MAS291 - HOMEWORK CHAP 3

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3-5

Determine the range (possible values) of the random variable.
A batch of 500 machined parts contains 10 that do not conform to customer requirements. Parts are selected successively, without replacement, until a nonconforming part is obtained. The random variable is the number of parts selected.

Solution:

500 machined parts contains:
+ 10 non-conforming parts
+ 490 conforming parts

Let X denote the random variable → X is the number of parts selected

- $X = 1$ → The first part selected is non-conforming
- $X = 2$ → The second part selected is non-conforming
- ...
- $X = 491$ → The 491^{th} part selected is non-conforming
  (All 490 parts selected earlier are conforming)

Hence, range of X is $\{1, 2, ..., 491\}$

3-75

Calculate the mean for the random variable in Exercise 3-37.

(3-37) Consider the circuit in Example 2-32. Assume that devices fail independently. What is the probability mass function of the number of failed devices?

(Example 2-32) The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently.

Solution:

The number of failed devices in total of 2 devices is the random varible.

Let X denote the random variable. Range of X is $\{0, 1, 2\}$

Calculate the PMF:

- $f(0) = P(X = 0) = P(x_1)P(x_2) = 0.8 \times 0.9 = 0.72$
- $f(2) = P(X = 2) = P(x_1{}')P(x_2{}') = (1 - P(x_1))(1 - P(x_2)) = 0.2 \times 0.1 = 0.02$
- $f(1) = P(X = 1) = 1 - P(X = 0) - P(X = 2) = 1 - 0.72 - 0.02 = 0.26$

Mean of the random variable:

$$\mu = E(X) = 0 \times f(0) + 1 \times f(1) + 2 \times f(2)$$
$$= 0 \times 0.72 + 1 \times 0.26 + 2 \times 0.02$$
$$= 0.3$$

3-95

The random variable X has a binomial distribution with n = 10 and p = 0.01. Determine the following probabilities.

(a) $P(X = 5)$

(b) $P(X \leqslant 2)$

(c) $P(X \geqslant 9)$

(d) $P(3 \leqslant X < 5)$

Solution:

$n = 10,\ p = 0.01$

(a) $P(X = 5) = \binom{10}{5} 0.01^5 (1 - 0.01)^{10-5} \approx 2.39 \times 10^{-8}$

(b) $P(X \leqslant 2) = P(0) + P(1) + P(2)$

$$= \binom{10}{0} 0.01^0 (1 - 0.01)^{10} + \binom{10}{1} 0.01^1 (1 - 0.01)^9 + \binom{10}{2} 0.01^2 (1 - 0.01)^8$$

$$\approx 0.9999$$

(c) $P(X \geqslant 9) = P(9) + P(10)$

$$= \binom{10}{9} 0.01^9 (1 - 0.01)^1 + \binom{10}{10} 0.01^{10} (1 - 0.01)^0$$

$$= 9.91 \times 10^{-18}$$

(d) $P(3 \leqslant X < 5) = P(3) + P(4)$

$$= \binom{10}{3} 0.01^3 (1 - 0.01)^7 + \binom{10}{4} 0.01^4 (1 - 0.01)^6$$

$$\approx 1.138 \times 10^{-4}$$