Assignment 9

Practice Problems

Sum to n

What is the sum of numbers 1 to 15

Solution

- Sum upto n = n* (n+1)/2

  = 15 * (15+1)/2 = 15*8 = 120

Larger sum to 'n'

What is the sum of integers up to 200000

Solution

Sum upto n = n* (n+1)/2

= 200000*(200001)/2

= 20000100000

Complexities

Big O notation for

(a)n

(b)5n+3

(c) 2002n+2003

Solution

All of them are of the form c*n+d

Growth rate of c*n is greater than growth rate of constant, d always. (Look at lecture notes)

Hence the O notation is O(n)

(a) c=1 , d=0

we get n+0=n.

(b) c=5 , d=3

we get 5n+3

(c) c=2002, d=2003

we get 2002n + 2003

Logarithms in complexity

Big O notation for

(a) n + lgn

(b) 2n + 3lgn

(c) 0.5n+ 400lgn

Solution

These are of the form c*n+d*lgn

Since, growth of n is greater than growth of lgn (Refere Lecture slides for Growth Rates)

The expression has Big O as O(n)

(a) c=1, d=1

(b) c=2, d=3

(c) c=0.5, d=400

N^2 with n lgn

Get Big O notation for

(a) $0.5*n^2 + 300\ n*lgn + 2000*n + 19999$

(b) $30\ n^2 + 0.5\ n*lgn + 0.01n + 0.0001$

Solution

These are of the form

$a*n^2 + b*n\log n + c*n + d$

The growth of n^2 is greater than nlogn,n,logn and constants(d is the constant here).

Hence the expression is $O(n^2)$

(a) a=0.5, b=300, c=2000,d=19999

(c) a= 30, b=0.5, d=0.01, d=0.0001

More complexity

Compute O(n) for

(a) (12+n) lgn + (12n+lgn)n + (n+1)25 +3

(b) (1+n)2n + (23+lgn)0.5n (Left as exercise)

Solution (contd)

The growth rate of n^2 is greater than nlogn,n,logn and constant.

Hence, we get Big O as

$O(n^2)$

Solution

Do the algebra

(a) $(12+n)\lg n + (12n+\lg n)n+(n+1)25+3$

$= 12\lg n+n\lg n+12n^2 +n\lg n+25n+25+3$

$= 12\lg n + 2n\lg n +12n^2 +25n+28$

(Rewriting)

$= 12n^2 +25n+2n\lg n+12\lg n+28$

This is of the form

$an^2 +bn+cn\lg n+d\lg n+e$

where a=12,b=25,c=2 d=12,e=28.

Syllable Lengths
Write Big O notation for this song with n verses

This old man, he played one,
He played knick, knack on my drum,

This old man he played two,
He played knick, knack on my shoe

This old man he played three,
He played knick, knack on my knee

This old man he played four,
He played knick, knack on my door

This old man he played five,
He played knick, knack on my hive

This old man he played six,
He played knick, knack on my sticks

This old man he played seven,
He played knick, knack up in heaven

This old man he played eight,
He played knick, knack on my gate

This old man he played nine,
He played knick, knack on my spine

… (till three hundred thirty three verses)
This old man he played three hundred thirty three
He played knick, knack on my keys

Solution

Consider n to be the number of verses.

- Lets say verse 3 has 10 syllables, without the word “three”

  That is

  This old man he played ~~**three**~~

  He played knick, knack on my knee

  has 10 syllables

- Now, Notice verse 333, here n=333.
- “Three thirty three” has approximately three syllables, which is lg(333).

This old man he played ~~**three hundred thirty three**~~

He played knick, knack on my keys

The other parts of the verse 333, except three thirty three have same number of syllables as verse 3, that is 10.

Solution(contd)

Totally we have (10+lg i) syllables for each verse i.

For n verses we have

10+(10+lg1)+(10+lg2)+ (10+lg3) … (10+lgn)

= 10n+(lg1+lg2+lg3 ...+lg(n-1) +lgn)

approximately = 10n + nlgn

This is O(nlgn)

(Hint: think of a song with 100 million verses. What is lgn for that?)

More Verses:
Consider Frost wrote a song of n verses of the same kind, how many syllables would he use in Big O notation

Whose woods these are I think I know.
His house is in the village though;
He will not see me stopping here
To watch his woods fill up with snow.

My little horse must think it queer
To stop without a farmhouse near
Between the woods and frozen lake
The darkest evening of the year.

He gives his harness bells a shake
To ask if there is some mistake.
The only other sound’s the sweep
Of easy wind and downy flake.

The woods are lovely, dark and deep.
But I have promises to keep,
And miles to go before I sleep,
And miles to go before I sleep.

Robert Frost

Solution

There are four verses in this song. Each has 20 syllables.

For a song with 'n' verses each with 20 syllables. We have 20n syllables

Since, there are four verses in this song n=4.

In Big O notation we write 20n as order O(n).

Another Solution

Each of the four verses have four lines and all of them almost the same number of syllables.

Hence, the song grows at the rate of number of verses 'n'

Growth of song

There's a hole at the bottom of the sea. There's a hole at the bottom of the sea. There's a hole, there's a hole, there's a hole at the bottom of the sea.

There's a log in the hole at the bottom of the sea. There's a log in the hole at the bottom of the sea. There's a log, there's a log, there's a log in the hole at the bottom of the sea.

There's a bump on the log in the hole at the bottom of the sea. There's a bump on the log in the hole at the bottom of the sea. There's a bump, there's a bump, there's a bump on the log in the hole at the bottom of the sea

There's a frog on the bump on the log in the hole at the bottom of the sea. There's a frog on the bump on the log in the hole at the bottom of the sea. There's a frog, there's a frog, there's a frog on the bump on the log in the hole at the bottom of the sea.

There's a leg on the frog on the bump on the log in the hole at the bottom of the sea. There's a leg on the frog on the bump on the log in the hole at the bottom of the sea. There's a leg, there's a leg, there's a leg on the frog on the bump on the log in the hole at the bottom of the sea

Solution

Consider the number of syllables in the first verse as 20, each line in the verse having five syllables.

There's a hole at the bottom of the (has 5 syllables)
sea. There's a hole at the bottom of the (5 syllables)
sea. There's a hole, there's a hole, (5 syllables)
there's a hole at the bottom of the sea. (5 syllables)

Notice second verse has one more line. So it will have 25 syllables
Third verse will have 30 syllables and so on.
For n verses we have

= 20+20+1*5+20+2*5+20+3*5 … 20+((n-1)*5) (See the growth of the function)
= 20n+5*(n)*(n-1)/2
This has order $O(n^2)$
In our song , when n=5,