LOGIC

LOGIC IS A SCIENCE OF THE NECESSARY LAWS OF THOUGHT ...

(KANT, 1785)

FOUNDATIONS OF LOGIC

- Propositional logic
  - Operators
  - Equivalence rules
- Predicate logic
- Rules of Inference

$\neg \quad \wedge \quad \vee \quad \rightarrow$

$\equiv$

- Design of digital electronic circuits.
- Express conditions in programs.
- Query to databases & search engines.

$\forall x P(x)$

- Express and manipulate statements in mathematics and computer science.

$\dfrac{P}{\therefore Q}$

- Produce valid arguments.

PROPOSITIONS

- **Proposition**: a declaration that is *true or false* but not both.

Proposition	**NOT a proposition**
p: Hanoi is the capital of Vietnam. q: 1 + 1 = 3. h: The moon is made of green cheese.	What time is it? Read this chapter carefully. x + 1 = 2. Oh no!

OPERATORS/CONNECTIVES

Compound Proposition	Symbol
My PC does not run Linux.	$\neg$
He is young and strong.	$\wedge$
You are going to take the final exam on Friday or Saturday.	$\oplus$
Experience with C++ or Java is required.	$\vee$
I will be shot if I know.	$\boxed{?}$
I go to cinema if and only if it rains.	$\boxed{?}$

NEGATION OF A PROPOSITION

$\neg$p: *negation* of p

Proposition	Negation
❑ p: Hanoi is the capital of Vietnam. ❑ q: 1 + 1 = 3.	❑ $\neg$p: Hanoi is not the capital of Vietnam. ❑ $\neg$q: 1 + 1 $\neq$ 3.

∧ CONJUNCTION OF PROPOSITIONS

$p \land q$: the conjunction of p and q Read: p and q

p	q	$p \land q$
T	T	T
T	F	F
F	T	F
F	F	F

The only **TRUE** case

EXAMPLES

- Nam is young (y) and strong (s).

  In symbols: $y \wedge s$

- I know (k) but I say nothing (s).

  In symbols: $k \wedge s$

∨ DISJUNCTION OF PROPOSITIONS

- p∨q: the disjunction of p and q

  Read: p or q

p	q	p ∨ q
F	F	F
T	F	T
F	T	T
T	T	T

The only **FALSE** case

p: 3 > 2

q: 1 > 2

p∨q: 3 > 2 or 1 > 2

⊕ EXCLUSIVE OR

- Nickname: XOR
- Symbol: ⊕

p	q	p ⊕ q
F	F	F
F	T	T
T	F	T
T	T	F

DIFERENT FROM ∨

CONDITIONAL STATEMENT

$p \Rightarrow r$

Read: if p, then r

Nickname: implication

s	h	s → h
It is sunny.	It is hot.	If it is sunny, then it is hot.

IF _ THEN

F ? F T

T ? T T

F ? T T

T ? F F

The only
FALSE case

A useful way to understand:
think of an obligation or a contract.

QUIZ

- Suppose p ⍰ r is false, find the value of each of these propositions:
  - r ⍰ p
  - $(\neg p) \vee r$
  - $p \oplus r$
  - $(\neg r)$ ⍰ p

BICONDITIONAL STATEMENT

$p \leftrightarrow q$

Read: p if and only if q

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

A shape is called a square
if and only if
it has 4 right angles.

PRECEDENCE

(1) In parentheses from inner to outer

(2) $\neg$

(3) $\wedge$

(4) $\vee$

(5) $\rightarrow$

$\neg p \vee q \wedge r$ means $(\neg p) \vee (q \wedge r)$

LOGICAL EQUIVALENCE

- He is young and strong $\equiv$ He is strong and young.
- Commutative laws: $p \wedge q \equiv q \wedge p$, $p \vee q \equiv q \vee p$

QUIZ

- $p \vee T \equiv$ **?**
- $p \wedge p \equiv$ **?**
- $p \wedge F \equiv$ **?**
- $p \wedge \neg p \equiv$ **?**
- $\neg(\neg p) \equiv$ **?**

DE MORGAN LAWS

A. De Morgan
(1806-1871)

- $\overline{p \wedge q} \equiv \overline{p} \vee \overline{q}$
- $\overline{p \vee q} \equiv \overline{p} \wedge \overline{q}$

Ex. Find the *negation* of the statement
Bob knows Python and Java.

In symbols: $P \wedge J$
Apply De Morgan law: $\overline{P \wedge J} \equiv \overline{P} \vee \overline{J}$

➔ *Bob does not know Python or Java.*

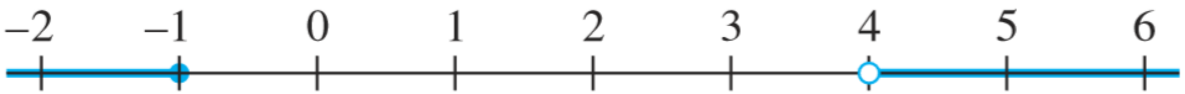
DE MORGAN LAWS

Write the negation of

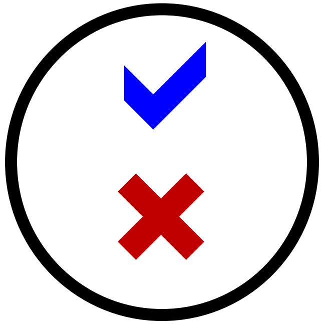
$$-1 < x \leq 4.$$

$-1 < x \leq 4$ means $-1 < x$ and $x \leq 4$.

Its negation is $-1 \nless x$ or $x \nleq 4$, which is eq. to $-1 \geq x$ or $x > 4$.



The negation is not $-1 \nless x$ and $x \nleq 4$.



QUIZ

- p ∨ q ≡ ¬p ⍰ q
- p ⍰ q ≡ q ⍰ p
- p ⍰ q ≡ ¬p ∨ q
- A ⍰ B ≡ ¬B ⍰ ¬A

DISTRIBUTIVE LAWS

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

IMPORTANT EQUIVALENCES

- De Morgan laws $$\begin{cases} \overline{p \wedge q} \equiv \overline{p} \vee \overline{q} \\ \overline{p \vee q} \equiv \overline{p} \wedge \overline{q} \end{cases}$$
- A → B ≡ ¬A ∨ B
- A → B ≡ ¬B → ¬A
- $\overline{\overline{p}} \equiv p$

PREDICATES. QUANTIFIERS

- 4 is a prime. // proposition
- x is a prime. // not a proposition

  x is a prime // a predicate
- Let P(x) be the statement *x is a prime.*

➔ P(3): 3 is a prime

QUIZ

Let $Q(x, y)$ be the statement $x + y = xy$, where x and y real numbers.

Find the truth value of each of these statements:

- $Q(2, 1)$
- $Q(3, 1)$
- $\forall y Q(0, y)$
- $\exists x Q(x, 3)$

NEGATING

- $\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$
- $\overline{\exists x P(x)} \equiv \forall x \overline{P(x)}$

$$\frac{P}{\therefore Q}$$

RULES OF INFERENCE

- Laws of thought
- Rules for producing valid arguments
- Rules for avoiding fallacies
- Rules for making draws from a hypothesis

$$\begin{array}{l} A \Rightarrow B \\ A \\ \hline \therefore B \end{array}$$

MODUS PONES

❶ If A, then B.

❷ A.

❸ Therefore, B.

If $\sqrt{2} > \frac{3}{2}$, then $\left(\sqrt{2}\right)^2 > \left(\frac{3}{2}\right)^2$.

We know that $\sqrt{2} > \frac{3}{2}$.

Therefore, $2 > \frac{9}{4}$.

Valid argument with false conclusion

$A \Rightarrow B$

$\neg B$

$\therefore \neg A$

MODUS TOLLENS

❶ If A, then B.

❷ ¬B

❸ Therefore, ¬A.

If this figure is a triangle, then the sum of its interior angles is 180°.
The sum of the interior angles of this figure is not 180°.
Therefore, this figure is not a triangle.

If logic is easy, then I am a monkey's uncle.
I am not a monkey's uncle.
Therefore, logic is not easy.

OTHER RULES

$$\begin{array}{l} ❶ \lor ❷ \\ \neg ❶ \\ \hline \therefore ❷ \end{array}$$

This real number is rational or it is irrational.
This real number is not rational.
Therefore, this real number is irrational.

$$\begin{array}{l} ❶ \land ❷ \\ \hline \therefore ❶ \end{array}$$

Sandra knows Java and Sandra knows C++.
Therefore, Sandra knows C++.

$$\begin{array}{l} ❶ \Rightarrow ❷ \\ ❷ \Rightarrow ❸ \\ \hline \therefore ❶ \Rightarrow ❸ \end{array}$$

If I go to the movies, I won't finish my homework.
If I don't finish my homework, I won't do well on the exam tomorrow.
Therefore, if I go to the movies, I won't do well on the exam tomorrow.

•≠● FALLACIES

If this number is larger than 2, then its square is larger than 4.
This number is not larger than 2.
Therefore, the square of this number is not larger than 4.

If Jules solved this problem correctly, then Jules obtained the answer 2.
Jules obtained the answer 2.
Therefore, Jules solved this problem correctly.

If apes are intelligent, then apes can solve puzzles.
Apes can solve puzzles.
Therefore, apes are intelligent.

QUIZ

If this number is larger than 2, then its square is larger than 4.
This number is not larger than 2.
Therefore, the square of this number is not larger than 4.

DIY

If Sarah knows Java, then she knows C++.
Sarah doesn’t know C++.
Therefore, Sarah doesn’t know Java.

If Lin eats banana every day, then she is healthy.
Lin is not healthy.
Therefore, Lin does not eat banana every day.

SUMMARY – P.28

- Propositional logic
- Logical equivalences
- Predicates & Quantifiers
- Rules of inference

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation l

TABLE 7 Logical Equivalenc
Involving Conditional Stateme

$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q$

$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q)$ –

$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q$

$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q)$ –

Assignment 1

THANKS

Chapter 1. Rosen Textbook

1.1 (x) , 1.2 (x + 7 mod 30),
1.3 (3x + 5 mod 31),
1.4 (11x - 5 mod 33)
1.5 (5x + 11 mod 35)
1.6 (3 - x mod 23)