Analysis of the Linguistic Aggregation Operator LAMA in Group Decision Making Process

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Abstract. At the present time different processes exist in group decision making processes which have been developed using different operators of linguistic aggregation. These operators satisfy a number of axioms that guarantee correct aggregation from a social point of view. The present work analyses, the operator of majority additive aggregation LAMA. Further, It is shown the rationality of its aggregation way; finally it present the properties of the LAMA operator, and how its operator is appropriate to solve group decision making problems from individual linguistic preference relations.

1. Introduction

Decision making is a usual task in human activities. It consists of finding the best option from a feasible set. Many decision making processes, in the real word, take place in an environment in which the goals, constraint and consequences of possible actions are not precisely known. In these cases, probability theory has always allowed one to deal quantitatively with that lack of precision. However, when the lack of precision is of a qualitative nature too, the use of other techniques like fuzzy logic is necessary [7, 8].

Fuzzy set theory applied to decision making allows a more flexible framework, where by it is possible to simulate humans' ability to deal with the fuzziness of human consistency or "human intelligence" in decision making models. Different fuzzy decision making models have been proposed, this models are classified depending on the number of stages before the decision reached [7].

A group decision making process may be defined as a decision situation in which (i) there are two or more individuals, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities, (ii) who recognize the existence of a common problem, and (iii) attempt to reach a collective decision.

In a fuzzy environment, a group decision problem is taken out of follows. It is assumed that there exists a finite set of alternatives $X = \{x_1, ..., x_n\}$ as well as a finite set of experts $E = \{e_1, ..., e_m\}$, and each expert $e_k \in E$ provides his preference relation on X, i.e., $p_k \subset X \times X$, and $\mu_{p_k}(x_i, x_j) \in [0, 1]$ denotes the degree of preference of alternative x_i over x_j.

Sometimes, an individual may have vague knowledge about the preference degree of the alternative x_i over x_j and cannot estimate his preference with an exact numerical value. Then, a more realistic approach may be to use linguistic assessments instead of numerical values [4]. A scale of certainty expressions (linguistically assessed) is presented to the individuals, who could then use it to describe their degrees of certainty a preference. In this environment we have linguistic preference relations for providing individuals' opinions.

One of the problems that we found in the group decision making process it is how to aggregate the experts opinion to obtain a result for the group that may be considerate as correct. Some works in classical theory of group making decision use the axioms proposed by Arrow [1] as beginning point which guarantees a correct decision. Arrow proposes a qualitative group composed by a group of axioms, which should be satisfied by any aggregation type in group decision making. The theorem of impossibility of Arrow introduces important results in this topic. In accordance with this theorem, it is impossible to add individual preferences inside a group of preferences in a completely rational way. This problem disappears in the cardinal groups in fuzzy contexts, introducing intensive preferences, which provide from additional degrees of freedom to the aggregation model [5].

The present work analyses the aggregation linguistic operator based in additive majority LAMA, and it's shown how the use of the LAMA operator is appropriated to solve group decision making problems with individual linguistic preferences relations.

The paper is structured as follows: Section 2 shows the use of linguistic preference relation and some properties and axioms; Section 3 present and analyses the LAMA operator; and finally, Section 4 present conclusions and future works.

2. Linguistic Preference Relations in Group Decision Making

Let X be a set of alternatives over which the fuzzy preference attitude of a decision-maker is defined. Then, according to Tanino [12], the fuzzy preference may be represented as:

1. *A fuzzy choice set to represent his total preference attitude.* It is described by a fuzzy subset of X, i.e., by a membership function μ on X, whose value $\mu(x)$ denotes the preference degree of x, or degree to which x is chosen as a desirable alternative.
2. *A fuzzy utility function.* It is described as fuzzy mapping v, which associates a fuzzy subset of the utility values space (usually the space of real numbers R) with each alternative x, v: $X \times R \rightarrow [0,1]$, where $v(x, t)$ denotes the degree to which the utility value of the alternative x is equal to t.
3. *A fuzzy preference relation.* It is described by a fuzzy binary relation R on X, i.e., a fuzzy set on the product set $X \times X$, characterized by a member-

ship function $\mu_R : X \times X \rightarrow [0,1]$, where (x_i, x_j) denotes the preference degree of the alternative x_i over x_j.

The use of fuzzy preference relations in decision making situations to voice experts' opinions about an alternative set, with respect to certain criteria, appears to be a useful tool in modelling decision processes. Among others, they appear in a very natural way when we want to aggregate experts' preferences into group ones, that is, in the processes of group decision making.

As we have mentioned above, in many cases an expert is not able to estimate his preference degrees with exact numerical values. Then, another possibility is to use linguistic labels, that is, to voice his opinions about alternatives by means of a Linguistic preference relation.

2.1. Linguistic Labels. Semantic and Properties

The semantic of the labels is given by fuzzy numbers defined on the [0,1] interval, which are described by membership functions. As the linguistic assessments are merely approximate ones given by the individuals, we can consider that linear trapezoidal membership functions are good enough to capture the vagueness of these linguistic assessments, since obtaining more accurate values may be impossible or unnecessary. This representation is achieved by the 4-tuple $(a_i,b_i,\alpha_i,\beta_i)$, where the first two parameters indicate the interval in which the membership value is 1.0; the third and fourth parameters indicate the left and right widths of the distribution.

We shall consider a finite and totally ordered label set $S=\{s_i\}$, $i \in H=\{0,..., T\}$, in the usual sense and with odd cardinality as in [2], where the middle label representing a probability of "approximately 0.5" and the remaining labels being placed symmetrically around it and the limit of granularity is 11 or not more than 13.

Each label s_i represents a possible value for a linguistic real variable, i.e., a vague property or constraint on [0,1]. We shall require the following properties:

The set is ordered: $s_i \geq s_j$ if $i \geq j$.
There is the negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = T - i$.
Maximization operator: $\max(s_i, s_j) = s_i$ if $s_i \geq s_j$.
Minimization operator: $\min(s_i, s_j) = s_i$ if $s_i \leq s_j$.

Assuming a linguistic framework and a finite set of alternatives $X = \{x_1, x_2, ..., x_n\}$, the experts' preference attitude over X can be defined as a $n \times n$ linguistic preference relation R, such that $R=(r_{ij})$, $i,j = 1, ..., n$, where $r_{ij} \in S$ denotes the preference degree of alternative x_i over x_j, linguistically assessed, with

$$s_o \leq r_{ij} \leq s_T \quad (i,j = 1, ..., n),$$

where:
$r_{ij} = s_T$ indicates the maximum degree of preference of x_i over x_j.
$s_{T/2} < r_{ij} < s_T$ indicates a definite preference of x_i over x_j.
$r_{ij} = s_{T/2}$ indicates the indifference between x_i and x_j.

So that the linguistic relation reflects a preference, it would be desirable to satisfy some of the following properties, proposed by Tanino in fuzzy environment [12], and interpreted here in a linguistic environment:

Reciprocity: $r_{ij} = \mathrm{Neg}(r_{ji})$, y $r_{ii} = s_0\ \forall\ i, j$.
max-min Transitivity: $r_{ik.} \min(r_{ij}, r_{jk})\ \forall\ i, j, k$.
max-max Transitivity: $r_{ik} \max(r_{ij}, r_{jk})\ \forall\ i, j, k$.
Restricted max-min transitivity: $r_{ij} \geq s_{T/2}, r_{ik} \geq s_{T/2} \Rightarrow r_{ik} \geq \min(r_{ij}, r_{jk})\ \forall\ i, j, k$.
Restricted max-max transitivity: $r_{ij.} \geq s_{T/2}, r_{jk} \geq s_{T/2} \Rightarrow r_{jk} \geq \max(r_{ij}, r_{jk})\ \forall\ i, j, k$.

In order to make good use of the linguistic preference relations for aggregating experts' preferences, an aggregation operator of linguistic information is needed. Various operators have been proposed [4, 6, 9, 10, 13]. It is important that these operators satisfy a well defined axiomatic for group decision making process.

In [9, 10] Pelaéz-Doña show how the most common operators used in the aggregation labels may produce distributed problems when the elements to aggregate have cardinality greater than one. This type of problems are consider as a variation of the cake cutting problems, where we need to divide a cake in a way that all fellow guests were satisfied with their portions [11]. To solve these types of problems have been proposed majority process [9, 10].

2.2. Preferences Aggregation Axiomatic

As it has been shown previously, an axiomatic to model the aggregation processes (crisp or fuzzy) is needed. Some axiomatic approaches have been partially taken by Cholewa [3] and offers a collection of axioms for the aggregation of fuzzy weighted opinions and indicates that the weighted mean satisfies these axioms.

In [5], a very detailed analysis about proposed axiomatic approaches to rational group fuzzy decision making is presented. A complete set of axioms in the fuzzy set setting for homogeneous groups is reviewed. These axioms are natural properties of a voting procedure that include the ones proposed by Arrow. Some of these are: unrestricted domain, unanimity, neutrality,..., three group classification has been established:

Imperative axioms, whose violation leads to counterintuitive aggregation modes, e.g.: neutrality.
Technical axioms, that facilitate the representation and the calculation of the aggregation operator, e.g.: unrestricted domain.
Facultative axioms, that are applied in special circumstances but are not universally accepted, e.g.: unanimity.

Obviously a particular aggregation operator ϕ does not have to satisfy all axioms together, it must satisfy those that its special application circumstances require.

In the next sections, we study some properties and axioms that the aggregation operator of linguistic opinions LAMA verifies.

3. The Linguistic Operator LAMA

The LAMA operator is an aggregation operator based on majorities process developed by Pélaez-Doña to solve the distribution problems [9, 11]. This operator is defined as a modification of the arithmetic mean, as a arithmetic mean of arithmetic mean, such that, the final result is a weighted arithmetic mean.

Let s_1, s_2,..., s_n, $\in P$, be such that $t > 0$ and let $\delta_1, \delta_2, ..., \delta_n \in N$, be the labels frequency or cardinality, where $\delta_i \leq \delta_{i+1}$ for all $1 \leq i \leq n$-1. The LAMA operator ϕ is the label s_m given by:

$$s_m = \phi((s_1, \Delta_1), (s_2, \Delta_2), \ldots, (s_n, \Delta_n)) = s_1 \otimes \lambda_1 \oplus s_2 \otimes \lambda_2 \oplus \ldots \oplus s_n \otimes \lambda_n \quad (1)$$

where:

$$\lambda_i = \begin{cases} \dfrac{1}{d_1} & \text{if } i=1 \\ \dfrac{1}{d_1} \cdot \dfrac{1-n^{\delta_2}}{1-n} & \text{if } i=2 \\ \lambda_{i-1} + \dfrac{1}{d_{i-1}} \cdot \dfrac{1-(n-i+2)^{\delta_i - \delta_{i-1}}}{1-(n-i+2)} & \text{if } i > 2 \end{cases} \quad (2)$$

with

$$d_i = \begin{cases} 1 & \text{if } i=1, n=1 \\ n^{\delta_2} & \text{if } i=1, n=2 \\ n^{\delta_2} \cdot \prod_{j=1}^{n-2} (n-j)^{\delta_{j+2} - \delta_{j+1}} & \text{if } i=1, n>2 \\ \prod_{j=i-1}^{n-2} (n-j)^{\delta_{j+2} - \delta_{j+1}} & \text{if } i > 1 \end{cases} \quad (3)$$

and

$\otimes$: the product of label by a positive real number [4].
$\oplus$: the sum of label [4].

The value of the cardinality of the labels may be calculated using two different methodologies:

1. Using the cardinality as the number of labels that represent the same information.
2. Using the cardinality as the number of labels that represent similar information.

Definition. Let s_i and s_j be two labels, with each one representing a counting of information, then if $i = j$ then represent the same information. If $i \neq j$ we can use a function to obtain the similarity between two labels or values to aggregate. A simple example of this function is the following:

$$\xi(s_i, s_j) = \begin{cases} 1 & \text{if } |s_i - s_j| < x \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(4)}$$

where x indicate the similarity degree that we want to use.

$\xi(s_i, s_j)$=1 then the labels are considered as similar.

Extend this new concept of similarity between labels or values to aggregate to others representation is easy.

3.1. Example.

Let the set of values: (1, 1, 1, 0.5, 0, 0) to aggregate of the range [0, 1], where the number of different values is three and the frequency or cardinality (ascender order) is the following:

$$\delta_1 = 1\,(0.5), \delta_2 = 2\,(0),\ \delta_3 = 3\,(1)$$

then

$$LAMA(0.5,\ 0,\ 1) = \ 0.5 \otimes \lambda_1 \oplus 0 \otimes \lambda_2 \oplus 1 \otimes \lambda_3 = 0.5 \otimes 0.055 \oplus 0 \otimes 0.222 \oplus 1 \otimes 0.722 = 0.75$$

where

$$\lambda_1 = \frac{1}{d_1} = \frac{1}{18} = 0.055$$

$$\lambda_2 = \frac{1}{d_1} \cdot \frac{1 - n^{\delta_2}}{1 - n} = \frac{1}{18} \cdot \frac{1 - 3^2}{1 - 3} = 0.222$$

$$\lambda_3 = \lambda_2 + \frac{1}{d_2} \cdot \frac{1 - (n - 3 + 2)^{\delta_3 - \delta_2}}{1 - (n - 3 + 2)} = \ 0.222 + \frac{1}{2} \cdot \frac{1 - (3 - 3 + 2)^{3-2}}{1 - (3 - 3 + 2)} = 0.722$$

and

$$d_1 = n^{\delta_2} \cdot \prod_{j=1}^{n-2} (n - j)^{\delta_{j+2} - \delta_{j+1}} = 3^2\left((3-1)^{3-2}\right) = 18$$

$$d_2 = \prod_{j=2-1}^{n-2} (n - j)^{\delta_{j+2} - \delta_{j+1}} = (3 - 1)^{3-2} = 2$$

and

$\otimes$: the product of positive real number.
$\oplus$: the sum of positive real number.

If we use other aggregation methods like arithmetic mean or OWA operator we will obtain the following results:

Arithmetic Mean:

$$\mathrm{AM}(1,1,1,0.5,0,0) = 1\otimes\frac{1}{6}\oplus 1\otimes\frac{1}{6}\oplus 1\otimes\frac{1}{6}\oplus 0.5\otimes\frac{1}{6}\oplus 0\otimes\frac{1}{6}\oplus 0\otimes\frac{1}{6} =$$
$$\frac{1+1+1+0+0+0.5}{6} = 0.583$$

OWA operator:

We use the OWA operator associated with the linguistic quantifier *most*; for six elements the weights of the weighting vector are:

W=(0, 0, 0.18, 0.34, 0.34, 0.14)

then

$$\mathrm{OWA}(1,1,1,0.5,0,0) = 1\otimes w_1 \oplus 1\otimes w_2 \oplus 1\otimes w_3 \oplus 0.5\otimes w_4 \oplus 0\otimes w_5 \oplus 0\otimes w_6 =$$
$$1\otimes 0\oplus 1\otimes 0\oplus 1\otimes 0.18\oplus 0.5\otimes 0.34\oplus 0\otimes 0.34\oplus 0\otimes 0.14 = 0.35$$

Clearly, the values obtained with method like OWA operators (which use the number of elements to aggregate without consider the cardinality of these elements) or traditional methods like arithmetic mean (which use the cardinality of the general set of elements to aggregate) not represent the values of the majority (the 50% of elements have the maximum possible value). The LAMA operator (0.75) represent the value of the majority and, at the same time, using the values of the minorities [9].

3.2. Properties of LAMA Operator

Property 1. The LAMA operator is increasing monotonous with respect to the argument values, in the following sense:

Let $A = [a_1, a_2, ..., a_n]$ be an ordered argument vector, let $B = [b_1, b_2, ..., b_n]$ be a second ordered argument vector, both with equal frequency vector F, such that $\forall\, j,\ a_i \geq a_j$ then

$$\phi\,(A) \geq \phi\,(B)$$

Property 2. The LAMA operator is commutative,

$$\phi\,(a_1, a_2, ..., a_n) = \phi\,(\pi(a_1), \pi(a_2), ..., \pi(a_n)),$$

where π is a permutation over the set of arguments.

Property 3. The LAMA operator is an "orand" operator. That is, for any weighting vector λ and ordered labels vector $A = [a_1, a_2, ..., a_n]$, then

$$\mathrm{Min}\,(a_1, a_2, ..., a_n) \leq \phi\,(a_1, a_2, ..., a_n) \leq \max(a_1, a_2, ..., a_n)$$

3.3. Axiomatic of the LAMA Operator

In what follows we are going to study some of the proposed axioms in fuzzy setting considering the LAMA operator which works with linguistically valued preferences. Before this, we include the following linguistic notation that we shall use.

Let $A = \{x_1, x_2, ..., x_n\}$ be a finite non-empty set of alternatives.
Let $E = \{e_1, \dots, e_m\}$ be a panel of experts.
Let $S = \{s_i; \mathrm{i} = 0, ..., T\}$ be a label set to voice experts' opinions.
Let $xij \in S$ be the linguistic rating of alternative x_i by expert e_j.
Let F_j be the linguistic rating set over alternatives by expert e_j.
Let μ_{F_j} be the linguistic membership function of F_j such that $x_{ij} = \mu_{F_j}(x_i)$.
Let F be the linguistic rating set such that $F = \phi(F_1, F_2, ..., F_m)$.

Axiom I. Unrestricted domain.
For any set of individual preference patterns $\{F_j, j = 1, ..., m\}$ there is a social preference pattern F, which may be constructed,

$$\forall F_1,...,F_n \in S^n, \exists F \in S^n \text{ such that } F = \phi(F_1, F_2, ..., F_m).$$

It is basically technical, and clearly it is satisfied in accordance with the LAMA operator definition.

Axiom II. Unanimity or idempotence.
If everyone agrees on a preference pattern, it must be chosen as the social choice pattern,

$$F_j = F, \forall j \Rightarrow F = \phi(F, F, ..., F).$$

Following this definition, the LAMA operator can immediately be verified.

Axiom III. Positive association of social and individual values.
If an individual increases his linguistic preference intensity for x_i, then the social linguistic preference for x_i cannot decrease. This means that if F'_j y F_j are such that $\mu_{F_j} \le \mu_{F'_j}$, then if $\phi(F_1, ..., F_j, ..., F_m) = F$ and $\phi(F_1, ..., F'_j, ..., F_m) = F'$, then $\mu_F \le \mu_{F'}$.

Clearly it is satisfied, because it is a consequence of increasing monotonicity property of the LAMA operator

Axiom IV. Independence of irrelevant alternatives.
The social preference intensity for x_i only depends on the individual preference intensity for x_i, and not for x_k, $k \neq i$,

$$\mu_{\varphi(F_1,\ldots,F_m)}(x_i) = \phi\,(x_{i_1},\ldots,x_{i_m}).$$

It is basically technical, and is satisfied by the definition of the LAMA operator. Clearly this axiom does not extend strictly speaking, since for preference relations the independence of irrelevant alternatives deals with pairs of alternatives.

Axiom V. Citizen sovereignty.
It means that any social preference pattern can be expressed by the society of individuals; in other words

$$\forall F, \exists\ F_1, \ldots, F_m \text{ such that } F = \phi\,(F_1, \ldots, F_m).$$

A weaker form of citizen sovereignty called NonDictatorship is as follows: there is no individual e_j such that

$$\phi\left(F_1, \ldots, F_j, \ldots, F_m\right) = F_j$$

This requirement prohibits any individual from acting as a veto or dictator under any circumstances, being this one of the characteristics base of the majority definition.

Obviously, this axiom is satisfied in its general form, because it is a consequence of axiom II (unanimity). Clearly, as the LAMA operator is commutative, then it also satisfies the weaker form of the axiom.

Axiom VI. *Neutrality.*

The neutrality axiom refers to the invariance properties of the voting procedure. There are three types:

Neutrality with respect to alternatives. If x_i and x_k are such that $x_{ij} = x_{kj}$, $\forall\ j$, then $\mu_{\varphi(F_1,\ldots,F_m)}(x_i) = \mu_{\varphi(F_1,\ldots,F_m)}(x_j)$.

Neutrality with respect to voters. In a homogeneous group, this is the anonymity property, i.e., the commutativity of ϕ.

Neutrality with respect to the intensity scale or Neutrality of Complement. If F_c^j is the complement to F_c, such that $F_c^j = Neg(F_j)$, the social pattern $\phi(F_1,\ldots,F_m)^c$ should be the complement of the social preference pattern,

$$\phi(F_1,\ldots,F_m)^c = \phi(F_1^c,\ldots,F_m^c)$$

Clearly it is verified for the form of neutrality respect to alternatives. As the LAMA operator is commutative, then it also verifies the neutrality with respect to voters.

4. Conclusions

In this work the aggregation operator LAMA is analysed from a social point of view, showing how the LAMA operator verifies the main axioms of social choice. The fulfilment of those axioms provides evidence of rational aggregation using the LAMA operator in particular frameworks (i.e. group decision making problems from individual linguistic preference relations). Also we compare the results obtained with LAMA operator with other type of aggregation operator. Finally the main properties of the operator have been presented.

Future Works

Actually exist great number of group decision making processes in linguistic environments that use the direct approach as resolution method. These processes have been designed using different aggregation operators. In future works we pretend to design these processes using the operator of linguistic aggregation of majority additive LAMA.

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