Exercise 9.4 (Part 4)

(a.) Are solutions to the ODE stable?

- eigenvalue = -5
  - negative value → solution decays exponentially
  - stable solutions

(b.) Is Euler's method stable for this ODE using this step size?

$= |1 + h\lambda|$

$= |1 + 0.5(-5)|$

$= 1.5 > 1$

Not stable.

c.) Compute the numerical value for the approximate solution at $t = 0.5$ given by Euler Method

$y = 1 + 0.5(-5)$

$y = -1.5$

d.) Is the backward Euler method stable for this ODE using this stepsize?

Backward Euler is unconditionally stable.

e.) Compute the numerical value for the approximate solution at $t = 0.5$ given backward Euler Method?

$y = 1 + 0.5(-5y)$

$y - 0.5(-5y) = 1$

$y = \frac{1}{1 - (0.5 \times -5)}$

$y = \frac{1}{3.5} \approx 0.285$