Parameter Estimation In Nonlinear Time-Varying Systems Through Takagi-Sugeno Fuzzy Models And Wavelets

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Abstract. In this paper, a parameter estimation problem for a Takagi-Sugeno (TS) fuzzy dynamical system is formulated under the assumption that the premises in the membership functions are known. A linear expression in consequent parameters is obtained under this assumption. If the system is time-varying (TV), the parameters can be determined by recursive estimation techniques. As an alternate approach, the use of multi-resolution wavelets is proposed. Furthermore, a parameter estimation toolbox for fuzzy dynamical models is developed which is then applied to a simple example and to the Mackey-Glass chaotic time series.

1 Introduction

There are two known methodologies for model based fault detection and diagnosis. The first one consists in the comparison of normal behavior patterns with respect to fault situations [1], [2]. The second consists in the analysis of signals generated (residues) in the different scenarios. For these purposes observers or Kalman filters are generally employed [3].

The design of such methodologies requires the availability of a dynamical model of the process, whose parameters are estimated from the input and output data [4]. If the process or the system is nonlinear, then polynomial (Volterra, Hammerstein or Wiener, [5]), neuronal [6], or fuzzy models [7] can be used.

This paper considers the case of nonlinear time-varying systems. In this situation, the literature presents two approachs. The first of them employs recursive estimation, such as recursive least square (RLS) methods or Kalman filters [8], [9], [10]. The second is based on the representation of TV parameters through series of time-invariant basis, which converts the TV problem into an invariant one [11], [12], [13].

In order to deal with system nonlinearity, it is proposed the use of Takagi-Sugeno dynamical fuzzy models [14], [18]; to take into consideration the time-varying characteristics of the system, it is assumed that the consequent parameters change with time. Wavelets [16] are used as an alternative to recursive estimation.

2 Takagi-Sugeno type dynamical models

Given a nonlinear dynamical system with input *u(t)* and output *y(t)*, a Takagi-Sugeno model is described by the following *R* rules [15]:

Rule r, r = 1, ..., R:

`if`

$y(k-1)$ `is` $\pi_{1,r}$ `and ... and` $y(k-n_a)$ `is` $\pi_{n_a,r}$

`and`

$u(k-1-d)$ `is` $\pi_{n_a+1,r}$ `and ...` $u(k-n_b-d)$ `is` $\pi_{n,r}$

`then`

$$y(k)=\sum_{i=1}^{n_a} a_{i,r}\,y(k-i)+\sum_{j=1}^{n_b} b_{j,r}\,u(k-j-d)$$

where d is the input delay of the sub-models, and $n=n_a+n_b$ is the total number of parameters in each submodel.

The term $\pi_{i,r}$ denotes the fuzzy set of the r^{th} rule and of the i^{th} term in the ARX (autoregressive with exogenous inputs) submodels. For each fuzzy set $\pi_{i,r}$ it corresponds a membership function $\mu_{\pi_{i,r}}$. The rule activation degree β_r of the rule r in an instant k is given by:

$$\beta_r(k)=\prod_{i=1}^{n_a}\mu_{\pi_{i,r}}(y(k-i))\prod_{j=1}^{n_b}\mu_{\pi_{na+j,r}}(u(k-j-d)) \tag{1}$$

where $\prod$ represents the fuzzy OR operator. Hence, using fuzzy inference we obtain [15]:

$$y(k)=\sum_{r=1}^{R}\omega_r(k)\left(\sum_{i=1}^{n_a} a_{i,r}\,y(k-i)+\sum_{j=1}^{n_b} b_{j,r}\,u(k-j-d)\right)$$
$$y(k)=Z(k)P \tag{2}$$

where:

$$\omega_r(k)=\frac{\beta_r(k)}{\sum_{r'=1}^{R}\beta_{r'}(k)}$$

$$Z(k) = [\omega_1(k)y(k-1)\cdots\omega_R(k)y(k-n_a)\cdots\omega_1(k)u(k-1-d)\cdots\omega_R(k)u(k-n_b-d)]$$
$$P = [a_{1,1}\cdots a_{n_a,R}\cdots b_{0,1}\cdots b_{n_b-1,R}]^T$$

If the membership functions $\mu_{\pi_{i,r}}$ are known and if a sufficient number of present and past input *u(k)* and output *y(k)* values are available, then it is possible to apply (2) and determine the consequent parameters vector *P* using a parameter estimation method [14].

3 Identification of time-varying fuzzy dynamical models

In fuzzy time-varying dynamical models, the coefficients of the consequents change with time. In this case, expression (2) can be used in order to estimate the parameters through recursive techniques [4].

This paper proposes the use of wavelets in order to estimate the time-varying parameters [16], [17]. The wavelet analysis allows representing signals in terms of coefficients that express variability in time as well as in the variation speed [18]. Thus, the analysis of changes in the signals can be done with a reduced number of coefficients of a wavelet decomposition. Another advantage of the wavelet analysis is that the nature of the approximation of signals through the wavelet basis is specially suited for characterizing the abrupt changes and faults in dynamical systems [20].

In the following section, the application of wavelets in parameter estimation is briefly described.

4 Time-varying parameter estimation through wavelets

4.1 Background in wavelets

Wavelet analysis emerges as a natural extension of Fourier analysis for the approximation of signals in $L^2(\Re)$. In this case, the "mother function" for the space basis is not a sinusoid but a *wavelet* [22,24].

Thus, it is possible to approximate signals through the shifts and scalings of a mother wavelet function ψ of the form $\psi_{j,i}(t) = 2^{j/2}\psi(2^j t - i),\ j,i \in Z$, which allows to express any function *f(t)* as a *wavelet series*:

$$f(t) = \sum_{j,i=-\infty}^{\infty} c_{j,i}\psi_{j,i}(t) \tag{3}$$

The selection of the mother function ψ is not trivial. Frequently, stability conditions are imposed upon the spectrum of functions $\psi_{j,i}(t)$, upon the convergence of the series (3), and upon the orthogonally relationships among the various generated spaces by $\psi_{j,i}(t)$ [23], [24].

4.2 Matrix representation of the multi-resolution analysis

The family function $\psi_{j,i}(t)$ described induces multiple levels of decomposition in spaces $V_j = span(\phi(2^j t - i))$. A function ϕ is called a *scaling function* if it is possible to obtain the decompositions over the spaces $V_j, j < j_0$ as a decomposition over $W_j = span(\psi(2^j t - i))$. In this manner, based on the functions ψ and ϕ, it is possible to decompose signals such as [22]:

$$f^{j_1}(t) = f^{j_0}(t) + \sum_{j=j_0}^{j_1-1} g^j(t) \tag{4}$$

where $f^j(t)$ is an approximation to a function $f(t)$, and $g^j(t)$ is an approximation of the same function $f(t)$ in W_j. The integers j_1 and j_2 define upper and lower levels for the decomposition.

We next consider time-discretized signals for which vectorial notation will be used. As $f^j(t)$ and $g^j(t)$ are projections over V_j and W_j respectively, we can write:

$$f^J(k) = \sum_i c_i^J \phi(2^J k - i) = \phi^J(k) c^J \tag{5}$$

$$g^J(k) = \sum_i d_i^J \psi(2^J k - i) = \psi^J(k) d^J \tag{6}$$

where the terms c^J and d^J are wavelet coefficients vectors, commonly called *approximation* and *detail* coefficients, respectively. Additionally, the terms ϕ^J and ψ^J are wavelet matrices. For simplicity, the details of its composition are omitted, but it is possible to deduct that they are of the form:

$$\phi^J = \prod_{j=1}^{J} A^j, \quad \psi^J = \prod_{j=1}^{J-1} A^j B^J \tag{7}$$

where A^j and B^j matrices contain scalings and shiftings of the functions ψ and ϕ respectively. Then:

$$f^{j_1}(k) = \phi^{j_0}(k) c^{j_0} + \sum_{j=j_0}^{j_1} \psi^j(k) d^j = \Phi(k) C \tag{8}$$

where:

$$\Phi(k) = \left[\phi^{j_0}(k)\psi^{j_0}(k)\cdots\psi^{j_1}(k)\right]$$
$$C^T = \left[c^{j_0\,T} d^{j_0\,T} \cdots d^{j_1\,T}\right]$$

4.3 Parameters in a fuzzy Takagi-Sugeno type system

If we assume that *f(t)* is a generic parameter of a time-varying system, then (8) is an approximation of *f(t)* by $f^{j1}(t)$, which corresponds to its multi-resolution decomposition at the j_1 level.

In a time-varying fuzzy model, each consequent parameter can be expressed as a wavelet decomposition. Applying the suggested decomposition for equation (8) over a fuzzy model expressed according equation (2), we obtain:

$$y(k) = Z(k)\Phi(k)C \tag{9}$$

Therefore, considering a time series in k, the problem of determining the parameter vector *P* can be reduced to a *regression problem in C*, given that *P* and *C* are related in an approximate manner by (8).

4.4 Selection of the decomposition structure for parameter estimation

From equation (9), it is possible to show that the matrix $Z\Phi$ is a $N \times R \cdot n \cdot n_p$ block, where n_p is the number of terms in each wavelet expansion. Since n_p can be expressed as a fraction $\alpha \in (0,1)$ of the length of data vectors, the regression problem (9) generates infinite solutions for *C*, which is numerically expressed as an ill-conditioned pseudoinverse matrix.

In this case, the conditioning norm for $\Theta = (Z\Phi)^T Z\Phi$ [25]:

$$cond(\Theta) = \|\Theta\|\|\Theta^{-1}\| \tag{10}$$

takes considerably big values, instead of values near 1. This can be interpreted as numerical deviations in the pseudo-inverse Θ^{-1} respect to a rectangular inverse for Θ. In (10), $\|\,\|$ is the norm obtained from taking the highest value of the diagonal matrix from the singular value decomposition (SVD) [25].

This shows that the use of the pseudo-inverse matrix in (8) is not an appropriated method for the estimation of *C*. In this way, the selection of coefficients in the wavelet decomposition is proposed as a solution for the conditioning problem [16].

The initial search space consists of the terms associated to the projections in j_0-j_1+1 spaces. Then, to reduce the search space, Multiple Hypothesis Tests are performed over different decomposition structures. In [20] various alternatives to perform such tests are mentioned.

The Hypothesis Test formulated between two models is:

$$H_0: \quad C = C_1 \tag{11}$$
$$H_1: \quad C = C_2 \tag{12}$$

where C_i is a vector of coefficients of length d_i, which defines the wavelet decomposition if the hypothesis H_i is true.

For two decompositions with the same number of coefficients, the hypothesis are evaluated using the mean square error $E_i = \sum_k (y(k) - \hat{y}_i(k))^2$ corresponding to the obtained parameters, with $\hat{y}_i(k)$ being the estimation of $y(k)$ using the decomposition C_i.

When two decompositions have different number of parameters, the test is decided through:

$$t(E_2, E_1, d_2, d_1) = \frac{N - d_2}{d_2 - d_1} \frac{E_1 - E_2}{E_2} \tag{13}$$

According what it was stated, Figure 1 shows the algorithm used to select between two models.

In this work, different schemes for selection of coefficients are tested. The following are among them:

- Successive addition of coefficients.
- Successive addition of coefficients with subtraction cycles.
- Monte Carlo search.
- Fixed size models.

The first three schemes are initialized with a model without coefficients; this is done in order to further increment the number of parameters in the structure. The fixed size technique does not require a search.

Finally, the decomposition structure obtained is fed into a multiple regression algorithm. The regression problem is solved through to the following equations [25]:

$$\begin{aligned}
\delta y &= y - AR^{-1}R^{T^{-1}}A^T y \\
\delta C &= R^{-1}R^{T^{-1}}A^T \delta y \\
C &= R^{-1}R^{T^{-1}}A^T y + \delta C
\end{aligned} \tag{13}$$

where R is the orthogonal matrix obtained by the QR decomposition of $Z\Phi$.

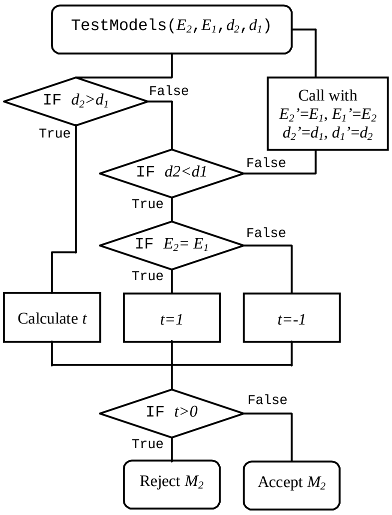

Fig. 1. Selection between two models

5 Toolbox for parameter estimation for time varying systems

A nonlinear time-varying parameter estimation MATLAB toolbox was developed based on the previous concepts. Its basic data structures are presented in Appendix A.1, while Appendix A.2 presents its main functionalities.

The following sections describe two applications of the toolbox oriented to nonlinear time-varying systems.

5.1 Example 1: Fuzzy dynamical system with time-varying parameters

This first example is included for testing purposes. Let's consider the following two rule TS fuzzy dynamical system:

Rule r, r = 1, 2:

```
if
  y(k-1) is π_{1,r}
then
  y(k) = a_{1,r} y(k-1) + b_{1,r} u(k-1) + e_r(k)
```

$$
\begin{aligned}
&\texttt{if} \\
&\quad y(k-1)\ \texttt{is}\ \pi_{1,r} \\
&\texttt{then} \\
&\quad y(k) = a_{1,r}\, y(k-1) + b_{1,r}\, u(k-1) + e_r(k)
\end{aligned}
$$

The parameters $a_{1,r}$ and $b_{1,r}$ vary in time; disturbances $e_r(k)$ are non-correlated white noise signals with a standard deviation $\sigma_{e_r} = 0.01$. The fuzzy sets $\pi_{1,1}$ and $\pi_{1,2}$ are characterized by two symmetric triangular membership functions on the interval [0,1].

Figure 2 shows the system's input and output signals obtained for an initial condition *y(0)=1*. The input *u(k)* is a PRBS (Pseudo Random Binary Sequence) signal with $u_{min}=0.05$, $u_{max}=0.45$, registry length 2^{11}, and clock time τ =1.

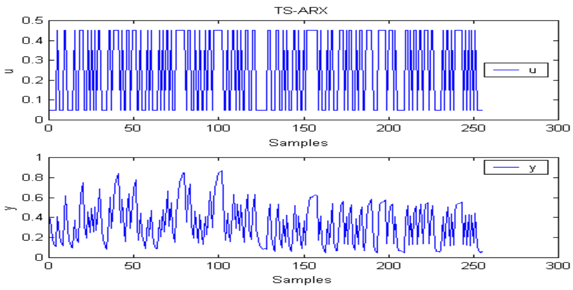

Fig. 2. Input and output signals

Figures 3 and 4 show the time evolution of the process and the estimated parameters. For such, the function `FuzzWavEst` present in the toolbox was employed, selecting in this case the wavelet family `db1` (First order Daubechies). Additionally, the limits of the decomposition were set at `Jmin=7` and `Jmax=10`. For structure selection, the fixed size models method has been used.

Table 1 presents the absolute and relative RMS errors of each estimated parameter, with the input and output signals shown in Figure 2. The mean relative RMS errors oscillate between 0.0171 and 0.086. Such indicates a good determination of the unknown parameters if is considered that such estimation has been performed by minimizing the estimation RMS error of the output *y(k)*.

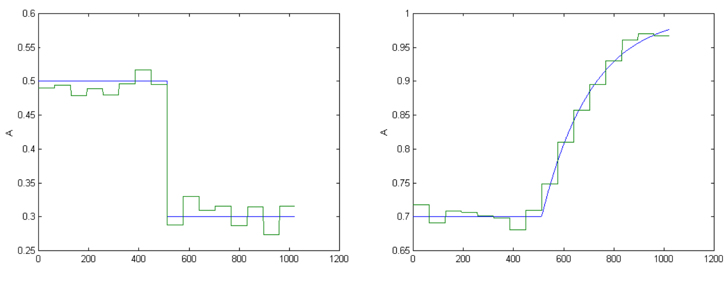

Fig. 3. Real (solid) and estimated (dashed) parameters $a_{1,1}$ and $a_{1,2}$

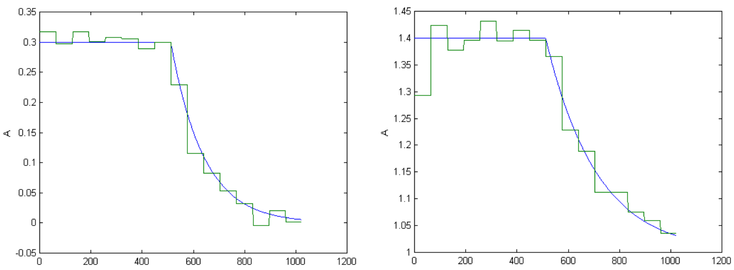

Fig. 4. Real (solid) and estimated (dashed) parameters $b_{1,1}$ and $b_{1,2}$

Table 1: Estimation errors in Example 1

Parameter	*Mean*	*RMS*	RMS_{REL}
$a_{1,1}$	0.397	0.0161	0.0405
$a_{1,2}$	0.737	0.0126	0.0171
$b_{1,1}$	0.186	0.0160	0.0860
$b_{1,2}$	1.350	0.0331	0.0245

5.2 Example 2: Mackey-Glass chaotic series

The Mackey-Glass chaotic series is defined by the following differential equation [7], [21]:

$$\dot{x}(t) = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.2x(t) \tag{14}$$

with $\tau=17$ and $x(0)=1.2$. For data generation, the $x(t)$ variation between $t=0$ and $t=1000$ with sample time $\Delta t=1$ is considered (see Figure 5).

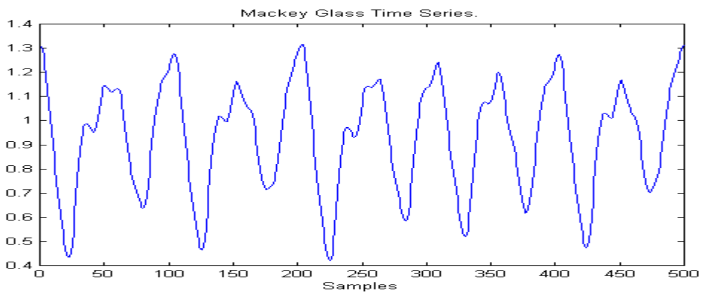

Fig. 5. Evolution of output of Mackey-Glass chaotic series

For estimation purposes, it is necessary to transform the original autonomous system to a discrete-time system with input u(k). In that sense, the following equation is considered as an input:

$$u(k) = [x(k-18)\ x(k-12)\ x(k-6)\ x(k)]$$

and as an output:

$$y(k) = [x(k+6)]$$

The generation of data for the estimation is made considering the first 500 samples starting from k=118.

In this case, the fuzzy sets for the model have been determined using fuzzy clustering (Fuzzy C-Means) through the `MakeFStr` Toolbox utility, using 3 clusters. In this manner, the structure of the obtained fuzzy model is the following:

Rule r, r = 1, ... , 3:

`if`
$x(k-18)$ `is` $\pi_{1,r}$ `and` $x(k-12)$ `is` $\pi_{2,r}$ `and`
$x(k-6)$ `is` $\pi_{3,r}$ `and` $x(k)$ `is` $\pi_{4,r}$
`then`
$y(k) = a_{1,r}x(k-18) + a_{2,r}x(k-12) + a_{3,r}x(k-6) + a_{4,r}x(k)$

Figure 6 shows the evolution in time of the estimated parameters for the first fuzzy rule using `FuzzWavEst`. The figure also shows the performance of the Toolbox for the `db2` wavelet family (Second order Daubechies) with decomposition limit levels `Jmin=7` and `Jmax=9`.

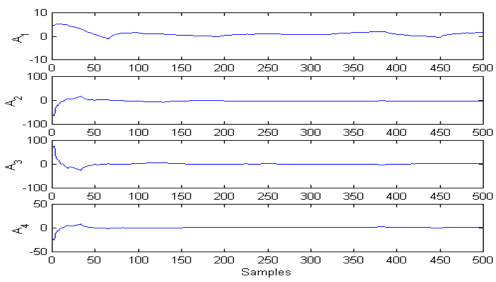

Fig. 6. Estimated parameters for the first fuzzy rule, for Mackey-Glass chaotic series

Figure 7 shows the estimation error variation. For the considered data, the RMS error is 0.0475, equivalent to a mean relative RMS error of 0.051. In this case the parameter estimation also performs a good fit, even-though the structure of the plant is unknown.

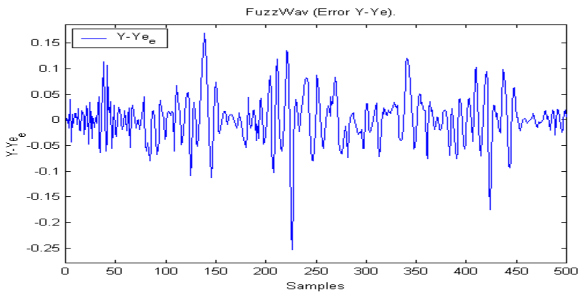

Fig. 7. Estimation error, for Mackey-Glass chaotic series

For validating the parameter estimation process, the initial condition of the autonomous system has been modified from x(0)=1.2 to x(0)=0.9. The results are shown in Figure 8 for the first 500 data samples from k=118. It is shown that the simulation with the new data based on the varying parameters generates a prediction RMS error of 0.190, which is equivalent to a relative RMS error of 0.204.

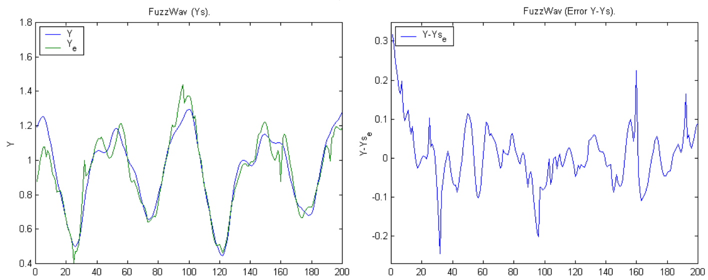

Fig. 8. Real (solid) and predicted (dashed) output, and prediction error, for Mackey-Glass chaotic series

6 Conclusions

This work introduces a parameter estimation method applicable to nonlinear time-varying dynamical systems. A toolbox was developed for such purpose that utilizes nonlinear fuzzy models and parameter estimation based on wavelets.

In the first example was shown that the time evolution of the estimated parameters corresponds to the parameters of the real TS model, which are supposedly unknown. In the second example, reduced estimation errors were obtained. The data validation indicates that the modelling through time-varying parameters represents reasonably the process dynamics independently of the initial condition introduced.

A topic under current research by the authors is the application of the method to on-line prediction using recursive parameter estimation. Also, applications to fault detection and diagnosis in simulated and real processes are under investigation.

7 Acknowledgements

The authors would like to thank FONDECYT for the financing of the project 1020141, Fault Detection and Diagnosis in Nonlinear Time-Varying Systems, and to Dr. Martin Murillo for his help in the translation of this paper.

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Appendix: Toolbox Structure

A.1 Toolbox basic data structures

The following paragraphs present the data structures utilized in this toolbox.

`N` : Number of samples considered for the data analysis.

`R` : Number of rules of the fuzzy model considered, and also number of clusters in a model built on fuzzy sets.

`n` : Number of variables on the Input Data.

`TH` : System input data matrix.

`Y` : System output data column vector.

`FStr` : Data structure for fuzzy sets representation in a Takagi-Sugeno model.

`WH` : Fuzzy activation degrees matrix for each sample and fuzzy rule.

`Str` : Wavelet family.

`Jmin, Jmax` : Minimum and maximum levels for a wavelet decomposition.

`Typestr` : Coefficient class. `typestr ='a'`: approximation coefficients. `typestr ='d'`: detail coefficients.

`C` : Conditioning number for the main problem matrix X, equal to $\|X\|\|X^{-1}\|$.

`lopt`, `laux`, `l` : Vectors for the obtained, the auxiliary and the initial Decomposition Structure, respectively.

Ye : Estimated output data.

Ae : Estimated parameters for the chosen Takagi-Sugeno structure.

A.2 Basic functions of the Toolbox

The following lines present the prototypes of the principal developed functions.

[Ye,Ae,c,lopt,l]=FuzzWavEst(TH,WH,Y,str,Jmin,Jmax):
Parameter estimation for Takagi-Sugeno models for nonlinear time-varying systems, using wavelet multi-resolution decomposition.

[Ye,Ae,c]=ArxWavEst(TH,Y,str,Jmin,Jmax):
Parameter estimation for ARX models for nonlinear time-varying systems, using one shoot least squares techniques.

[TH,Y,A] = MakeExample<i>(N):
Data generation routines of nonlinear time-varying systems. <i>=1 corresponds to an ARX system, <i>=2 to a TS-ARX system and <i>=3 to the Mackey-Glass autonomous system.

[FStr,WH]=MakeFStr(TH,R):
Generation of the premises structure for a Takagi-Sugeno model based on input data and desired number of rules. Fuzzy Clustering (C-Means) is applied using the fcm utility, exported from the MATLAB© Fuzzy Logic Toolbox©.

Pdemo<i>:
Demos for FuzzWavEst tool.

[h,t]=TestModels(MSE2,MSE1,d2,d1,N):
Perform an hypothesis test between two wavelet decomposition models based on its mean square errors MSE1 and MSE2 and its number of parameters d1 and d2. h=0 implies that the model 2 is rejected, and h=1 indicates that such model is accepted.