MAS291 - HOMEWORK CHAP 9

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9-25

The proportion of adults living in Tempe, Arizona, who are college graduates is estimated to be p = 0.4. To test this hypothesis, a random sample of 15 Tempe adults is selected. If the number of college graduates is between 4 and 8, the hypothesis will be accepted; otherwise, you will conclude that p ≠ 0.4.
(a) Find the type I error probability for this procedure, assuming that p = 0.4.

(b) Find the probability of committing a type II error if the true proportion is really p = 0.2.

a)

$$\begin{aligned}\alpha &= P(\overline{x} < 4) + P(\overline{x} > 8),\ p = 0.4\\ &= P(\frac{\overline{x} - np}{\sqrt{np(1-p)}} < \frac{4 - 15 \times 0.4}{\sqrt{15 \times 0.4 \times (1-0.4)}}) + P(\frac{\overline{x} - np}{\sqrt{np(1-p)}} > \frac{8 - 15 \times 0.4}{\sqrt{15 \times 0.4 \times (1-0.4)}})\\ &= P(z < -1.054) + P(z > 1.054)\\ &= 2 \times P(z < -1.054)\\ &= 0.292\end{aligned}$$

b)

$$\begin{aligned}\alpha &= P(4 \leq \overline{x} \leq 8),\ p = 0.2\\ &= P(\frac{4 - 15 \times 0.4}{\sqrt{15 \times 0.2 \times (1-0.2)}} \leq \frac{\overline{x} - np}{\sqrt{np(1-p)}} \leq \frac{8 - 15 \times 0.2}{\sqrt{15 \times 0.2 \times (1-0.2)}})\\ &= P(0.64 \leq z \leq 3.23)\\ &= P(z < 3.23) - P(z < 0.64)\\ &= 0.257\end{aligned}$$

9-39

Output from a software package follows:

One-Sample Z:
Test of mu = 20 vs mu > 20
The assumed standard deviation = 0.75

Variable	N	Mean	StDev	SE Mean	Z	P
x	10	19.889	?	0.237	?	?

(a) Fill in the missing items. What conclusions would you draw?
(b) Is this a one-sided or a two-sided test?
(c) Use the normal table and the preceding data to construct a 95% two-sided CI on the mean.
(d) What would the P-value be if the alternative hypothesis is $H_1 : \mu \neq 20$?

Solution

a)

$$\sigma = SE\ Mean \times \sqrt{n} = 0.237 \times \sqrt{10} = 0.7495$$

$$Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{19.889 - 20}{0.75/\sqrt{10}} = -0.468$$

$$P = 1 - \Phi(z_0) = 1 - \Phi(-0.468) = 1 - 0.32 = 0.68$$

b)

This is one-sided test because $H_1 : \mu > 20$

c)

A 95% two-sided CI on the mean:

$$\overline{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$\Leftrightarrow 19.889 - 1.96\frac{0.7495}{\sqrt{10}} \le \mu \le 19.889 + 1.96\frac{0.7495}{\sqrt{10}}$$
$$\Leftrightarrow 19.4245 \le \mu \le 20.354$$

d)

With $H_1 : \mu \neq 20$, P-value is

$P = 2 \times (1 - \Phi(z_0)) = 2 \times (1 - \Phi(-0.468)) = 2 \times (1 - 0.32) = 1.36$

9-53

A hypothesis will be used to test that a population mean equals 10 against the alternative that the population mean is greater than 10 with unknown variance. What is the critical value for the test statistic T_0 for the following significance levels?
(a) $\alpha = 0.01$ and $n = 20$
(b) $\alpha = 0.05$ and $n = 12$
(c) $\alpha = 0.10$ and $n = 15$

Solution

$H_0 : \mu = 10$
$H_1 : \mu > 10$

a) $t_{0.01,19} = 2.539$

a) $t_{0.05,11} = 1.796$

a) $t_{0.10,15} = 1.345$