

## 2. Realne funkcije realne varijable - 1. dio

1. Odredite limese funkcije  $f(x) = \frac{1}{x}$  kada  $x \rightarrow 0 \pm$  i  $x \rightarrow \pm\infty$ .
2. Odredite limese funkcije  $f(x) = \frac{1}{x^2}$  kada  $x \rightarrow 0 \pm$  i  $x \rightarrow \pm\infty$ .
3. Izračunajte

- (a)  $\lim_{x \rightarrow 1} \frac{x^2 + 1}{x};$
- (b)  $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{\sqrt{x+2}} - \frac{1}{x} \right);$
- (c)  $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 4}{\sqrt{x^4 + 1}};$
- (d)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{x}};$
- (e)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}};$
- (f)  $\lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2 + 1}}{x + 1};$
- (g)  $\lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2 - 3x} - x + 1}{x}.$

### 4. Izračunajte

- (a)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1};$
- (b)  $\lim_{x \rightarrow 1} \left( \frac{1}{1-x} - \frac{3}{1-x^3} \right);$
- (c)  $\lim_{x \rightarrow \infty} \left( \sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right);$
- (d)  $\lim_{x \rightarrow \pm\infty} \left( \sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x + 3} \right);$
- (e)  $\lim_{x \rightarrow \pm\infty} \left( x - \sqrt{x^2 + 1} \right);$
- (f)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1+x^2}}{\sqrt{1+x} - 1};$
- (g)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + b^2} + x};$

$$(h) \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt[4]{x} - 1};$$

$$(i) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1}.$$

5. Izračunajte

$$(a) \lim_{x \rightarrow \infty} x \sin \frac{1}{x};$$

$$(b) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3};$$

$$(c) \lim_{x \rightarrow a} \frac{\cos x - \cos a}{x - a};$$

$$(d) \lim_{x \rightarrow 0} \frac{2 \arcsin x}{3x};$$

$$(e) \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}.$$

6. Izračunajte

$$(a) \lim_{x \rightarrow 0} \frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}};$$

$$(b) \lim_{x \rightarrow 0} \sqrt[x]{1 + \sin x};$$

$$(c) \lim_{x \rightarrow a} \frac{\ln x - \ln a}{x - a};$$

$$(d) \lim_{x \rightarrow \infty} x(\ln(1+x) - \ln x);$$

$$(e) \lim_{x \rightarrow 0} \frac{e^x - 1}{x};$$

$$(f) \lim_{x \rightarrow \infty} x(e^{\frac{1}{x}} - 1).$$

7. Izračunajte

$$(a) \lim_{x \rightarrow \infty} \left( \frac{x^2 + 2}{2x^2 + 1} \right)^{x^2};$$

$$(b) \lim_{x \rightarrow 0} \left( \frac{\sin 3x}{x} \right)^{x+2}.$$

8. Izračunajte

$$(a) \lim_{x \rightarrow \infty} \frac{2x + 3}{x + \sqrt[3]{x^2}};$$

$$(b) \lim_{x \rightarrow 7} \frac{2 - \sqrt{x-3}}{x^2 - 49};$$

$$(c) \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{\cos 3x};$$

$$(d) \lim_{x \rightarrow -1 \pm} x e^{\frac{1}{x^2-1}}.$$

9. Odredite kako se funkcija  $f(x) = e^{\frac{1}{x}}$  ponaša u točki 0 i  $\pm\infty$ .

10. Odredite kako se funkcija  $f(x) = e^{\frac{1}{x^2}}$  ponaša u točki 0 i  $\pm\infty$ .

11. Kakve prekide imaju funkcije  $f$  u točki  $x_0$ :

$$(a) f(x) = \frac{a^{\frac{1}{x}} - 1}{a^{\frac{1}{x}} + 1}, a > 1 \quad x_0 = 0;$$

$$(b) f(x) = \frac{x}{|x|}, \quad x_0 = 0;$$

$$(c) f(x) = \sqrt{\frac{x^2}{|\sin x|}}, \quad x_0 = 0;$$

$$(d) f(x) = \frac{1 + x^2}{1 + x}, \quad x_0 = -1;$$

$$(e) f(x) = \frac{\sqrt{7+x} - 3}{x^2 - 4}, \quad x_0 = -2;$$

$$(f) f(x) = \frac{\sqrt{7+x} - 3}{x^2 - 4}, \quad x_0 = 2.$$

12. Odredite parametar  $\lambda$  tako da funkcija  $f$  bude neprekidna:

$$(a) f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ \lambda, & x = 2 \end{cases}$$

$$(b) f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ 5x^2 - 2x + \lambda, & x \geq 0 \end{cases},$$

$$(c) f(x) = \begin{cases} e^{-x} + 1, & x \geq 0 \\ x + \lambda, & x < 0 \end{cases}$$