NONLINEAR MODELS FOR RESIDUAL GENERATION IN FAULT DETECTION AND DIAGNOSIS SYSTEMS APPLIED TO THE PENDUBOT DYNAMIC SYSTEM

Aldo Levrini[1], and Aldo Cipriano[1]

[1] College of Engineering, Pontificia Universidad Católica de Chile, PO Box 306, Santiago 22, Chile
Phone: 56-2-6864281, Fax: 56-2-5522563,
E-mail: aciprian@ing.puc.cl

Abstract. This paper presents a comparison of three nonlinear models used for residual generation. The residual generation is part of a simple fault detection and diagnostics scheme applied to the Pendubot dynamic system operating in closed loop. The compared models are: Hammerstein, neural NARMAX, and Takagi-Sugeno Fuzzy models.

1 Introduction

Fault detection and diagnosis in industrial processes gained a great deal of interest in the last years due to the notorious development and spread of automation systems. In processes characterized by strong interactions and nonlinear behavior, it is required the support and aid tools that assure a safe process operation for which eventual anomalies have to be detected and discriminated.

This work is the first of a set of studies to be done on fault detection and diagnosis in the Pendubot dynamic system. Here we compare the performance of three nonlinear models utilized for the generation of residuals. Based on these residuals a simple fault detection criterion is used due to illustrate this performance and the differences between the compared models. This plant, given its inherent instability, needs to be operated in closed loop. The nonlinear models to be compared are:

- The Hammerstein Polynomial Model
- The Neural NARMAX Model
- The Takagi-Sugeno Fuzzy Model

The comparison is done based on indexes that define the predictive capacity of the corresponding model.

The work starts with a review of the theory and a description of the process and its dynamic model. Next, the models being compared, the results of the identification, and the conclusions of their analysis are presented.

2 The Pendubot System

Fault detection and diagnosis includes two main tasks: the generation of residuals that reflect the fault conditions, and its consequent evaluation for detection and diagnosis. The most utilized methods for fault detection in mechanical systems are based on observers and on parameter estimation [1]. In both cases a model of the process is required in order to characterize its behavior. Such model can be deduced from the phenomenology of the process or by model identification using input-output data.

As shown in Figure 1, the Pendubot dynamic system corresponds to a double inverted pendulum that has an actuator in the base of its first arm which applies a torque that allows the movement of both arms to a vertical position ($q_1 = \pi/2$, $q_2 = 0$). This position corresponds to the only stable operation point of the system, which can be verified by means of the Lyapunov's stability criteria [2]. Nevertheless, there exists a small range around this point where the system remains stable and controllable; q_1 or q_2 light deviations in the range of $\pm$ 5° do not cause system instability but a small but recognizable deviation from the reference.

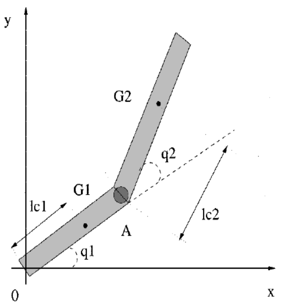

Fig.1. Schematic of the Pendubot System

The control that allows reaching the equilibrium position has two parts: a PD controller to raise the arm, and a linear quadratic regulator to keep the arm in that position. The transition from one controller to another is carried out when predetermined values of the manipulated variable *u* and the controlled variable q_1 are reached [3].

Even though the phenomenological model of the Pendubot system is known [3], this work presents a different approach to characterize its behavior through the use of models that represent its strong nonlinear dynamics; for that purpose, it is required to estimate the parameters of such models using input-output data.

3 Nonlinear Models

When a system presents strong nonlinear dynamics, it is reasonable to consider nonlinear models in order to represent its behavior. Then several nonlinear structures such as the Hammerstein, neural, and fuzzy models have been formulated in order to characterize a specific system. These models were chosen for being some of the most representative and standard nonlinear models used in characterizising a nonlinear process.

3.1 The Hammerstein Model

The Hammerstein model corresponds to the structure shown in Figure 2 [4]. The function *f* that relates the intermediate variable *v[n]* with the input *u[n]* is polynomial, while *G(z)* is a transfer function of the linear relation between the output *y[n]* and the intermediate variable *v[n]*.

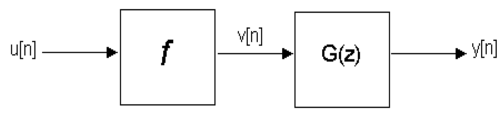

Fig. 2. Structure of the Hammerstein Model

The parameters of this model correspond to the coefficients of the polynomial function *f* and those of the transference function *G(z)*. The expression that relates *y[n]* to *u[n]* is nonlinear in the parameters, for which a numerical optimization method is required in order to estimate them, for example, the minimization of the mean quadratic error.

3.2 Neural Models

The parameters of a neural model correspond to the weights of a neural network that is trained with measures of the input and output variables. There exist several tools for neural networks development. In this study, an identification toolbox based of neural networks was applied [5]. A two-layer, ten-neuron NARMAX model (Nonlinear Auto Regressive Moving Average model with exogenous inputs) has the structure shown in Figure 3.

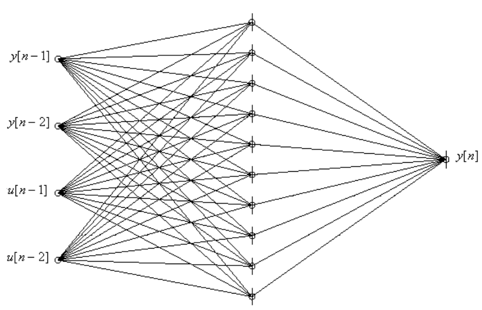

Fig. 3. NARMAX Neural Model

3.3 Fuzzy Models

Among the several formulations for fuzzy nonlinear models are the Takagi-Sugeno and the ANFIS models.

An ANFIS model consists of a neural network in which each layer corresponds to a fuzzy rule base [6].

A Takagi-Sugeno model consists on a set of *If / Then* rules in which each consequence corresponds to an ARMA submodel:

$$y[n] = \sum_i a_i y[n-i] + \sum_j b_j u[n-j-d] \tag{1}$$

where d is the delay. The premise of each rule includes fuzzy sets associated to possible values of the input and output variables. This type of models requires the identification of the membership functions with the corresponding parameters and the coefficients of the local consequence models. In order to accomplish such, the input and output data are grouped together in cells or "clusters" (technique known as clustering). In this study, a clustering based identification toolbox was utilized [7].

4 Fault Detection Structure

Figure 4 shows a residual generation scheme for the Pendubot system that uses nonlinear models as the ones formerly described [8].

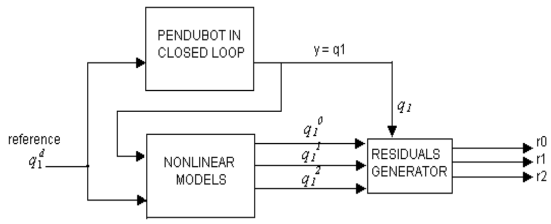

Fig. 4. Residual generation scheme for model comparison

The nonlinear model block consists on a model for each one of the three assumed operating conditions: normal operation, type 1 fault of the q_1 angle sensor, and type 2 fault of the q_1 angle sensor. Each model gives a prediction for the output variable q_1: q_1^0 is the unit step prediction for normal operation, q_1^1 is the prediction for sensor fault type 1, and q_1^2 is the prediction for sensor fault type 2. In this study, for simplicity reasons, only angle q_1 sensor faults have been considered: type 1 fault corresponds to a 4° deviation and type 2 fault corresponds to a 2° deviation. Higher deviations (over 5°) produce the unstability of the system, so only deviations between 0° and 5° can be considered as a possible fault to be detected, being mainly this the reason for the choice of 4° and 2° as faulty conditions. Then, it is clear that a multimodel scheme is being used for the residual generation.

The study also considers a closed loop Pendubot with LQR control due to two reasons: the instability of the open loop system and the assumption that access to the manipulated variable is not granted. Figure 5 describes the scheme utilized to identify the nonlinear models employed for the generation of residuals.

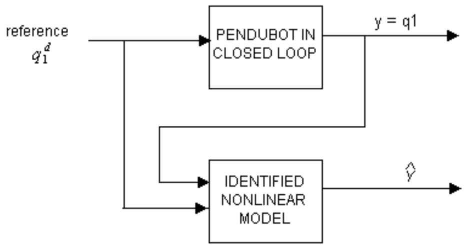

Fig. 5. Block diagram of the closed loop plant identification scheme

5 Tests Definition

The tests carried out are aimed to specify how indicative are the residuals of the presence of a plant operating condition: normal operation, fault 1 condition (4º deviation in angle q_1 sensor), and fault 2 condition (2° deviation in angle q_1 sensor). The residuals are defined as:

$$r_i[n] = |e_i[n]| \tag{2}$$

in which:

$$e_i[n] = y[n] - \hat{y}_i[n] \tag{3}$$

where:
e: Prediction error.
y: Measured output of the plant ($y = q_1$).
$\hat{y}_i$: Predicted output for the model "i" (i=0, model for normal operation, i=1, model for operation with 4° fault in sensor 1, i=2 model for operation with 2° fault in sensor 1).

A small value of the residual implies that the measured angle q_1 and the predicted angle are similar; the smallest residual indicates the presence of the corresponding operating condition.

5.1 Identification Procedure

For the identification of the models, 3000 samples of the input (reference q_1^d) and the output (q_1) measured each 1 mseg are used. In the fault cases, deviations of 4° and 2° respectively are considered, being 4° the maximum possible deviation without the system becoming unstable.

The determination of the parameters for each model is carried out by the minimization of the adjustment error between the output of the model and the output of the process. The error is determined as the square root of the mean quadratic error (RMS), defined as:

$$RMS = \sqrt{\frac{\sum_{n=0}^{N} (y[n+1] - \hat{y}[n+1])^2}{N+1}} \tag{4}$$

The best adjustment of the Hammerstein model is achieved by considering four regressors of *u[n]*, $u^2[n]$ and $u^3[n]$, and six regressors of *y[n]*, from *y[n-1]* to *y[n-6]*.

By using the NARMAX model, the best adjustment is obtained with two regressors of *y[n]*, two regressors of *u[n],* and two regressors of the prediction error.

In the identification of the Takagi-Sugeno fuzzy model, the best adjustment was obtained with 32 clusters. Only one regressor of the input *u[n]* was considered, although four regressors of the output *y[n]*, from *y[n-1]* to *y[n-4]* also were considered. Local linear models in function of the input reference variable were constructed over these fuzzy sets.

The election of the amount of clusters was carried out on a trial and error basis; a quite minor quantity would have given a model with an inferior adjustment.

5.2 Performance Indicators

Two indicators were considered for the comparative analysis of the nonlinear models:

- The quadratic mean error of the prediction.
- The variability of the residuals given by each model before several operating conditions.

6 Analysis of the Results

6.1 Normal Operating Condition

Using validation data, the RMS of the three nonlinear models were first calculated for the normal operation case. Table 1 presents these values. Figures 6 to 8 shows the comparison between the real output and the respective nonlinear model prediction for the steady state operation.

Table 1 : RMS values for the models identified in normal operation

	Hammerstein	NARMAX	Fuzzy
RMS [rad]	0.0035	0.003	0.013

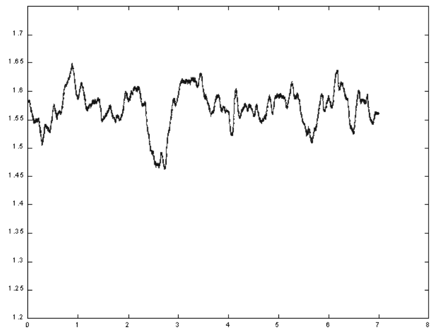

Fig. 6. Prediction with the Hammerstein model

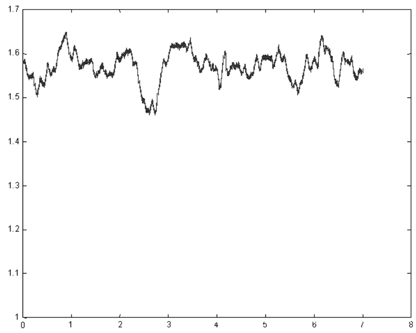

Fig. 7. Prediction with the NARMAX model

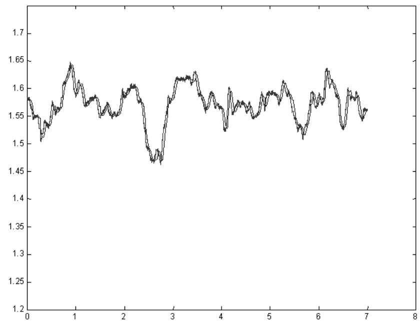

Fig. 8. Prediction with the fuzzy model

It can be observed that the three models give similar qualitative predictions, although the error produced by the fuzzy model is four times bigger than the ones produced by the other models.

6.2 Fault Condition 1 (4° deviation)

Table 2 shows the RMS values obtained for the three nonlinear models in fault condition 1 in the sensor of the q_1 angle.

Table 2 : RMS values for the identified models in fault condition 1 (4° deviation)

	Hammerstein	NARMAX	Fuzzy
RMS [rad]	0.0027	0.0033	0.0153

From these values it is observed that the smallest error in the characterization of the fault is made by the Hammerstein model. Also in this case the fuzzy model leads to the highest error.

Figures 9 to 11 show the residuals obtained for fault 1 activated at t = 4 sec.

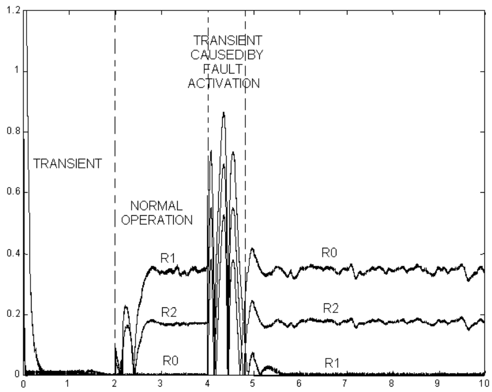

Fig. 9. Generated residuals for Hammerstein models. The smallest residual is r_1

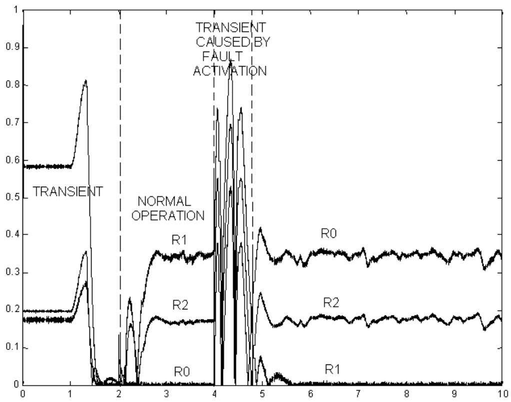

Fig. 10. Generated residuals for NARMAX models. The smallest residual is r_1

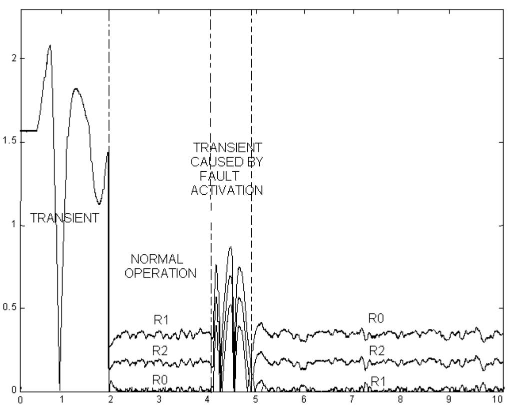

Fig. 11. Generated residuals for Fuzzy models. The smallest residual is r_1

From the three models it is observed that the smallest residual is the one associated with the model representing the fault condition 1, achieving thus the 4° fault detection. A variable behavior before the system arriving to a steady state is observed in the three models. Also a short transient is noted when the fault is triggered which quickly finishes as the system becomes stable around the new operating point; being this operating point 4° deviated from the correct set point ($q_1 = \pi/2$)

Table 3 presents the mean and standard deviation values of the residuals once the fault acts. It can be seen again the better performance of the Hammerstein and NARMAX models compared to the fuzzy model.

Table 3: Mean (μ) and standard deviation (σ) of the residuals generated by fault condition 1 (4° deviation)

Fault 1	**Hammerstein**	**NARMAX**	**Fuzzy**
μ_0	0.3465	0.3465	0.3458
σ_0	0.0098	0.0010	0.0174
μ_1	0.0022	0.0027	0.0125
σ_1	0.0016	0.0021	0.0105
μ_2	0.1760	0.1770	0.1753
σ_2	0.0078	0.0074	0.0105

6.3 Fault Condition 2 (2° deviation)

Table 4 presents the RMS values obtained for the three nonlinear models in fault condition 2 in the q_1 angle sensor. Note that in this case, the Hammerstein and NARMAX models also show the best results.

Table 4 : RMS values for the identified models in fault condition 2 (2° deviation)

	Hammerstein	NARMAX	Fuzzy
RMS [rad]	0.0018	0.0019	0.0140

Figures 12 to 14 show the residuals obtained for fault condition 2 triggered at t = 4 sec.

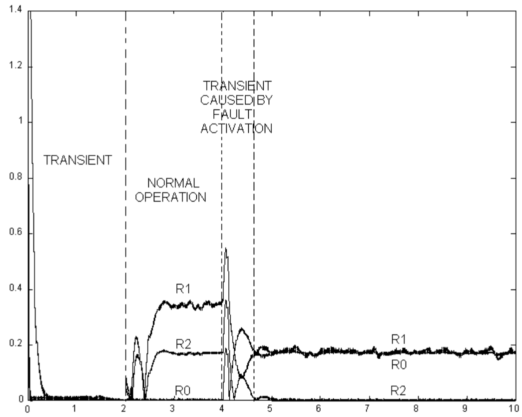

Fig. 12. Residuals for Hammerstein models. The smallest residual is r_2

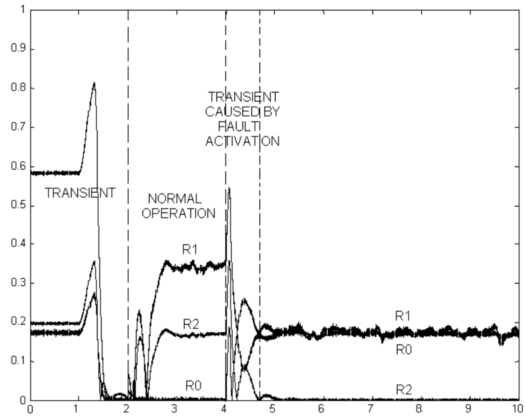

Fig. 13. Residuals for NARMAX models. The smallest residual is r_2

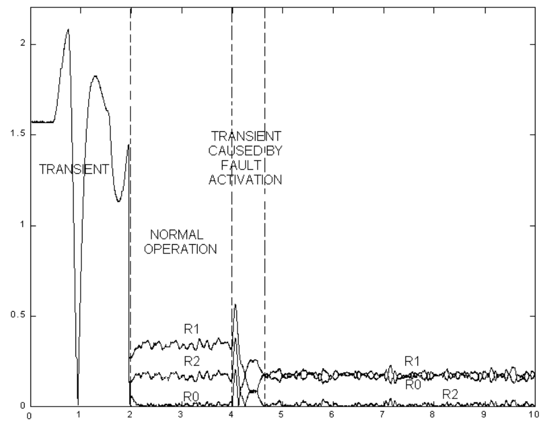

Fig. 14. Residuals for Fuzzy models. The smallest residual is r_2

The situation in this case is similar to the former. It can be seen, in the response given by the three models, that the 2° fault residual is the smallest one once the fault has been triggered. Also, considering the respective standard deviations, the fuzzy model again shows to be the weakest one. Table 5 shows the mean and the standard deviations of the residuals once the fault acts.

Table 5 : Mean (μ) and standard deviation (σ) of the residuals generated by fault condition 2 (2° deviation)

Fault 2	Hammerstein	NARMAX	Fuzzy
μ_0	0.1705	0.1701	0.1707
σ_0	0.0033	0.0032	0.0143
μ_1	0.1762	0.1761	0.1758
σ_1	0.0079	0.0081	0.0162
μ_2	0.0014	0.0015	0.0108
σ_2	0.0011	0.0011	0.0090

6.4 Fault Conditions 3 and 4 (1° and 3° deviations)

As it was mentioned, the normal operation condition and faults 1 and 2 were diagnosed correctly. It is interesting to know how the detection and diagnosis system reacts before other faults, by instance, 1° and 3° deviations in the q_1 angle act. In Table 6 the results for these conditions are showed:

Table 6 : Mean (μ) and standard deviation (σ)of the residuals generated with the three nonlinear models for faults conditions 3 and 4 (1° and 3° deviations)

Fault 3	Hammerstein	NARMAX	Fuzzy
μ_0	0.0849	0.0849	0.0851
σ_0	0.0024	0.0032	0.0137
μ_1	0.2617	0.2617	0.2615
σ_1	0.0100	0.0101	0.0166
μ_2	0.0856	0.0856	0.0854
σ_2	0.0026	0.0027	0.0138
Fault 4	**Hammerstein**	**NARMAX**	**Fuzzy**
μ_0	0.2577	0.2577	0.2579
σ_0	0.0063	0.0067	0.0154
μ_1	0.0889	0.0889	0.0887
σ_1	0.0056	0.0059	0.0156
μ_2	0.0872	0.0872	0.0874
σ_2	0.0036	0.0037	0.0147

It is observed that the smallest residuals in fault condition 3 are r_0 and r_2, which indicates that the systems indeed detect an intermediate situation between normal operation and fault condition 2. Also, when fault 3 is presented, the smallest residuals reach for fault condition 2 and 1. Then, in this case, the system diagnoses an intermediate situation between such operation scenarios.

7 Conclusions

This work studied the performance of three nonlinear models in residual generation. The residual generation is part of a simple fault detection scheme in a nonlinear closed-loop mechanical system.

It was shown through the analysis, that the application of the Hammerstein and Narmax models, considering only one output (angle q_1), generate residuals that allow to detect and diagnose quite well, using a simple criteria, 2° and 4° deviations faults in the q_1 angle sensor. The fuzzy dynamical models, on the contrary, always present a greater prediction error, becoming clear that it is not quite a good model for the residual generation scheme. In the intermediate cases, that is, deviations of 1° and 3°, the system diagnoses intermediate situations that correspond to a normal and 2° fault operation conditions in the first case, and a 4° and 2° fault operation conditions in the second case. Although this means that the simple criterion used does not allow detection of the intermediate values 1° and 3°, the residual generation scheme as it was described could allow the use of a diagnosis module based on the values of the residuals to detect also these deviations.

8 Acknowledgments

The authors wish to thank FONDECYT for the financing of the project 1020741, Fault Detection and Diagnosis in Nonlinear Time-varying Dynamic Systems, and to Dr. Martin Murillo for his help in the translation of this paper.

References

1. R. Isermann, R. Bralle, "Trends in the application of model based fault deteection and diagnosis of technical process (survey)" . *Proceedings of the 13th IFAC World Congress*, San Francisco, 1996.
2. I. Fantoni, R. Lozano, M.W. Spong, "Energy based control of the Pendubot," *IEEE Trans. on Automatic Control*, Vol. 45, N°4, pp 725-729, 2000.
3. D. J. Block "Mechanical design and control of the Pendubot," *Proceedings of the SAE Earthmoving Industry Conference*, Peoria, IL, April 1995.
4. D. Westwick, R. Kearney, "Identification of a Hammerstein model of the stretch reflex EMG using separable last squares," *Proceedings of EMBS22*, Chicago, 2000.
5. M. Norgaard, "Neural network based system identification toolbox," Internal Report, *Department of Automation, Technical University of Denmark*, 2000.

6. J. Shing, R. Jang, "Anfis: Adaptive-Network-Based Fuzzy Inference System," *Internal Report*, University of California, Berkeley.
7. R. Babuska. "Fuzzy modeling and identification toolbox." *Delft Technical University*, Holland, 1998.
8. A. Levrini, J. F. Araya, A. Cipriano, "Fault detection and diagnosis in the Pendubot dynamic system by using observers and fuzzy inference," *Proceedings of the XV Congress of the Chilean Automatic Control Society*, Santiago, Oct. 2002, pp 43-49 (in Spanish).