Robotics: Science and Systems (R:SS)
Course Assignment 1

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Notes regarding this assignment:

- This assignment is due at 4 pm on 4th November, 2016. Coursework submission is by hand to the ITO mailbox.
- The School's policy on late submission and academic misconduct are to be found at: http://web.inf.ed.ac.uk/infweb/student-services/ito/admin/coursework-projects http://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct
- This coursework is worth 10% of total marks in the course. The assignment will be marked out of 100 and then, scaled.

1 Forward and inverse kinematics (30 marks)

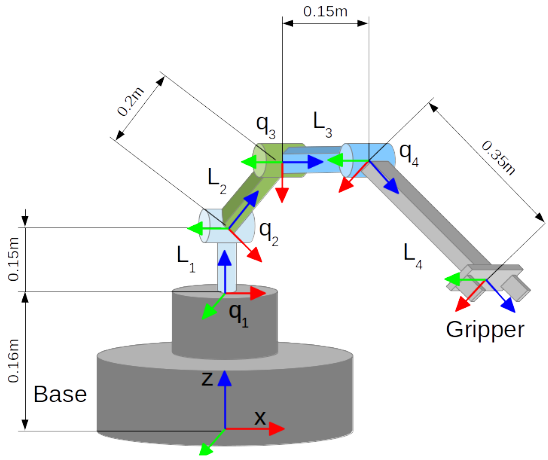

Figure 1: 4 DoF robotic arm. Joint q_1 rotates around z axis (blue), joints q_2, q_3 and q_4 rotate around the y axis (green).

Let's consider a robot with 4 Degrees of Freedom (DoF) and a gripper attached to the tip of its last link (see Figure 1). The transformation frame of each joint with respect to it's parent frame is as follows:

- $T_{\text{base}\rightarrow L_1}$ — The first joint (q_1) is translated along the z axis by $0.16m$ relative to the base and it rotates around the z axis.

- $T_{L_1 \to L_2}$ — The second joint (q_2) is translated along the z axis by $0.15m$ relative to the first joint and it rotates around the y axis.
- $T_{L_2 \to L_3}$ — The third joint (q_3) is translated along the z axis by $0.20m$ relative to the second joint and it rotates around the y axis.
- $T_{L_3 \to L_4}$ — The fourth joint (q_4) is translated along the z axis by $0.15m$ relative to the third joint and it rotates around the y axis.
- $T_{L_4 \to \text{gripper}}$ — The gripper is rigidly attached to the fourth joint at an offset of $0.35m$ along the z axis.

1.1 (10 marks)

Calculate and report the position part of the homogeneous transformation from the base to the gripper $T_{\text{base}\to\text{gripper}}$ for the following joint angles:

$$q = \begin{pmatrix} 0.66 \\ -0.44 \\ 1.06 \\ 1.33 \end{pmatrix}. \tag{1}$$

The joint angles are in radians. We will denote the position element of this transformation as y.

Now, let's assume that there is an object at the coordinates

$$y^* = \begin{pmatrix} -0.37 \\ 0.34 \\ 0.16 \end{pmatrix} \tag{2}$$

relative to the base (in meters). If we wanted move the gripper towards this position, we would iterate the inverse kinematics equations. However, we will only consider a one-shot solution from our current joint configuration. This solution will provide us with the optimal direction to move $\triangle q^*$, rather than the exact joint angles q^* for reaching the target.

1.2 (10 marks)

Compute the the Jacobian matrix at the joint configuration $q = \begin{pmatrix} 0.66 & -0.44 & 1.06 & 1.33 \end{pmatrix}$:

$$J = ? \tag{3}$$

1.3 (10 marks)

Compute the optimal joint motion $\triangle q^*$ of the one-shot IK, where $q_{t+1} = q_t + \triangle q^*$. Don't use any q-space metric ($W = I$) but use the following regularisation:

$$C = \begin{pmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{pmatrix}. \tag{4}$$

This is the value of C not C^{-1}!

For all answers report numbers with at least 5 decimal places.

2 Signal filtering & State Estimation (40 marks)

2.1 (10 marks)

Explain: 1. why signal filtering is necessary in real world applications; 2. what types of filters are commonly used; 3. how to choose them.

Suppose a scenario that the wheels of a mobile robot are following a trajectory of $A\sin(\omega t)$, if you have noisy measurements (Gaussian noise) of the position encoders of the wheels, what type of filter you need to choose to get a clean feedback signal? Can you explain how you will choose the cut-off frequency in Hz? Note that the unit of ω is rad/s.

2.2 (10 marks)

What is your understanding of state estimation? Why and in what cases do we need it? (2 marks)

Following the same scenario that the wheels of a mobile robot are following a trajectory of $A\sin(\omega t)$, your measurements start at $t_0 = 0$ for a period of time and stop at t_f. We know the frequency of the reference ($A\sin(\omega t)$), and the wheels track the frequency well but not the amplitude. Now with the collected noise measurement y, how would you estimate the real amplitude $\bar{A}$ of your sinusoidal measurements? Given the known parameter ω, the measurement y and t over t_0 to t_f with sampling time Δt, please formulate your solution. (Math formulation, pseudo code or real code demo are all acceptable.) (8 marks)

2.3 (20 marks)

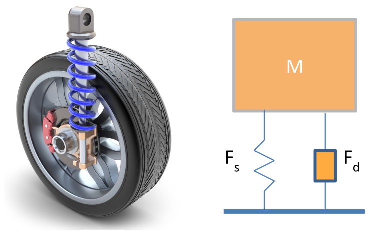

Figure 2: Modelling a car suspension system by a mass-spring-damper model.

The model of a car suspension system is given in Fig. 2, assume the mass m is known, and we would like to identify the real stiffness k and damping γ. Can you formulate the dynamics of the system using these parameters?

Now we have multiple measurements of the vertical position z, one is from the on-board transducer with high accuracy, another is the external vision sensor based on markers. The on-board measurement has good resolution, its standard deviation is α_1. But it may drift slowly during a long measurement. Obviously, it won't affect the velocity $\dot{z}$ and acceleration $\ddot{z}$ much. The external vision system has absolute positional information, but with lower resolution, its standard deviation is α_2.

Our task now is to fuse these two measurements together, and perform system identification of stiffness k and damping γ. Can you: 1. solve the sensor fusion problem; 2. discuss if filtering is needed or not, if yes, then where to apply it, give your reasons; 3. formulate your identification problem expressed by k and γ, and explain how to use the data in your formula.

3 Computer Vision (30 marks)

3.1 Camera Geometry (10 marks)

You have a camera with a focal length of 10 mm perfectly focused on a statue 10 meters away. The statue is observed in the image as 100 pixels tall. The image sensor is 10mm x 10mm and gives 1000x1000 pixels. How tall is the observed statue in metres?

Explain with a sketch why you need to know how far away the person is.

3.2 Image Processing (10 marks)

You have an outdoor car counting camera that looks straight down onto a road, and acquires images of only 1 lane. The image is large enough to see a full car, but not a long lorry/truck. Assume that you know the maximum and minimum dimensions of all possible cars in pixels. Write the pseudocode that describes the main steps in the counting process. Write the pseudocode that gives enough details of each step that another could implement your pseudocode in a suitable programming language (*e.g.* matlab, C, or python).

What conditions does your algorithm handle and what will cause it to fail?

3.3 Shape Recognition (10 marks)

You are building a playing card recogniser. Assume that the deck only consists of the 2...9 value cards (*i.e.* only have 2, 3, ... or 9 spots on the card). There are also the number and symbols in the upper left and lower right corners. Ignore these numbers and symbols (*i.e.* those in the corners of the cards). There are the 4 usual suits { spades, hearts, clubs, diamonds }. Design an algorithm to recognise each card, using a Bayes classifier as part of the solution. Write the algorithm in pseudocode to the level of detail that another person could code up the algorithm. Assume that the cards are well lighted and viewed directly from above by a camera. The cards are mainly white and are sitting on a black background, but in an unknown position.

What properties will you use in the feature vector and how are they computed? What could go wrong with your method?