MAS291 - HOMEWORK CHAP 4

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4-49

Integration by parts is required. The probability density function for the diameter of a drilled hole in millimeters is $10e^{-10(x-5)}$ for $x > 5mm$. Although the target diameter is 5 millimeters, vibrations, tool wear, and other nuisances produce diameters greater than 5 millimeters.
(a) Determine the mean and variance of the diameter of the holes.
(b) Determine the probability that a diameter exceeds 5.1 millimeters.

Solution:

The diameter of the drilled hole is the random variable, denote as X.

The probability density function of random variable X is: $f(x) = 10e^{-10(x-5)}, x > 5$

a)

- $E(X) = \int_5^\infty x10e^{-10(x-5)}dx \ = e^{50}\int_5^\infty x10e^{-10x}dx$

Let $u = -e^{-10x} \Rightarrow du = 10e^{-10x}dx,$ then:

$$
\begin{aligned}
E(X) &= e^{50}\int_5^\infty xdu \\
&= e^{50}\left(ux\Big|_5^\infty - \int_5^\infty udx\right) \\
&= e^{50}\left(-e^{-10x}x\Big|_5^\infty - \int_5^\infty -e^{-10x}dx\right) \\
&= e^{50}\left(0 + 5e^{-50}\right) + e^{50}\int_5^\infty e^{-10x}dx \\
&= 5 + e^{50}\tfrac{-1}{10}e^{-10x}\Big|_5^\infty \\
&= 5 + \tfrac{1}{10} \\
&= 5.1
\end{aligned}
$$

- $E(X^2) = \int_5^\infty x^2 10e^{-10(x-5)}dx \ = e^{50}\int_5^\infty x^2 10e^{-10x}dx$

Let $u = -e^{-10x} \Rightarrow du = 10e^{-10x}dx,\ v = x^2 \Rightarrow dv = 2xdx$

$$
\begin{aligned}
E(X^2) &= e^{50}\int_5^\infty vdu \\
&= e^{50}\left(uv\Big|_5^\infty - \int_5^\infty udv\right) \\
&= e^{50}\left(-e^{-10x}x^2\Big|_5^\infty - \int_5^\infty -e^{-10x}2xdx\right) \\
&= e^{50}\left(-e^{-10x}x^2\Big|_5^\infty\right) + \tfrac{2}{10}e^{50}\int_5^\infty 10e^{-10x}xdx \\
&= e^{50}\left(0 + 25e^{-50}\right) + \tfrac{2}{10}E(X) \\
&= 25 + 0.2 \times 5.1 \\
&= 26.02
\end{aligned}
$$

- $V(X) = E(X^2) - E(X)^2 = 26.02 - 5.1^2 = 0.01$

b)

$$P(X > 5.1) = \int_{5.1}^\infty 10e^{-10(x-5)}dx \ = e^{50}\int_{5.1}^\infty 10e^{-10x}dx \ = e^{50}(-e^{-10x}\Big|_{5.1}^\infty) = e^{50} \times e^{-51} = e^{-1} \approx 0.36788$$

4-71

The compressive strength of samples of cement can be modeled by a **normal distribution** with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.

(a) What is the probability that a sample's strength is less than $6250 kg/cm^2$?

(b) What is the probability that a sample's strength is between 5800 and $5900 kg/cm^2$?

(c) What strength is exceeded by 95% of the samples?

Solution: $\mu = 6000,\ \sigma = 100$

Let X denote the normal random variable (the compressive strength of samples of cement). Standardize: $Z = \dfrac{X - 6000}{100}$

a)

$$P(X < 6250) = P(Z < \frac{6250 - 6000}{100}) = P(Z < 2.5) = 0.993790$$

b)

$$\begin{aligned} P(5800 < X < 5900) &= P(\frac{5800 - 6000}{100} < Z < \frac{5900 - 6000}{100}) \\ &= P(-2 < Z < -1) \\ &= P(Z < -1) - P(Z < -2) \\ &= 0.158655 - 0.022750 \\ &= 0.135905 \end{aligned}$$

c)

$$P(X > x) = 0.95 \Leftrightarrow P(Z > \frac{x - 6000}{100}) = 0.95$$
$$\Rightarrow P(Z \leqslant \frac{x - 6000}{100}) = 0.05 \Rightarrow \frac{x - 6000}{100} \approx -1.64 \Rightarrow x \approx 5836$$

4-93

An article in *International Journal of Electrical Power & Energy Systems* ["Stochastic Optimal Load Flow Using a Combined Quasi–Newton and Conjugate Gradient Technique" (1989, Vol.11(2), pp. 85–93)] considered the problem of optimal power low in electric power systems and included the effects of uncertain variables in the problem formulation. The method treats the system power demand as a normal random variable with 0 mean and unit variance.

(a) What is the power demand value exceeded with 95% probability?

(b) What is the probability that the power demand is positive?

(c) What is the probability that the power demand is more than – 1 and less than 1?

Solution:

Let X be power demand with mean = 0, variance = 1 ⇒ X is standard normal distribution

a)

$$P(X > x) = 0.95 \Rightarrow P(X \leqslant x) = 1 - P(X > x) = 1 - 0.95 = 0.05$$
$$\Rightarrow x \approx -1.64$$

b)

$$P(X > 0) = 1 - P(X \leqslant 0) = 1 - 0.5 = 0.5$$

c)

$$P(-1 < X < 1) = P(X < 1) - P(X < -1) = 0.841345 - 0.158655 = 0.68269$$