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UAS Struktur Diskrit 1

1. $A = \{a, b, \{a,c\}, \{\}\}$. $B = \{a, \{a\}, d, e\}$

a) $\{\{a,c\}\} - A = \{\}$

b) $\{a\} - \{A\} = \{a\}$

c) $\{\} - A = \{\}$

d) $B^2 = \{(a,a), (a,\{a\}), (a,d), (a,e), (\{a\},a), (\{a\},\{a\}), (\{a\},d), (\{a\},e), (d,a), (d,\{a\}), (d,d), (d,e), (e,a), (e,\{a\}), (e,d), (e,e)\}$

e) $A \cap P(A) = \{\}$

2. Andaikan $p(n)$ menyatakan proposi bahwa untuk $n \geq 0$, $1^2 + 3^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$.

(1) Basis Induksi : $p(0)$ benar, karena untuk $n=0$ diperoleh

$1^2 = (0+1)(0+1)(0+3)/3$

$1 = 1$ (terbukti)

(2) Langkah Induksi : misalkan $p(n)$ benar, yaitu mengasumsikan bahwa

$1^2 + 3^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$

adalah benar. Kita juga harus memperlihatkan $p(n+1)$ juga benar.

Hal ini dapat ditunjukkan dengan :

$$1^2 + 3^2 + \ldots + (2n+1)^2 + (2(n+1)+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3} + (2n+3)^2$$

$$= \frac{(2n^2+3n+1)(2n+3) + 3(2n+3)(2n+3)}{3}$$

$$= \frac{(2n^2+3n+1)(2n+3) + (6n+9)(2n+3)}{3}$$

$$= \frac{(2n^2+9n+10)(2n+3)}{3}$$

$$= \frac{(n+2)(2n+3)(2n+5)}{3}$$

$$= \frac{((n+1)+1)(2(n+1)+1)(2(n+1)+3)}{3}$$

(terbukti)

Karena langkah 1 & 2 benar, maka untuk $n \geq 0$, $1^2 + 3^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$ adalah benar.

3) FPB (220 , 1400) = ?

$m = 1400$, $n = 220$.

$1400 = 6 \times 220 + 80$ (i) → $80 = 1400 - 6 \times 220$ (v)

$m=220, n=80 \rightarrow 220 = 2 \times 80 + 60$ (ii) → $60 = 220 - 2 \times 80$ (vi)

$m=80, n=60 \rightarrow 80 = 1 \times 60 + 20$ (iii)

$m=60, n=20 \rightarrow 60 = 3 \times 20 + 0$. (iv)

$m = 20$, $n = 0$ → karena $n = 0$, maka FPB (220, 1400) = 20.

Bentuk kombinasi lanjarnya :

$20 = 80 - 1 \times 60$.

Substitusikan pers (vi) $20 = 80 - (220 - 2 \times 80)$

$20 = -1 \times 220 + 3 \times 80$

Substitusikan pers (V) $20 = -1 \times 220 + 3 \times (1400 - 6 \times 220)$

$= 3 \times 1400 - 19 \times 220$

maka bentuk kombinasi lanjarnya

FPB(220,1400) = 20 = $3 \times 1400 + (-19) \times 220$)

4. $R = \{(1,2), (1,3), (2,3), (2,4), (3,1)\}$

$A = \{1, 2, 3, 4\}$

a. Refleksif : Tidak, karena (1,1), (2,2), (3,3), dan (4,4) $\notin R$

b. Simetri : tidak, karena $(1,2) \in R$ namun $(2,1) \notin R$.

c. tolak setangkup : Tidak, karena $(1,3) \in R$ dan $(3,1) \in R$, namun $3 \neq 1$.

d. Transitif : Tidak, karena $(1,2) \in R$, $(2,4) \in R$, namun $(1,4) \notin R$.

pasangan berbentuk		
(a,b)	(b,c)	(a,c)
(1,2)	(2,4)	$\boxed{(1,4)}$

→ $(1,4) \notin R$

5. $R = \{(2,1), (2,3), (3,1), (3,4), (4,1), (4,3)\}$

Himpunan {1, 2, 3, 4}.

a. Klosur refleksif $R = \{ (1,1), (2,1), (2,2), (2,3), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)\}$

~~b. $R^2 = R \circ R = \{$~~

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} , R^2 = R \circ R = M_R \cdot M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$R^3 = R^2 \circ R = M_{R^2} \cdot M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$R^4 = R^3 \circ R = M_{R^3} \cdot M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Klosur transitif $R = R \cup R^2 \cup R^3 \cup R^4$

$$M_R = M_R \vee M_{R^2} \vee M_{R^3} \vee M_{R^4}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Maka klosur transitif $R = \{(2,1), (2,3), (2,4), (3,1), (3,3), (3,4), (4,1), (4,3), (4,4)\}$.