

5. Linearna algebra, 2. dio

1. Izračunajte rang matrice

$$(a) \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix};$$

$$(b) \begin{bmatrix} 2 & -3 & 16 & 1 \\ 1 & 6 & -2 & 3 \\ 1 & 3 & 2 & 2 \end{bmatrix};$$

$$(c) \begin{bmatrix} 1 & -1 & 3 \\ 2 & -1 & 3 \\ 3 & 1 & 3 \end{bmatrix};$$

$$(d) \begin{bmatrix} 3 & 1 & 1 & 4 \\ 0 & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{bmatrix}.$$

2. Izračunajte determinantu matrice

$$(a) \begin{bmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & 1 & 8 \end{bmatrix};$$

$$(b) \begin{bmatrix} 1 & 5 & -1 & 1 \\ 2 & 0 & 1 & -1 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 0 & -1 \end{bmatrix};$$

$$(c) \begin{bmatrix} 3 & -5 & 1 & 4 \\ 1 & 3 & 0 & -2 \\ -3 & 5 & 2 & 1 \\ -1 & -3 & 5 & 7 \end{bmatrix};$$

$$(d) \begin{bmatrix} -2 & 5 & 0 & -1 & 3 \\ 1 & 0 & 3 & 7 & -2 \\ 3 & -1 & 0 & 5 & -5 \\ 2 & 6 & -4 & 1 & 2 \\ 0 & -3 & -1 & 2 & 3 \end{bmatrix}.$$

3. Izračunajte inverznu matricu matrice primjenom Gauss-Jordanovog postupka:

$$(a) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

$$(b) \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{bmatrix};$$

$$(c) \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}.$$

4. Izračunajte inverznu matricu matrice primjenom Cramerova pravila:

$$(a) \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 5 \\ 4 & 2 & 7 \end{bmatrix};$$

$$(b) \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix};$$

$$(c) \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}.$$

5. Izračunajte inverznu matricu matrice

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ ako je } (ad - bc) \neq 0.$$

6. Riješite matricne jednadžbe

$$(a) (AX)^{-1} + X^{-1} = B, A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 1 & -3 \end{bmatrix};$$

$$(b) AX + 2B = C + BX, A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & -3 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix};$$

$$(c) \quad A(A+B)BX = I, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$(d) \quad AX^{-1}B - C = AX^{-1}, \quad A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

7. Riješite nehomogen sustav

$$\begin{cases} -x_2 + 3x_3 = 7 \\ -2x_1 + x_2 - x_3 = -3 \\ 3x_1 - 2x_2 = 2 \end{cases}$$

- (a) s pomoću Cramerova pravila;
- (b) rješavanjem matrične jednadžbe;
- (c) Gaussovom metodom eliminacije.

8. Primjenom Cramerova pravila riješite sustav

$$\begin{cases} 2x_1 + x_2 + x_3 = 2 \\ x_1 + 2x_2 + x_3 = 3 \\ x_1 + x_2 + 2x_3 = -1 \end{cases}$$

9. Izračunajte $x \in \mathbb{R}$ takav da matrica

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{bmatrix}$$

bude singularna.

10. Za koju vrijednost $x \in \mathbb{R}$ je realna matrica

$$\begin{bmatrix} \ln(x-3) & -2 & 6 \\ x & -2 & 5 \\ 0 & -1 & 3 \end{bmatrix}$$

invertibilna?