# MAS291 - HOMEWORK CHAP 10

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## 10-31

An article in Radio Engineering and Electronic Physics [1984, Vol. 29 No. (3), pp. 63–66] investigated the behavior of a stochastic generator in the presence of external noise. The number of periods was measured in a sample of 100 trains for each of two different levels of noise voltage, 100 and 150 mV. For 100 mV, the mean number of periods in a train was 7.9 with
s = 2.6 For 150 mV, the mean was 6.9 with s = 2.4.
(a) It was originally suspected that raising noise voltage would reduce the mean number of periods. Do the data support this claim? Use α= 0.01 and assume that each population is normally distributed and the two population variances are equal. What is the P-value for this test?
(b) Calculate a confidence interval to answer the question in part (a).

***Solution:***

$n_1 = 100,\ \overline{x}_1 = 7.9,\ s_1 = 2.6$
$n_2 = 100,\ \overline{x}_2 = 6.9,\ s_2 = 2.4$

a)

**Parameter of interest:** the difference in mean number of periods, $\mu_1 - \mu_2$ and $\Delta_0 = 0$

**Null hypothesis:** $H_0 : \mu_1 = \mu_2$

**Alt. hypothesis:** $H_1 : \mu_1 > \mu_2$

**Test statistic:** $t_0 = \dfrac{\overline{x_1} - \overline{x_2} - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{\overline{x_1} - \overline{x_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

**Computations:**

+ The pooled estimator of $\sigma^2$:

$$S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(100-1)2.6^2 + (100-1)2.4^2}{100+100-2}} = 2.5$$

+ Test statistic: $t_0 = \dfrac{\overline{x_1} - \overline{x_2} - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{7.9 - 6.9}{2.5\sqrt{\frac{1}{100} + \frac{1}{100}}} = 2.828$

+ Degree of freedom:

$$v = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}{\dfrac{(s_1^2/n_1)^2}{n_1-1} + \dfrac{(s_2^2/n_2)^2}{n_2-1}} = \frac{\left(\dfrac{2.6^2}{100} + \dfrac{2.4^2}{100}\right)}{\dfrac{(2.6^2/100)^2}{100-1} + \dfrac{(2.4^2/100)^2}{100-1}} \approx 196$$

+ $t_\alpha = t_{0.01,196} = 2.345$

**Conclusion:**

$t_0 = 2.828 > t_\alpha = 2.345 \Rightarrow$ reject $H_0$

Hence, the data support the claim that raising noise voltage would reduce the mean number of periods.

**P-value:** $P = P(T > |t_0|) = P(T > 2.828) \approx 0.01$

b)

$t_{\alpha/2,v} = t_{0.01/2,196} = 2.601$

$\overline{x_1} - \overline{x_2} - t_{\alpha/2,n_1+n_2-2}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \overline{x_1} - \overline{x_2} + t_{\alpha/2,n_1+n_2-2}\, s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$7.9 - 6.9 - 2.601 \times 2.5\sqrt{\frac{1}{100} + \frac{1}{100}} \le \mu_1 - \mu_2 \le 7.9 - 6.9 + 2.601 \times 2.5\sqrt{\frac{1}{100} + \frac{1}{100}}$

$0.0804 \le \mu_1 - \mu_2 \le 1.9196$

## 10-45

Consider the pipe deflection data in Exercise 10-22.
(a) Use the Wilcoxon rank-sum test for the pipe deflection temperature experiment.

If α = 0.05, what are your conclusions?
(b) Use the normal approximation for the Wilcoxon rank-sum test. Assume that α = 0.05. Find the approximate P-value for this test.

**10-22**

The deflection temperature under load for two different types of plastic pipe is being investigated. Two random samples of 15 pipe specimens are tested, and the deflection temperatures observed are as follows (in °F):
Type 1: 206, 188, 205, 187, 194, 193, 207, 185, 189, 213, 192, 210, 194, 178, 205
Type 2: 177, 197, 206, 201, 180, 176, 185, 200, 197, 192, 198, 188, 189, 203, 192

***Solution:***

## 10-88

A random sample of 500 adult residents of Maricopa County indicated that 385 were in favor of increasing the highway speed limit to 75 mph, and another sample of 400 adult residents of Pima County indicated that 267 were in favor of the increased speed limit.
(a) Do these data indicate that there is a difference in the support for increasing the speed limit for the residents of the two counties? Use α = 0.05. What is the P-value for this test?
(b) Construct a 95% conidence interval on the difference in the two proportions. Provide a practical interpretation of this interval

***Solution:***

$n_1 = 500,\ x_1 = 385$
$n_2 = 400, x_2 = 267$
$$\widehat{p_1} = \frac{x_1}{n_1} = \frac{385}{500} = 0.77,\ \widehat{p_2} = \frac{x_2}{n_2} = \frac{267}{400} = 0.6675$$
$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{385 + 267}{500 + 400} = 0.724$$

a)

**Parameter of interest:** the difference in 2 proportions, $p_1$ and $p_2$

**Null hypothesis:** $H_0 : p_1 = p_2$

**Alt. hypothesis:** $H_1 : p_1 \neq p_2$

**Test statistic:** $z_0 = \dfrac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

**Computation:**

+ $z_0 = \dfrac{\widehat{p_1} - \widehat{p_2}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{0.77 - 0.6675}{\sqrt{0.724(1-0.724)\left(\frac{1}{500} + \frac{1}{400}\right)}} = 3.4182$
+ $z_{\alpha/2} = z_{0.025} = 1.96$

**Conclusion:**

$z_0 = 3.4182 > 1.96 = z_{\alpha/2} \Rightarrow$ Reject $H_0$

**P-value:** $P = 2(1 - \Phi(3.4182)) \approx 0.001$

b)

A two-sided 95% CI on the difference of the 2 proportions:

$$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \leq p_1 - p_2 \leq \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$\Leftrightarrow 0.77 - 0.6675 - 1.96\sqrt{\frac{0.77(1-0.77)}{500} + \frac{0.6675(1-0.6675)}{400}} \leq p_1 - p_2 \leq 0.77 - 0.6675 -$
$1.96\sqrt{\frac{0.77(1-0.77)}{500} + \frac{0.6675(1-0.6675)}{400}}$

$\Leftrightarrow 0.0434 \leq p_1 - p_2 \leq 0.1616$