Marlo Diagrams: representation and operations

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Abstract

Some notes to fix the structure of the diagrams and behaviour of the operations prior to the implementation of a visual reasoning assistant.

1 Diagram structure

Definition 1 (Literals) *Let P be a set of propositional variables. Given $p \in P$, both p and $\neg p$ are* literals. *Literals p and $\neg p$ are* complementary literals. *Given a literal λ (like p or $\neg p$), $\overline{\lambda}$ denotes its complementary literal ($\neg p$ or p, respectively). The set of literals built from P is $Lit(P)$. A set of literals is* consistent *if it does not contain complementary literals.*

Definition 2 (Diagram) *A* diagram $D = \langle S, A, I, O \rangle$ *has the following structure:*

- $S \in Lit(P)$*, the* "subject" *of D.*
- $A \subset Lit(P) - \{S\}$ *with* $|A| \in \mathbb{N}$*, the* "all" *of D.*
- $I \subset \mathcal{P}\left(Lit(P) - (A \cup \{S\})\right)$ *with* $|I| \in \mathbb{N} - \{0\}$ *and* $|\cup I| \in \mathbb{N}$*, the* "in" *of D.*
- $O \subset A \cup (\cup I)$ *with* $|O| \in \mathbb{N}$*, the* "out" *of D.*

Definition 3 (Consistent Diagram) *A diagram $D = \langle S, A, I, O \rangle$ is* consistent *iff for all $R \in I$, the set of literals $\{S\} \cup A \cup R$ is consistent.*

Definition 4 (Scope) *Given a diagram $D = \langle S, A, I, O \rangle$, the* scope *of D, denoted as $Sc(D)$ is the set of propositions appearing in the diagram, that is,*

$$Sc(D) = \{S\} \cup A \cup (\cup I).$$

1.1 Examples

Example 5 (All a are b) *The diagram for "All a (universal) are b (particular)" is $D = \langle a, \{b\}, \{\varnothing\}, \{b\} \rangle$.*

Example 6 (Some a are b) *The diagram for "Some a (particular) are b (particular)" is $D = \langle a, \varnothing, \{\{b\}, \varnothing\}, \{b\} \rangle$.*

1.2 Definición de las conectivas por Marcos

** Revisar, en las partes O se confunde $\varnothing$ con $\{\varnothing\}$. Para hablar de equivalencia (y, sobre todo, justificarlas) es necesaria una semántica. O bien, una vez dadas las operaciones probar que se puede ir de cada variante a las demás mediante transformación y conversión.

Example 7 (*a* iff b) *The diagrams for biconditional "$a \leftrightarrow b$" are*

$$\begin{aligned} D &= \langle\ a, \{b\}, \{\varnothing\}, \varnothing\rangle \quad \equiv \quad \langle\ b, \{a\}, \{\varnothing\}, \varnothing\rangle \\ &\equiv \langle\neg b, \{\neg a\}, \{\varnothing\}, \varnothing\rangle \quad \equiv \quad \langle\neg a, \{\neg b\}, \{\varnothing\}, \varnothing\rangle\,. \end{aligned}$$

Example 8 ($a \veebar b$) *The diagrams for exclusive disjunction "$a \veebar b$" are*

$$\begin{aligned} D &= \langle\ a, \{\neg b\}, \{\varnothing\}, \varnothing\rangle \quad \equiv \quad \langle\neg b, \{a\}, \{\varnothing\}, \varnothing\rangle \\ &\equiv \langle\ b, \{\neg a\}, \{\varnothing\}, \varnothing\rangle \quad \equiv \quad \langle\neg a, \{\ b\}, \{\varnothing\}, \varnothing\rangle\,. \end{aligned}$$

Example 9 ($a \to b$ (conditional)) *The diagram for "$a \to b$" (conditional) is*

$$\begin{aligned} D &= \langle\ a, \{b\}, \{\varnothing\}, \{b\}\rangle \quad \equiv \quad \langle\ b, \{\varnothing\}, \{a\}, \varnothing\rangle \\ &\equiv \langle\neg a, \{\neg b\}, \{\varnothing\}, \{\neg b\}\rangle \quad \equiv \quad \langle\neg b, \{\varnothing\}, \{a\}, \varnothing\rangle\,. \end{aligned}$$

Example 10 ($a \vee b$ (inclusive)) *The diagram for inclusive disjunction "$a \vee b$" are*

$$\begin{aligned} D &= \langle\neg a, \{b\}, \{\varnothing\}, \{b\}\rangle \quad \equiv \quad \langle\ b, \{\varnothing\}, \{\neg a\}, \varnothing\rangle \\ &\equiv \langle\neg b, \{a\}, \{\varnothing\}, \{a\}\rangle \quad \equiv \quad \langle\ a, \{\varnothing\}, \{\neg b\}, \varnothing\rangle\,. \end{aligned}$$

Example 11 (a Nand b) *The diagrams for "a Nand b" are*

$$\begin{aligned} D &= \langle\ a, \{\neg b\}, \{\varnothing\}, \{\neg b\}\rangle \quad \equiv \quad \langle\neg b, \{\varnothing\}, \{a\}, \varnothing\rangle \\ &\equiv \langle\ b, \{\neg a\}, \{\varnothing\}, \{\neg a\}\rangle \quad \equiv \quad \langle\neg a, \{\varnothing\}, \{b\}, \varnothing\rangle\,. \end{aligned}$$

Example 12 (a or b (inclusive)) *The diagram for "a or b" (inclusive disjunction) is $D = \langle\neg a, \{b\}, \{\varnothing\}, \{b\}\rangle$.*

Example 13 (or a or b (exclusive)) *The diagram for "a or b" (exclusive disjunction) is $D = \langle\neg a, \{b\}, \{\varnothing\}, \varnothing\rangle$.*

1.3 Graphical representation

** TODO: Explain the representation of the diagrams.

Figure 1 shows the graphical representation of the diagrams in examples 5 and 6.

1.4 Translation into FOL

** TODO: Explain the general idea of the translation.

Idea: Existence for $x \cup S$ when $x \in I$ and $x \cap O \neq \varnothing$.

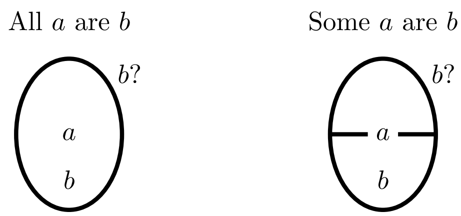


Figure 1: Diagrams for examples 5 and 6.

The translation of the diagram in example 5 into FOL is:

$$\forall x(Ax \to Bx) \wedge \exists x Ax$$

The translation of the diagram in example 6 into FOL is:

$$\exists x(Ax \wedge Bx)$$

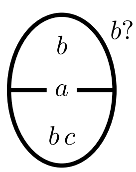
The translation of the diagram $D = \langle a, \varnothing, \{\{b\}, \varnothing\}, \varnothing\rangle$ (like the one in the right of Fig.1 but without $b?$ in the outer part) is

$$\exists x(Ax \wedge Bx) \wedge \forall x(Bx \to Ax)$$

The translation of the diagram $D = \langle a, \{b\}, \{\varnothing, \{c\}\}, \{b\}\rangle$ is

$$\forall x(Ax \to Bx) \wedge \exists x(Ax \wedge Cx) \wedge \forall x(Cx \to Ax)$$

And the representation:



2 Operations on Marlo Diagrams

2.1 Conversion

*** Esto hay que revisarlo con cuidado y poner muchos ejemplos.

Definition 14 (Conversion) *Given $D = \langle S, A, I, O\rangle$, the conversion of D for*

S' *produces* $Conv(D, S') = \langle S', A', I', O' \rangle$ *iff*

$$
\begin{array}{rcl}
S' & \in & A \cup (\cup I) \\
A' & = & \left\{ \begin{array}{ll} \varnothing & if \quad S' \in O \\ \{S\} \cup (A - \{S'\}) \cup & \\ ((\cup I - \{S'\}) \cap (\cap \{x - \{S'\} \mid x \in I \,\&\, S' \in x\})) & otherwise \end{array} \right. \\
I' & = & \left\{ \begin{array}{ll} \{((x - \{S'\}) \cup \{S\} \cup A) - A' \mid x \in I\} & if \quad S' \in A \\ \{((x - \{S'\}) \cup \{S\} \cup A) - A' \mid x \in I \,\&\, S' \in x\} & otherwise \end{array} \right. \\
 & \cup & \left\{ \begin{array}{ll} \{\varnothing\} & if \quad S' \in O \\ \varnothing & otherwise \end{array} \right. \\
O' & = & ((O - \{S'\}) \cup (\cup \{x \cup A \cup \{S\} \mid x \in I \,\&\, S' \notin (x \cup A)\})) \\
 & \cap & (A' \cup (\cup I'))
\end{array}
$$

2.2 Transformation

Definition 15 (Transformation) *Given a diagram D, the* transformation *of D produces $Tr(D)$ in two cases:*

- $D = \langle \gamma, \{\lambda\}, \{\varnothing\}, O \rangle$. *Then,*

$$Tr(D) = \left\langle \overline{\lambda}, \{\overline{\gamma}\}, \{\varnothing\}, O' \right\rangle,$$

  with

$$O' = \left\{ \begin{array}{ll} \varnothing & \textit{if} \quad O = \varnothing \\ \{\overline{\gamma}\} & \textit{otherwise (that is, } O = \{\lambda\}\textit{)} \end{array} \right.$$

- $D = \langle \gamma, \varnothing, \{\{\lambda\}, \varnothing\}, \varnothing \rangle$. *Then,*

$$Tr(D) = \left\langle \overline{\lambda}, \varnothing, \{\{\overline{\gamma}\}, \varnothing\}, \varnothing \right\rangle.$$

2.3 Inference

Definition 16 (Inference) *Given the diagrams*

$$D_1 = \langle s, A_1, I_1, O_1 \rangle \qquad \textit{and} \qquad D_2 = \langle s, A_2, I_2, O_2 \rangle,$$

the inference with D_1 and D_2 produces

$$D_3 = D_1 \oplus D_2 = \langle s, A_1 \cup A_2, I_3, O_3 \rangle,$$

with

$$
\begin{array}{rcl}
I_3 & = & \{x - A_2 \mid x \in I_1\} \cup \{x - A_1 \mid x \in I_2\} \\
O_3 & = & (O_1 - (Sc\,(D_2) - O_2)) \cup (O_2 - (Sc\,(D_1) - O_1)).
\end{array}
$$

Example 17 *Consider the following diagrams:*

$$
\begin{array}{rcl}
D_1 & = & \langle a, \{b\}, \{\{c\}, \varnothing\}, \{b, c\} \rangle \\
D_2 & = & \langle a, \varnothing, \{\{b, d\}, \varnothing\}, \{d\} \rangle.
\end{array}
$$

The inference with D_1 and D_2 (view Figure 2) produces

$$D_1 \oplus D_2 \quad = \quad \langle a, \{b\}, \{\{c\}, \{d\}, \varnothing\}, \{c, d\} \rangle.$$

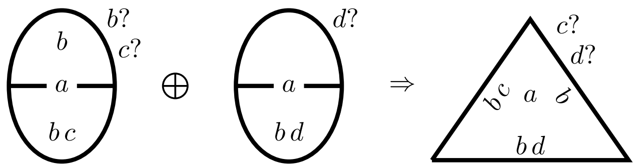


Figure 2: Example of inference (example 17).

2.4 Extraction

Definition 18 (Extraction) *Given the diagram* $D = \langle S, A, I, O \rangle$ *and* $\lambda \in A \cup (\cup I)$*, the* extraction of λ in D *produces*

$$Elim(D, \lambda) = \langle S, A - \{\lambda\}, \{x - \{\lambda\} \mid x \in I\}, O - \{\lambda\} \rangle .$$