```python
import warnings ; warnings.filterwarnings('ignore')

import itertools
import gym, gym_walk
import numpy as np
from tabulate import tabulate
from pprint import pprint
from tqdm import tqdm_notebook as tqdm

from itertools import cycle, count

import random
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.pylab as pylab
SEEDS = (12, 34, 56, 78, 90)

%matplotlib inline
```

```python
pip install git+https://github.com/mimoralea/gym-walk#egg=gym-walk
```

```
    Collecting gym-walk
      Cloning https://github.com/mimoralea/gym-walk to /tmp/pip-install-gckuwi1_/gym-walk_fd06d6ca8165426babd6c446b977be84
      Running command git clone --filter=blob:none --quiet https://github.com/mimoralea/gym-walk /tmp/pip-install-gckuwi1_/gym-walk_fd06
      Resolved https://github.com/mimoralea/gym-walk to commit b915b94cf2ad16f8833a1ad92ea94e88159279f5
      Preparing metadata (setup.py) ... done
    Requirement already satisfied: gym in /usr/local/lib/python3.11/dist-packages (from gym-walk) (0.25.2)
    Requirement already satisfied: numpy>=1.18.0 in /usr/local/lib/python3.11/dist-packages (from gym->gym-walk) (2.0.2)
    Requirement already satisfied: cloudpickle>=1.2.0 in /usr/local/lib/python3.11/dist-packages (from gym->gym-walk) (3.1.1)
    Requirement already satisfied: gym-notices>=0.0.4 in /usr/local/lib/python3.11/dist-packages (from gym->gym-walk) (0.0.8)
    Building wheels for collected packages: gym-walk
      Building wheel for gym-walk (setup.py) ... done
      Created wheel for gym-walk: filename=gym_walk-0.0.2-py3-none-any.whl size=5377 sha256=a1eeffcef0b9e01f14ab90bbb92058c7151461205e03
      Stored in directory: /tmp/pip-ephem-wheel-cache-4dthdan1/wheels/60/02/77/2dd9f31df8d13bc7c014725f4002e29d0fc3ced5e8ac08e1cf
    Successfully built gym-walk
    Installing collected packages: gym-walk
    Successfully installed gym-walk-0.0.2
```

```python
plt.style.use('fivethirtyeight')
params = {
    'figure.figsize': (15, 8),
    'font.size': 24,
    'legend.fontsize': 20,
    'axes.titlesize': 28,
    'axes.labelsize': 24,
    'xtick.labelsize': 20,
    'ytick.labelsize': 20
}
pylab.rcParams.update(params)
np.set_printoptions(suppress=True)
```

```python
def value_iteration(P, gamma=1.0, theta=1e-10):
    V = np.zeros(len(P), dtype=np.float64)
    while True:
        Q = np.zeros((len(P), len(P[0])), dtype=np.float64)
        for s in range(len(P)):
            for a in range(len(P[s])):
                for prob, next_state, reward, done in P[s][a]:
                    Q[s][a] += prob * (reward + gamma * V[next_state] * (not done))
        if np.max(np.abs(V - np.max(Q, axis=1))) < theta:
            break
        V = np.max(Q, axis=1)
    pi = lambda s: {s:a for s, a in enumerate(np.argmax(Q, axis=1))}[s]
    return Q, V, pi
```

```python
def print_policy(pi, P, action_symbols=('<', 'v', '>', '^'), n_cols=4, title='Policy:'):
    print(title)
    arrs = {k:v for k,v in enumerate(action_symbols)}
    for s in range(len(P)):
        a = pi(s)
        print("| ", end="")
        if np.all([done for action in P[s].values() for _, _, _, done in action]):
            print("".rjust(9), end=" ")
        else:
            print(str(s).zfill(2), arrs[a].rjust(6), end=" ")
        if (s + 1) % n_cols == 0: print("|")
```

```python
def print_state_value_function(V, P, n_cols=4, prec=3, title='State-value function:'):
    print(title)
    for s in range(len(P)):
        v = V[s]
        print("| ", end="")
        if np.all([done for action in P[s].values() for _, _, _, done in action]):
            print("".rjust(9), end=" ")
        else:
            print(str(s).zfill(2), '{}'.format(np.round(v, prec)).rjust(6), end=" ")
        if (s + 1) % n_cols == 0: print("|")
```

```python
def print_action_value_function(Q,
                                optimal_Q=None,
                                action_symbols=('<', '>'),
                                prec=3,
                                title='Action-value function:'):
    vf_types=('',) if optimal_Q is None else ('', '*', 'err')
    headers = ['s',] + [' '.join(i) for i in list(itertools.product(vf_types, action_symbols))]
    print(title)
    states = np.arange(len(Q))[..., np.newaxis]
    arr = np.hstack((states, np.round(Q, prec)))
    if not (optimal_Q is None):
        arr = np.hstack((arr, np.round(optimal_Q, prec), np.round(optimal_Q-Q, prec)))
    print(tabulate(arr, headers, tablefmt="fancy_grid"))
```

```python
def get_policy_metrics(env, gamma, pi, goal_state, optimal_Q,
                       n_episodes=100, max_steps=200):
    random.seed(123); np.random.seed(123) ; env.seed(123)
    reached_goal, episode_reward, episode_regret = [], [], []
    for _ in range(n_episodes):
        state, done, steps = env.reset(), False, 0
        episode_reward.append(0.0)
        episode_regret.append(0.0)
        while not done and steps < max_steps:
            action = pi(state)
            regret = np.max(optimal_Q[state]) - optimal_Q[state][action]
            episode_regret[-1] += regret

            state, reward, done, _ = env.step(action)
            episode_reward[-1] += (gamma**steps * reward)

            steps += 1

        reached_goal.append(state == goal_state)
    results = np.array((np.sum(reached_goal)/len(reached_goal)*100,
                        np.mean(episode_reward),
                        np.mean(episode_regret)))
    return results
```

```python
def get_metrics_from_tracks(env, gamma, goal_state, optimal_Q, pi_track, coverage=0.1):
    total_samples = len(pi_track)
    n_samples = int(total_samples * coverage)
    samples_e = np.linspace(0, total_samples, n_samples, endpoint=True, dtype=np.int)
    metrics = []
    for e, pi in enumerate(tqdm(pi_track)):
        if e in samples_e:
            metrics.append(get_policy_metrics(
                env,
                gamma=gamma,
                pi=lambda s: pi[s],
                goal_state=goal_state,
                optimal_Q=optimal_Q))
        else:
            metrics.append(metrics[-1])
    metrics = np.array(metrics)
    success_rate_ma, mean_return_ma, mean_regret_ma = np.apply_along_axis(moving_average, axis=0, arr=metrics).T
    return success_rate_ma, mean_return_ma, mean_regret_ma
```

```python
def rmse(x, y, dp=4):
    return np.round(np.sqrt(np.mean((x - y)**2)), dp)
```

```python
def moving_average(a, n=100) :
    ret = np.cumsum(a, dtype=float)
    ret[n:] = ret[n:] - ret[:-n]
    return ret[n - 1:] / n
```

```python
def plot_value_function(title, V_track, V_true=None, log=False, limit_value=0.05, limit_items=5):
    np.random.seed(123)
    per_col = 25
    linecycler = cycle(["-","--",":","-."])
    legends = []

    valid_values = np.argwhere(V_track[-1] > limit_value).squeeze()
    items_idxs = np.random.choice(valid_values,
                                  min(len(valid_values), limit_items),
                                  replace=False)
    # draw the true values first
    if V_true is not None:
        for i, state in enumerate(V_track.T):
            if i not in items_idxs:
                continue
            if state[-1] < limit_value:
                continue

            label = 'v*({})'.format(i)
            plt.axhline(y=V_true[i], color='k', linestyle='-', linewidth=1)
            plt.text(int(len(V_track)*1.02), V_true[i]+.01, label)

    # then the estimates
    for i, state in enumerate(V_track.T):
        if i not in items_idxs:
            continue
        if state[-1] < limit_value:
            continue
        line_type = next(linecycler)
        label = 'V({})'.format(i)
        p, = plt.plot(state, line_type, label=label, linewidth=3)
        legends.append(p)

    legends.reverse()

    ls = []
    for loc, idx in enumerate(range(0, len(legends), per_col)):
        subset = legends[idx:idx+per_col]
        l = plt.legend(subset, [p.get_label() for p in subset],
                       loc='center right', bbox_to_anchor=(1.25, 0.5))
        ls.append(l)
    [plt.gca().add_artist(l) for l in ls[:-1]]
    if log: plt.xscale('log')
    plt.title(title)
    plt.ylabel('State-value function')
    plt.xlabel('Episodes (log scale)' if log else 'Episodes')
    plt.show()
```

```python
def decay_schedule(init_value, min_value, decay_ratio, max_steps, log_start=-2, log_base=10):
    decay_steps = int(max_steps * decay_ratio)
    rem_steps = max_steps - decay_steps
    values = np.logspace(log_start, 0, decay_steps, base=log_base, endpoint=True)[::-1]
    values = (values - values.min()) / (values.max() - values.min())
    values = (init_value - min_value) * values + min_value
    values = np.pad(values, (0, rem_steps), 'edge')
    return values
```

```python
env = gym.make('FrozenLake-v1')
init_state = env.reset()
goal_state = 15
gamma = 0.99
n_episodes = 3000
P = env.env.P
n_cols, svf_prec, err_prec, avf_prec=9, 4, 2, 3
action_symbols=('<', 'v', '>', '^')
limit_items, limit_value = 5, 0.0
cu_limit_items, cu_limit_value, cu_episodes = 10, 0.0, 100
```

```
/usr/local/lib/python3.11/dist-packages/gym/core.py:317: DeprecationWarning: WARN: Initializing wrapper in old step API which return
  deprecation(
/usr/local/lib/python3.11/dist-packages/gym/wrappers/step_api_compatibility.py:39: DeprecationWarning: WARN: Initializing environmen
  deprecation(
```

```python
plt.plot(decay_schedule(0.5, 0.01, 0.5, n_episodes),
         '-', linewidth=2,
         label='Alpha schedule')
plt.plot(decay_schedule(1.0, 0.1, 0.9, n_episodes),
```
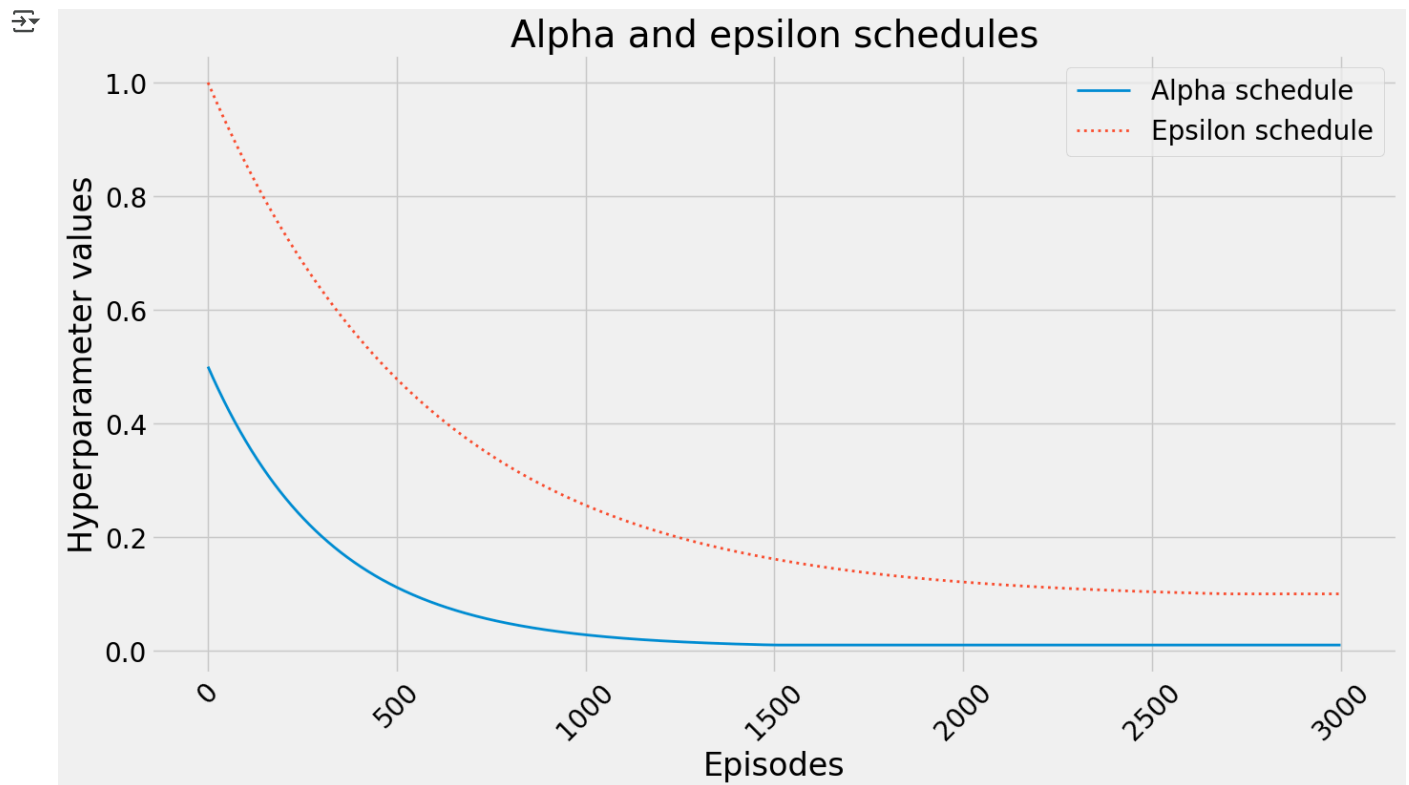
```
        ':', linewidth=2,
        label='Epsilon schedule')
plt.legend(loc=1, ncol=1)

plt.title('Alpha and epsilon schedules')
plt.xlabel('Episodes')
plt.ylabel('Hyperparameter values')
plt.xticks(rotation=45)

plt.show()
```



```
optimal_Q, optimal_V, optimal_pi = value_iteration(P, gamma=gamma)
print_state_value_function(optimal_V, P, n_cols=4, prec=svf_prec, title='Optimal state-value function:')
print()

print_action_value_function(optimal_Q,
                            None,
                            action_symbols=action_symbols,
                            prec=avf_prec,
                            title='Optimal action-value function:')
print()
print_policy(optimal_pi, P, action_symbols=action_symbols, n_cols=4)
success_rate_op, mean_return_op, mean_regret_op = get_policy_metrics(
    env, gamma=gamma, pi=optimal_pi, goal_state=goal_state, optimal_Q=optimal_Q)
print('\n\nReaches goal {:.2f}%. Obtains an average return of {:.4f}. Regret of {:.4f}'.format(
    success_rate_op, mean_return_op, mean_regret_op))
```

```
Optimal state-value function:
| 00  0.542 | 01 0.4988 | 02 0.4707 | 03 0.4569 |
| 04 0.5585 |           | 06 0.3583 |           |
| 08 0.5918 | 09 0.6431 | 10 0.6152 |           |
|           | 13 0.7417 | 14 0.8628 |           |

Optimal action-value function:
```

s	<	v	>	^
0	0.542	0.528	0.528	0.522

1	0.343	0.334	0.32	0.499
2	0.438	0.434	0.424	0.471
3	0.306	0.306	0.302	0.457
4	0.558	0.38	0.374	0.363
5	0	0	0	0
6	0.358	0.203	0.358	0.155
7	0	0	0	0
8	0.38	0.408	0.397	0.592
9	0.44	0.643	0.448	0.398
10	0.615	0.497	0.403	0.33
11	0	0	0	0
12	0	0	0	0
13	0.457	0.53	0.742	0.497
14	0.733	0.863	0.821	0.781
15	0	0	0	0

```
Policy:
| 00      < | 01      ^ | 02      ^ | 03      ^ |
| 04      < |           | 06      < |           |
| 08      ^ | 09      v | 10      < |           |
|           | 13      > | 14      v |           |
```

Reaches goal 69.00%. Obtains an average return of 0.4808. Regret of 0.0000

```python
def generate_trajectory(select_action, Q, epsilon, env, max_steps=200):
    done, trajectory = False, []
    while not done:
        state = env.reset()
        for t in count():
            action = select_action(state, Q, epsilon)
            next_state, reward, done, _ = env.step(action)
            experience = (state, action, reward, next_state, done)
            trajectory.append(experience)
            if done:
                break
            if t >= max_steps - 1:
                trajectory = []
                break
            state = next_state
    return np.array(trajectory, object)
```

```python
def mc_control(env,
               gamma=1.0,
               init_alpha=0.5,
               min_alpha=0.01,
               alpha_decay_ratio=0.5,
               init_epsilon=1.0,
               min_epsilon=0.1,
               epsilon_decay_ratio=0.9,
               n_episodes=3000,
               max_steps=200,
               first_visit=True):
    nS, nA = env.observation_space.n, env.action_space.n
    discounts = np.logspace(0,
                            max_steps,
                            num=max_steps,
                            base=gamma,
                            endpoint=False)
    alphas = decay_schedule(init_alpha,
                            min_alpha,
                            alpha_decay_ratio,
                            n_episodes)
    epsilons = decay_schedule(init_epsilon,
                              min_epsilon,
                              epsilon_decay_ratio,
                              n_episodes)
    pi_track = []
```

```python
    Q = np.zeros((nS, nA), dtype=np.float64)
    Q_track = np.zeros((n_episodes, nS, nA), dtype=np.float64)
    select_action = lambda state, Q, epsilon: np.argmax(Q[state]) \
        if np.random.random() > epsilon \
        else np.random.randint(len(Q[state]))

    for e in tqdm(range(n_episodes), leave=False):

        trajectory = generate_trajectory(select_action,
                                         Q,
                                         epsilons[e],
                                         env,
                                         max_steps)
        visited = np.zeros((nS, nA), dtype=np.bool)
        for t, (state, action, reward, _, _) in enumerate(trajectory):
            if visited[state][action] and first_visit:
                continue
            visited[state][action] = True

            n_steps = len(trajectory[t:])
            G = np.sum(discounts[:n_steps] * trajectory[t:, 2])
            Q[state][action] = Q[state][action] + alphas[e] * (G - Q[state][action])

        Q_track[e] = Q
        pi_track.append(np.argmax(Q, axis=1))

    V = np.max(Q, axis=1)
    pi = lambda s: {s:a for s, a in enumerate(np.argmax(Q, axis=1))}[s]
    return Q, V, pi, Q_track, pi_track
```

```python
Q_mcs, V_mcs, Q_track_mcs = [], [], []
for seed in tqdm(SEEDS, desc='All seeds', leave=True):
    random.seed(seed); np.random.seed(seed) ; env.seed(seed)
    Q_mc, V_mc, pi_mc, Q_track_mc, pi_track_mc = mc_control(env, gamma=gamma, n_episodes=n_episodes)
    Q_mcs.append(Q_mc) ; V_mcs.append(V_mc) ; Q_track_mcs.append(Q_track_mc)
Q_mc, V_mc, Q_track_mc = np.mean(Q_mcs, axis=0), np.mean(V_mcs, axis=0), np.mean(Q_track_mcs, axis=0)
del Q_mcs ; del V_mcs ; del Q_track_mcs
```

```
<ipython-input-21-a266861b13d1>:2: TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
 Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`
  for seed in tqdm(SEEDS, desc='All seeds', leave=True):
All seeds: 100%                                          5/5 [00:09<00:00,  1.91s/it]
```

```python
print_state_value_function(V_mc, P, n_cols=4,
                           prec=svf_prec, title='\n\nState-value function found by FVMC:')
print_state_value_function(optimal_V, P, n_cols=4,
                           prec=svf_prec, title='\n\nOptimal state-value function:')
print_state_value_function(V_mc - optimal_V, P, n_cols=4,
                           prec=err_prec, title='\n\nState-value function errors:')
print('\nState-value function RMSE: {}'.format(rmse(V_mc, optimal_V)))
print()
print_action_value_function(Q_mc,
                            optimal_Q,
                            action_symbols=action_symbols,
                            prec=avf_prec,
                            title='FVMC action-value function:')
print('Action-value function RMSE: {}'.format(rmse(Q_mc, optimal_Q)))
print()
print_policy(pi_mc, P, action_symbols=action_symbols, n_cols=4)
success_rate_mc, mean_return_mc, mean_regret_mc = get_policy_metrics(
    env, gamma=gamma, pi=pi_mc, goal_state=goal_state, optimal_Q=optimal_Q)
print('\n\nReaches goal {:.2f}%. Obtains an average return of {:.4f}. Regret of {:.4f}'.format(
    success_rate_mc, mean_return_mc, mean_regret_mc))
```

```
| 08 0.5918 | 09 0.6431 | 10 0.6152 |           |
|           | 13 0.7417 | 14 0.8628 |           |


State-value function errors:
| 00  -0.32 | 01  -0.34 | 02  -0.32 | 03  -0.38 |
```

1	0.082	0.071	0.068	0.152	0.343	0.334	0.32	0.499	0.262	0.263	0.252	0.346
2	0.125	0.086	0.091	0.077	0.438	0.434	0.424	0.471	0.313	0.348	0.333	0.394
3	0.049	0.029	0.009	0.036	0.306	0.306	0.302	0.457	0.257	0.277	0.293	0.421
4	0.24	0.149	0.139	0.143	0.558	0.38	0.374	0.363	0.318	0.23	0.235	0.22
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0.175	0.044	0.065	0.016	0.358	0.203	0.358	0.155	0.183	0.159	0.294	0.14
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0.128	0.161	0.137	0.28	0.38	0.408	0.397	0.592	0.251	0.246	0.259	0.312
9	0.143	0.332	0.238	0.159	0.44	0.643	0.448	0.398	0.297	0.311	0.209	0.239
10	0.318	0.24	0.177	0.093	0.615	0.497	0.403	0.33	0.298	0.257	0.226	0.237
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0.176	0.249	0.509	0.215	0.457	0.53	0.742	0.497	0.281	0.28	0.232	0.282
14	0.339	0.573	0.614	0.438	0.733	0.863	0.821	0.781	0.394	0.289	0.207	0.343
15	0	0	0	0	0	0	0	0	0	0	0	0

```
Action-value function RMSE: 0.2383

Policy:
| 00      < | 01      v | 02      > | 03      < |
| 04      < |           | 06      < |           |
| 08      ^ | 09      v | 10      v |           |
|           | 13      > | 14      > |           |


Reaches goal 53.00%. Obtains an average return of 0.3942. Regret of 0.2561
```

```python
def q_learning(env,
               gamma=1.0,
               init_alpha=0.5,
               min_alpha=0.01,
               alpha_decay_ratio=0.5,
               init_epsilon=1.0,
               min_epsilon=0.1,
               epsilon_decay_ratio=0.9,
               n_episodes=3000):
    nS, nA = env.observation_space.n, env.action_space.n
    pi_track = []
    Q = np.zeros((nS, nA), dtype=np.float64)
    Q_track = np.zeros((n_episodes, nS, nA), dtype=np.float64)

    # Write your code here
    select_action=lambda state, Q, epsilon: np.argmax(Q[state]) \
        if np.random.random() > epsilon \
        else np.random.randint(len(Q[state]))
    alphas=decay_schedule(init_alpha,min_alpha,alpha_decay_ratio,n_episodes)
    epsilons=decay_schedule(init_epsilon,min_epsilon,epsilon_decay_ratio,n_episodes)
    for e in tqdm(range(n_episodes), leave=False):
      state,done=env.reset(),False
      while not done:
        action=select_action(state,Q,epsilons[e])
        next_state,reward,done,_=env.step(action)
        td_target=reward+gamma*Q[next_state].max()*(not done)
        td_error=td_target-Q[state][action]
        Q[state][action]=Q[state][action]+alphas[e]*td_error
        state=next_state
      Q_track[e]=Q
      pi_track.append(np.argmax(Q,axis=1))
    V=np.max(Q,axis=1)
    pi=lambda s:{s:a for s,a in enumerate(np.argmax(Q,axis=1))}[s]

    return Q, V, pi, Q_track, pi_track
```

```python
Q_qls, V_qls, Q_track_qls = [], [], []
for seed in tqdm(SEEDS, desc='All seeds', leave=True):
    random.seed(seed); np.random.seed(seed) ; env.seed(seed)
    Q_ql, V_ql, pi_ql, Q_track_ql, pi_track_ql = q_learning(env, gamma=gamma, n_episodes=n_episodes)
    Q_qls.append(Q_ql) ; V_qls.append(V_ql) ; Q_track_qls.append(Q_track_ql)
Q_ql = np.mean(Q_qls, axis=0)
V_ql = np.mean(V_qls, axis=0)
```

```
Q_track_ql = np.mean(Q_track_qls, axis=0)
del Q_qls ; del V_qls ; del Q_track_qls
```

➔ <ipython-input-24-3604a42f7f45>:2: TqdmDeprecationWarning: This function will be removed in tqdm==5.0.0
    Please use `tqdm.notebook.tqdm` instead of `tqdm.tqdm_notebook`
      for seed in tqdm(SEEDS, desc='All seeds', leave=True):

All seeds: 100%   5/5 [00:14<00:00, 3.17s/it]
◀   ▶

```
print("Name: NARESH.R \nRegister No: 212223240104")
print_state_value_function(V_ql, P, n_cols=4,
                        prec=svf_prec, title='\nState-value function found by Q-learning:\n')

print_state_value_function(optimal_V, P, n_cols=4,
                        prec=svf_prec, title='\n\nOptimal state-value function:\n')

print_state_value_function(V_ql - optimal_V, P, n_cols=4,
                        prec=err_prec, title='\n\nState-value function errors:\n')

print('\n\nState-value function RMSE: {}'.format(rmse(V_ql, optimal_V)))
print()
print_action_value_function(Q_ql,
                        optimal_Q,
                        action_symbols=action_symbols,
                        prec=avf_prec,
                        title='Q-learning action-value function:')
print("Name:NARESH.R \nRegister No: 212223240104")
print('Action-value function RMSE: {}'.format(rmse(Q_ql, optimal_Q)))
print()
print_policy(pi_ql, P, action_symbols=action_symbols, n_cols=4)
success_rate_ql, mean_return_ql, mean_regret_ql = get_policy_metrics(
    env, gamma=gamma, pi=pi_ql, goal_state=goal_state, optimal_Q=optimal_Q)
print("Name:NARESH.R \nRegister No: 212223240104")
print('\n\nReaches goal {:.2f}%. Obtains an average return of {:.4f}. Regret of {:.4f}'.format(
    success_rate_ql, mean_return_ql, mean_regret_ql))
```

➔ Name: NARESH.R
   Register No: 212223240104

   State-value function found by Q-learning:

00 0.4977	01 0.41	02 0.3404	03 0.2021
04 0.5138		06 0.2904	
08 0.5477	09 0.6046	10 0.5733	
	13 0.7108	14 0.8449	

   Optimal state-value function:

00 0.542	01 0.4988	02 0.4707	03 0.4569
04 0.5585		06 0.3583	
08 0.5918	09 0.6431	10 0.6152	
	13 0.7417	14 0.8628	

   State-value function errors:

 | 00 -0.04 | 01 -0.09 | 02 -0.13 | 03 -0.25 |
04 -0.04		06 -0.07	
08 -0.04	09 -0.04	10 -0.04	
	13 -0.03	14 -0.02	

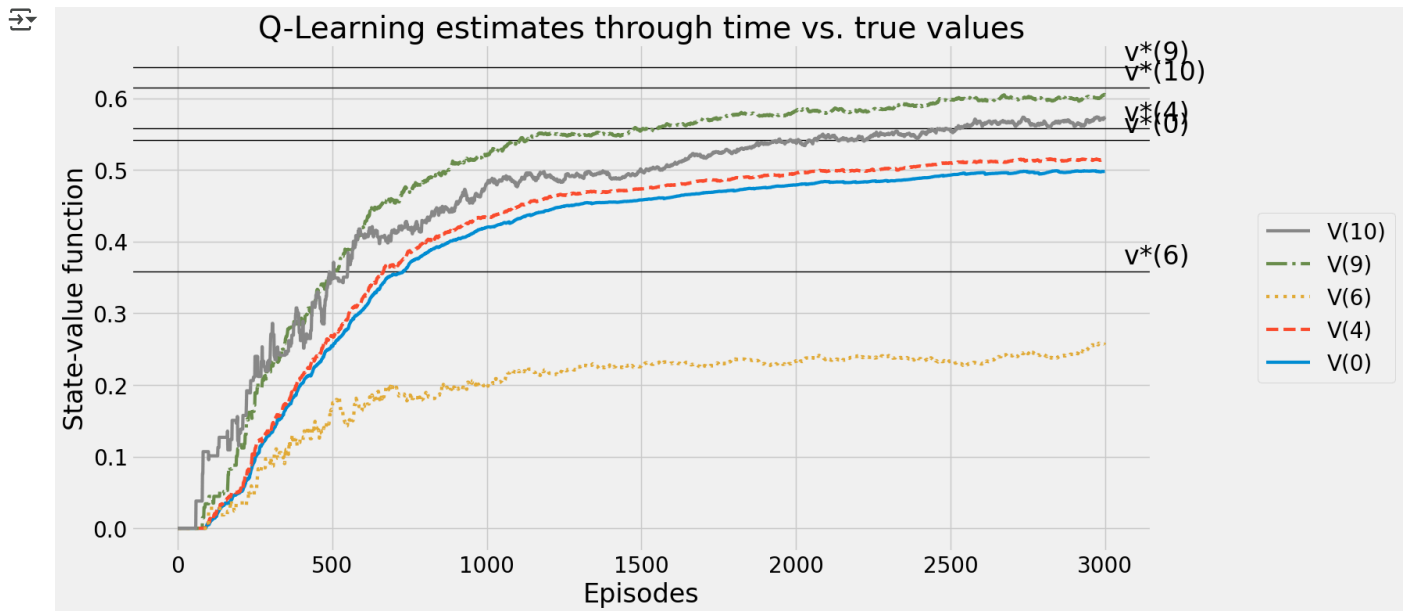   State-value function RMSE: 0.0809

   Q-learning action-value function:

s	<	v	>	^	* <	* v	* >	* ^	err <	err v	err >	err ^
0	0.498	0.462	0.464	0.454	0.542	0.528	0.528	0.522	0.044	0.066	0.064	0.068
1	0.24	0.22	0.187	0.41	0.343	0.334	0.32	0.499	0.104	0.114	0.133	0.089
2	0.322	0.226	0.206	0.244	0.438	0.434	0.424	0.471	0.116	0.208	0.218	0.227
3	0.116	0.076	0.071	0.18	0.306	0.306	0.302	0.457	0.191	0.23	0.231	0.276
4	0.514	0.348	0.341	0.316	0.558	0.38	0.374	0.363	0.045	0.031	0.033	0.047
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0.229	0.119	0.256	0.076	0.358	0.203	0.358	0.155	0.129	0.084	0.102	0.08
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0.331	0.366	0.344	0.548	0.38	0.408	0.397	0.592	0.049	0.042	0.053	0.044

9	0.387	0.605	0.372	0.342	0.44	0.643	0.448	0.398	0.054	0.038	0.076	0.056
10	0.573	0.393	0.318	0.216	0.615	0.497	0.403	0.33	0.042	0.104	0.085	0.115
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0

```
plot_value_function(
    'FVMC estimates through time vs. true values',
    np.max(Q_track_mc, axis=2),
    optimal_V,
    limit_items=limit_items,
    limit_value=limit_value,
    log=False)
```
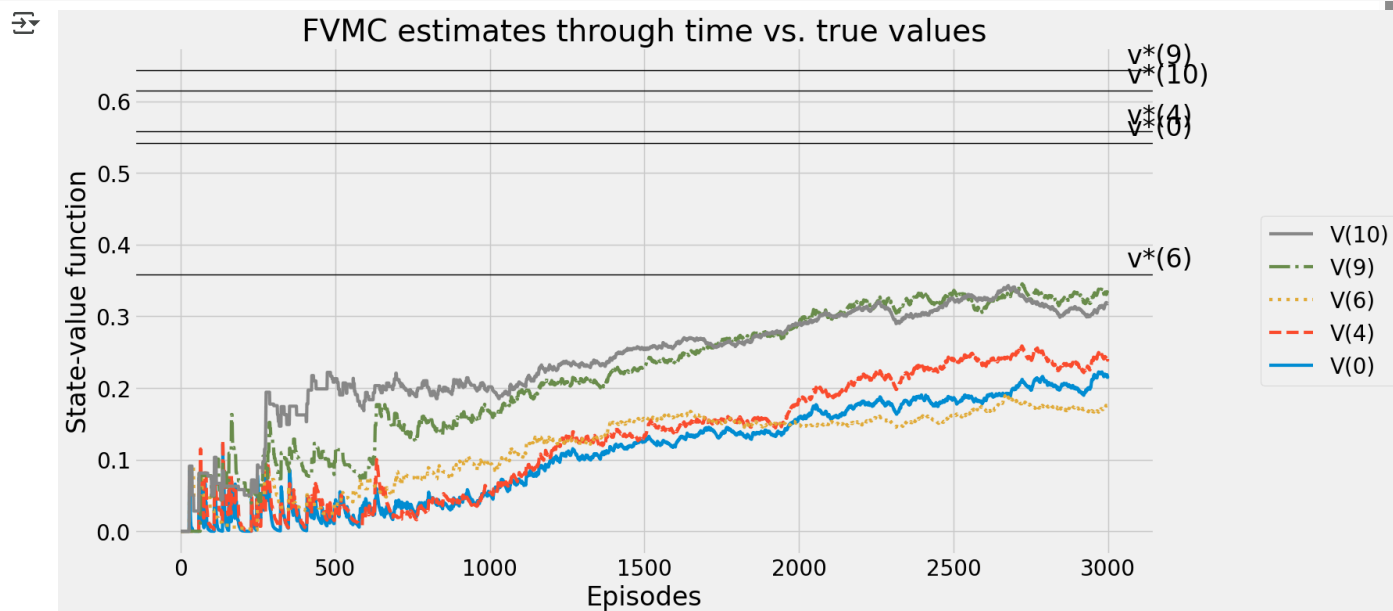


```
plot_value_function(
    'Q-Learning estimates through time vs. true values',
    np.max(Q_track_ql, axis=2),
    optimal_V,
    limit_items=limit_items,
    limit_value=limit_value,
    log=False)
```

Q-Learning estimates through time vs. true values