ResRows, ResCols, ResultRegions):-
    fetchResultRegions(Board, Result, ResRows, ResCols, [], 1, ResultRegions).

fetchResultRegions(_, _, ResRows, ResCols, ResultRegions, Pos, ResultRegions):-
    Pos #= ResRows * ResCols + 1.
fetchResultRegions(Board, Result, ResRows, ResCols, ResultRegionsSoFar, Pos, ResultRegions):-
    % calculating row and col of result to access
    Row #= (Pos - 1) // ResCols + 1,
    Col #= ((Pos - 1) mod ResCols) + 1,

    % get the value of result[Row][Col], which is the column where a star is placed
    getMatrixOfListElemAt(Result, ResRows, ResCols, Row, Col, StarCol),

    % get line Row of the board
    getListElemAt(Board, Row, Line),

    % get the region of that position - board[Row][StarCol]
    element(StarCol, Line, Region),

    % push value to ResultRegionsSoFar
    listPushBack(ResultRegionsSoFar, Region, NewResultRegionsSoFar),

    % fetch next element
    Pos1 #= Pos + 1,
    fetchResultRegions(Board, Result, ResRows, ResCols, NewResultRegionsSoFar, Pos1, ResultRegions).


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noAdjacentStars(Result, S, N):-
    noAdjacentStars(Result, S, N, 1).

noAdjacentStars(Result, S, N, 1):-
    noAdjacentStarsOnRow(Result, S, 1),
    noAdjacentStars(Result, S, N, 2).
noAdjacentStars(_, _, N, Row):-
    Row #= N + 1.
noAdjacentStars(Result, S, N, Row):-
    Row #> 1,
    noAdjacentStarsOnRow(Result, S, Row),
    noAdjacentStarsWithPreviousRow(Result, S, Row),
    Row1 #= Row + 1,
    noAdjacentStars(Result, S, N, Row1).


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noAdjacentStarsOnRow(Result, S, Row):-
    StartPos #= (Row - 1) * S + 1,
    EndPos #= StartPos + S,
    validateStarsFromStartToEnd(Result, StartPos, EndPos).
validateStarsFromStartToEnd(Result, Start, End):-
    Next #= Start + 1,
    validateStarsFromStartToEnd(Result, Start, Next, End).

validateStarsFromStartToEnd(_, Start, _, End):-
    Start #= End - 1.
validateStarsFromStartToEnd(Result, Start, End, End):-
    Start1 #= Start + 1,
    Next #= Start1 + 1,
    validateStarsFromStartToEnd(Result, Start1, Next, End).
validateStarsFromStartToEnd(Result, Start, Next, End):-
    validateHorizontalDistanceBetweenStars(Result, Start, Next),
    Next1 #= Next + 1,
    validateStarsFromStartToEnd(Result, Start, Next1, End).


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noAdjacentStarsWithPreviousRow(Result, S, Row):-
    StartPos #= (Row - 1) * S + 1,
    EndPos #= StartPos + S,
    noAdjacentStarsWithPreviousRow(Result, S, Row, StartPos, EndPos).

noAdjacentStarsWithPreviousRow(_, _, _, EndPos, EndPos).
noAdjacentStarsWithPreviousRow(Result, S, Row, CurrentPos, EndPos):-
    % for each star of the row being validated,
    % validate horizontal distance to each star of the previous row
    PrevRow #= Row - 1,
    starIsNotAdjacentWithAnyOfThePreviousRow(Result, S, CurrentPos, PrevRow),
    % procceed to next row
    CurrentPos1 #= CurrentPos + 1,
    noAdjacentStarsWithPreviousRow(Result, S, Row, CurrentPos1, EndPos).

starIsNotAdjacentWithAnyOfThePreviousRow(Result, S, PivotStar, PrevRow):-
    FirstStarPos #= (PrevRow - 1) * S + 1,
    LastStarPos #= FirstStarPos + S,
    starIsNotAdjacentToAnyOtherStarFromFirstToLastPos(Result, PivotStar, FirstStarPos, LastStarPos).

starIsNotAdjacentToAnyOtherStarFromFirstToLastPos(_, _, LastStarPos, LastStarPos).
starIsNotAdjacentToAnyOtherStarFromFirstToLastPos(Result, PivotStar, CurrentStarPos, LastStarPos):-
    validateHorizontalDistanceBetweenStars(Result, PivotStar, CurrentStarPos),
    NextStarPos #= CurrentStarPos + 1,
    starIsNotAdjacentToAnyOtherStarFromFirstToLastPos(Result, PivotStar, NextStarPos, LastStarPos).
```

```
%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%

validateHorizontalDistanceBetweenStars(Result, Pos1, Pos2):-
    element(Pos1, Result, Col1),
    element(Pos2, Result, Col2),
    Dist #= abs(Col2 - Col1),
    Dist #> 1.
```

printer.pl

```
%===============================%
%= @@ board printing functions =%
%===============================%
printBoard(Board):-
    getBoardSize(Board, N),
    printBoardTopBorder(N),
    printBoard(Board, 1, N),
    nl, !.
printResultBoard(Board, Result, S):-
    getBoardSize(Board, N),
    printBoardTopBorder(N),
    printBoard(Board, 1, N, Result, S),
    nl, !.

printBoardTopBorder(N):-
    N1 is N - 1, createSeparatorN(N1, '______', TopBorder),
    write(' '), printList(TopBorder), write('_____'), nl.

printBoard(Board, N, N):-
    printBoardRow(Board, N, N).
printBoard(Board, I, N):-
    printBoardRow(Board, I, N), !,
    I1 is I + 1,
    printBoard(Board, I1, N).
%-%-%-%-%-%-%
printBoard(Board, N, N, Result, S):-
    printBoardRow(Board, N, N, Result, S).
printBoard(Board, I, N, Result, S):-
    printBoardRow(Board, I, N, Result, S), !,
    I1 is I + 1,
    printBoard(Board, I1, N, Result, S).

%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%
```

```
printBoardRow(Board, N, N):-
    write('|'), printBoardRowTop(Board, N, N, 1), nl, !,
    write('|'), printBoardRowMiddle(Board, N, N, 1), nl, !,
    write('|'), printBoardLastRowBottom(Board, N, N, 1), nl, !.
printBoardRow(Board, I, N):-
    write('|'), printBoardRowTop(Board, I, N, 1), nl, !,
    write('|'), printBoardRowMiddle(Board, I, N, 1), nl, !,
    write('|'), printBoardRowBottom(Board, I, N, 1), nl, !.
%-%-%-%-%-%-%
printBoardRow(Board, N, N, Result, S):-
    write('|'), printBoardRowTop(Board, N, N, 1), nl, !,
    write('|'), printBoardRowMiddle(Board, N, N, 1, Result, S), nl, !,
    write('|'), printBoardLastRowBottom(Board, N, N, 1), nl, !.
printBoardRow(Board, I, N, Result, S):-
    write('|'), printBoardRowTop(Board, I, N, 1), nl, !,
    write('|'), printBoardRowMiddle(Board, I, N, 1, Result, S), nl, !,
    write('|'), printBoardRowBottom(Board, I, N, 1), nl, !.

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printBoardRowTop(_, _, N, N):-
    write('     |').
printBoardRowTop(Board, I, N, Col):-
    getListElemAt(Board, I, Row),
    Col1 is Col + 1,
    element(Col, Row, V1),
    element(Col1, Row, V2),
    printCellTop(V1, V2),
    printBoardRowTop(Board, I, N, Col1).

% @@@ swap comment to toggle region display
%printBoardRowMiddle(Board, I, N, N):-
%   getListElemAt(Board, I, Row),
%   element(N, Row, V1),
%   write('  '), write(V1), write('  |').
printBoardRowMiddle(_, _, N, N):-
    write('     |').
printBoardRowMiddle(Board, I, N, Col):-
    getListElemAt(Board, I, Row),
    Col1 is Col + 1,
    element(Col, Row, V1),
    element(Col1, Row, V2),
    printValue(V1, V2),
    printBoardRowMiddle(Board, I, N, Col1).
%-%-%-%-%-%-%
```

```
printBoardRowMiddle(_, I, N, N, Result, S):-
    starExistsIn(Result, S, I, N),
    write('  *  |').
printBoardRowMiddle(_, _, N, N, _, _):-
    write('     |').
printBoardRowMiddle(Board, I, N, Col, Result, S):-
    starExistsIn(Result, S, I, Col),

    getListElemAt(Board, I, Row),
    Col1 is Col + 1,
    element(Col, Row, V1),
    element(Col1, Row, V2),
    printStar(V1, V2),
    printBoardRowMiddle(Board, I, N, Col1, Result, S).
printBoardRowMiddle(Board, I, N, Col, Result, S):-
    getListElemAt(Board, I, Row),
    Col1 is Col + 1,
    element(Col, Row, V1),
    element(Col1, Row, V2),
    printValue(V1, V2),
    printBoardRowMiddle(Board, I, N, Col1, Result, S).

printBoardRowBottom(Board, I, N, N):-
    getListElemAt(Board, I, Row),
    I1 is I + 1, getListElemAt(Board, I1, NextRow),

    element(N, Row, V1),
    element(N, NextRow, V3),

    printCellBottom(V1, V3).
printBoardRowBottom(Board, I, N, Col):-
    getListElemAt(Board, I, Row),
    I1 is I + 1, getListElemAt(Board, I1, NextRow),
    NextCol is Col + 1,

    element(Col, Row, V1),
    element(NextCol, Row, V2),
    element(Col, NextRow, V3),

    printCellBottom(V1, V2, V3),
    printBoardRowBottom(Board, I, N, NextCol).

printBoardLastRowBottom(_, _, N, N):-
    write('_____|').
printBoardLastRowBottom(Board, I, N, Col):-
```

```
    getListElemAt(Board, I, Row),
    NextCol is Col + 1,

    element(Col, Row, V1),
    element(NextCol, Row, V2),

    printLastRowCellBottom(V1, V2),
    printBoardLastRowBottom(Board, I, N, NextCol).

%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%

printCellTop(V1, V1):-
    write('     .').
printCellTop(_, _):-
    write('     |').

% @@@ swap comment to toggle region display
%printValue(V, V):-
%   write('  '), write(V), write('  .').
printValue(V, V):-
    write('     .').

% @@@ swap comment to toggle region display
%printValue(V, _):-
%   write('  '), write(V), write('  |').
printValue(_, _):-
    write('     |').

printStar(V, V):-
    write('  *  .').
printStar(_, _):-
    write('  *  |').

printCellBottom(V, V, V):-
    write(' . . .').
printCellBottom(V, V, _):-
    write('_____.').
printCellBottom(V, _, V):-
    write(' . . |').
printCellBottom(_, _, _):-
    write('_____|').

printCellBottom(V, V):-
    write(' . . |').
printCellBottom(_, _):-
```

```
    write('_____|').

printLastRowCellBottom(V, V):-
    write('______').
printLastRowCellBottom(_, _):-
    write('_____|').


%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%

createSeparatorN(0, _, []).
createSeparatorN(N, SS, [SS | Ls]):-
    N1 is N-1,
    createSeparatorN(N1, SS, Ls).


%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%-%

starExistsIn(Result, S, Row, StarCol):-
    StartPos is (Row - 1) * S + 1,
    EndPos is StartPos + S,
    starExistsSomewhereBetween(Result, StartPos, EndPos, StarCol).

starExistsSomewhereBetween(Result, CurrentPos, _, StarCol):-
    element(CurrentPos, Result, ScanRes),
    StarCol =:= ScanRes.
starExistsSomewhereBetween(Result, CurrentPos, EndPos, StarCol):-
    NextPos is CurrentPos + 1,
    NextPos < EndPos,
    starExistsSomewhereBetween(Result, NextPos, EndPos, StarCol).


%================================%
%= @@ result printing functions =%
%================================%
printResult(Result, N, S):-
    write('Result:'), nl,
    printResultRow(Result, N, S, 1).


printResultRow(Result, N, S, N):-
    write('\t'), printResultRowValues(Result, N, S, N, 1).

printResultRow(Result, N, S, Row):-
    write('\t'), printResultRowValues(Result, N, S, Row, 1),
    Row1 is Row + 1,
    printResultRow(Result, N, S, Row1).


printResultRowValues(Result, _, S, Row, S):-
    Pos is (Row - 1) * S + S,
    getListElemAt(Result, Pos, Elem),
    write(Elem), nl.

printResultRowValues(Result, N, S, Row, Column):-
    Pos is (Row - 1) * S + Column,
    getListElemAt(Result, Pos, Elem),
    write(Elem), write(', '),

    Column1 is Column + 1,
    printResultRowValues(Result, N, S, Row, Column1).
```

containers.pl

```
%=================%
%= @@ containers =%
%=================%
% containers are indexed starting at 1, not 0.

%%% 1. matrix; 2. element row; 3. element column; 4. query element.
getMatrixElemAt([ListAtTheHead|_], 1, ElemCol, Elem):-
    getListElemAt(ListAtTheHead, ElemCol, Elem).
getMatrixElemAt([_|RemainingLists], ElemRow, ElemCol, Elem):-
    ElemRow > 1,
    ElemRow1 is ElemRow - 1,
    getMatrixElemAt(RemainingLists, ElemRow1, ElemCol, Elem).

% treats list as if it was a matrix of NRows x NCols and returns the Elem at ElemRow, ElemCol
getMatrixOfListElemAt(List, NRows, NCols, ElemRow, ElemCol, Elem):-
    ElemRow =< NRows, ElemCol =< NCols,
    Pos is (ElemRow - 1) * NCols + ElemCol,
    element(Pos, List, Elem).

%%% 1. list; 2. element position; 3. query element.
getListElemAt([ElemAtTheHead|_], 1, ElemAtTheHead).
getListElemAt([_|RemainingElems], Pos, Elem):-
    Pos > 1,
    Pos1 is Pos - 1,
    getListElemAt(RemainingElems, Pos1, Elem).

listPushBack([], Elem, [Elem]).
listPushBack([Head|Tail], Elem, [Head|NewTail]):-
    listPushBack(Tail, Elem, NewTail).

printList([]).
printList([Head|Tail]):-
    write(Head), printList(Tail).
```

utilities.pl

```
%================%
%= @@ utilities =%
%================%
clearConsole:-
    clearConsole(40), !.
clearConsole(0).
clearConsole(N):-
    nl,
    N1 is N-1,
    clearConsole(N1).

pressEnterToContinue:-
    write('Press <Enter> to show the solution.'), nl,
    waitForEnter, !.
waitForEnter:-
    get_char(_).

exactly(_, [], 0).
exactly(X, [Y|L], N) :-
    X #= Y #<=> B,
    N #= M + B,
    exactly(X, L, M).
```

starBattleTestBoards.pl

```
%======================================%
%= @@ function to retrieve test boards =%
%======================================%
getBoard(N, Board):-
    (
        N =:= 1 -> testBoard4x4_1(Board);
        N =:= 2 -> testBoard5x5_1(Board);
        N =:= 3 -> testBoard5x5_2(Board);
        N =:= 4 -> testBoard8x8_1(Board);
        N =:= 5 -> testBoard8x8_2(Board);
        N =:= 6 -> testBoard10x10_1(Board);
        N =:= 7 -> testBoard10x10_2(Board);

        nl,
        write('Error: the specified board does not exist.'),
        fail
    ).

%==================%
%= @@ test boards =%
%==================%
% expected answer:2413
testBoard4x4_1([
    [1, 2, 1, 1],
    [1, 1, 1, 3],
    [4, 1, 1, 1],
    [1, 1, 1, 1]]).

% expected answer: 14253
testBoard5x5_1([
    [1, 1, 2, 2, 2],
    [1, 2, 2, 3, 2],
    [1, 2, 2, 2, 2],
    [4, 2, 4, 2, 5],
    [4, 4, 4, 5, 5]]).

testBoard5x5_2([
    [1, 1, 1, 2, 2],
    [1, 3, 3, 3, 4],
    [1, 1, 3, 3, 4],
    [1, 5, 5, 5, 5],
    [1, 1, 1, 5, 5]]).

% expected answer: 2468246813571357
testBoard8x8_1([
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8],
    [1, 2, 3, 4, 5, 6, 7, 8]]).

% expected answer: 2468246813571357
testBoard8x8_2([
    [1, 1, 1, 1, 1, 1, 1, 1],
    [2, 2, 2, 2, 2, 2, 2, 2],
    [3, 3, 3, 3, 3, 3, 3, 3],
    [4, 4, 4, 4, 4, 4, 4, 4],
    [5, 5, 5, 5, 5, 5, 5, 5],
    [6, 6, 6, 6, 6, 6, 6, 6],
    [7, 7, 7, 7, 7, 7, 7, 7],
    [8, 8, 8, 8, 8, 8, 8, 8]]).

testBoard10x10_1([
    [1,  1,  1,  2,  2,  3,  3,  3,  3,  3],
    [1,  4,  4,  4,  2,  5,  3,  5,  3,  6],
    [1,  1,  1,  4,  2,  5,  3,  5,  6,  6],
    [1,  4,  4,  4,  2,  5,  5,  5,  6,  6],
    [1,  4,  7,  7,  7,  8,  9,  5,  9,  6],
    [1,  4,  4,  4,  7,  8,  9,  5,  9,  6],
    [1,  1,  7,  7,  7,  8,  9,  9,  9,  6],
    [10, 10, 7,  8,  8,  8,  8,  8,  9,  6],
    [10, 10, 7,  7,  7,  10, 6,  8,  9,  6],
    [10, 10, 10, 10, 10, 10, 6,  6,  6,  6]]).

testBoard10x10_2([
    [1,  1,  1,  2,  2,  2,  2,  2,  2,  2],
    [3,  3,  1,  1,  1,  1,  2,  2,  2,  2],
    [3,  4,  4,  4,  4,  5,  5,  5,  5,  2],
    [3,  4,  4,  4,  4,  5,  5,  5,  6,  2],
    [3,  7,  7,  7,  7,  7,  7,  5,  6,  2],
    [3,  7,  8,  6,  6,  6,  6,  6,  6,  2],
    [3,  7,  8,  8,  8,  9,  9,  9,  9,  2],
    [3,  8,  8,  8,  8,  9,  9,  9,  9,  2],
    [3,  3,  10, 10, 10, 10, 10, 10, 2,  2],
    [10, 10, 10, 10, 10, 10, 10, 10, 10, 10]]).
```

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Lemmas, Propositions, and Theorems. The numbers accorded to lemmas, propositions, and theorems, etc. should appear in consecutive order, starting with Lemma 1, and not, for example, with Lemma 11.

6.1 Figures

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Figura 3. One kernel at x_s (*dotted kernel*) or two kernels at x_i and x_j (*left and right*) lead to the same summed estimate at x_s. This shows a figure consisting of different types of lines. Elements of the figure described in the caption should be set in italics, in parentheses, as shown in this sample caption.

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6.2 Formulas

Displayed equations or formulas are centered and set on a separate line (with an extra line or halfline space above and below). Displayed expressions should be numbered for reference. The numbers should be consecutive within each section or within the contribution, with numbers enclosed in parentheses and set on the right margin – which is the default if you use the *equation* environment, e.g.,

$$\psi(u) = \int_{o}^{T} \left[\frac{1}{2}\left(\Lambda_{o}^{-1}u, u\right) + N^{*}(-u)\right] dt \; . \qquad (1)$$

Equations should be punctuated in the same way as ordinary text but with a small space before the end punctuation mark.

6.3 Footnotes

The superscript numeral used to refer to a footnote appears in the text either directly after the word to be discussed or – in relation to a phrase or a sentence – following the punctuation sign (comma, semicolon, or period). Footnotes should appear at the bottom of the normal text area, with a line of about 2 cm set immediately above them.[1]

[1] The footnote numeral is set flush left and the text follows with the usual word spacing.