MAS291 - HOMEWORK CHAP 8

Nguyen Dang Loc - SE160199

8-53

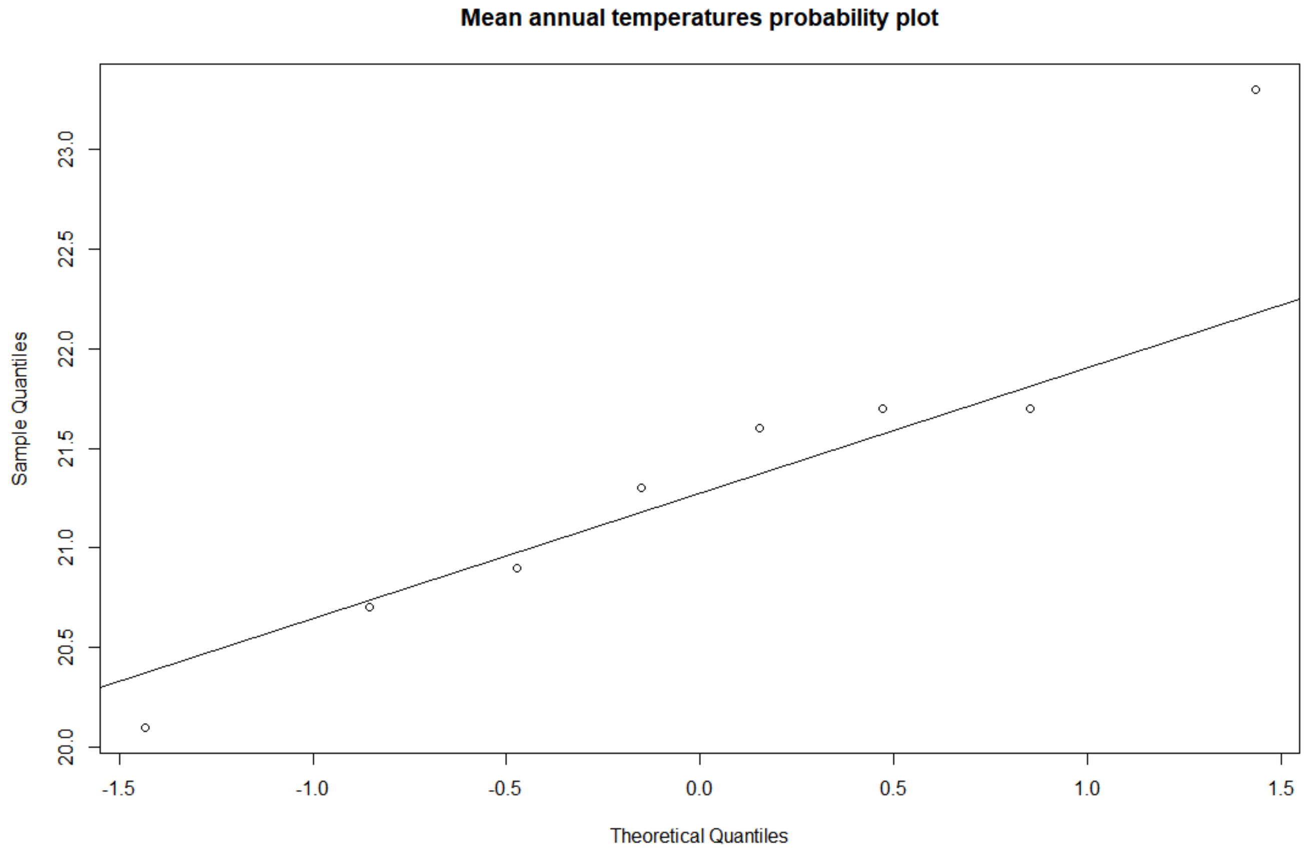
An article in Urban Ecosystems, “Urbanization and Warming of Phoenix (Arizona, USA): Impacts, Feedbacks and Mitigation” (2002, Vol. 6, pp. 183–203), mentions that Phoenix is ideal to study the effects of an urban heat island because it has grown from a population of 300,000 to approximately 3 million over the last 50 years, which is a period with a continuous, detailed climate record. The 50-year averages of the mean annual temperatures at eight sites in Phoenix follow. Check the assumption of normality in the population with a probability plot. Construct a 95% confidence interval for the standard deviation over the sites of the mean annual temperatures.

Site	Average Mean Temperature (°C)
Sky Harbor Airport	23.3
Phoenix Greenway	21.7
Phoenix Encanto	21.6
Waddell	21.7
Litchi eld	21.3
Laveen	20.7
Maricopa	20.9
Harlquahala	20.1

Solution:

Code in R

```
# plot
dt <- c(23.3, 21.7, 21.6, 21.7, 21.3, 20.7, 20.9, 20.1)
qqnorm(dt, main = 'Mean annual temperatures probability plot')
qqline(dt)
```



The pattern of normal probability plot is roughly linear and close to the line, we can say that the the distribution of observed values is approximately normal.

Sample size $n = 8$, the sample mean and sample standard deviation:

$$\overline{x} = \frac{\sum_{i=1}^{8} x_i}{8} = 21.4125$$

$$s = \frac{\sum_{i=1}^{8} (x_i - \overline{x})}{7} = 0.8955$$

The CI is 95% $\Rightarrow 1 - \alpha = 0.95 \Leftrightarrow \alpha = 0.05$

$$\frac{\alpha}{2} = 0.025 \Rightarrow \chi^2_{\alpha/2,n-1} = \chi^2_{0.025,7} = 16.01$$

$$1 - \frac{\alpha}{2} = 0.975 \Rightarrow \chi^2_{1-\frac{\alpha}{2},n-1} = \chi^2_{0.975,7} = 1.69$$

A 95% confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} \Leftrightarrow \frac{7 \times 0.8955}{16.01} \leq \sigma^2 \leq \frac{7 \times 0.8955}{1.69}$$

$$\Leftrightarrow 0.392 \leq \sigma^2 \leq 3.709$$

$$\Leftrightarrow 0.626 \leq \sigma \leq 1.926$$

8-55

An article in Technometrics (1999, Vol. 41, pp. 202–211) studied the capability of a gauge by measuring the weight of paper. The data for repeated measurements of one sheet of paper are in the following table. Construct a 95% one-sided upper confidence interval for the standard deviation of these measurements. Check the assumption of normality of the data and comment on the assumptions for the confidence interval.

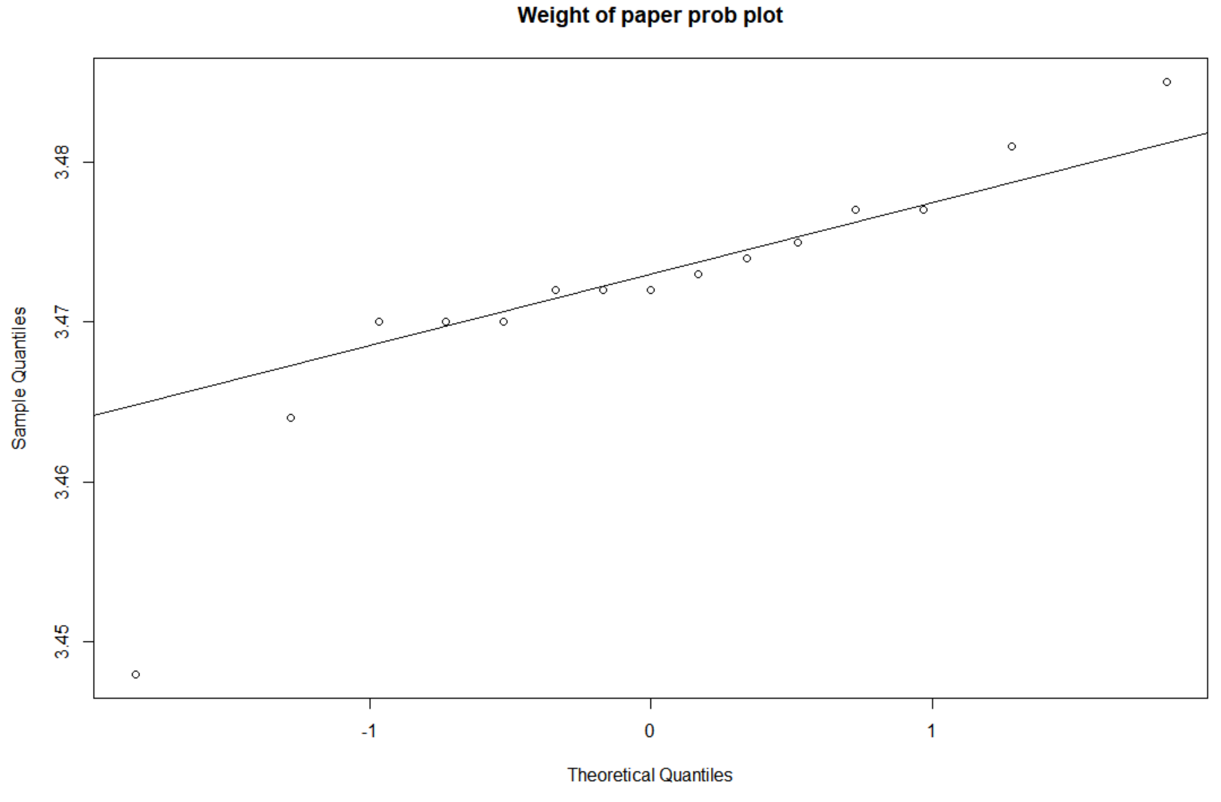
Observations

3.481, 3.448, 3.485, 3.475, 3.472,
3.477, 3.472, 3.464, 3.472, 3.470,
3.470, 3.470, 3.477, 3.473, 3.474

Solution:

Code in R

```
# plot
dt <- c(3.481, 3.448, 3.485, 3.475, 3.472,
        3.477, 3.472, 3.464, 3.472, 3.470,
        3.470, 3.470, 3.477, 3.473, 3.474)
qqnorm(dt, main = 'Weight of paper prob plot')
qqline(dt)
```



The pattern of normal probability plot is roughly linear and close to the line, we can say that the the distribution of observed values is approximately normal.

Sample size $n = 15$, the sample mean and sample standard deviation:

$$\overline{x} = \frac{\sum_{i=1}^{15} x_i}{15} = 3.472$$

$$s = \frac{\sum_{i=1}^{1} 5(x_i - \overline{x})}{14} = 0.0083$$

The CI is 95% $\Rightarrow 1 - \alpha = 0.95 \Rightarrow \chi^2_{1-\alpha,n-1} = \chi^2_{0.975,7} = 6.57$

A 95% one-sided upper confidence interval for σ^2 is:

$$\sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}} \Rightarrow \sigma^2 \leq \frac{14 \times 0.0083^2}{6.57} \approx 0.000147$$

$$\Rightarrow \sigma \leq 0.01212$$

8-67

The U.S. Postal Service (USPS) has used optical character recognition (OCR) since the mid-1960s. In 1983, USPS began deploying the technology to major post ofices throughout the country (www.britannica.com). Suppose that in a random sample of 500 handwritten zip code digits, 466 were read correctly.

(a) Construct a 95% confidence interval for the true proportion of correct digits that can be automatically read.

(b) What sample size is needed to reduce the margin of error to 1%?

(c) How would the answer to part (b) change if you had to assume that the machine read only one-half of the digits correctly?

Solution:

A point estimate of p: $\hat{p} = \dfrac{x}{n} = \dfrac{466}{500} = 0.932$

$$np = 500 \times 0.932 = 466 > 5$$
$$n(1-p) = 500 \times (1 - 0.932) = 34 > 5$$

a)

$$\frac{\alpha}{2} = \frac{1 - 0.95}{2} = 0.025 \Rightarrow z_{0.025} = 1.96$$

A 95% confidence interval of the true proportion:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Leftrightarrow 0.932 - 1.96\sqrt{\frac{0.932(1-0.932)}{500}} \leq p \leq 0.932 + 1.96\sqrt{\frac{0.932(1-0.932)}{500}}$$

$$\Leftrightarrow 0.9099 \leq p \leq 0.9541$$

b)

Error $E = 0.01$

$$n \geq \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.01}\right)^2 \times 0.932 \times (1 - 0.932) = 2434.65$$

Sample size required $n = 2435$

c)

$$p = 0.5 \Rightarrow p(1-p) = 0.25$$

$$n \geq \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.01}\right)^2 \times 0.25 = 9604$$

Sample size required $n = 9604$