MAS291 - HOMEWORK CHAP 7

Nguyen Dang Loc - SE160199

7-10

Suppose that the random variable X has the continuous uniform distribution

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose that a random sample of n = 12 observations is selected from this distribution. What is the approximate probability distribution of $\overline{X} - 6$? Find the mean and variance of this quantity.

Solution:

Mean and variance of continuous unifrom distribution:

$$\mu = E(X) = \frac{0+1}{2} = 0.5,$$

$$\sigma^2 = V(X) = \frac{(1-0)^2}{12} = \frac{1}{12} \approx 0.0833$$

The sampling distribution of the sample mean $\overline{X}$ is approximately normal with

+ Mean: $\mu_{\overline{X}} = E(\overline{X}) = \mu$

+ Variance: $\sigma^2_{\overline{X}} = V(\overline{X}) = \frac{\sigma^2}{n}$

Mean and variance of the probability distribution $\overline{X} - 6$:

$$E(\overline{X} - 6) = E(\overline{X}) - 6 = 0.5 - 6 = -5.5$$

$$V(\overline{X} - 6) = V(\overline{X}) = \frac{\sigma^2}{n} = \frac{1/12}{12} \approx 0.00694$$

7-3

PVC pipe is manufactured with a mean diameter of 1.01 inch and a standard deviation of 0.003 inch. Find the probability that a random sample of n = 9 sections of pipe will have a sample mean diameter greater than 1.009 inch and less than 1.012 inch.

Solution:

Let $X_1, X_2, ..., X_9$ is a random sample of size $n = 9$ of pipe's diameter with $\mu = 1.01,\ \sigma = 0.003$

The sampling distribution of $\overline{X}$ is normal with

+ Mean $\mu_{\overline{X}} = \mu = 1.009$,

+ Standard deviation $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{0.003}{\sqrt{9}} = 0.001$

Then:

$$\begin{aligned} P(1.009 < \overline{X} < 1.012) &= P\left(\frac{1.009 - 1.01}{0.003/\sqrt{9}} < Z < \frac{1.012 - 1.01}{0.003/\sqrt{9}}\right) \\ &= P(-1 < Z < 2) \\ &\approx 0.8185946 \end{aligned}$$

7-17

From the data in Exercise 6-21 on the pH of rain in Ingham County, Michigan:

5.47	5.37	5.38	4.63	5.37	3.74	3.71	4.96	4.64	5.11	5.65
5.39	4.16	5.62	4.57	4.64	5.48	4.57	4.57	4.51	4.86	4.56
4.61	4.32	3.98	5.70	4.15	3.98	5.65	3.10	5.04	4.62	4.51
4.34	4.16	4.64	5.12	3.71	4.64					

What proportion of the samples has pH below 5.0?

Solution:

A sample size $n = 39,$ a subsample size $x = 26$ which contain values less than 5.

The proportion of the sample has pH below 5.0 is: $P(X < 5.0) = \frac{x}{n} = \frac{26}{39} \approx 0.667$

About 67% of the sample has pH below 5.0.

Use R:

```
data <- read.table("D://MAS//717.txt", sep="", header=F,
                   na.strings="", stringsAsFactors=F) # read data
dt <- as.numeric(data)
res <- length(dt[dt<5])/length(dt) # 0.6666667
```