EXERCISE 9.1 (Part 1)

1. (a.) $y'' = t + y + y'$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= t + u_1 + u_2 \end{aligned}}$$

(b) $y''' = y'' + ty$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= u_3 + tu_1 \end{aligned}}$$

(3) $y''' = y'' - 2y' + y - t + 1$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= u_3 - 2u_2 + u_1 - t + 1 \end{aligned}}$$

Exercise 9.2

b.) (1) $y'' = y'(1-y^2) - y$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= u_2(1 - (u_1)^2) - u_1 \end{aligned}}$$

(2.) $y''' = -yy''$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= -u_1 u_3 \end{aligned}}$$

(3) $y_1'' = -GMy_1/(y_1^2 + y_2^2)^{3/2}$ $\quad y_2'' = -GM\, y_2/((y_1^2 + y_2^2)^{3/2}$

$$\boxed{\begin{aligned} u_1' &= u_2 \\ u_2' &= -GMu_1/((u_1)^2 + (u_3)^2)^{3/2} \qquad u_3' = u_4 \\ u_4' &= -GM\, u_3/(u_1^2 + u_3^2)^{3/2} \end{aligned}}$$

Exersise 9.4 (Part 4)

(a) Are solutions to the ODE stable?

- eigenvalue $= -5$
  - negative value → solution decays exponentially
  - stable solutions

(b) Is Euler's method stable for this ODE using this step size?

$$= |1 + h\lambda|$$
$$= |1 + 0.5(-5)|$$
$$= 1.5 > 1$$

Not stable.

c.) Compute the numerical value for the approximate solution at $t = 0.5$ given by Euler Method

$$y = 1 + 0.5(-5)$$
$$y = -1.5$$

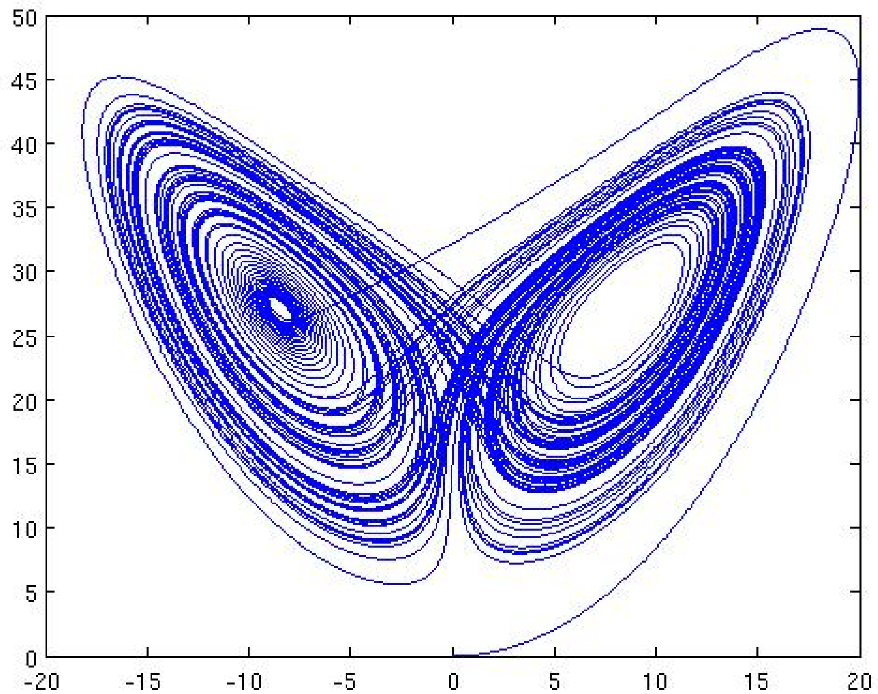
d.) Is the backward Euler method stable for this ODE using this stepsize?

Backward Euler is unconditionally stable.

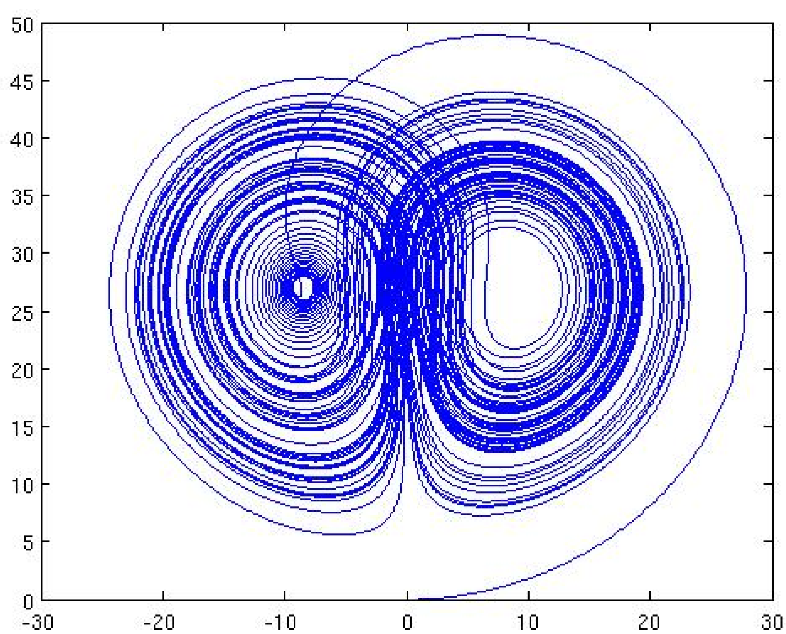
2.) Compute the numerical value for the approximate solution at $t = 0.5$ given backward Euler Method?

$$y = 1 + 0.5(-5y)$$
$$y - 0.5(-5y) = 1$$
$$y = \frac{1}{1 - (0.5 \times -5)}$$
$$y = \frac{1}{3.5} \approx 0.285$$

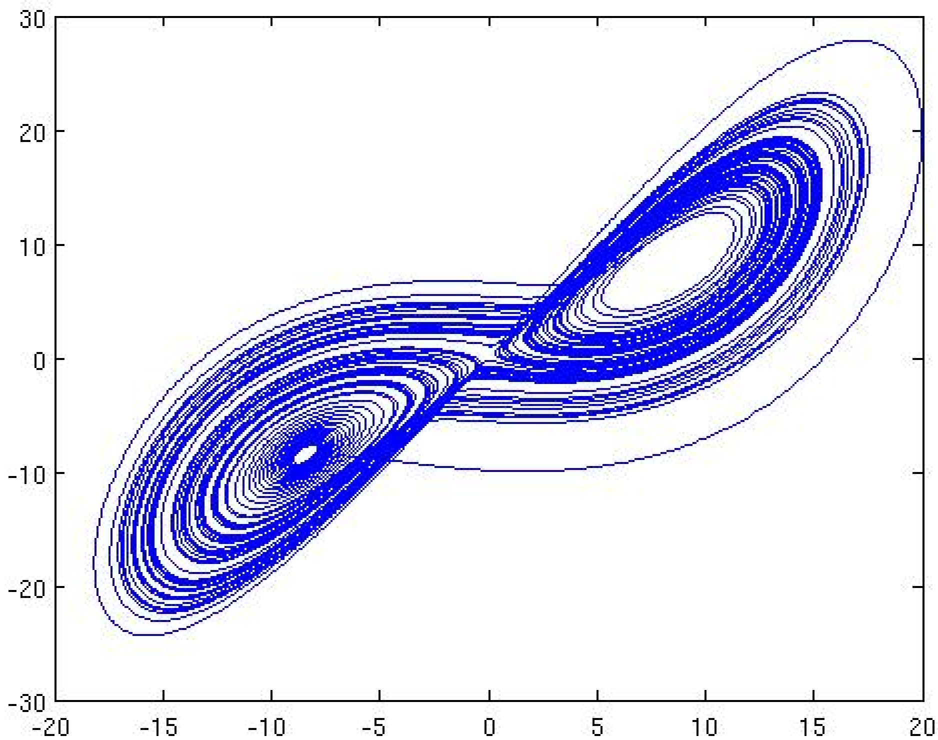
Part 2
y1 vs y3



y2 vs y3

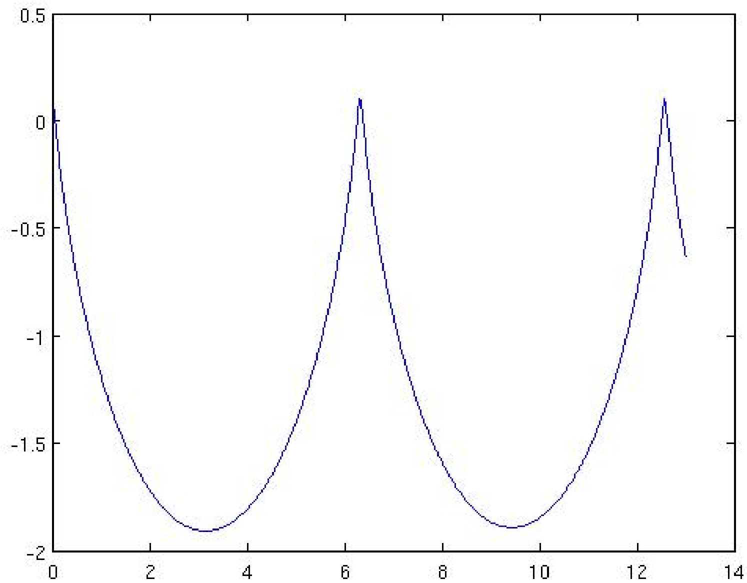


y1 vs y2

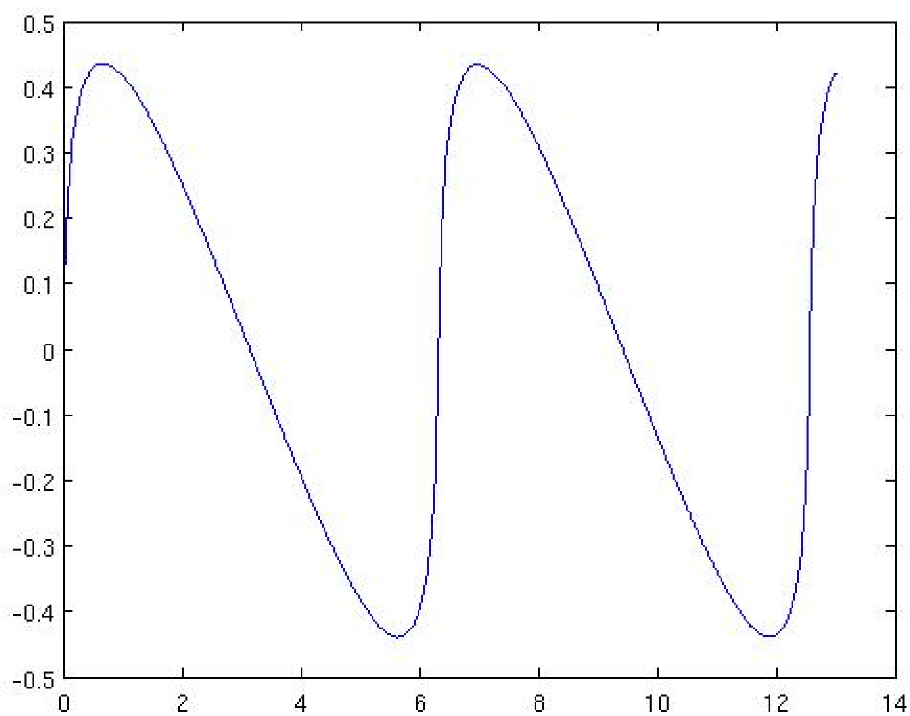


Part 3: plots for e = 0 , 0.5 , 0.9

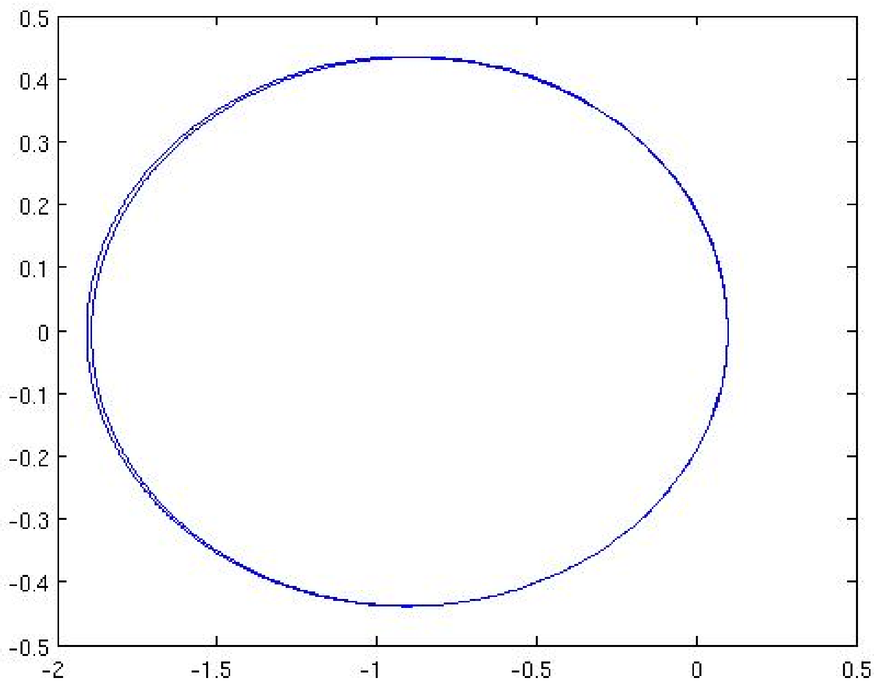
x vs t:



y vs t:



y vs x:



b.)

energy for each tolerance and eccentricity(Standard deviation):
0.5 = 2.086e-03
0.9 = 1.933e-03
angular momentum for each tolerance and eccentricity(Standard deviation):
0.5 = 4.169e-04
0.9 = 2.393e-04