Can climbers take the uphill slope again? — A fitness landscape on weight space of an application using spiking neurons under rate coding.

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Abstract. We simulate an associative memory model using spiking neurons instead of McCulloch-Pitts neurons. To store a set of patterns, we employ a Hebbian-like learning algorithm. The learning behavior, however, is somewhat of a different one of the traditional Hopfield model. To study the difference we explore the fitness landscape defined on synaptic weights space when they evolve searching for the optimal learning.

1 Introduction

Associative memory is a memory system in which we can store information and recall it later from its partial and/or imperfect stimuli. An information is stored as a number of stable states with a domain of attraction around each of the stable states. If the system starts with any stimulus within the domain it will converge to the attractor following a trajectory, hopefully a short one. This models human memory in the sense that we can recognize our friend's face even without meeting for a long time, or we can recall a song immediately after listening to a very beginning part of the song. Hopfield [1] proposed a fully connected neural network model of associative memory in which a set of patterns is stored distributedly among neurons as attractors. Since then the model had been fairly well studied for more than a decade, and we now know it is not so practical, partly due to its small storage capacity. Though we still have some interesting unknown issues in the model such as the number and distribution of the weight solutions which give a network a function of associative memory and these issues still attract interests of some physicists, we study another model using spiking neurons instead of McCulloch-Pitts neurons [3] . The goal is to overcome issues like small storage capacity and, more importantly, to look for more biologically plausible models of human memory.

2 Spiking Neuron Model

Some regions in our brain such as *neocortex* or *hippocampus* are said to be made up of two categories of neurons, that is, *pyramidal cells* and *interneurons*. Typically, the pyramidal cells communicate with each other via *excitatory synapses* (positive influences), while interneurons send signals to pyramidal cells via *inhibitory synapses* (negative influences).

As Wilson [4] wrote in his book, Marr [5] was one of the first to propose this hippocampal model involving both recurrent excitation via Hebbian [2] synapses and inhibition.

2.1 A description of the model by differential equations.

Probably one of the most fundamental differential equations is

$$\frac{dr}{dt} = -\frac{1}{\tau}r \tag{1}$$

where r is assumed to be a function of time and τ is referred to as *time constant*.

In this paper the variable of our interest is the number of spikes emitted from one neuron in unit time as a function of time. We call it *spiking ratio* and denote it $r(t)$. In his book, Wilson [4] wrote that a single neuron which emits spike train when it receives an external stimulus $P(t)$ could be modeled simply by adding $S(P)$ to the right hand side of Eq. (3) without describing explicitly shape and timing of each individual spike. That is,

$$\frac{dr(t)}{dt} = \frac{1}{\tau}(-r(t) + S(P)) \tag{2}$$

As stimulus $S(P)$, Wilson [4] proposed to employ, among many alternatives, Naka-Rushton [6] function:

$$S(P) = \begin{cases} mP^n/(\sigma^n + P^n) & \text{if } P \geq 0 \\ 0 & \text{if } P < 0 \end{cases} \tag{3}$$

where m and σ are called *saturation* and *semi-saturation constant*, respectively, and n is a integer parameter for its graph to fit a phenomena.

We now assume N pyramidal cells and implicit number of interneurons. We simulate these pyramidal cells with spiking neurons which interact with each other using electric current via plastic synapses. Pyramidal cells are also interacted by interneurons by global inhibition. To be more specific, stimuli to one pyramidal cell are given from all the other pyramidal cells as well as interneuron cells whose number is reduced to only one in this paper, via synaptic strength. See Fig. 1 bellow.

The synaptic strength from pyramidal cell j to i is denoted as w_{ij} and all the inhibitory synapses from interneuron is assumed to have a value g. Then stimulus to the i-th pyramidal cell P_i is described as

$$P_i = (\sum_{j=1}^{N} w_{ij} \cdot R_j - g \cdot G)_+^2$$

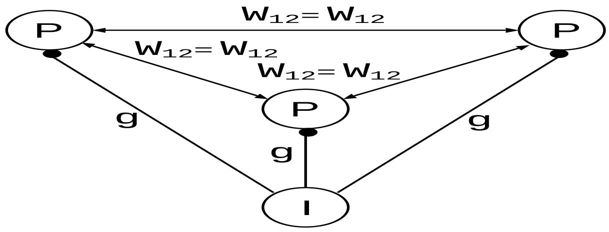

Fig. 1. Schematic diagram with three, instead of N, pyramidal cells (P) and one global interneuron cell (G). Connections with arrow indicate excitatory synapses and with closed circle inhibition.

where $(\cdot)_+$ means that we use the value if and only if inside the parentheses is positive and zero otherwise. Following Wilson [4] we experiment with $\sigma = 10$, $M = 100$, and $n = 2$ in Eq. (3).

Thus, our equation of spiking ratio of the i-th pyramidal cell R_i and the one of the interneuron G is given as

$$\tau_R \frac{dR_i}{dt} = -R_i + \frac{100(\sum_{j=1}^{N} w_{ij} R_j - 0.1G)_+^2}{100 + (\sum_{j=1}^{N} w_{ij} R_j - 0.1G)_+^2}$$

$$\tau_G \frac{dG}{dt} = -G - 0.07 \sum_{j=1}^{N} R_i \tag{4}$$

where, both τ_R and τ_G are to be set to 10. Also note that w_{ii} $(i = 1, \cdots N)$ should be set to all zero.

2.2 To encode patterns.

We have a fair amount of ways to encode N-bit patterns using N spiking neurons. (1) *Firing-rate* of a neuron within certain time window could express one bit of a continuous real number, or binary number according to whether the rate exceeds a threshold or not. (2) *Time-to-first-spike* could express one bit of continuous real number. In this case, we neglect informations about spike trains after the first one. (3) *Spike-timing* also could encode real number on the condition that emergence of spike of every neuron is periodic and only one spike in one period. Or, in this scheme binary bit could also be encoded according to presence/absence of spike in the period. In this paper, we exploit the first scheme.

2.3 To store & recall patterns.

In what he calls *CA3 network* in his book, Wilson [4] employed 256 pyramidal cells so that these cells represent a pattern constructed by 16×16 array of pixels.

The network also incorporates one interneuron cell to provide pyramidal cells feedback inhibition. The task of the network is to recognize 4 given patterns from its noisy input. Each of the 4 patterns is represented by 32 active cells plus other 224 quiet cells. Network has learned to recognize these 4 patterns by modifying the synapses according to

$$w_{ij} = k \cdot \mathrm{sgn}(R_i - 0.5M) \cdot \mathrm{sgn}(R_j - 0.5M) \qquad (5)$$

if the previous value of $w_{ij} = 0$, otherwise $w_{ij} = 1$ is not modified. Here k is set to 0.016 and note that $\mathrm{sgn}(x) = 1$ if $x > 0$ and 0 otherwise. A noisy input of a pattern is constructed by randomly picking up about one-third of the active cells of the selected pattern with adding them other 20 quiet cells, also chosen at random, after turning them active.

Then one of these 4 patterns is given to the network, that is, network starts the dynamics with the pattern as the initial configuration of its neurons' state. Network updates the state according to Eq. (4). The dynamics is observed during a total of 100ms (assuming step of dt of dr/dt 1ms), while the noisy input remains ON for the 1st 20 ms.

3 Experiment

In his book, Wilson [4] assigns values of w_{ij} employing the Hebbian algorithm, and succeeded in storing 4 patterns and recalling each of them from each of their noisy input. We tested Wilson's experiment with the same set of patterns and succeeded in obtaining the similar results when we also apply the Hebbian algorithm to store the patterns.
The synaptic weights plasticity is very robust. If we modify lots of weights, the network still recall the pattern correctly. That is to say, the top region of the Hebbian peak in weight space is broad. However if explore outside edge of the top region of the peak, all neurons become immediately quiescent or fire at their maximum rate.

3.1 Are Hebbian weights unique?

Then the question is "Are Hebbian weights unique solution?" Or, "Are Hebbian weights biologically plausible?" In order to tackle this issue, we search for other configuration of weight values. For the purpose we employ an evolutionary computation.

3.2 Searching for other solutions.

The number of synapses among pyramidal cells is N^2, including the one to itself. We represent one candidate solution of a weight configuration by one dimensional binary string of length $N^2 = 256 \times 256 = 65,536$ called chromosome. Fitness, the degree to which how good each corresponding phenotype performs, is evaluated as the ratio of how many pixels out of 256 are correctly recovered when it's noisy input is given as the initial state, using Wilson's example of 4 patterns. Result, however, is unpredictably horrible. It is as if a search for only one needle in a large haystack. What we observed is a series of fitness zero even after observing thousands of randomly created chromosomes.

3.3 Downhill walk from the top of Hebbian peak.

In order to know what is going on, we learn what does the fitness landscape on the weight space look like in which the hight of the land is almost everywhere zero except for already known Hebbian peak.

Downhill walk is in two different ways. The one is by flipping only the bit that is one. The other is opposite, that is, by flipping the bit one to zero. Both are by flipping bit by bit.

3.4 Can climbers take the uphill slope again?

To learn what does the side wall of the peak look like, hill-climbers try to climb the peak starting from the point that the downhill walker above walked three steps further after the walker reached the flat land of fitness zero. Even from this very close point to the edge of the peak, it is a very difficult task to find the point which has a fitness higher than zero. One of out experiment shows that 1411 trials are needed to find 20 no-zero-fitness points, for instance.

Thus we have a population of non-zero-fitness points and the maximum-fitness-point of the population is the first step of the walker. Then by flipping only one gene out of 256×256, again a population of points are obtained. The point of the maximum fitness of the population is the next point that the walker proceeds. Thus the walker, hopefully climb the top of the hill again. The population is set here to 20.

4 Results & Discussion

Two traces of downhill walkers from the top of the Hebbian peak are shown in Fig. 2 and Fig. 3.
As shown in both figures, the top region of the peak is not extremely narrow. Of course the ratio of area of top region of the peak is a kind of infinitesimal if we compare it to the whole area of search space. However, this is not so special case. Generally, the solution to a problem does not have a top region but just a point. What is problematic here is a very steep slope of side wall of the peak. It is terribly difficult to find a-little-better-solutions-than-others.

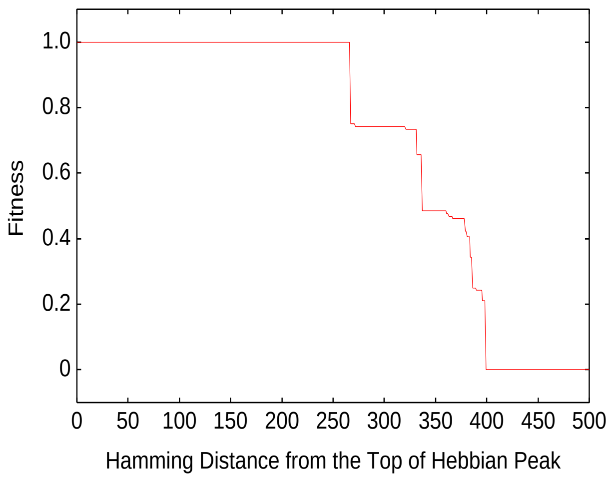

Fig. 2. A downhill walk from top of the Hebbian peak with flipping only a gene whose allele is one.

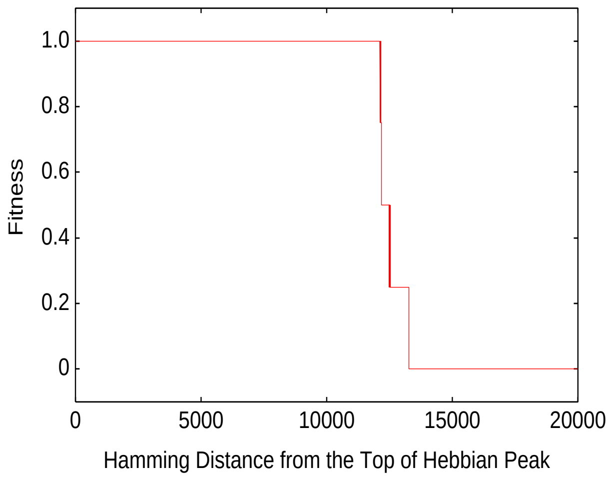

Fig. 3. A downhill walk from top of the Hebbian peak with flipping only a gene whose allele is zero.

The results are very different from the similar experiment in Hopfield model. As our previous report the solutions are not unique at all and it is quite easy to find one of the other solutions. See Imada [7], for example.

The traces of the hillclimber which start from the point that the above mentioned downhill walker walked three steps further after the walker reached the flat land of fitness zero is shown in Fig. 4.

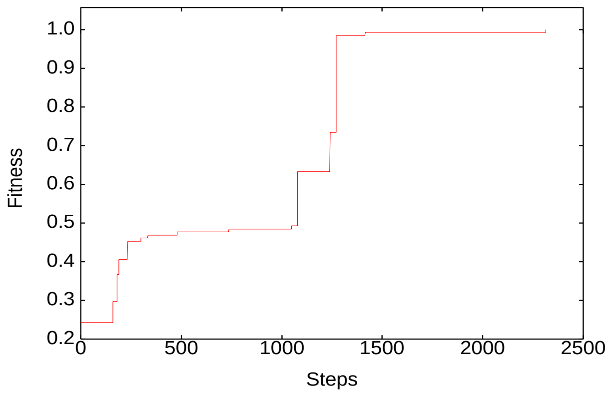

Fig. 4. A trace of hillclimber who tried to climb the peak again from the edge of the peak.

5 Conclusion

We have described a neural network model of associative memory using spiking neurons. We explore the Wilson's model [4] in which each neuron emits spike train and when spiking ratio exceeds a threshold the corresponding pixel is ON and otherwise OFF. Wilson [4] specified the strength (weight) of communication among neurons via synapses by Hebbian principle [2]. It was ascertained here too, however, we doubt this is not the only solution. In order to other possible configurations of weight values, we have explored, in this paper, the search space by evolutionary computations, but the result was worse than a-needle-in-hay-stack. We study why and conclude this is due to the shape of the peak in the fitness landscape. That is to say, side walls of the peak that we already know the location (Hebbian peak) is too steep to be climbed.

As a next step, We plan to experiment of an evolution in which each individual employs his/her learning during lifetime, like the famous experiment by Hinton and Nowlan [8], so that we expect that we will find a tiny needle from hay stack.

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