REGION-BASED HOUGH-INVERSION TRANSFORM FOR CIRCLES

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Abstract. The Hough Transform is a robust algorithm, intended to detect lines, circles or even more complex shapes within an image. A weakness of the algorithm is that it requires an important processing time, in particular if the shape to be detected is not a straight line. In many practical applications this constraint could not be acceptable. A solution to this problem has been called the Fast Hough Transform (FHT) -[8] and [5]-. The FHT approaches addresses the problem using specialized parallel hardware. Instead of this, this paper proposes an algorithmic approach for circle detection, which yields an acceptable processing time, without the need of any specialized hardware. To reach this goal the inversion transform [2] is used.

1 Introduction

In many visual inspection applications, a high time consuming algorithm is either not desirable or not acceptable. For example, if visual inspection is used in a computer integrated manufacturing environment, in which each part must be checked one by one by an inspection automata, the available time to inspect each part is severely constrained in order to avoid manufacturing bottlenecks.

The Hough Transform (HT) has been proposed as a robust method intended to detect lines, circles or even more complex shapes [6]. The method is used in a wide range of image processing applications. The weakness of the method is that processing time increases with the complexity of the shape to be detected in the image [3].

This paper discusses a modified approach, which is called the Hough/Inversion Transform (HIT). The method is based on the inversion-transform graphical-properties which allows to transform circles into lines and lines into circles.

2 The Hough Transform

The problem of shape detection in images requires to determine if a set of borderline points belongs to a region or not. Given an image composed by *n* points, straight lines can be detected by considering a discrete set of all possible lines which gets through each pair of points and then finding all the point subsets composed by the nearest points to each line.. This approach implies an unacceptable processing time, since it requires to find $n(n-1)/2 \cong n^2$ lines, and to make $n(n(n-1))/2 \cong n^3$ comparisons. A different approach to solve this problem is to employ classification schemes. A relative disadvantage of classification schemes is that they need to be trained and tested which is also not always desirable -or even possible- regarding industrial application purposes. A different approach is the use of the Hough Transform (HT).

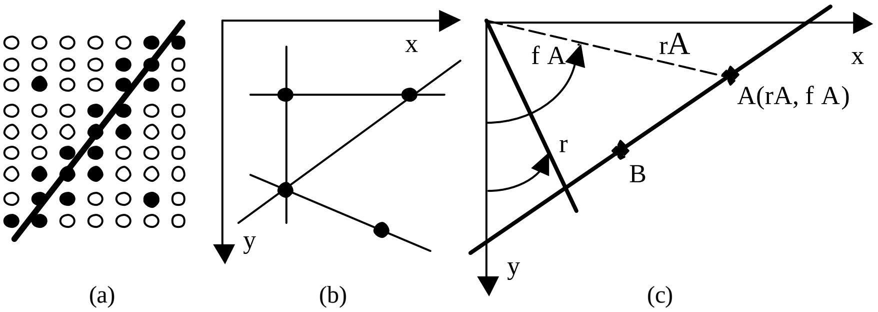

Fig. 1. a) Straight line within a cluster of points. b) Possible straight lines defined by them. c) Straight line defined using Hessian parameters

The HT is well suited for industrial inspection applications due to the following reasons:

- Parts show frequently lines or regular curves.
- HT is a robust method. It exhibits high noise insensitivity and fault tolerance regarding shape description.
- The global description nature of HT.

The underlying idea for line detection using the HT is that each input measure (Usually each pixel of a binary image) contributes to a globally consistent solution. (e.g. Physical Line through some point, Fig. 1a).

As an example let us consider the set of lines shown in Fig. 1b. Each line corresponds to a given point set.

In this case –straight line- results are given as a pair of parameters which define a line expressed by the normalized Hessian form (r, f) and the number of pixels on the line.

Then, parameters (r, f) satisfy the following equation:

$$r / r_A = \cos(j - j_A) \tag{1}$$

where (r_A, j_A) are the polar coordinates of the point A (Fig. 1c).

In an image analysis context, if the point coordinates belonging to the edges are known, then they can be used as the constant values in the parametric equation, in which *r* and q are unknown.

Let A and B be two borderline pixels; then the problem is to find the line trough A and B.

Considering a bunch of lines trough A and B, each of them can be described by the angle f and the module r -normal to the line-. To obtain a limited set L of lines, the parameters (r_A, j_A) must be quantified. -For example $\Delta r = 5 \;\; y \;\; \Delta j = 15^\circ$ -

In general, for a given shape to be detected the algorithm quantifies the parameter space on finite intervals or accumulation cells (r_i, j_i) e.g. $(L = 180 / 15 = 12)$

creating a multidimensional array, where for every point (x, y) on the line, the constants (r_i, j_i) correspond to a line of the set.

Selecting an accumulator step K, implies to divide one of the axis (e.g. *r*) of the parameter space in K segments. Due this quantification process each line of the bunch do not goes exactly trough A. For the corresponding set of B a similar accumulation array is obtained (r_B, j_B)

For each point *(r, ?) K* values of q are obtained. Those values correspond to the *K* values of *r*. Then the number of operations that must be repeated is *n x K* where *n* is the number of pixels in the image.

A local maximum in the multidimensional array indicates the presence of a line in the image.

The HT for straight lines detection requires to consider the application of R^2 *(x, y)* on the Hough *parameter space (r,* q *).*

Using the same process is possible to detect a different shape regarding its analytical description. E.g. if the process intend to detect circles, the equation is:

$$(x - a)^2 + (y - b)^2 = r^2 \tag{2}$$

in which *a* and *b* are the circle center coordinates, and *r* is the radius. In this case the computing cost is increased due that now there are three coordinates in the parameter space and the application is $R^2 \to R^3$.

3 The Generalized Hough Transform

The generalization of the Hough Transform approach allows to extend it for such a situation in which a simple analytical definition of the perimeter of a shape must be evaluated [Huss91], [1]. In this generalized approach a Look Up Table (LUT) is used to define the relationships between perimeter position and orientation and Hough parameters instead of an analytical description of the shape.

Given the shape definition and orientation (Fig. 2), and provided an arbitrary reference point (x_{ref}, y_{ref}) lying inside the feature, the shape is defined respect to this reference point (distance r and angle ω of normal lines). The LUT is then composed by pairs of distances/directions, indexed by the orientation w of the perimeter. The Hough parameter space is now defined in terms of the possible positions of the shape in the image, i.e. possible ranges of (x_{ref}, y_{ref}). Then,

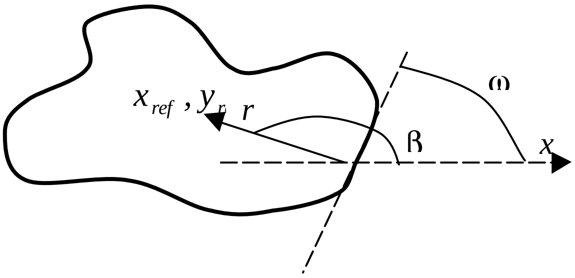

Fig. 2. The generalized Hough Transform is based on a reference point definition to encode the shape

$$x_{ref} = x + r\ cos b \tag{3}$$

$$y_{ref} = y + r\ sen b \tag{4}$$

(Where r and b are derived from the accumulator for a given set of orientations w). In case of the shape orientation is unknown, the accumulator must be extended by adding an additional parameter to consider the possible orientations. This extension implies a more complex and time consuming process.

4 Inversion

The equation

$$g(u, v) = \frac{1}{f(x, y)} \tag{5}$$

defines a one-to-one correspondence between points –non null- of planes (*u*, *v*) and (*x*, *y*). To use compact expressions the inversion transform is frequently discussed in terms of complex numbers notation, that is:

$$w = \frac{1}{z} \tag{6}$$

where (6) describes the inversion with respect the unit circle $|z| = 1$. The image w of a non null point z has the following properties;

$$|w| = \frac{1}{|z|} \quad \text{and} \quad \arg w = -\frac{1}{\arg z} \tag{7}$$

Then, the external points of a circle $|z| = 1$ are mapped on the inner points of the same circle, except (0, 0), and reciprocally. Every point on the circle is transformed univocally. The second transformation is simply a reflection over the real axis.

Given that:

$$\lim_{x\to 0} \frac{1}{z} = \infty \quad and \quad \lim_{x\to\infty} \frac{1}{z} = 0 \tag{8}$$

It follows that

$$T(0) = \infty\,,\ T(\infty) = 0 \quad and \quad T(z) = \frac{1}{|z|} \tag{9}$$

for the rest of z values.

Given the point $z = x + iy$, the image point $w = u + iv$ and the inversion transform $w = \frac{1}{z}$,

$$\text{Let } w = \frac{z}{|z|^2} \tag{10}$$

Then;

$$u = \frac{x}{x^2 + y^2} \quad , v = \frac{-y}{x^2 + y^2} \tag{11}$$

Considering $$z = \frac{1}{w} \ \ and \ \ z = \frac{w}{|w|^2}\,; \tag{12}$$

$$x = \frac{u}{u^2 + v^2}\,,\quad y = \frac{-v}{u^2 + v^2} \tag{13}$$

The following equation

$$a(x^2 + y^2) + bx - cy + d = 0$$
$$with \ \ a,b,c,d \in R \ \ / \ \ b^2 + c^2 > 4ad \tag{14}$$

represents circles and lines on the z plane. $-a \neq 0$ for circles and $a = 0$ for straight lines-.

Substituting eq. (13) in eq. (14) yields

$$d\,(u^2 + v^2) + bu - cv + a = 0 \qquad \textbf{(15)}$$

which also represents an equation of a circle or a straight line when the first term is zero,. Reciprocally if u and v satisfy eq. (11), then x and y satisfy eq. (13).

From eqs. (14) and (15) the following properties arise:

a) A circle on plane z, ($a \neq 0$) which does not get through the origin ($d \neq 0$), is transformed on plane w in a circle which also does not pass through the origin.

b) A circle on plane z, ($a \neq 0$) which gets through the origin ($d = 0$), is transformed on plane w in a straight line which also goes through the origin.

c) A straight line on plane z, ($a = 0$) which does not get through the origin ($d \neq 0$), is transformed on plane w in a circle which goes through the origin.

d) A straight line on plane z, ($a \neq 0$) which gets through the origin ($d = 0$), is transformed on plane w in a straight line which goes through the origin.

The graphical representation of these inversion properties is shown on Fig. 3.

An additional property is that for modules r>0, the inverted modulus result a real value 1/r < 0, and reciprocally if modules r<0, the inverted modulus result a value 1/r > 0. This property is important due in a image processing context distances are basically measured in pixel units which are integer values.

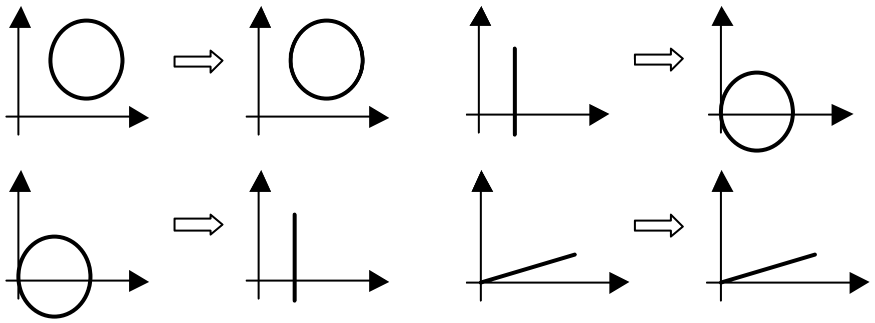

Fig. 3. If the values of f(x, y) describe a line -through the origin- on the (x, y) plane (reference plane), then the corresponding points of g(u, v) describe a circle -through the origin- on the (u, v) plane (target plane). Correspondingly if f describes a circle -through the origin- on the original plane, then g describes a line -through the origin- on the target plane

5 Hough/Inversion Transform

The proposal is to reduce the computational cost of the HT for circles by means the Inverse Transform. Because not every IT property seems to be appropriate for the present goal, some constraints must be considered. In order to explain how to combine the IT with the HT let us consider a set of points which correspond to a circle-shaped edge in a binary image.

From the above mentioned properties of the IT only b) and c) transform circles in straight lines and vice versa. Then a circle can be transformed into a straight line only if it gets trough the origin of the coordinate system in which the circle is defined.

Obviously, this condition is not true for every possible circle on the image plane. Then this condition must be imposed by the process itself. The only way to do this is to define a relative coordinate system centered on a point which belongs to the expected circle. To reach this goal the image must be segmented as a first step, and then a relative coordinate system is defined for each region. One point (x, y) of the region is selected as the origin of the local region coordinate system.

This Region-based approach imposes the constraint that the circular border must be *compact*. In this case compact means that a representative amount of points belonging to the circular shape must be able to be grouped into a segmented region.

Each region –expected circle- is translated to be defined in terms of its relative coordinate system. For each point on the current region the inverse transform of the complex value *z* is computed: *1/z=(u,jv)*. The transformed points *(u, v)* are obtained in terms of the relative coordinate system.

The set of points describes a new shape on the plane w. The new distribution corresponds to a line on this plane if the original shape corresponds to a circle in the image plane. Since distances are measured in pixels, the diameter value on the image plane is a natural number 1,2,3,…,n.

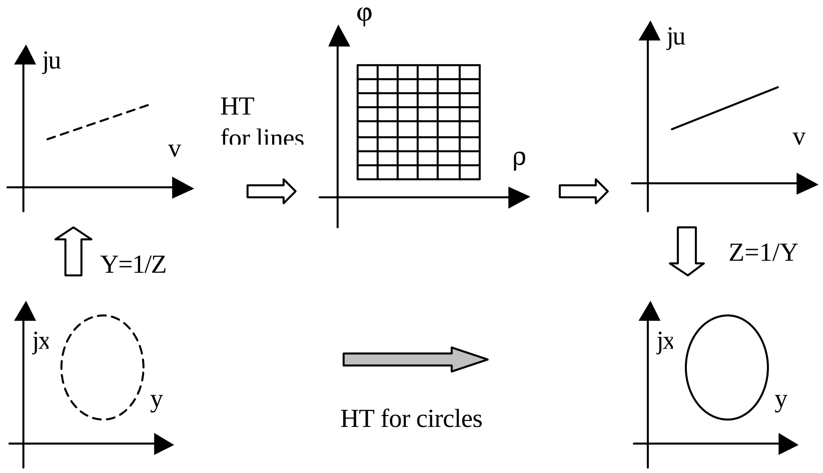

Fig. 4. Main steps of the HIT algorithm. (White arrows)

In order to apply the HT for straight lines, the inverse of this diameter value is mapped on to a discrete array –e.g. a plane with the same size as the original image plane-. In case of using a plane with the same size as the original image plane is not possible to recover the original diameter value by doing the inverse-anti-transformation of the straight line detected using the HT. To avoid this problem a scale factor k is used. The authors have reached remarkable results using values of k=500

Due to the computing cost of the HT for circles is larger than the HT computing cost for lines (given the same processing conditions), transforming a point set of a circle to a point set of a line implies a substantial reduction in processing time.

From the transformed plane w the line can be extracted using the HT for lines. After this the mathematical description of the line is known. (function parameters). Finally it is possible to anti transform the lineal function to obtain circle parameters in the original plane Z (image plane). Fig. 4 describes this process.

Point to point transform from plane z to plane w=1/z:

Anti transform of line f(φ, ρ) in plane w to a circle f(cx, cy, r) on plane Z:

$$y = \frac{u}{u^2 + v^2} = a\,t + b \qquad \textbf{(16)}$$

$$x = \frac{v}{u^2 + v^2} = t\,, \qquad \textbf{(17)}$$

Substituting (16) in (17):

$$\frac{u}{u^2 + v^2} = a\left(\frac{v}{u^2 + v^2}\right) + b = \frac{a\,v + b\left(u^2 + v^2\right)}{u^2 + v^2} \qquad \textbf{(18)}$$

then;

$$u = av + b\left(u^2 + v^2\right) = av + bu^2 + bv^2 \qquad \textbf{(19)}$$

then;

$$bu^2 - u + bv^2 + av = 0 \qquad \textbf{(20)}$$

and,

$$u^2 - \frac{u}{b} + v^2 + a\,\frac{v}{b} = 0 \qquad \textbf{(21)}$$

adding the term $\left(\frac{1}{2b}\right)^2$ y $\left(\frac{1a}{2b}\right)^2$ to both members of eq. (21):

$$u^2 + \left(\frac{1}{2b}\right)^2 - \frac{u}{b} + v^2 + \left(\frac{1a}{2b}\right)^2 + a\,\frac{v}{b} = \left(\frac{1}{2b}\right)^2 + \left(\frac{1a}{2b}\right)^2 \qquad \textbf{(22)}$$

then, the resulting equation is:

$$\left(u \;\; -\frac{1}{2b}\right)^2+\left(v+\frac{1b}{2a}\right)^2=\frac{1}{4b^2}\left(1+a^2\right). \qquad \textbf{(23)}$$

6 Steps of the Hit Algorithm

```
1. Image segmentation.
2. for each region in the image:
  2.1. Random selection of a region point as the origin of a relative coordinate system
  2.2. Translation of the region to the RCS
  2.3. Region points inversión.
  2.4. Inverted region points scaling.
  2.5. Straight lines detection through HT
    2.5.1. Parameters definition $\Delta r\ y\ \Delta j$
    2.5.2. For each $(r_p, j_p)$
      2.5.2.1 $z(r_p, j_p) = z(r_p, j_p) + 1$
      2.5.2.2 Maximun of accumulation cells
3. Line inversion parameters (r, cx, cy)
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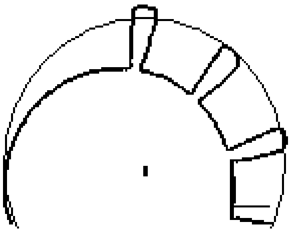

Fig. 5. A region containing a noisy image of a circle produces an important shift of the resulting circle coordinates and a false radius value

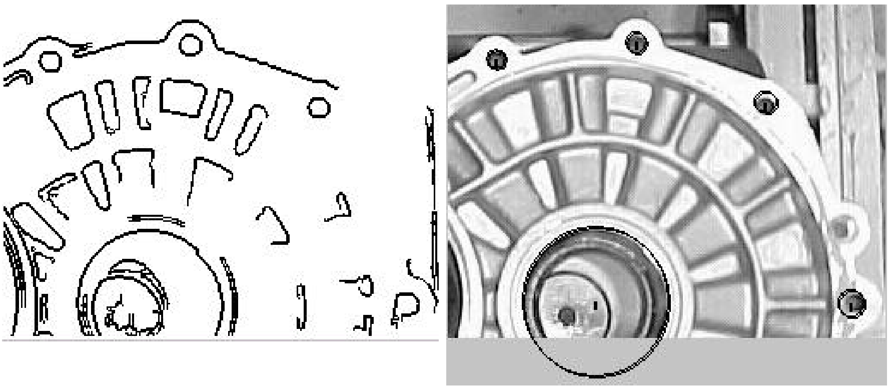

Fig. 6. Simple tests using HIT have shown a remarkable level of noise insensitivity and accuracy. In this example the algorithm has yield a significant shift for circles with larger radius

7 Results

A battery of tests was made using a selected set of five different parts. Each part was controlled a hundred of times. After initial calibration to avoid some operating faults the system has successfully recognized the whole set of circles.

The method has proved to be highly reliable (99.6 %) in those images in which circular shapes are isolated. Many industrial applications exist in which circles can be segmented and isolated, and therefore this method can be successfully applied. For circles that intersect borders of another shapes results are shown in Fig. 6, which shows a view of complex part and the resulting image after applying a Sobel operator. The detected circles -overlapped on the image- have been selected using a small and medium diameter range of variation. The shift of one of the lower circles –medium size diameter- is due the involved region contains points which lies not near the circle (Fig. 5). An important aspect of applying the HIT is that it allows a short processing time (a reduction of 1/100 for a 100 x 100 pixel region) using simple and low cost equipment against more expensive hardware and complex mechanical equipment.

8 Acknowledgments

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