The Office DVD Problem

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Screensavers have captivated [this] man since the 1990s. If watched long enough, what will the spirits of the machine tell us?

Specifically, the question of whether a bouncing rectangle will slide *exactly* into the corner of the screen, for a satisfying, perfectly diametric rebound, was even addressed on *The Office* (link).

However, though these characters reportedly watched this sleep-mode drama play out for years until payoff, we ask - under what conditions will the rectangle *definitely* perfectly bounce into the screen's corner?

0.1 Statement

Suppose we have a continuous screen of length l, height h, containing an axis-aligned rectangle of length j and height k centered at point (x, y).

Suppose this rectangle is launched at direction $\langle 1, m \rangle$ [1] and "bounces" according to billiard

[1] Think of this as slope m

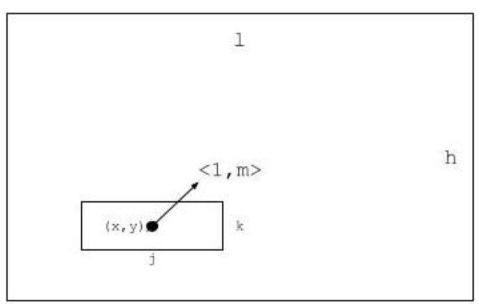

Figure 1: The Office DVD problem's most generic setup

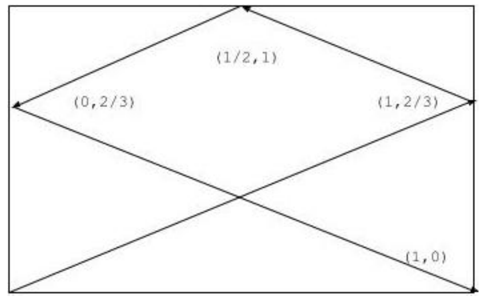

Figure 2: Success for $m = \frac{2}{3}, j, k = 0, h, l = 1$ (not to scale)

rules [2].

Given $l, h, j, k, m \in \mathbb{R}$, can we tell whether the rectangle ever bounce perfectly into a corner?

We can approach this problem from the simplest version to the most complex.

0.2 Problem 1

Suppose $j = k = 0$ and $x = y = 0$. In other words, suppose we have a *point* starting at the bottom left corner (origin). Under what conditions (i.e. choice of m) does this bounce into a corner?

0.3 Problem 2

Suppose $j, k > 0, x = \frac{j}{2}, y = \frac{k}{2}$. In other words, suppose we have a rectangle starting at the bottom left corner. Under what conditions does this bounce into a corner?

0.4 Problem 3

Suppose we have maximally open (reasonable) conditions, with $x \in [\frac{j}{2}, l - \frac{j}{2}], y \in [\frac{k}{2}, h - \frac{k}{2}]$ (that is, a $j \times k$ rectangle fitting entirely in the screen). Under what conditions does this bounce into a corner?

0.5 Problem 4

Some clowns [3] have come along demanding a version of the setup respecting the discrete (pixellated) nature of digital screens. Very well.

[2] Glancing off a horizontal boundary, our trajectory goes from $\langle 1, m \rangle$ to $\langle 1, -m \rangle$, with $\langle \pm 1, m \rangle$ to $\langle \mp 1, m \rangle$ for a horizontal one

[3] J. H. Wang, N. H. Talbert, L. F. Waldman

For each of problem 1, 2, and 3, how does the answer change if the screen comprises square pixels of length $p \in \mathbb{N}$ [4], where $p|j, k, h, l$, and "meeting a corner" means a corner of the small (continuous) rectangle meets a wall within length p of the corner point?

[4] If p is not a whole number, this can be normalized