4. Nizovi i redovi - 1. dio

1. Izračunajte limese sljedećih nizova

(a) $a_n = \dfrac{2n^3 - 1}{2 - n^3}$;

(b) $a_n = \dfrac{3}{2 \operatorname{arctg} n}$;

(c) $a_n = \dfrac{n^2(n^2+1)}{2^n(n^2-1)}$;

(d) $a_n = \dfrac{(n+1)(3^n+1)}{2 \cdot 3^n + 1}$;

(e) $a_n = \left(1 - \dfrac{1}{3n}\right)^n$;

(f) $a_n = \left(\dfrac{n+1}{n-1}\right)^n$;

(g) $a_n = \left(\dfrac{2n+3}{2n+1}\right)^{(n+1)}$;

(h) $a_n = n[\ln(n+3) - \ln n]$;

(i) $a_n = \dfrac{2^n}{\ln\left(1 + \frac{1}{2n}\right)^{n+1}}$;

(j) $a_n = n\sqrt[n]{\sin\dfrac{2}{n}}$;

(k) $a_n = \dfrac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$;

(l) $a_n = \dfrac{(-3)^n + 4^n}{(-3)^{n+1} + 4^{n+1}}$.

2. Izračunajte limese sljedećih nizova

(a) $a_n = \dfrac{1}{n^3} \displaystyle\sum_{k=1}^{n} k(k+1)$;

(b) $a_n = \dfrac{1}{n^2} \displaystyle\sum_{k=1}^{n} k$;

(c) $a_n = \dfrac{1}{n^3} \displaystyle\sum_{k=1}^{n} k^2$;

(d) $a_n = \dfrac{1}{n+1} \displaystyle\sum_{k=1}^{n} (2k-1) - \dfrac{2n+1}{2}$;

(e) $a_n = \dfrac{1}{n^3} \displaystyle\sum_{k=1}^{n} \sum_{i=1}^{k} i$.

3. Izračunajte limese sljedećih nizova

(a) $a_n = \displaystyle\prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right)$;

(b) $a_n = \displaystyle\prod_{k=2}^{n} \frac{k^3 - 1}{k^3 + 1}$;

(c) $a_n = \displaystyle\prod_{k=2}^{n} \frac{k^2 + k - 2}{k(k+1)}$;

(d) $a_n = \displaystyle\sum_{k=1}^{n} \frac{1}{k \cdot (k+1)}$;

(e) $a_n = \displaystyle\sum_{k=2}^{n+1} \ln\left(1 - \frac{1}{k^2}\right)$;

(f) $a_n = n\left(\ln\dfrac{1}{2} + \displaystyle\sum_{k=1}^{n} \ln\frac{(k+1)^2}{k(k+2)}\right)$.

4. Odredite sva gomilišta niza

(a) $a_n = \dfrac{3n^2 + 2n}{n^2 - 1} \cdot \dfrac{1 + (-1)^n}{2} + \dfrac{1 - (-1)^n}{n}$;

(b) $a_n = \left(1 + \dfrac{1}{n}\right) \cos n\pi.$

5. Zadan je niz

$$a_n = \frac{(1-a)n^2 + 2n + b}{an^2 + n + 1}.$$

Odredite a i b takve da je $\lim\limits_{n\to\infty} a_n = 2$ i $a_1 = 0$.

6. Zadani su nizovi

$$a_n = \frac{cn^2 + 1}{(c+3)n^2 + cn + 4} \quad \text{i}$$

$$b_n = \frac{1}{c(c+1)} + \frac{1}{(c+1)(c+2)} + \cdots + \frac{1}{(c+n-1)(c+n)}.$$

Odredite $c > 0$ takvo da je $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = 4$.

7. Odredite limese sljedećih nizova

(a) $a_n = \dfrac{\sqrt[3]{n^2}\sin n}{n+1}$;

(b) $a_n = \dfrac{n \sin(n!)}{n^2+1}$.

8. Odredite limese sljedećih nizova

(a) $a_n = \dfrac{1}{\sqrt[3]{n^3+1}} + \dfrac{1}{\sqrt[3]{n^3+2}} + \cdots + \dfrac{1}{\sqrt[3]{n^3+n}}$;

(b) $a_n = \dfrac{2}{\sqrt{n^2+1}} + \dfrac{2}{\sqrt{n^2+\frac{1}{2}}} + \cdots + \dfrac{2}{\sqrt{n^2+\frac{1}{n}}}$.