

#### 4. Nizovi i redovi, 3. dio - Rješenja

1. (a)  $\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{n+1}} (x-3)^n, x \in (0, 6);$

(b)  $\ln(1+2x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{n} x^n, x \in \left(-\frac{1}{2}, \frac{1}{2}\right];$

(c)  $\frac{2}{1+2x} = \sum_{n=0}^{\infty} (-1)^n 2^{n+1} x^n, x \in \left(-\frac{1}{2}, \frac{1}{2}\right);$

(d)  $\sin \frac{\pi}{4} x = 1 + \sum_{k=1}^{\infty} (-1)^k \left(\frac{\pi}{4}\right)^{2k} \frac{(x-2)^{2k}}{(2k)!}, x \in \mathbb{R};$

(e)  $\cos(2x-2) = \sum_{k=0}^{\infty} \frac{(-1)^k 2^{2k}}{(2k)!} (x-1)^{2k}, x \in \mathbb{R};$

(f)  $\ln \sqrt{x^2+3x-4} = \frac{1}{2} \ln 6 + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \left(\frac{1}{6^n} + 1\right) (x-2)^n,$   
 $x \in (1, 3];$

(g)  $\ln \frac{1}{\left(\frac{1}{3}x+2\right)^4} = 4 \ln \frac{3}{5} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n 5^n} (x+1)^n, x \in (-6, 4];$

(h)  $\frac{x^2}{1-x} = \sum_{n=0}^{\infty} x^{n+2}, x \in (-1, 1);$

(i)  $\frac{e^x + e^{-x}}{2} = \sum_{k=0}^{\infty} \frac{1}{(2k)!} x^{2k}, x \in \mathbb{R}.$

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4.  $\sum_{n=0}^{\infty} \frac{1}{2^n n!} = \sqrt{e}$