Name:..

Class:..

Applied Statistics and Probability for Engineers

Exercise Book

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Probability

Chapter 1: The Role of Statistics in Engineering

1. Explain the statistical jargons as listed below:

1.1 Population - Sample

1.2 Parameter - Statistic

1.3 Observational study - Experiment

1.4 The type of observational study: Cross - sectional, Retrospective and Prospective

1.5 Quantitative data - Qualitative data

1.6 Discrete data - Continuous data

1.7 Mechanistic model - Empirical model

2. The US government wants in know how American citizens feel about the war in Iraq. They randomly select 500 citizens from each state and ask them about their feeling. What are the population and the sample?

3. Determine whether the given value is a statistic or a parameter.

3.1 A sample of 120 employees of a company is selected, and the average is found to be 37 years.

3.2 After inspecting all of 55,000 kg of meat stored at the Wurst Sausage Company, it was found that 45,000 kg of the meat was spoiled.

4. Is the study experimental or observational?

4.1 A marketing firm does a survey to find out how many people use a product. Of the one hundred people contacted, fifteen said they use the product.

4.2 A clinic gives a drug to a group of ten patients and a placebo to another group of ten patients to find out if the drug has an effect on the patients' illness.

5. Identify the type of observational study.

5.1 A statistical analyst obtains data about ankle injuries by examining a hospital's records from the past 3 years.

5.2 A researcher plans to obtain data by following those in cancer remission since January of 2015.

5.3 A town obtains current employment data by polling 10,000 of its citizens this month.

6. Identify the number as either continuous or discrete.

6.1 The total number of phone calls a sales representative makes in a month is 425.

6.2 The average height of all freshmen entering college in a certain year is 68.4 inches.

6.3 The number of stories in a Manhattan building is 22.

7. Classify each set of data as discrete or continuous.

7.1 The number of suitcases lost by an airline.

7.2 The height of corn plants.

7.3 The number of ears of corn produced.

7.4 The time it takes for a car battery to die.

Chapter 2: Probability

1. Tossing a six-sided die and a coin. What is the sample space?

2. The Ski Patrol at Criner Mountain Ski Resort has determined the following probability distribution for the number of skiers that are injured each weekend:

Injured Skiers	0	1	2	3	4
Probability	0.05	0.15	0.4	0.3	0.1

What is the probability that the number of injuries per week is at most 3?

3. The probability of a New York teenager owning a skateboard is 0.37, of owning a bicycle is 0.81 and of owning both is 0.36.

3.1 If a New York teenager is chosen at random, what is the probability that the teenager owns a skateboard or a bicycle?

3.2 If a New York teenager is chosen at random, what is the probability that the teenager owning a skateboard but not owning a bicycle.

3.3 Find the probability that the teenager owns a bicycle given that the teenager owns a skateboard.

4. Let $P(A) = 0.4$, $P(B) = 0.5$ and $P(A+B) = 0.7$. Find

4.1 $P(AB)$

4.2 $P(\overline{A}B)$

4.3 $P(B \mid A)$

5. If the last digit of a weight measurement is equally likely to be any of the digits 0 through 9.

5.1 What is the probability that the last digit is 0?

5.2 What is the probability that the last digit is greater than or equal to 5?

6. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

	Shock Resistance		
Scratch Resistance		High	Low
	High	70	9
	Low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

6.1 $P(A)$ and $P(B)$

6.2 $P(AB)$ and $P(A+B)$

6.3 $P(A|B)$ and $P(B|A)$

7. Samples of a cast aluminum part are classified on the basis of surface finish (in microinches) and length measurements. The results of 100 parts are summarized as follows:

	Length		
Surface Finish		Excellent	Good
	Excellent	80	2
	Good	10	8

Let A denote the event that a sample has excellent surface finish, and let B denote the event that a sample has excellent length. Determine:

7.1 $P(A)$ and $P(B)$

7.2 $P(AB)$ and $P(A+B)$

7.3 $P(A|B)$ and $P(B|A)$

8. A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch.

8.1 What is the probability that the second one selected is defective given that the first one was defective?

8.2 What is the probability that both are defective?

8.3 What is the probability that both are acceptable?

Three containers are selected, at random, without replacement, from the batch.

8.4 What is the probability that the third one selected is defective given that the first and second ones selected were defective?

8.5 What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?

8.6 What is the probability that all three are defective?

9. Suppose that $P(A|B)=0.4$ and $P(B)=0.5$. Determine the following:

9.1 $P(AB)$ and $P(A\overline{B})$

9.2 $P(A+B)$ and $P(B|A)$

10. Suppose 2% of cotton fabric rolls and 3% of nylon fabric rolls contain flaws. Of the rolls used by a manufacturer, 70% are cotton and 30% are nylon. What is the probability that a randomly selected roll used by the manufacturer contains flaws?

11. In the 2012 presidential election, exit polls from the critical state of Ohio provided the following results:

Total	Obama	Romney
No college degree (60%)	52%	45%
College graduate (40%)	47%	51%

What is the probability a randomly selected respondent voted for Obama?

12. The probability that a lab specimen contains high levels of contamination is 0.1. Five samples are checked, and the samples are independent.

12.1 What is the probability that none contain high levels of contamination?

12.2 What is the probability that exactly one contains high levels of contamination?

12.3 What is the probability that at least one contains high levels of contamination?

13. An e-mail filter is planned to separate valid e-mails from spam. The word free occurs in 60% of the spam messages and only 4% of the valid messages. Also, 20% of the messages are spam. Determine the following probabilities:

13.1 The message contains free.

13.2 The message is spam given that it contains free.

13.3 The message is valid given that it does not contain free.

14. Decide whether a discrete or continuous random variable is the best model for each of the following variables:

14.1 The time until a projectile returns to earth.

14.2 The number of times a transistor in a computer memory changes state in one operation.

14.3 The volume of gasoline that is lost to evaporation during the filling of a gas tank.

14.4 The outside diameter of a machined shaft.

Chapter 3: Discrete Random Variables and Probability Distributions

1. The sample space of a random experiment is {a, b, c, d, e, f}, and each outcome is equally likely. A random variable is defined as follows:

Outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Use the probability mass function to determine the following probabilities:

1.1 $P(X=1.5)$ 1.2 $P(0.5<X<2.7)$

1.3 $P(0\le X<2)$ 1.4 $P(X=0 \text{ or } X=2)$

2. Verify that the following functions are probability mass functions, and determine the requested probabilities.

x	-2	-1	0	1	2
f(x)	0.2	0.4	0.1	0.2	0.1

2.1 $P(X\le 2)$ 2.2 $P(X>-2)$ 2.3 $P(-1\le X \text{ or } X=2)$

2.4 Calculate $E(X)$, $V(X)$ and σ_X

3. The thickness of wood paneling (in inches) that a customer orders is a random variable with the following cumulative distribution function:

$$F(x)=\begin{cases}0 & ,x<1/8\\ 0.2 & ,1/8\le x<1/4\\ 0.9 & ,1/4\le x<3/8\\ 1 & ,3/8\le x\end{cases}$$

Determine the following probabilities:

3.1 $P(X\le 1/4)$ 3.2 $P(X\le 5/16)$ 3.3 $P(X>1/2)$

4. Let the random variable X have a discrete uniform distribution on the integers $1\le x\le 3$. Determine the mean and variance of X.

5. Let the random variable X have a discrete uniform distribution on the integers $0 \le x \le 99$. Determine the mean and variance of X.

6. Thickness measurements of a coating process are made to the nearest hundredth of a millimeter. The thickness measurements are uniformly distributed with values 0.15, 0.16, 0.17, 0.18, and 0.19. Determine the mean and variance of the coating thickness for this process.

7. The random variable X has a binomial distribution with n = 10 and p = 0.5. Determine the following probabilities:

7.1 $P(X=5)$ 7.2 $P(X \le 2)$ 7.3 $P(X > 7)$

7.2 Determine the mean and variance of X.

8. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.

8.1 What is the probability that for exactly three calls, the lines are occupied?

8.2 What is the probability that for at least one call, the lines are not occupied?

8.3 What is the expected number of calls in which the lines are all occupied?

9. A multiple-choice test contains 25 questions, each with four answers. Assume that a student just guesses on each question.

9.1 What is the probability that the student answers more than 20 questions correctly?

9.2 What is the probability that the student answers fewer than 5 questions correctly?

10. The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected?

11. Suppose that the random variable X has a geometric distribution with p = 0 5.

11.1 Determine the following probabilities: $P(X=4)$, $P(X=5)$, $P(X>3)$

11.2 Determine the mean and variance of X.

12. Suppose that X is a negative binomial random variable with p = 0.2 and r = 4. Determine the following:

12.1 $E(X)$ and $V(X)$

12.2 $P(X=3)$ and $P(X=5)$

12.3 $P(X>5)$

13. The probability of a successful optical alignment in the assembly of an optical data storage product is 0.8. Assume that the trials are independent.

13.1 What is the probability that the first successful alignment requires exactly four trials?

13.2 What is the probability that the first successful alignment requires at most four trials?

13.3 What is the probability that the first successful alignment requires at least four trials?

14. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1.

14.1 What is the probability that four or more people need to be tested to detect two with the gene?

14.2 What is the expected number of people to test to detect two with the gene?

15. A batch of parts contains 100 from a local supplier of tubing and 200 from a supplier of tubing in the next state. If four parts are selected randomly and without replacement.

15.1 What is the probability they are all from the local supplier?

15.2 What is the probability that two or more parts in the sample are from the local supplier?

16. Suppose that X has a hypergeometric distribution with $N=100, n=4$ and $K=20$. Determine the following:

16.1 $P(X=4)$ and $P(X=6)$

16.2 $P(4 \le X < 7)$ and $P(X \ge 1)$

16.3 Mean and variance of X

17. A research study uses 800 men under the age of 55. Suppose that 30% carry a marker on the male chromosome that indicates an increased risk for high blood pressure.

17.1 If 10 men are selected randomly and tested for the marker, what is the probability that exactly 1 man has the marker?

17.2 If 10 men are selected randomly and tested for the marker, what is the probability that more than 1 has the marker?

18. The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by the type of transformation completed:

	Total Textural Transformation		
Total Color Transformation		Yes	No
	Yes	243	26
	No	13	18

A naturalist randomly selects three leaves from this set without replacement. Determine the following probabilities.

18.1 Exactly one has undergone both types of transformations.

18.2 At least one has undergone both transformations.

18.3 Exactly one has undergone one but not both transformations.

18.4 At least one has undergone at least one transformation.

19. On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

19.1 Exactly 5 accidents will occur?

19.2 Fewer than 3 accidents will occur?

19.3 At least 2 accidents will occur?

20. On average, a textbook author makes two word processing errors per page on the first draft of her textbook. What is the probability that on the next page she will make

20.1 Four or more errors?

20.2 No errors?

21. Suppose that X has a Poisson distribution with a mean of 4. Determine the following probabilities:

21.1 $P(X = 0)$ and $P(X = 4)$

21.2 $P(3 \leq X \leq 5)$ and $P(X > 3)$

22. The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.

22.1 What is the probability that there are two flaws in one square meter of cloth?

22.2 What is the probability that there is one flaw in 10 square meters of cloth?

22.3 What is the probability that there are at least two flaws in 10 square meters of cloth?

22.4 What is the probability that there are no flaws in 20 square meters of cloth?

23. Let X denote the number of bits received in error in a digital communication channel, and assume that X is a binomial random variable with p = 0.001. If 1000 bits are transmitted, determine the following:

23.1 $P(X = 1)$ and $P(X \geq 2)$

23.2 Mean and variance of X

Chapter 4: Continuous Random Variables and Probability Distributions

1. Suppose that $f(x) = e^{-x}$ for $x > 0$. Determine the following:

1.1 $P(X = 1)$ and $P(X = 2 \text{ or } X = 3)$

1.2 $P(X > 2)$ and $P(1 \le X \le \ln 5)$

1.3 Mean and variance of X

1.4 The cumulative distribution function $F(x)$

1.5 x such that $P(X \le x) = 0.1$

2. The probability density function of the weight of packages delivered by a post office is $f(x) = \dfrac{70}{69x^2}$ for $1 < x < 70$ pounds.

2.1Determine the mean and variance of weight.

2.2 If the shipping cost is $2.50 per pound, what is the average shipping cost of a package?

2.3Determine the probability that the weight of a package exceeds 50 pounds

3. The diameter of a particle of contamination (in micrometers) is modeled with the probability density function $f(x) = \dfrac{c}{x^3}$ for x > 1. Determine the following:

3.1 c and $P(X < 2)$

3.2 $P(X > 4)$ and $P(2 < X < 8)$

3.3 Mean and variance of X

3.4 x such that $P(X < x) = 0.95$

3.5 The cumulative distribution function $F(x)$

4. Suppose that X has a continuous uniform distribution over the interval $[1.5; 5.5]$

Determine the following:

4.1Mean, variance, and standard deviation of X

4.2 $P(X < 2.5)$

4.3 Cumulative distribution function

5. Suppose X has a continuous uniform distribution over the interval $[-1;1]$. Determine the following:

5.1 Mean, variance, and standard deviation of X

5.2 Value for x such that $P(-x < X < x) = 0.9$

5.3 Cumulative distribution function

6. The thickness of a flange on an aircraft component is uniformly distributed between 0.95 and 1.05 millimeters. Determine the following:

6.1 Cumulative distribution function of flange thickness

6.2 Proportion of flanges that exceeds 1.02 millimeters

6.3 Thickness exceeded by 90% of the flanges

6.4 Mean and variance of flange thickness

7. Assume that Z has a standard normal distribution. Determine the following:

7.1 $P(Z < 1.32)$ and $P(Z \le 3)$

7.2 $P(Z > -2.15)$ and $P(-2 < Z < 1.2)$

7.3 $P(-1.96 < Z < 1.96)$ and $P(0 < Z < 1)$

Determine the value for z that solves each of the following:

7.4 $P(Z > z) = 0.01$ and $P(Z < z) = 0.97$

7.5 $P(Z > z) = 0.05$ and $P(Z < z) = 0.9$

7.6 $P(-z < Z < z) = 0.95$ and $P(-z < Z < z) = 0.99$

8. Assume that X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:

8.1 $P(X < 13)$ and $P(X > 9)$

8.2 $P(6 < X < 14)$ and $P(-2 < X < 8)$

8.3 x such that $P(X > x) = 95\%$

9. Assume that the current measurements in a strip of wire follow a normal distribution with a mean of 10 milliamperes and a variance of 4 (milliamperes)^2. What is the probability that a measurement exceeds 13 milliamperes?

10. The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.

10.1 What is the probability that a fill volume is less than 12 fluid ounces?

10.2 If all cans less than 12.1 or more than 12.6 ounces are scrapped, what proportion of cans is scrapped?

10.3 Determine specifications that are symmetric about the mean that include 99% of all cans.

11. Suppose that X is a binomial random variable with $n = 200$ and $p = 0.4$. Approximate the following probabilities:

11.1 $P(X \leq 70)$

11.2 $P(70 < X < 90)$

11.3 $P(X=80)$

12. Suppose that X is a Poisson random variable with λ = 6.

12.1 Compute the exact probability that X is less than four.

12.2 Approximate the probability that X is less than four and compare to the result in 12.2

12.3 Approximate the probability that $8<X<12$

13. The manufacturing of semiconductor chips produces 2% defective chips. Assume that the chips are independent and that a lot contains 1000 chips. Approximate the following probabilities:

13.1 More than 25 chips are defective.

13.2 Between 20 and 30 chips are defective.

14. Suppose that X has an exponential distribution with λ = 2. Determine the following:

14.1 $P(X\leq 0)$ and $P(X\geq 2)$

14.2 $P(X\leq 1)$ and $P(1<X<2)$

14.3 Find the value of x such that $P(X<x)=0.95$

14.4 $P(X<5|X>2)$ and $P(X<3)$

15. Suppose that the counts recorded by a Geiger counter follow a Poisson process with an average of two counts per minute.

15.1 What is the probability that there are no counts in a 30-second interval?

15.2 What is the probability that the first count occurs in less than 10 seconds?

15.3 What is the probability that the first count occurs between one and two minutes after start-up?

Chapter 6: Random Sampling and Data Description

1. Explain the statistical jargons as listed below:

1.1 Sample size - Range - Midrange

1.2 Mean - Variance - Standard deviation

1.3 Median - Mode

1.4 Quartile - Interquartile range - Percentiles

1.5 Frequency Polygon - Ogive - Dotplots - Stemplots - Bar Graphs - Pareto Charts - Scatterplots - Time Series Graph - Pie Charts - Boxplot

1.6 Outliers - z Score - Unusual Values

2. The lengths of time, in minutes, that 10 patients waited in a doctor's office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Treating the data as a random sample. Find:

2.1 The mean

2.2 The median

2.3 The mode

2.4 The range

2.5 The variance

2.6 The standard deviation.

3. A random sample of employees from a local manufacturing plant pledged the following donations, in dollars, to the United Fund: 100, 40, 75, 15, 20, 100, 75, 50, 30, 10, 55, 75, 25, 50, 90, 80, 15, 25, 45, and 100. Calculate

3.1 The mean

3.2 The median

3.3 The mode

3.4 The range

3.5 The variance

3.6 The standard deviation.

4. Wayne Nelson presents the breakdown time of an insulating fluid between electrodes at 34 kV. The times, in minutes, are as follows: 0.19, 0.78, 0.96, 1.31, 2.78, 3.16, 4.15, 4.67, 4.85, 6.50, 7.35, 8.01, 8.27, 12.06, 31.75, 32.52, 33.91, 36.71, and 72.89.

4.1 Find the median and quartiles for the data

4.2 Find the interquartile range (IQR) and the outliers for data

4.3 Calculate the sample mean and sample standard deviation.

5. The lengths of time, in minutes, that 10 patients waited in a doctor's office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Treating the data as a random sample, find

5.1 The mean;

5.2 The median;

5.3 The mode.

Chapter 7: Point Estimation of Parameters and Sampling Distributions

1. An electronics company manufactures resistors that have a mean resistance of 100 ohms and a standard deviation of 10 ohms. The distribution of resistance is normal. Find the probability that a random sample of $n = 25$ resistors will have an average resistance of fewer than 95 ohms.

2. A synthetic fiber used in manufacturing carpet has tensile strength that is normally distributed with mean 75.5 psi and standard deviation 3.5 psi.

2.1 Find the probability that a random sample of $n = 6$ fiber specimens will have sample mean tensile strength that exceeds 75.75 psi.

2.2 How is the standard deviation of the sample mean changed when the sample size is increased from $n = 6$ to $n = 49$?

3. The compressive strength of concrete is normally distributed with $\mu = 2500$ psi and $\sigma = 50$ psi. Find the probability that a random sample of $n = 5$ specimens will have a sample mean diameter that falls in the interval from 2499 psi to 2510 psi.

4. Data on pull-off force (pounds) for connectors used in an automobile engine application are as follows: 79.3, 75.1, 78.2, 74.1, 73.9, 75.0, 77.6, 77.3, 73.8, 74.6, 75.5, 74.0, 74.7, 75.9, 72.9, 73.8, 74.2, 78.1, 75.4, 76.3, 75.3, 76.2, 74.9, 78.0, 75.1, 76.8.

4.1 Calculate a point estimate of the mean pull-off force of all connectors in the population.

4.2 Calculate point estimates of the population variance and the population standard deviation.

4.3 Calculate a point estimate of the proportion of all connectors in the population whose pull-off force is less than 73 pounds.

Chapter 8: Statistical Intervals for a Single Sample

1. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours.

1.1 If a sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

1.2 How large a sample is needed in Exercise 9.2 if we wish to be 96% confident that our sample mean will be within 10 hours of the true mean.

2. The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Construct a 98% confidence interval for the mean height of all college students.

3. A random sample of 100 automobile owners in the state of Virginia shows that an automobile is driven on average 23,500 kilometers per year with a standard deviation of 3900 kilometers. Assume the distribution of measurements to be approximately normal. Construct a 99% confidence interval for the average number of kilometers an automobile is driven annually in Virginia.

4. An efficiency expert wishes to determine the average time that it takes to drill three holes in a certain metal clamp. How large a sample will she need to be 95% confident that her sample mean will be within 15 seconds of the true mean? Assume that it is known from previous studies that $\sigma = 40$ seconds.

5. A random sample of 10 chocolate energy bars of a certain brand has, on average, 230 calories per bar, with a standard deviation of 15 calories. Construct a 99% confidence interval for the true mean calorie content of this brand of energy bar. Assume that the distribution of the calorie content is approximately normal.

6. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint: 3.4 2.5 4.8 2.9 3.6 2.8 3.3 5.6 3.7 2.8 4.4 4.0 5.2 3.0 4.8. Assuming that the measurements represent a random sample from a normal population, find a 95% prediction interval for the drying time for the next trial of the paint.

7. Past experience has indicated that the breaking strength of yarn used in manufacturing drapery material is normally distributed and that $\sigma = 2$ psi. A random sample of nine

specimens is tested, and the average breaking strength is found to be 98 psi. Find a 95% two-sided confidence interval on the true mean breaking strength.

8. A manufacturer produces piston rings for an automobile engine. It is known that ring diameter is normally distributed with σ = 0.001 millimeters. A random sample of 15 rings has a mean diameter of $\bar{x} = 74\ 036$ millimeters.

8.1 Construct a 99% two-sided confidence interval on the mean piston ring diameter.

8.2 Construct a 99% lower-confidence bound on the mean piston ring diameter.

9. An Izod impact test was performed on 20 specimens of PVC pipe. The sample mean is $\bar{x} = 1.25$ and the sample standard deviation is s = 0.25. Find a 99% lower confidence bound on Izod impact strength.

10. Determine the χ^2 percentile that is required to construct each of the following CIs:

10.1 Confidence level = 95%, degrees of freedom = 24, one-sided (upper)

10.2 Confidence level = 99%, degrees of freedom = 9, one-sided (lower)

10.3 Confidence level = 90%, degrees of freedom = 19, two-sided.

11. A rivet is to be inserted into a hole. A random sample of n = 15 parts is selected, and the hole diameter is measured. The sample standard deviation of the hole diameter measurements is s = 0.008 millimeters.

11.1 Construct a 99% lower confidence bound for σ^2.

11.2 Find a 99% lower confidence bound on the standard deviation.

12. The sugar content of the syrup in canned peaches is normally distributed. A random sample of n = 10 cans yields a sample standard deviation of s = 4.8 milligrams. Calculate a 95% two-sided confidence interval for σ.

13. The fraction of defective integrated circuits produced in a photolithography process is being studied. A random sample of 300 circuits is tested, revealing 13 defectives.

13.1 Calculate a 95% two-sided CI on the fraction of defective circuits produced by this particular tool.

13.2 Calculate a 95% upper confidence bound on the fraction of defective circuits.

14. An article in Knee Surgery, Sports Traumatology, Arthroscopy ["Arthroscopic Meniscal Repair with an Absorbable Screw: Results and Surgical Technique" (2005, Vol. 13, pp. 273–279)] showed that only 25 out of 37 tears (67.6%) located between 3 and 6 mm from the meniscus rim were healed.

14.1 Calculate a two-sided 95% confidence interval on the proportion of such tears that will heal.

14.2 Calculate a 95% lower confidence bound on the proportion of such tears that will heal.

15. Of 1000 randomly selected cases of lung cancer, 823 resulted in death within 10 years.

15.1 Calculate a 95% two-sided confidence interval on the death rate from lung cancer.

15.2 Using the point estimate of p obtained from the preliminary sample, what sample size is needed to be 95% confident that the error in estimating the true value of p is less than 0.03?

15.3 How large must the sample be if you wish to be at least 95% confident that the error in estimating p is less than 0.03, regardless of the true value of p?

16. An article in the Journal of the American Statistical Association (1990, Vol. 85, pp. 972–985) measured the weight of 30 rats under experiment controls. Suppose that 12 were underweight rats.

16.1 Calculate a 95% two-sided confidence interval on the true proportion of rats that would show underweight from the experiment.

16.2 Using the point estimate of p obtained from the preliminary sample, what sample size is needed to be 95% confident that the error in estimating the true value of p is less than 0.02?

16.3 How large must the sample be if you wish to be at least 95% confident that the error in estimating p is less than 0.02, regardless of the true value of p?

17. The percentage of titanium in an alloy used in aerospace casting is measured in 31 randomly selected parts. The sample standard deviation is $s = 0.34$. Construct a 98% two - side confidence interval for σ.

18. The sugar content of the syrup in canned peaches is normally distributed. A random sample of $n = 10$ cans yields a sample standard deviation of $s = 4.8$ milligrams. Find a 95% two-sided confidence interval for σ.

Chapter 9: Tests of Hypotheses for a Single Sample

1. State the null and alternative hypothesis in each case.

1.1 A hypothesis test will be used to potentially provide evidence that the population mean is more than 10.

1.2 A hypothesis test will be used to potentially provide evidence that the population mean is not equal to 7.

(1.3 A hypothesis test will be used to potentially provide evidence that the population mean is less than 5.

2. A hypothesis will be used to test that a population mean equals 7 against the alternative that the population mean does not equal 7 with known variance σ. What are the critical values for the test statistic Z_0 for the following significance levels?

2.1 0.01

2.2 0.05

2.3 0.10

3. For the hypothesis test H_0: $\mu = 7$ against H_1: $\mu \neq 7$ and variance known, calculate the P-value for each of the following test statistics.

3.1 $z_0 = 2.05$

3.2 $z_0 = -1.84$

3.3 $z_0 = 0.4$

4. For the hypothesis test H_0: $\mu = 10$ against H_1: $\mu > 10$ and variance known, calculate the P-value for each of the following test statistics.

4.1 $z_0 = 2.05$

4.2 $z_0 = -1.84$

4.3 $z_0 = 0.4$

5. For the hypothesis test H_0: $\mu = 5$ against H_1: $\mu < 5$ and variance known, calculate the P-value for each of the following test statistics.

5.1 $z_0 = 2.05$

5.2 $z_0 = -1.84$

5.3 $z_0 = 0.4$

6. Output from a software package follows:

One-Sample Z:

Test of mu = 35 vs not = 35

The assumed standard deviation = 1.8

Variable	N	Mean	StDev	SE Mean	Z	P
x	25	35.710	1.475	?	?	?

6.1 Fill in the missing items. What conclusions would you draw?

6.2 Is this a one-sided or a two-sided test?

6.3 Use the normal table and the preceding data to construct a 95% two-sided CI on the mean.

6.4 What would the P-value be if the alternative hypothesis is H_1: $\mu > 35$?

7. Output from a software package follows:

One-Sample Z:

Test of mu = 20 vs > 20

The assumed standard deviation = 0.75

Variable	N	Mean	StDev	SE Mean	Z	P
x	10	19.889	?	0.237	?	?

7.1 Fill in the missing items. What conclusions would you draw?

7.2 Is this a one-sided or a two-sided test?

7.3 Use the normal table and the preceding data to construct a 95% two-sided CI on the mean.

7.4 What would the P-value be if the alternative hypothesis is H_1: $\mu \neq 20$?

8. The mean water temperature downstream from a discharge pipe at a power plant cooling tower should be no more than 100°F. Past experience has indicated that the standard deviation of temperature is 2°F. The water temperature is measured on nine randomly chosen days, and the average temperature is found to be 98°F.

8.1 Is there evidence that the water temperature is acceptable at $\alpha = 0.05$?

8.2 What is the P-value for this test?

9. A hypothesis will be used to test that a population mean equals 7 against the alternative that the population mean does not equal 7 with unknown variance. What are the critical values for the test statistic T_0 for the following significance levels and sample sizes?

9.1 $\alpha = 0.01$ and n = 20

9.2 $\alpha = 0.05$ and n = 12

9.3 $\alpha = 0.10$ and n = 15

10. For the hypothesis test H_0: $\mu = 7$ against H_1: $\mu \neq 7$ with variance unknown and n = 20, approximate the P-value for each of the following test statistics.

10.1 $t_0 = 2.05$

10.2 $t_0 = -1.84$

10.3 $t_0 = 0.4$

11. Output from a software package follows:

One-Sample Z:

Test of mu = 91 vs > 91

Variable	N	Mean	StDev	SE Mean	95% Lower Bound	T	P
x	20	92.379	0.717	0.237	?	?	?

11.1 Fill in the missing values. You may calculate bounds on the P-value. What conclusions would you draw?

11.2 Is this a one-sided or a two-sided test?

11.3 If the hypothesis had been H_0: $\mu = 90$ versus H_1: $\mu > 90$, would your conclusions change?

12. Consider the test of H_0: $\sigma^2 = 7$ against H_1: $\sigma^2 \neq 7$. What are the critical values for the test statistic χ_0^2 for the following significance levels and sample sizes?

12.1 $\alpha = 0.01$ and n = 20

12.2 $\alpha = 0.05$ and n = 12

12.3 $\alpha = 0.1$ and n = 15

13. Data from an Izod impact test was described. The sample standard deviation was 0.25 and n = 20 specimens were tested. Test the hypothesis that $\sigma = 0.10$ against an alternative specifying that $\sigma \neq 0.10$, using $\alpha = 0.01$, and draw a conclusion.

14. Reconsider the percentage of titanium in an alloy used in aerospace castings from Exercise 8-52. Recall that s = 0.37 and n = 51. Test the hypothesis H_0: $\sigma = 0.25$ versus H_1: $\sigma \neq 0.25$ using $\alpha = 0.05$.

15. Consider the following computer output:

Test and Cl for One Proportion					
Test of p = 0.4 vs p not = 0.4					
X	N	Sample p	95% CI	Z-Value	P-Value
98	275	?	(0.299759, 0.412968)	?	?

15.1 Is this a one-sided or a two-sided test?

15.2 Complete the missing items.

16. Suppose that of 1000 customers surveyed, 850 are satisfied or very satisfied with a corporation's products and services.

16. 1 Test the hypothesis H_0: p = 0.9 against H_1: $p \neq 0.9$ at $\alpha = 0.05$.

16.2 Find the P-value.

17. Suppose that 500 parts are tested in manufacturing and 10 are rejected.

17. 1 Test the hypothesis H_0: p = 0.03 against H_1: $p < 0.03$ at $\alpha = 0.05$.

17.2 Find the P-value.

18. Compute the standardized test statistic to test the claim $\sigma^2 = 34.4$ if n =12; $s^2 = 28.8$ and α=0.05.

Chapter 11: Analysis of Simple Linear Regression and Correlation

1. Use the given data to find the equation of the regression line and the value of the linear correlation coefficient r.

1.1

x	2	4	5	6
y	7	11	13	20

1.2

Cost	9	2	3	4	2	5	9	10
Number	85	52	55	68	67	86	83	73

1.3

x	-4	2	8	6	11	9	-2	-1	-4
y	3	6	12	10	10	7	7	2	3

2. Four pairs of data yield r = 0.942 and the regression equation $y = 3x$. Also, $\bar{y} = 12.75$. What is the best predicted value of y for x = 2.5?

3. Suppose data is obtained from 27 pairs of *(x, y)* and the sample correlation coefficient is 0.85. Test the hypothesis that H_0:p = 0 against H_1:p ≠ 0 with α = 0.05.

4. Given a sample with r = 0.823, n=10 and α=0.05, determine the standardized test statistic t necessary to test the claim ρ = 0.

5. A Company has just brought out an annual report in which the capital investment and profits were given for the past few years.

Capital Investment	10	16	18	24	36	48	57

Profits	12	14	13	18	26	38	62

5.1 Find the coefficient of correlation.

5.2 Test H_0: $\beta_1 = 1$ using $\alpha = 0.01$. Let $Se((\hat{\beta}_1)) = 0.145$.

5.3 Test H_0: $\beta_0 = 0.5$ using $\alpha = 0.05$. Let $Se((\hat{\beta}_0)) = 4.95$.

6. A study was conducted to find whether there is any relationship between the weight and blood pressure of an individual. The following set of data was arrived at from a clinical study.

weight	78	86	72	82	80	86	84	89	68	71
Blood pressure	140	160	134	144	180	176	174	178	128	132

6.1 Find the equation of estimated linear regression line of Blood pressure on weight.

6.2 Find the residual of the third observation.

6.3 Find the sum of square error; sum of square regression.

6.4 Find the best predicted value of blood pressure of a person who weigh 90 kilograms.

7.

Years	1965	1970	1975	1980	1985	1990
Raw cotton import	42	60	112	98	118	132
Cotton manufacture exports	68	79	88	86	106	114

Based on sample, test for significance of regression using $\alpha = 0.05$.

8. Which of the following are examples of positive correlation and negative correlation?

8.1 Heights and weights

8.2 Volume and pressure of perfect gas

8.3 Current and resistance (keeping the voltage constant)

8.4 Price and demand of goods

8.5 Household income and expenditure

8.6 Price and supply of commodities

8.7 Amount of rainfall and yield of crops

9. Let $n = 20$, $\sum y_i = 12.75, \sum y_i^2 = 8.86$, $\sum x_i = 1478, \sum x_i^2 = 143215.8$, $\sum x_i y_i = 1083.67$

Find the regression line and correlation coefficient.