

3. Derivacije i primjene, 1. dio - Rješenja

1. (a) $2x + 3x^2 + \cos x$;
(b) $6x^6 - 5x^4 + 4x^3 + 4x - 2$;
(c) $\frac{1 - x^2}{x^4 + 2x^2 + 1}$;
(d) $\frac{1}{2\sqrt{x}}$;
(e) $\frac{2}{3\sqrt[3]{x}} - \frac{3}{4\sqrt[4]{x}}$;
(f) $-\frac{\frac{1}{2\sqrt{x}} + \frac{1}{6\sqrt[6]{x}} + \frac{1}{3\sqrt[3]{x^2}}}{(1 + \sqrt[3]{x})^2}$.
2. (a) $\cos x + \frac{1}{\cos^2 x}$;
(b) $-\sin x - \frac{\cos x}{\sin^2 x} - \cos x$;
(c) $-\cos x \cdot \frac{\sin x \cos x + \sin^2 x + 1}{2 \sin x \cos x + 1}$;
(d) 0;
(e) $\cos x \cdot \operatorname{arctg} x + \sin x \cdot \frac{1}{1 + x^2}$.
3. (a) $e^x + 2^x \ln 2 + \left(\frac{2}{3}\right)^x \ln \frac{2}{3}$;
(b) $\frac{1}{x}(1 + \log e)$, $\log e = \frac{1}{\ln 10}$;
(c) $\frac{\cos x}{x^3} - \frac{3 \sin x}{x^4} + e^x \cos x - e^x \sin x - 3x^2 \log x - x^2 \log e - \frac{2 \log e}{x}$;
(d) $-\frac{1}{(\sin^2 x)x \ln x} - \frac{\operatorname{ctg} x(\ln x + 1)}{x^2 \ln^2 x} + 3e^x + 3xe^x$.
4. (a) $6x^3 + 12x$;
(b) $\operatorname{ctg} x$;
(c) $\frac{1}{2\sqrt{xe^x}}(e^x + xe^x)$;
(d) $\frac{4x - 1}{2x\sqrt{4x - 1}}$;

- (e) $\frac{2x - e^x}{2\sqrt{x^2 - e^x}} - \frac{1}{x\sqrt{x^2 - 1}};$
- (f) $\frac{\sin^3 t}{\cos^5 t} - \frac{\sin t}{\cos^3 t} + \frac{\sin t}{\cos t};$
- (g) $\frac{1}{x\sqrt{2x - 1}};$
- (h) $\frac{4\ln(2x + 1)}{2x + 1};$
- (i) $\frac{1}{1 - x^2};$
- (j) $\frac{x(2x + 2\cos(2x)(\ln x + \cos(2x + 3)) - (x^2 + \sin 2x)(1 - 2x\sin(2x + 3)))}{x(\ln x + \cos(2x + 3))^2};$
- (k) $-e^{-x} + \frac{\ln 2}{2} \cdot 2^{\sin \frac{x}{2}} \left(\cos \frac{x}{2} \right) + 2\sin x \cos x;$
- (l) $\frac{\ln 2}{2\sqrt{x}(1 + x)} 2^{\arctg \sqrt{x}};$
- (m) $-\frac{1}{\sqrt{x^2 + 1}};$
- (n) $\frac{2\sin x \cos x + x}{2\sqrt{\operatorname{tg} x} \cos^2 x};$
- (o) $\frac{5}{9} \cdot \frac{(t - 2)^8}{(2t + 1)^{10}};$
- (p) $\frac{1}{x(1 + \ln^2 x)} + \frac{x}{(x^2 + 1)\sqrt{\ln(x^2 + 1)}};$
- (q) $-\frac{2x \ln 5}{5x^2};$
- (r) $\frac{e^x(1 + x)}{2\sqrt{xe^x}} e^{\sqrt{xe^x}}.$
5. (a) $x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right);$
- (b) $\frac{(x^2 + 2x + 3)^{15}(2x + 5)^{10}}{(5x - 9)^{13}} \cdot \left(15 \cdot \frac{2x + 2}{x^2 + 2x + 3} + 10 \cdot \frac{2}{2x + 5} - 13 \cdot \frac{5}{5x - 9} \right);$
- (c) $(\ln x)^x \left[\ln(\ln x) + \frac{1}{\ln x} \right];$
- (d) $\frac{(\cos x)^{\sin x}}{x^2 + 3} \left[\cos x \ln(\cos x) - \frac{\sin^2 x}{\cos x} - \frac{2x}{x^2 + 3} \right];$

- (e) $\frac{\cos x}{x \sin x} - \frac{\ln(\sin x)}{x^2}$;
- (f) $e^{\cos x}(-\sin x) + (\cos x)^x [\ln(\cos x) - x \operatorname{tg} x]$;
- (g) $\frac{\sqrt{(x-2)(x-4)}}{(x+1)(x+3)} \left[\frac{1}{2(x-2)} + \frac{1}{2(x-4)} - \frac{1}{x+1} - \frac{1}{x+3} \right]$;
- (h) $\sqrt{x} \cdot \frac{1 - \ln x}{x^2}$.
6. (a) $\frac{e^x - 3x^2y - y^3}{x^3 + 3xy^2}$;
- (b) $\frac{e^{x+y} - y}{x + \cos y - e^{x+y}}$;
- (c) $-\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$;
- (d) $-\frac{\sqrt{y}}{\sqrt{x}}$;
- (e) $\frac{2ax - 2xy^2}{4y^3 + 2x^2y}$.
7. (a) $f^{(n)}(x) = 0$, za $n \geq 6$, $f^{(n)}(0) = \begin{cases} 5!, & n = 5 \\ 0, & n \neq 5 \end{cases}$
- (b) $f^{(n)}(x) = (-1)^n \cdot n! \cdot x^{-(n+1)}$, $\forall n$, $f^{(n)}(-1) = -n!$
- (c) $f^{(n)}(x) = \cos\left(x + \frac{n\pi}{2}\right)$, $f^{(n)}(\pi) = \begin{cases} 0, & n \text{ neparan} \\ -1, & n = 4k \\ 1, & n = 4k + 2 \end{cases}$
- (d) $f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right)$, $f^{(n)}\left(\frac{\pi}{2}\right) = \begin{cases} 0, & n \text{ neparan} \\ 1, & n = 4k \\ -1, & n = 4k + 2 \end{cases}$
- (e) $f^{(n)}(x) = -(n-1)!(1-4x)^{-n} \cdot 4^n + (-1)^n(n-1)!(1+4x)^{-n} \cdot 4^n$
 $f^{(n)}(0) = 4^n(n-1)! [(-1)^n - 1]$
8. (a) $\frac{\sin t}{1 - \cos t}$;
- (b) $\frac{(t^2 - 1)(t + 2)^2}{t^2(t^2 + 4t + 2)}$;
- (c) $-\frac{20(t+3)^4}{(t-3)^6} \cdot \frac{\sqrt{\sin(3t)}}{\cos(3t)}$.