Solutions to Chapter 3

1. Suppose the size of an uncompressed text file is 1 megabyte.

Solutions follow questions: **[4 marks – 1 mark each for a & b, 2 marks c]**

a. How long does it take to download the file over a 32 kilobit/second modem?

T_{32k} = 8 (1024) (1024) / 32000 = 262.144 seconds

b. How long does it take to take to download the file over a 1 megabit/second modem?

T_{1M} = 8 (1024) (1024) bits / $1x10^6$ bits/sec = 8.38 seconds

c. Suppose data compression is applied to the text file. How much do the transmission times in parts (a) and (b) change?

If we assume a maximum compression ratio of 1:6, then we have the following times for the 32 kilobit and 1 megabit lines respectively:

T_{32k} = 8 (1024) (1024) / (32000 x 6) = 43.69 sec
T_{1M} = 8 (1024) (1024) / ($1x10^6$ x 6) = 1.4 sec

2. A scanner has a resolution of 600 x 600 pixels/square inch. How many bits are produced by an 8-inch x 10-inch image if scanning uses 8 bits/pixel? 24 bits/pixel? **[3 marks – 1 mark for pixels per picture, 1 marks each representation]**

Solution:

The number of pixels is 600x600x8x10 = $28.8x10^6$ pixels per picture.
With 8 bits/pixel representation, we have: $28.8x10^6$ x 8 = 230.4 Mbits per picture.
With 24 bits/pixel representation, we have: $28.8x10^6$ x 24 = 691.2 Mbits per picture.

6. Suppose a storage device has a capacity of 1 gigabyte. How many 1-minute songs can the device hold using conventional CD format? using MP3 coding? **[4 marks – 2 marks each]**

Solution:

A stereo CD signal has a bit rate of 1.4 megabits per second, or 84 megabits per minute, which is approximately 10 megabytes per minute. Therefore a 1 gigabyte storage will hold 1 gigabyte/10 megabyte = 100 songs.

An MP3 signal has a lower bit rate than a CD signal by about a factor of 14, so 1 gigabyte storage will hold about 1400 songs.

8. How many HDTV channels can be transmitted simultaneously over the optical fiber transmission systems in Table 3.3? **[2 marks]**

Solution:

Suppose that an optical fiber carries 1600 x 10^9 bps, and an HDTV channel is about 38 Mbps, then the fiber can carry about 1600000/38 = 40,000 HDTV channels.

60. Let $g(x)=x^3+x+1$. Consider the information sequence 1001.

Solutions follow questions:

a. Find the codeword corresponding to the preceding information sequence.

Using polynomial arithmetic we obtain: **[3 marks]**

```
            1010
1011 | 1001000
       1011
       -------
        01000
         1011
        -------
          00110
```

Codeword = 1001110

b. Suppose that the codeword has a transmission error in the first bit. What does the receiver obtain when it does its error checking? **[2 marks]**

```
            0001
1011 | 0001110
          1011
       -------
           101
```

CRC calculated by Rx = 101 → error

62. Suppose a header consists of four 16-bit words: (11111111 11111111, 11111111 00000000, 11110000 11110000, 11000000 11000000). Find the Internet checksum for this code. **[3 marks]**

Solution:

b_0 = 11111111 11111111 = $2^{16} - 1$ = 65535

b_1 = 11111111 00000000 = 65280

b_2 = 11110000 11110000 = 61680

b_3 = 11000000 11000000 = 49344

$x = b_0 + b_1 + b_2 + b_3$ modulo 65535 = 241839 modulo 65535 = 45234

$b_4 = -x$ modulo 65535 = 20301

So the Internet checksum = 01001111 01001101

63. Let $g_1(x) = x + 1$ and let $g_2(x) = x^3 + x^2 + 1$. Consider the information bits (1,1,0,1,1,0).

a. Find the codeword corresponding to these information bits if $g_1(x)$ is used as the generating polynomial. **[2 marks]**

```
          100100
   11  | 1101100
         11
         ------
          0011
            11
          ----
            0000
```

Codeword = 1101100

b. Find the codeword corresponding to these information bits if $g_2(x)$ is used as the generating polynomial. **[2 marks]**

```
             100011
   1101  | 110110000
           1101
           ---------
             01000
              1101
             -----
              1010
              1101
             -----
               111
```

Codeword = 110110111

c. Can $g_2(x)$ detect single errors? double errors? triple errors? If not, give an example of an error pattern that cannot be detected. **[2 marks – 0.5 each]**

Single errors can be detected since $g_2(x)$ has more than one term. Double errors *cannot* be detected even though $g_2(x)$ is primitive because the codeword length exceeds $2^{n-k}-1=7$. An example of such undetectable error is 1000000010. Triple errors cannot be detected since $g_2(x)$ has only three terms.

d. Find the codeword corresponding to these information bits if $g(x) = g_1(x)\, g_2(x)$ is used as the generating polynomial. Comment on the error-detecting capabilities of $g(x)$. **[4 marks – 2 marks for the codeword and 2 for the comment]**

```
              111101
10111 | 1101100000
        10111
        -----
         11000
         10111
         -----
          11110
          10111
          -----
           10010
           10111
           -----
            010100
             10111
             -----
              0011
```

Codeword = 1101100011

The new code can detect all single and all odd errors. It cannot detect double errors. It can also detect all bursts of length $n - k = 4$ or less. All bursts of length 5 are detected except for the burst that equals $g(x)$. The fraction $1/2^{n-k} = 1/16$ of all bursts of length greater than 5 are detectable.

67. Consider the $m = 4$ Hamming code.

a. What is n, and what is k for this code? **[2 marks]**

$n = 2^m - 1 = 15$; $k = n - m = 11$

(15,11) Hamming code

b. Find parity check matrix for this code. **[2 marks – 0.5 for each]**

$$\mathbf{H} = \begin{bmatrix} 0&0&0&0&1&1&1&1&1&1&1&1&0&0&0 \\ 1&1&1&0&0&0&0&1&1&1&1&0&1&0&0 \\ 0&1&1&1&0&1&1&0&0&1&1&0&0&1&0 \\ 1&0&1&1&1&0&1&0&1&0&1&0&0&0&1 \end{bmatrix}$$

c. Give the set of linear equations for computing the check bits in terms of the information bits. **[2 marks – 0.5 for each]**

$b_{12} = b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11}$
$b_{13} = b_1 + b_2 + b_3 + b_8 + b_9 + b_{10} + b_{11}$
$b_{14} = b_2 + b_3 + b_4 + b_6 + b_7 + b_{10} + b_{11}$
$b_{15} = b_1 + b_3 + b_4 + b_5 + b_7 + b_9 + b_{11}$