Q, Study sets, textbooks, questions

Explanations

Question

In a test of a printed circuit board using a random test pattern, an array of 10 bits is equally likely to be 0 or 1. Assume the bits are independent. a. What is the probability that all bits are 1s? b. What is the probability that all bits are 0s? c. What is the probability that exactly 5 bits are 1s and 5 bits are 0s?

Explanation

Verified

Step 1

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Denote events

$$A_i = \{i^{th} \text{ bit is } 1\};$$

for $i = 1, 2, \ldots, 10$. We are given

$$P(A_i) = \frac{1}{2} = 0.5;$$
$$P(A_i') = 1 - P(A_i) = \frac{1}{2} = 0.5,$$

because the bits is equally likely to be 0 or 1.

(a):

Event that all bits are 1s is actually intersection of events A_i, $i = 1, 2, \ldots, 10$, therefore

$$\begin{aligned} P(A_1 \cap A_2 \cap \ldots \cap A_{10}) &\overset{(1)}{=} P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_{10}) \\ &= 0.5 \cdot 0.5 \cdot \ldots \cdot 0.5 \\ &= 0.5^{10} = 0.00098, \end{aligned}$$

(1) : events A_i, $i = 1, 2, \ldots, 10$ are independent (given in the exercise), so we can use the multiplication property given below.

Multiplication Property:

For events $A_1, A_2, \ldots, A_n$, $n \in \mathbb{N}$ we say that they are mutually independent if

$$P(A_{i_1} \cap A_{i_2} \ldots A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \ldots \cdot P(A_{i_k})$$

for every $k \in \{2, 3, \ldots, n\}$, and every subset of indices $i_1, i_2, \ldots, i_k$.

Step 2

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(b):

Event that all bits are 0s can be represented as intersection of complement of events A_i, $i = 1, 2, \ldots, 10$. Remember that complement of event A_i is that i^{th} bit is 0. Therefore,

$$\begin{aligned} P(A_1' \cap A_2' \cap \ldots \cap A_{10}') &\overset{(1)}{=} P(A_1') \cdot P(A_2') \cdot \ldots \cdot P(A_{10}') \\ &= 0.5 \cdot 0.5 \cdot \ldots \cdot 0.5 \\ &= 0.5^{10} = 0.00098, \end{aligned}$$

(1) : events A_i, $i = 1, 2, \ldots, 10$ are independent (given n the exercise) implies that events A_i', $i = 1, 2, \ldots, 10$ are independent (see proposition below), so we can use the multiplication property given above.

Proposition: If A and B are independent, then

- A' and B are independent;
- A' and B' are independent;
- A and B' are independent.

which is true for n events too (each combination of events A_i and their complements are independent events).

Step 3

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(c):

Event that exactly 5 bits are 1s and 5 bits are 0s can be represented as union of events such as

$$0101010101, 0000011111, \ldots$$

where each of those events is intersection of 5 events A_i and 5 events $A'_{i'}$, and all of them have the same probability. Since these events are disjoint (mutually exclusive), probability of their union is the sum of their probabilities. Denote

$$E_5 = \{\text{exactly 5 bits are 1s and 5 bits are 0s}\}.$$

Event E_5 is going to be the sum of probabilities of disjoint events.

We know that event E_5 is union of disjoint events, but how many? Since we do not care about the order, we will use combinations to determine this.

A combination is an unordered subset. For n individuals in a group, the number of combinations size k is denoted as $C_{k,n}$ or

$$\binom{n}{k}$$

which we read "from n elements choose k". The proposition:

$$C_{k,n} = \binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

In our case, $n = 10$ (we have 10 bits) and we want group size $k = 5$ (five 1s), therefore, there are

$$C_{5,10} = \binom{10}{5} = \frac{10!}{5!(10-5)!} = 252$$

different size 5 groups of 1s to put in 10 bit array.

Step 4

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Now we just need to determine probability of intersections. Since probabilities of all 252 intersection are the same, and we have some of those probabilities (explained above), we will calculate one (first five 1s, others 0s)

$$\begin{aligned} &P(A_1 \cap A_2 \cap \ldots \cap A_5 \cap A_6' \cap A_7' \cap \ldots \cap A_{10}') \\ &\overset{(1)}{=} P(A_1) \cdot \ldots \cdot P(A_5) \cdot \ldots \cdot P(A_6') \cdot \ldots \cdot P(A_{10}') \\ &\overset{(2)}{=} 0.5^{10} = 0.00098, \end{aligned}$$

(1) : here we use proposition given in (b), so events in the intersection are independent, and than we use the multiplication property given in (a);

(2) : all probabilities are the same.

Therefore, probability that exactly 5 bits are 1s and 5 bits are 0s is

$$\begin{aligned} P(E_5) &\overset{(1)}{=} 0.00098 + \ldots + 0.00098 \\ &= 252 \cdot 0.00098 = 0.246, \end{aligned}$$

Result

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a. $P(A_1 \cap A_2 \cap \ldots \cap A_{10}) = 0.00098$; b. $P(A_1' \cap A_2' \cap \ldots \cap A_{10}') = 0.00098$; c. $P(E_5) = 0.246$.

Related questions

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