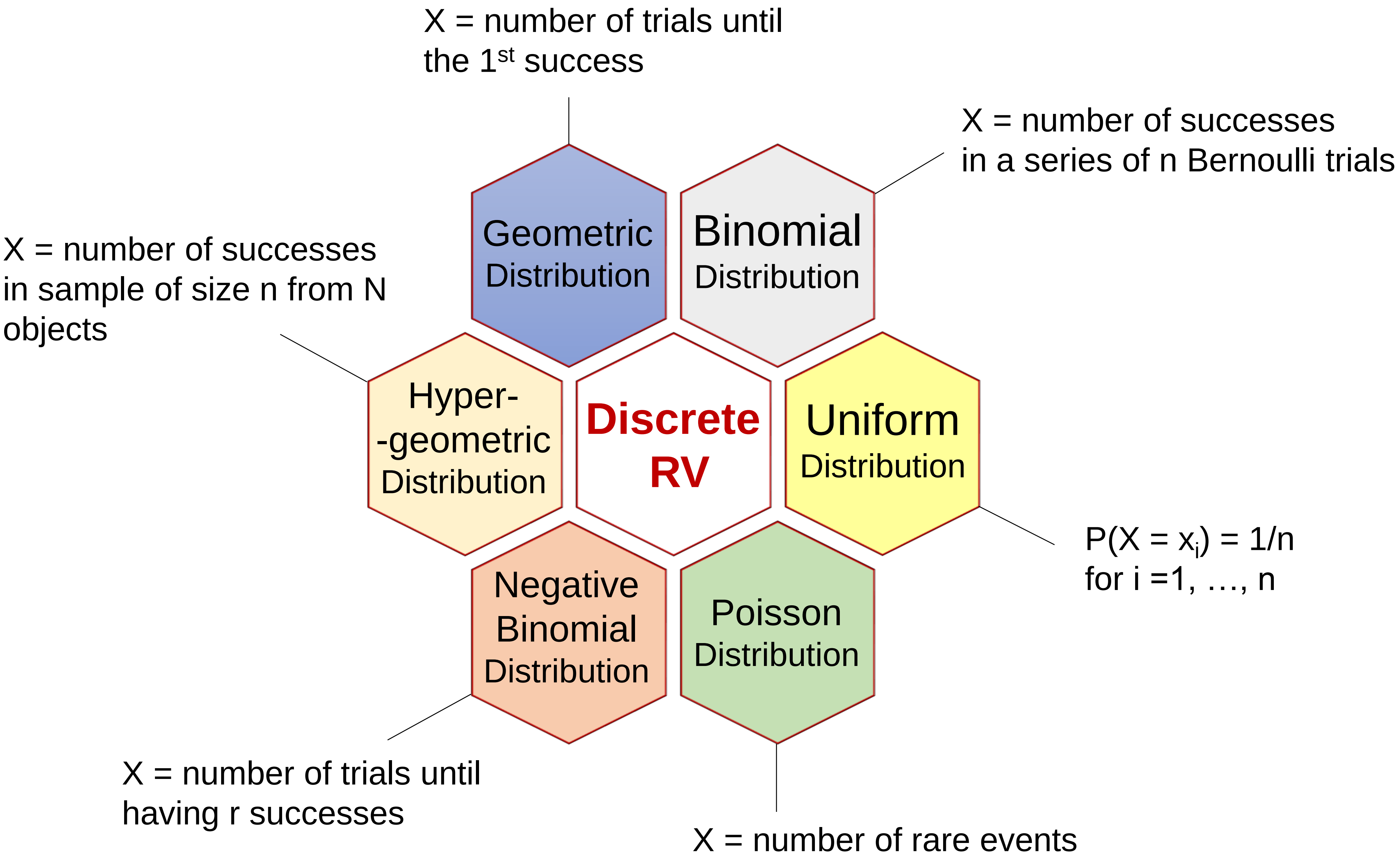
3

Discrete Random Variables and Probability Distributions

LO

- Describe a discrete ***random variable***
- Check if a function is a ***probability mass function*** and use it to calculate probability
- Find the ***cumulative distribution function*** of a discrete random variable
- Compute the ***mean*** and ***variance*** of a discrete random variable
- Determine the probability, mean and variance of ***uniform***, ***binomial***, ***geometric*** and ***negative binomial***, ***hypergeometric*** and ***Poisson distributions***


X = number of trials until the 1st success
X = number of successes in a series of n Bernoulli trials
X = number of successes in sample of size n from N objects
Geometric Distribution
Binomial Distribution
Hyper--geometric Distribution
Discrete RV
Uniform Distribution
P(X = x_i) = 1/n for i =1, …, n
Negative Binomial Distribution
Poisson Distribution
X = number of trials until having r successes
X = number of rare events

Random Variables

- A random variable is a function that assigns a real number to each outcome in the sample space of a random experiment.

$$X : S \to \mathbb{R}$$

$$X(\omega) \in \mathbb{R}$$

If $X(S) = \{x_1, x_2, \ldots, x_n\}$ or $X(S) = \{x_1, x_2, \ldots, x_n, \ldots\}$,

X is called *discrete*.

Random variables – Ex

Flipping a coin twice.

➔ The sample space is S = {HH, HT, TH, TT}.

$$X : S \to \mathbb{R}$$

$X(\omega)$ = *number of heads* in each outcome ω

$\omega \in S$	HH	HT	TH	TT
$X(\omega)$	2	1	1	0

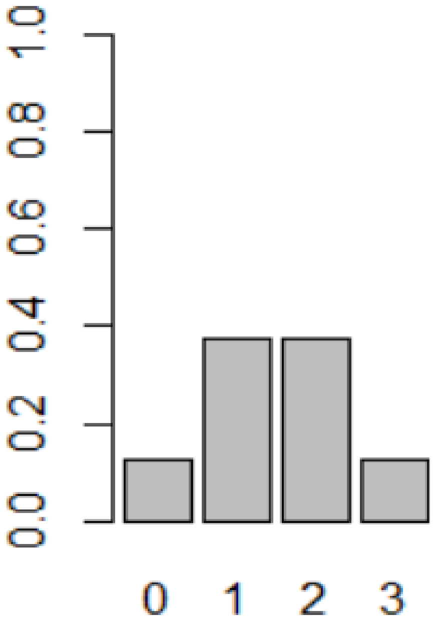
Discrete random variable

Probability Distribution – Ex1

Toss a fair coin three times and let *X be the number of Heads* observed, $X(\omega) \in \{0, 1, 2, 3\}$. Then we have the following *probabilities*

	TTT	HTT THT TTH	HHT HTH THH	HHH
X	0	1	2	3
P(X = x)	1/8	3/8	3/8	1/8

$$\Sigma_x P(x) = 1$$



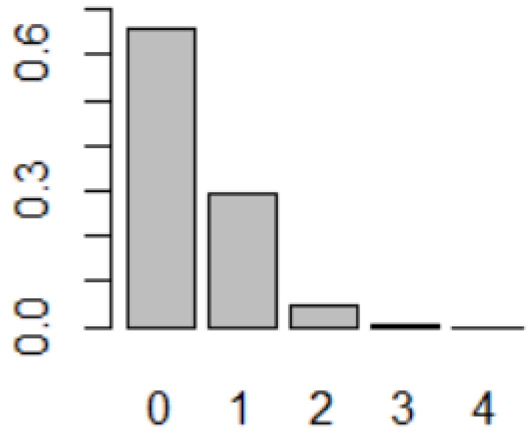
Probability distribution for number of heads.

Probability Distribution - Ex2

- **<u>Ex.</u>** (Digital Channel) There is a chance that a bit transmitted through a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are {0, 1, 2, 3, 4}. Suppose that the probabilities are

X	0	1	2	3	4
P(X = x)	0.6561	0.2916	0.0486	0.0036	0.0001

The *probability distribution* of a random variable X is a description of the probabilities associated with the possible values of X.



Probability distribution for bits in error.

Probability Mass Functions (pmf)

For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$, a *probability mass function* is a function f such that

(1) $f(x_i) \geqslant 0$

(2) $f(x_i) = P(X = x_i)$

(3) $\Sigma_i f(x_i) = 1$

Ex. Verify that the following function is a *pmf*.

$$f(x) = \frac{2x+1}{25}, \quad x = 0, 1, 2, 3, 4$$

(1) $f(x) \geqslant 0$

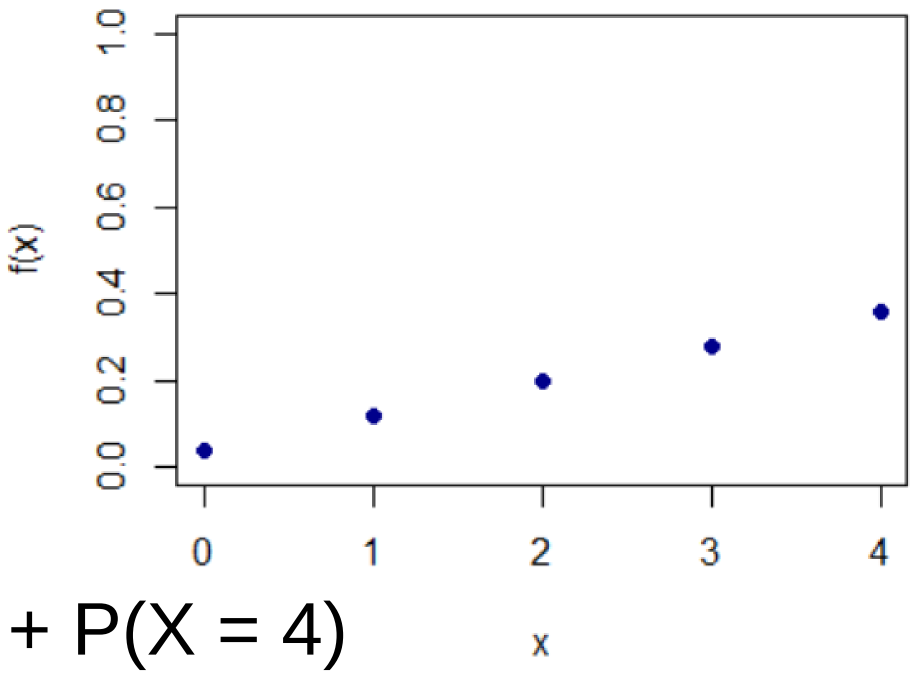
(2) $f(x) = P(X = x)$, and $P(X = 4) = f(4) = 9/25$

(3) $\Sigma_i f(x_i) = f(0) + f(1) + f(2) + f(3) + f(4) = 1$

Probability Mass Functions (pmf) - Ex

Given the *pmf* f(x), determine the probabilities.

$f(x) = \dfrac{2x + 1}{25}$, x = 0, 1, 2, 3, 4



a/ $P(X = 4)$ b/ $P(X \leqslant 3)$

c/ $P(2 \leqslant X \leqslant 4)$ d/ $P(X > -3)$

---

c/ $P(2 \leqslant X \leqslant 4) = P(X = 2) + P(X = 3) + P(X = 4)$

$= (2 \cdot 2 + 1)/25 + (2 \cdot 3 + 1)/25 + (2 \cdot 4 + 1)/25 = 21/25$

Cumulative Distribution Function (cdf)

The *cumulative distribution function (cdf)* of a discrete random variable X, denoted as F(x), is

$$F(x) = P(X \leqslant x)$$

For discrete random variable X, F(x) satisfies

(1) $F(x) = \sum_{x_i \leqslant x} f(x_i)$

(2) $0 \leqslant F(x) \leqslant 1$

(3) If $x \leqslant y$, then $F(x) \leqslant F(y)$

x	**-1**	**0**	**1**	**2**	**otherwise**
f(x)	0.2	0.5	0	0.3	0

Find F(-1), F(1), F(1.9)

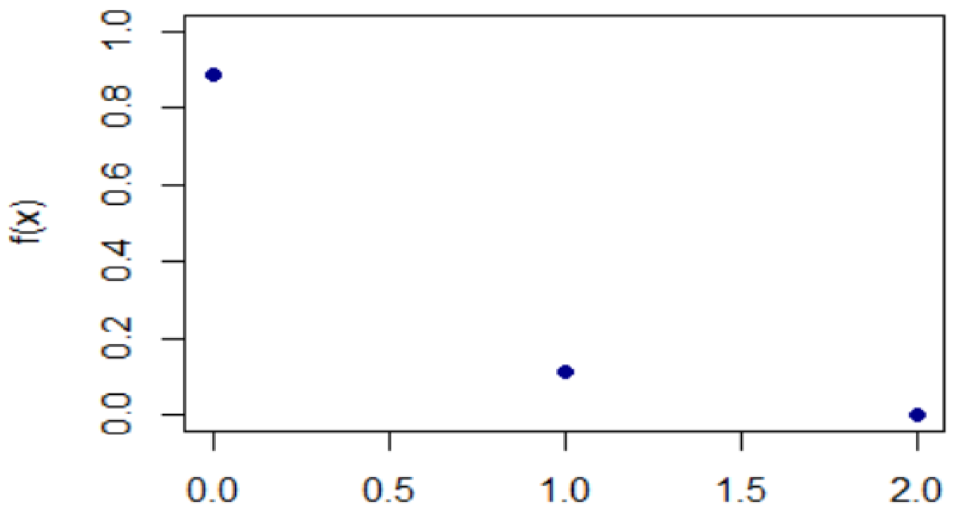
F(-1) = f(-1) = 0.2, F(1) = f(-1) + f(0) + f(1) = 0.7

F(1.9) = f(-1) + f(0) + f(1) = 0.7

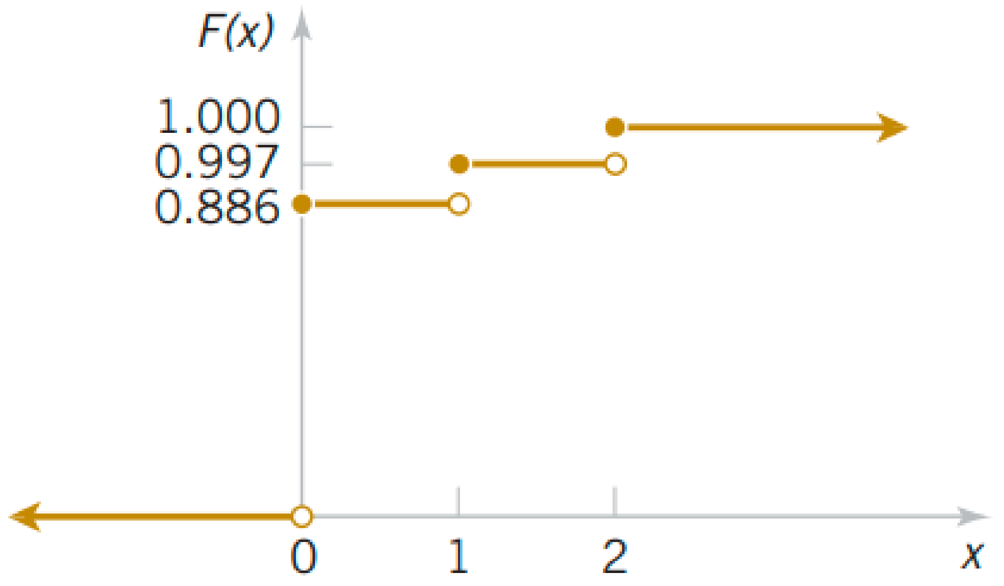
Pmf vs cdf

x	0	1	2
f(x)	0.886	0.111	0.003

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.886 & 0 \le x < 1 \\ 0.997 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



Probability mass function pmf

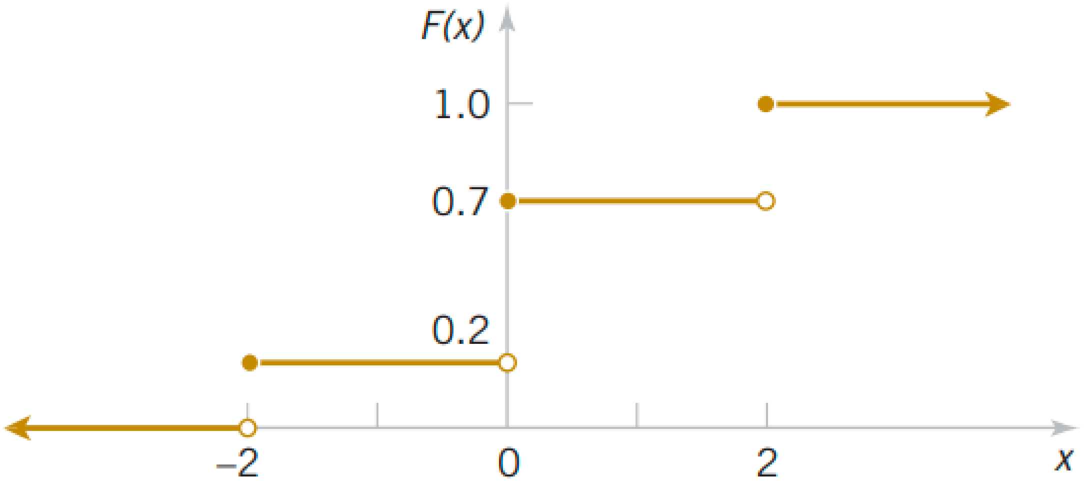


Cumulative distribution function cdf

Cdf - Ex

Determine the *pmf* of X from the following *cdf*

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \le x < 0 \\ 0.7 & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$



Find f(x) from F(x):
$f(x) = F(x) - F(x^-)$

x	-3	-2	-1	0	1	2	3
F(x)	0	0.2	0.2	0.7	0.7	1	1
f(x)	0	0.2	0	0.5	0	0.3	0

Mean and Variance

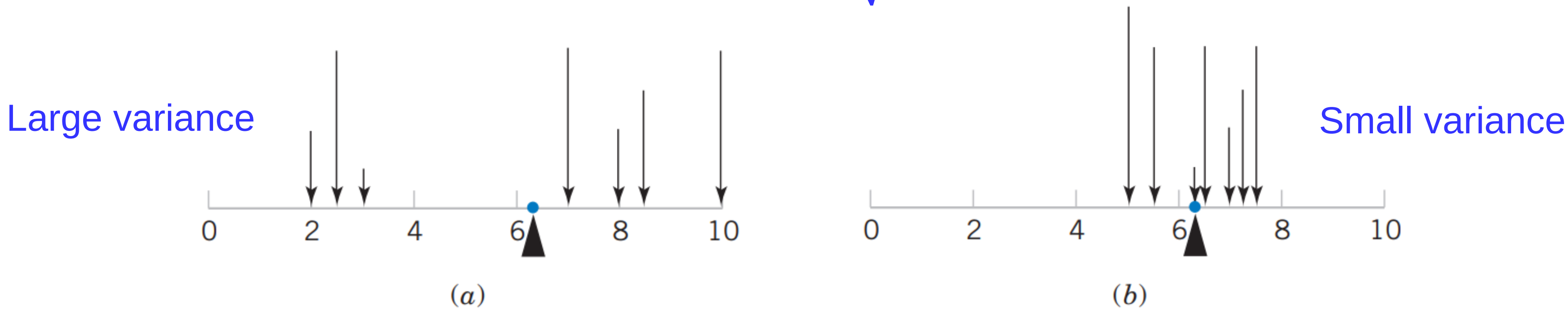
- The *mean* or *expected value* of X, denoted as μ or E(X) is

$$\mu = E(X) = \Sigma_x x f(x)$$

- The *variance* of X, denoted as σ^2 or V(X), is

$$V(X) = \sigma^2 = \Sigma_x (x - \mu)^2 f(x) = \Sigma_x x^2 f(x) - \mu^2$$

- The *standard deviation* of X is $\sigma = \sqrt{\sigma^2}$



Parts (a) and (b) illustrate equal means, but Part (a) illustrates a larger variance

Mean and Variance - Ex

Ex. (Digital Channel) There is a chance that a bit transmitted through a digital transmission channel is received in error. Let *X equal the number of bits* in error in the next four bits transmitted. Suppose that the probabilities are

x	0	1	2	3	4
f(x) = P(X = x)	0.6561	0.2916	0.0486	0.0036	0.0001

x	$x - 0.4$	$(x - 0.4)^2$	$f(x)$	$f(x)(x - 0.4)^2$
0	−0.4	0.16	0.6561	0.104976
1	0.6	0.36	0.2916	0.104976
2	1.6	2.56	0.0486	0.124416
3	2.6	6.76	0.0036	0.024336
4	3.6	12.96	0.0001	0.001296

$\mu = E(X)$

$= \Sigma_x x f(x) = 0.4$

$\sigma^2 = V(X)$

$= \Sigma_x (x - \mu)^2 f(x) = 0.36$

Mean and Variance - Ex

Ex. Given the pmf of a discrete r. v. X.

x	0	1	2	3
f(x)	0.5	0.3	0.1	0.1

Find E(X), V(X).

E(X) = 0*0.5 + 1*0.3 + 2*0.1 + 3*0.1 = 0.8

V(X) = $0.5*(0 – 0.8)^2 + 0.3*(1 – 0.8)^2 + 0.1*(2 – 0.8)^2 + 0.1*(3 – 0.8)^2 = 0.96$

Expected Value of a Function of a Discrete Random Variable

If X is a discrete random variable with *probability mass function* f(x),

$$E[h(X)] = \Sigma_x h(x)f(x)$$

Ex. Given the pmf of a discrete r. v. X.

x	0	1	2	3
f(x)	0.5	0.3	0.1	0.1

Find E(X), E(X + 2), E(3X), E(3X + 2), $E(X^2)$

$E(X^2) = 0^2(0.5) + 1^2(0.3) + 2^2(0.1) + 3^2(0.1) = 1.6$

Some Useful properties

$$E(aX+b) = aE(X) + b$$

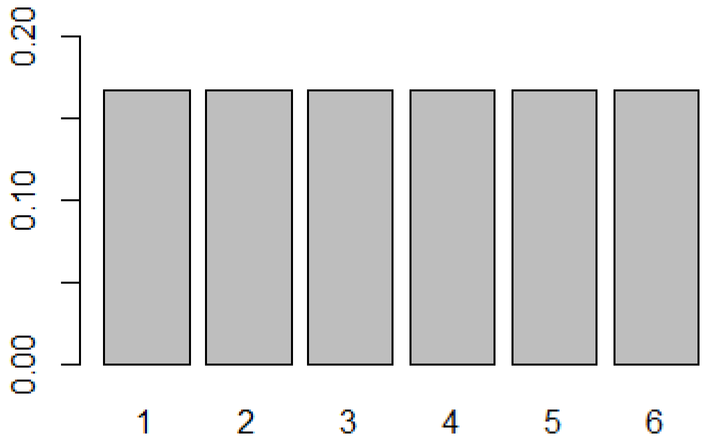
$$V(aX+b) = a^2V(X)$$

$$\begin{aligned}
\sigma^2 &= V(X) \\
&= \Sigma_x(x - \mu)^2 f(x) \\
&= E[(X - \mu)^2] \\
&= E[X^2 - 2\mu X + \mu^2] \\
&= E(X^2) - \mu^2 \\
&= E(X^2) - E(X)^2
\end{aligned}$$

Discrete Uniform Distribution

A random variable X has a discrete uniform distribution if each of the n values in its range, say $x_1, x_2, \ldots, x_n$, has *equal probability*. Then,

$$f(x_i) = 1/n$$



Ex. Roll a fair die.

Let X be the number shown.

Then X is discrete uniform range of 1 to 6.

$$f(1) = f(2) = \ldots = f(6) = 1/6$$

Discrete Uniform Distribution

- Suppose that X is a discrete uniform random variable on the ***consecutive integers*** a, a + 1, a + 2, …, b for $a \leqslant b$.
  - The *mean* of X is $$\mu = E(X) = \frac{a + b}{2}$$
  - The *variance* of X is $$\sigma^2 = V(X) = \frac{(b - a + 1)2 - 1}{12}$$

Ex. Suppose the discrete uniform random variable Y has range 5, 10, …, 30. Let Y = 5X, where X has range 1, 2, …, 6. Then,

$$E(Y) = 5E(X) = 5(1 + 6)/2 = 17.5,$$

$$V(Y) = 5^2V(X) = 25[(6 - 1 + 1)^2 - 1]/12 = 72.92$$

Bernoulli trials

- Bernoulli trial: A trial with only two possible outcomes (*success* or *failure*).

Ex. The following random experiments are series of Bernoulli trials:

  - Flip a coin 10 times.
  - Guess each question of a multiple-choice exam with 50 questions, each with four choices.

- Independence: The outcome from one trial has *no effect* on the outcome to be obtained from any other trial.

Bernoulli trials – Ex

For each question of a quiz, without preparation, you select at random an answer from 4 options. Suppose the quiz has 5 questions.

What is the probability that you get 2 correct answers?

Binomial Distribution
(bi = two: two outcomes {success, failure})

- A random experiment consists of n Bernoulli trials such that

(1) The trials are *independent*.

(2) Each trial results in only two possible outcomes, labeled as "success" and "failure".

(3) The probability of a *success* in each trial, denoted as *p*, remains constant.

- The random variable X that equals the number of trials that result in a success has a binomial random variable with parameters $0 < p < 1$ and $n = 1, 2, \ldots$ The probability mass function of X is

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

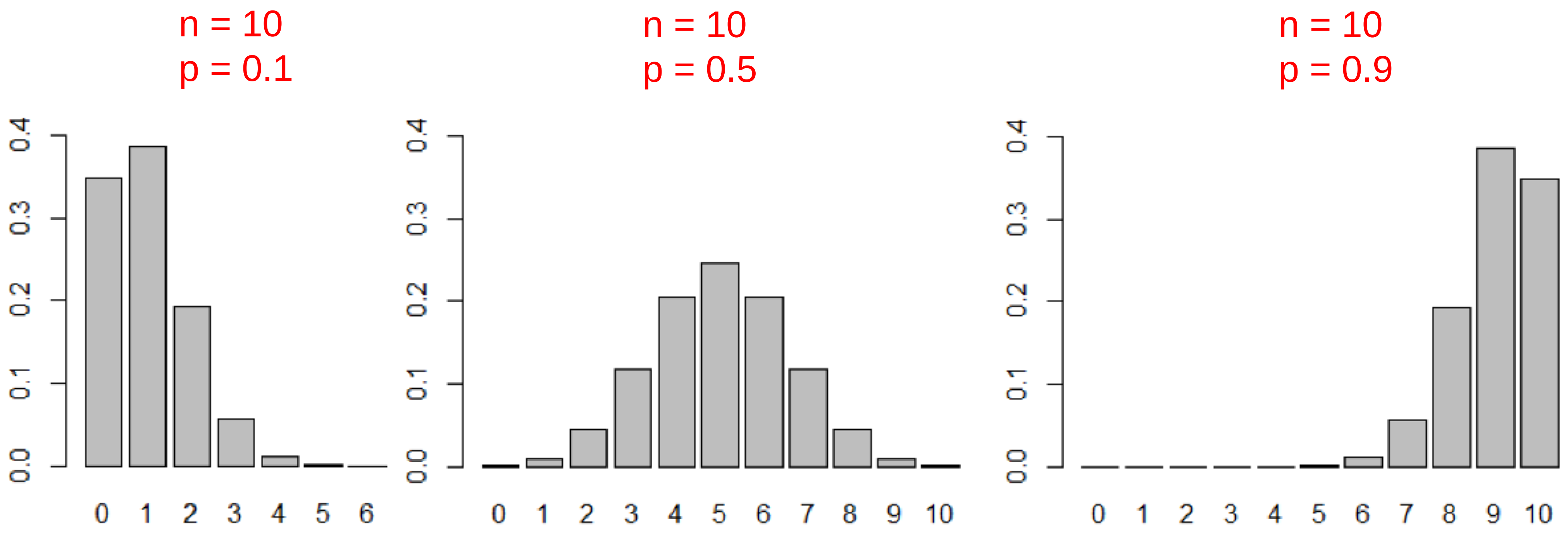
Binomial Distribution – Ex

A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other.

Find the probability of exactly 3 defective computers in a shipment of twenty.

$P(X = 3) = \binom{20}{3}(0.05)^3(0.95)^{17} = 0.0596$

Binomial distribution - specific cases



Binomial distribution – R-Ex

The random variable X has a *binomial distribution* with n = 10 and p = 0.2.

Then,

(a) $P(X = 4) = f(4) = \binom{10}{4} 0.2^4 (1-0.2)^{10-4} \approx 0.088$

(b) $P(X = 6) = f(6) = \binom{10}{6} 0.2^6 (1-0.2)^{10-6} \approx 0.0055$

(c) $P(X \leqslant 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) \approx 0.879$

(d) $P(X \geqslant 4) = 1 - P(X \leqslant 3) \approx 0.121$

Binomial Distribution - Mean and Variance

Consider random variables

$$X_i = \begin{cases} 1 & \text{if ith trial is a success} \\ 0 & \text{otherwise} \end{cases}$$

X_i	0	1
$P(X_i)$	1-p	p

$E(X_i) = p$
$V(X_i) = p(1-p)$

$X = X_1 + X_2 + \ldots + X_n$

$E(X) = np$
$V(X) = np(1-p)$

R-Ex. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the expected number of computers attacked by this virus.

Let X = "number of computers attacked by the virus"

➔X ~ binom(n=20, p=0.4)

➔ E(X) = np = 20(0.4) = 8 computers

Binomial distribution

Binomial Distribution

n	=	number of trials
X	=	number of successes
p	=	probability of success
$P(x)$	=	$\binom{n}{x}p^x(1-p)^{n-x}$
$E(X)$	=	np
$V(X)$	=	$np(1-p)$

Geometric Distribution

In a series of Bernoulli trials (independent trials with constant probability ***p of a success***), let the random variable X denote the number of trials *until the first success*. Then X is a *geometric* random variable with parameter $0 < p < 1$ and

P(the 1st success occurs on the x-th trial) is

$$f(x) = \underbrace{(1-p)^{x-1}}_{\text{x-1 trials with failures}}\,\underbrace{p}_{\text{the last trial (x-th trial) with a success}}, \quad x = 1, 2, \ldots$$

Ex. A search engine goes through a list of sites looking for a given key phrase. Suppose the search terminates as soon as the key phrase is found. The number of sites visited has geometric distribution.

Geometric Distribution - Ex

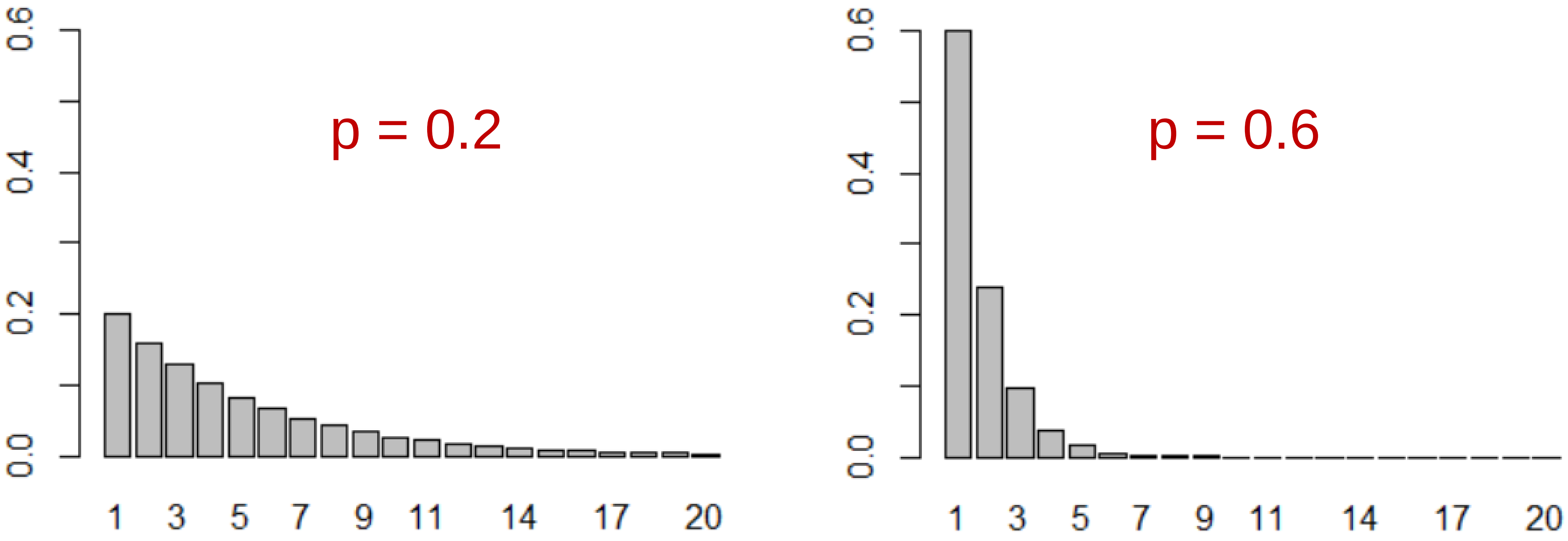
The probability that a bit transmitted through a digital transmission channel is received in *error* is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted *until* the first error. Find $P(X = 5)$

$$P(X = 5) = P(OOOOE) = 0.9^4 0.1 = 0.0656$$

O: Okey bit

E: error bit

Geometric Distribution



Geometric Distribution – Mean

x	1	2	…	k	…
f(x)	p	(1-p)p	…	$(1\text{-}p)^{k\text{-}1}p$	

The mean of X is

$$\mu = E(X) = \sum_{k=1}^{\infty} kp(1-p)^{k-1} = p\sum_{k=1}^{\infty} kq^{k-1}$$

$$= p\frac{\partial}{\partial q}\left(\sum_{k=1}^{\infty} q^k\right) = p\frac{\partial}{\partial q}\left(\frac{q}{1-q}\right) = p\frac{1}{(1-q)^2} = \frac{1}{p}$$

Ex. Geometric Distribution - Mean

The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted *until* the first error. What is the *expected number* of bits transmitted until the first error?

$E(X) = 1/p = 10$ bits

```
> rgeom(5,0.1)+1
[1] 8 8 1 4 1

> mean(rgeom(1000,0.1)+1)
[1] 10.264
```

Geometric Distribution - Variance

$$\sigma^2 = V(X) = (1 - p)/p^2$$

Ex. The probability that a bit transmitted through a digital transmission channel is received in error is 0.1. Assume the transmissions are independent events, and let the random variable X denote the number of bits transmitted *until* the first error.

$$\sigma^2 = V(X) = (1 - p)/p^2 = (1 - 0.1)/(0.1)^2 \Rightarrow \sigma = 9.49$$

Practical Interpretation when p is small:

- $\sigma \approx \mu = 1/p$, which is *large*
- The number of trials until the first success may be *much different* from the mean.

```
> rgeom(5,0.1) + 1
[1]  8  1  5  4 14
```

Lack of Memory Property

The probability that a bit is transmitted in error is equal to 0.1.

For example, if 100 bits are transmitted, the probability that the first error, after bit 100, occurs on bit 105 is the probability that the next six outcomes are OOOOE. This probability is $(0.9)^5(0.1) = 0.0656$, which is identical to the probability that the initial error occurs on bit 5.

$P(X = 5 \text{ after } 100^{th} \text{ bit}) = P(X = 5 \text{ at the beginning})$

And the mean number of bits until the next error is $1/0.1 = 10$.

Geometric Distribution

Geometric Distribution

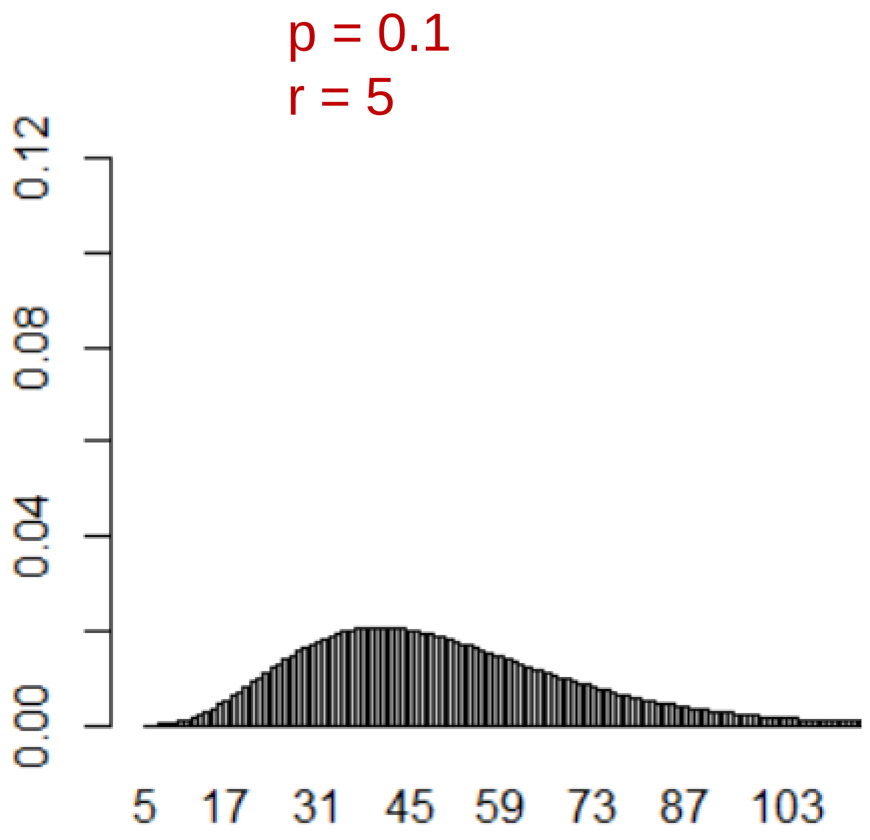
X	=	number of trials
p	=	probability of success
P(x)	=	$p(1 - p)^{x-1}$, $x = 1, 2, \ldots$
E(X)	=	$1/p$
V(X)	=	$(1 - p)/p^2$

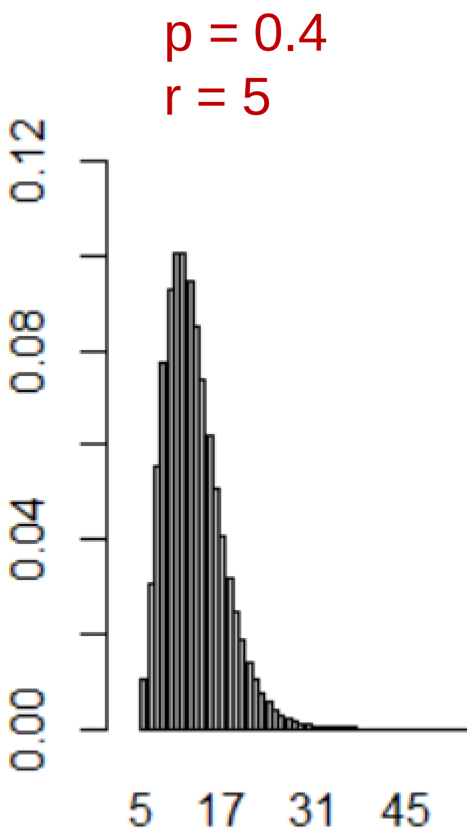
Negative binomial distribution

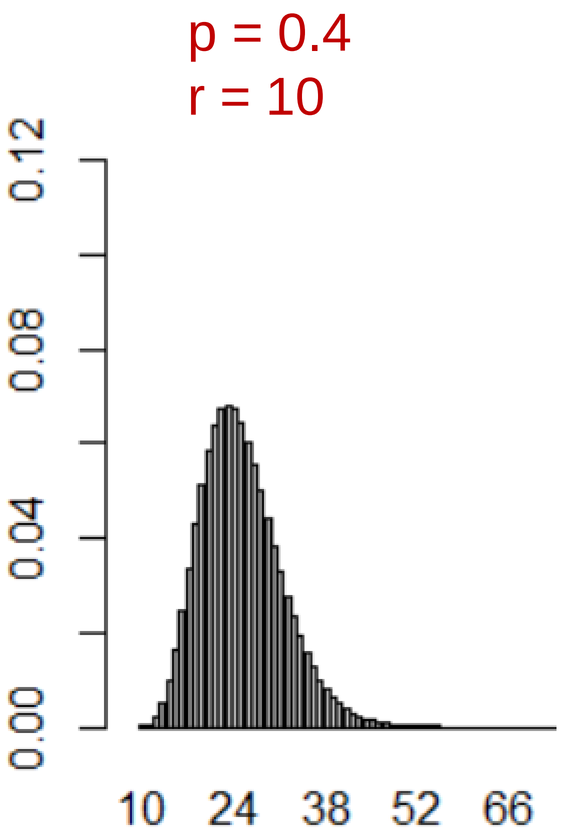
In a series of Bernoulli trials (independent trials, Prob(success) = p = constant), let the random variable X denote the number of trials until *r successes* occur. Then X is a *negative binomial* random variable with parameters $0 < p < 1$ and $r = 1, 2, 3, \ldots$, and

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad x = r, r + 1, \ldots$$

- r = 1: a negative binomial distribution becomes a geometric distribution







Smaller value of p, larger number of trials

Larger value of r, larger number of trials

Negative binomial distribution – Ex

Ex. Applicants for a new student internship are accepted with probability p = 0.2 independently from person to person. Several hundred people are expected to apply. Find the probability that it will take no more than 100 applicants to find 10 students for the program.

Let X be the number of people who apply for the internship until the 10th student is accepted. Then X has a *negative binomial distribution* with parameters *r = 10* and *p = 0.2*.

The desired probability is $P(X \le 100) = \sum_{k=10}^{100} P(X = k)$

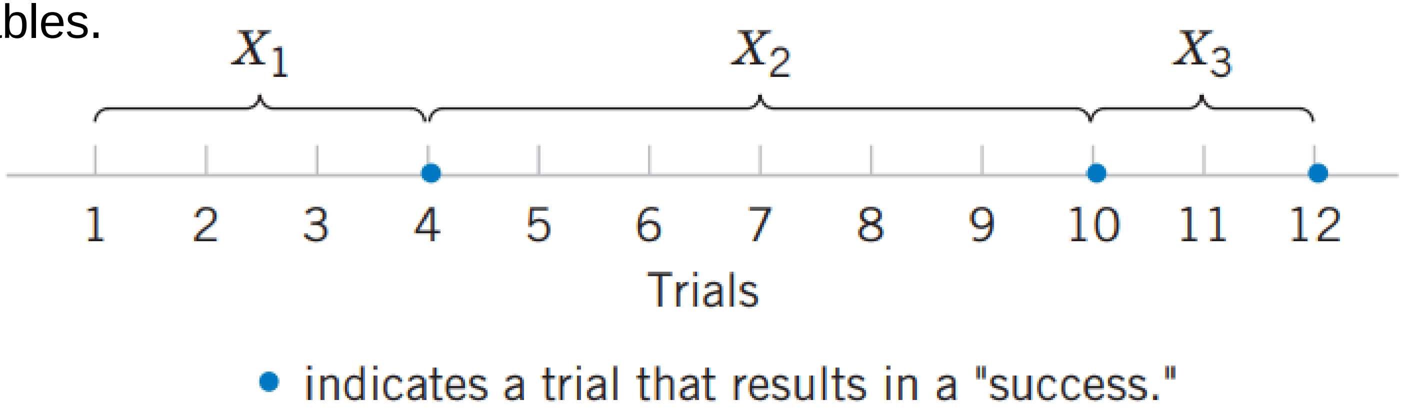
$$= \sum_{k=10}^{100} \binom{k-1}{r-1} p^r (1-p)^{k-r}$$

```
> pnbinom(100-10,10,0.2)
[1] 0.9976664
```

Negative binomial distribution

Negative binomial random variable represented as a sum of geometric random variables.

$$X = X_1 + X_2 + X_3$$



Mean and Variance

$E(X) = r/p$

$V(X) = r(1-p)/p^2$

Ex. (Web Servers) A Web site contains three identical computer servers. Only one is used to operate the site, and the other two are spares that can be activated in case the primary system fails. The probability of a failure in the primary computer (or any activated spare system) from a request for service is 0.0005. Assuming that each request represents an independent trial, what is the mean number of requests until failure of all three servers?

$E(X) = r/p = 3/(0.0005) = 6000$ requests.

Negative binomial distribution – Ex

A Web site randomly selects among 10 products to discount each day. The color printer of interest to you is discounted today. (a) What is the expected number of days until this product is again discounted? (b) What is the probability that this product is first discounted again exactly 10 days from now? (c) If the product is not discounted for the next five days, what is the probability that it is first discounted again 15 days from now? (d) What is the probability that this product is first discounted again within three or fewer days?

Negative Binomial Distribution

Negative Binomial Distribution

X	=	number of trials until r successes
p	=	probability of success
P(x)	=	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$, $x = r, r+1, \ldots$
E(X)	=	r/p
V(X)	=	$r(1-p)/p^2$

Hypergeometric Distribution

A set of N objects contains

K objects classified as *successes*

N – K objects classified as *failures*

A sample of size n objects is selected randomly (**without replacement**) from the N objects.

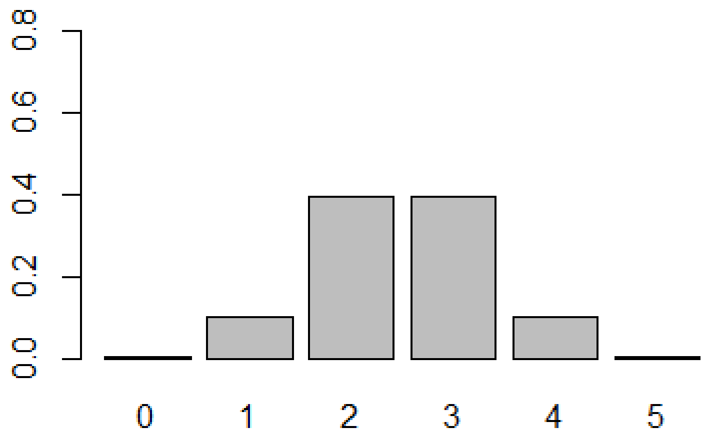
Let *X denote the number of successes in the sample*. Then X is a *hypergeometric random variable* and

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

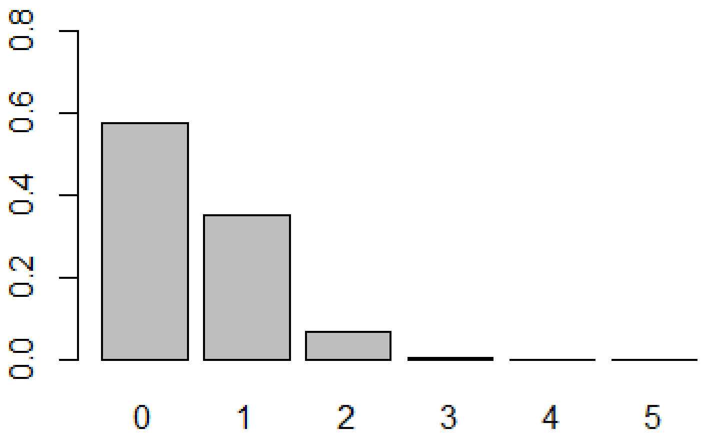
for $\max\{0, n - (N - K)\} \leqslant x \leqslant \min\{n, K\}$

Hypergeometric Distribution – Selected cases

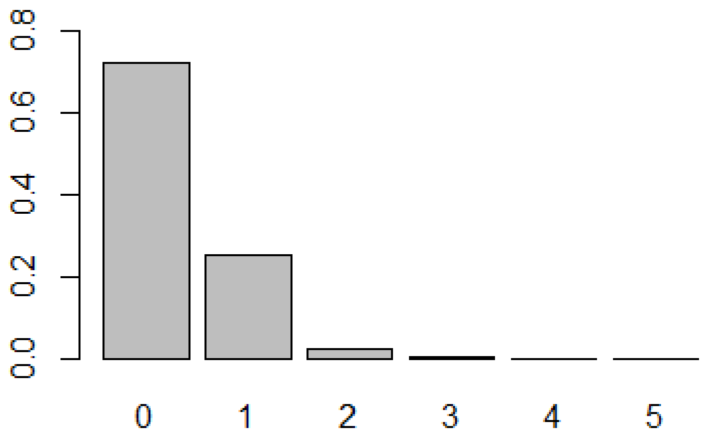
$N = 10, K = 5, n = 5$



$N = 50, K = 5, n = 5$



$N = 50, K = 3, n = 5$



Hypergeometric Distribution - Ex

Ex. A shipment of 50 computers contains 4 defective ones. Ten are bought at random. What is the probability that two of them will be defective?

X = the number of defective computers ➔ X is a *hypergeometric random variable* with parameters N = 50, K = 4, n = 10.

The desired probability is

$$P(X=2)=f(2)=\frac{\binom{4}{2}\binom{50-4}{10-2}}{\binom{50}{10}}$$

```
> dhyper(2,4,50-4,10)
[1] 0.1524099
```

Hypergeometric Distribution

If X is a hypergeometric random variable with parameters N, K, and n, then

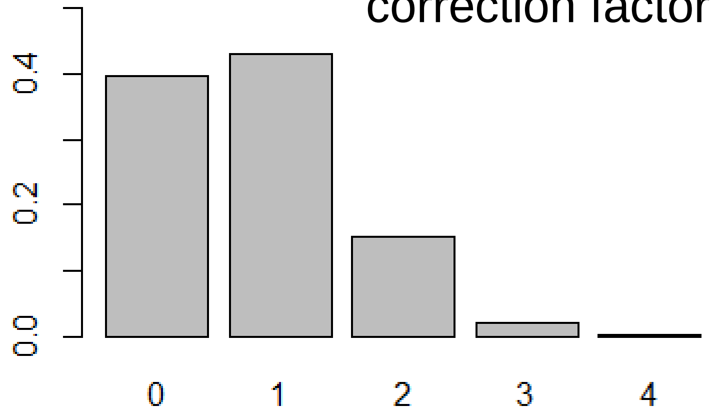
$$\mu = E(X) = np \qquad \text{and} \qquad \sigma^2 = V(X) = np(1-p)\boxed{\left(\frac{N-n}{N-1}\right)}$$

where $p = K/N$.

p: the proportion of successes in the set of N objects

finite population correction factor

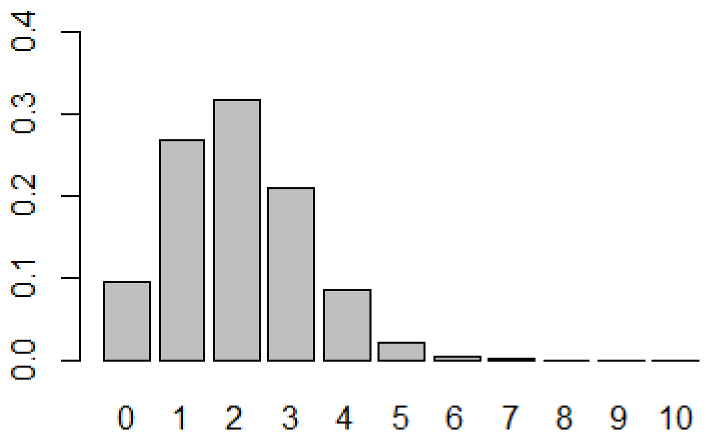
Ex. In the previous example, n = 10, p = 4/50.
➔ E(X) = 10(4/50) = 0.8
and V(X) = 10(4/50)(46/50)(40/49) = 0.601



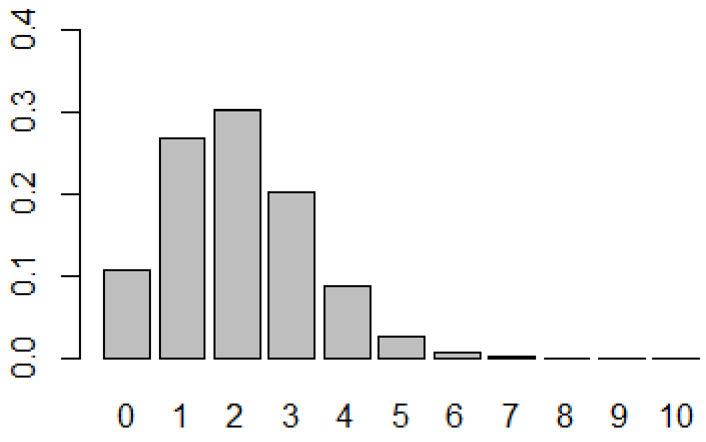
Hypergeometric vs Binomial

$n \ll N$, $p = K/N$ is not too close to 0 or 1

Hypergeometric distribution
$N = 100$, $K = 20$, $n = 10$



Binomial Distribution
$p = K/N = 0.2$, $n = 10$



Binomial distribution approximates Hypergeometric Distribution - Example

Ex. Suppose a shipment of 100 computers contains 20 defective ones. Ten are selected at random. Find the probability that 3 of them will be defective.

Use hypergeometric distribution (N = 100, K = 20, n = 10):

$$P(X=3)=\frac{\binom{20}{3}\binom{80}{7}}{\binom{100}{10}}=0.209$$

Use binomial distribution ($10 = n \ll N = 100$, p = 20/100 = 0.2):

$$P(X=3)=\binom{10}{3}(0.2)^3(0.8)^7=0.201$$

Hypergeometric Distribution

Hypergeometric Distribution

N	=	number of objects
K	=	number of success-objects
n	=	sample size
$P(x)$	=	$\dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$
$E(X)$	=	np, where $p = K/N$
$V(X)$	=	$np(1-p)\dfrac{N-n}{N-1}$

Poisson Distribution

The number of *rare events* occurring within a fixed period of time has *Poisson distribution* with parameter $\lambda > 0$.

Poisson Distribution

λ: frequency, average number of events

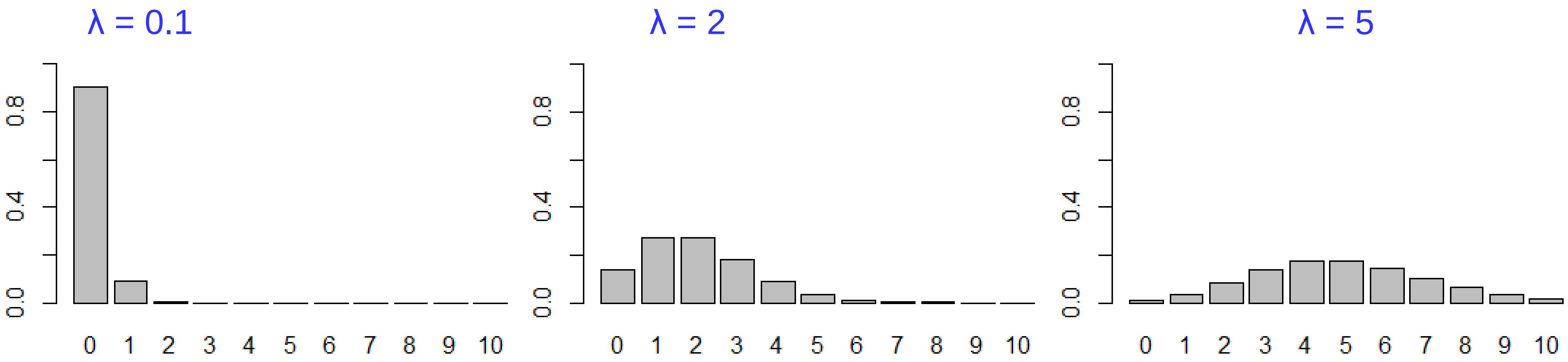
$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \ x = 0, 1, 2, \ldots$$

$$\mu = \lambda$$

$$\sigma^2 = \lambda$$

Examples of *rare events*: telephone calls, e-mail messages, traffic accidents, network blackouts, virus attacks, errors in software, floods, earthquakes, soldiers killed by horse kick, etc.

Poisson Distribution



Poisson distributions for selected values of the parameters.

Poisson Distribution - Ex

(New accounts) The number of new accounts of an internet service provider has a Poisson distribution with a mean of 5 accounts per day.

a/ What is the probability that there more than 4 new accounts in one day?

b/ What is the probability that there are 15 new accounts in 2 days?

a/ $P(X > 4) = 1 – P(X \leqslant 4) = 1 – P(X = 0) – P(X = 1) – P(X = 2) – P(X = 3) – P(X = 4) = 0.56$

b/ Let X denote the number of new accounts in 2 days. Then X has a Poisson distribution with $\lambda = E(X) = 2(5) = 10$.

$\Rightarrow P(X = 15) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-10}10^{15}}{15!} = 0.0347$

Poisson Distribution – Exercises

The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

(a) What is the probability that there are exactly five calls in one hour?

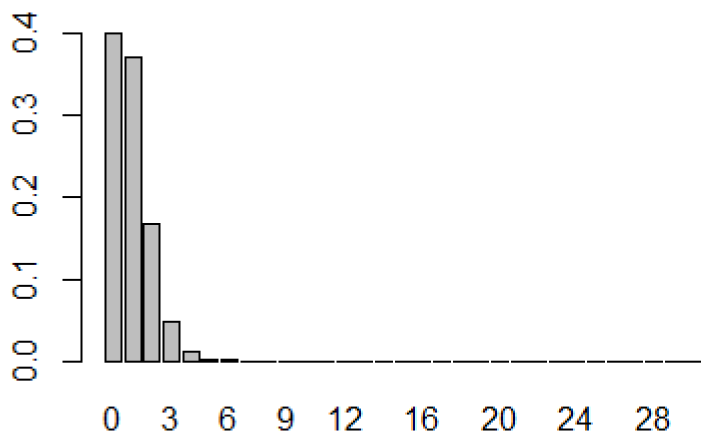
(b) What is the probability that there are three or fewer calls in one hour?

(c) What is the probability that there are exactly 15 calls in two hours?

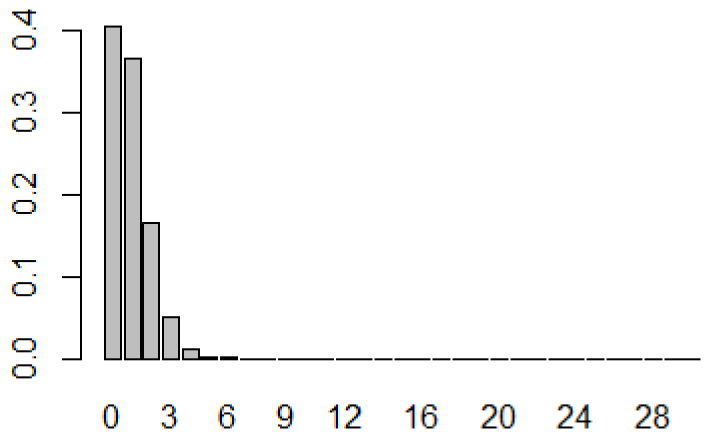
(d) What is the probability that there are exactly five calls in 30 minutes?

Poisson approximation to Binomial

- Poisson distribution ($\lambda = np$) can be effectively used to approximate Binomial probabilities when
  - n is large (e.g., $n \geqslant 30$)
  - p is small (e.g., $p \leqslant 0.05$)



Binom(n = 30, p = 0.03)



Pois(λ = np = 0.9)

Poisson approximation to Binomial

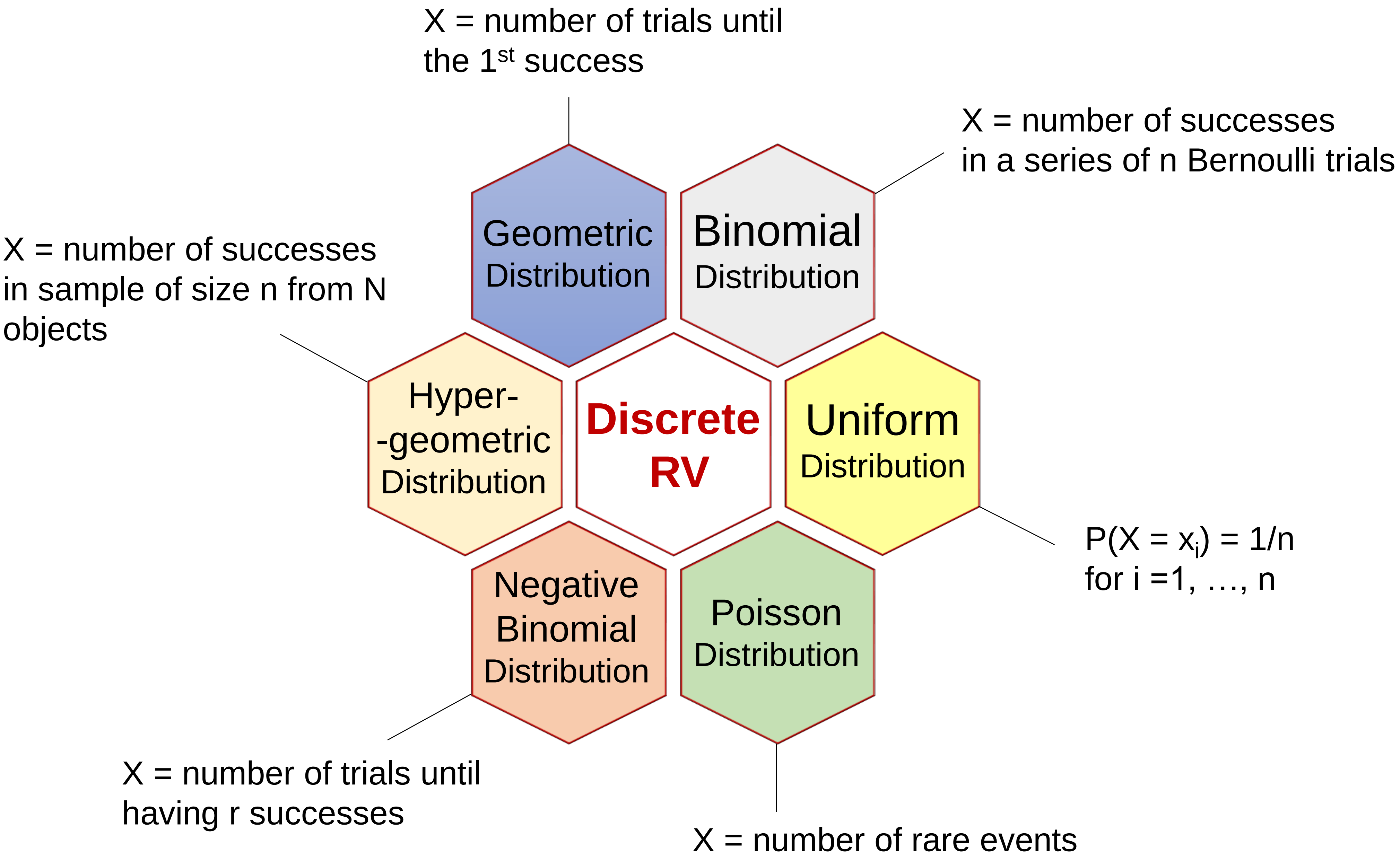
Ex. 3% of messages are transmitted with errors. What is the probability that out of 200 messages, exactly 5 will be transmitted incorrectly?

- Let X be the number of messages with errors.

➔ X ~ Binom(n = 200, p = 0.03) and P(X = 5) = 0.162

- Use Poisson distribution with λ = np = 6:

$$P(X = 5) = \frac{e^{-6}6^5}{5!} = 0.161$$



X = number of trials until the 1st success
Geometric Distribution
Binomial Distribution
X = number of successes in a series of n Bernoulli trials
X = number of successes in sample of size n from N objects
Hyper- -geometric Distribution
Discrete RV
Uniform Distribution
$P(X = x_i) = 1/n$ for i =1, …, n
Negative Binomial Distribution
Poisson Distribution
X = number of trials until having r successes
X = number of rare events

THANKS