Brilliant: Vector Calculus

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Note: Latex reference: http://tug.ctan.org/info/undergradmath/undergradmath.pdf

1 Chapter 1.2: Calculus of Motion

Consider vectors of motion against t of the form $\overrightarrow{x}(t) = \langle x(t), y(t), \ldots \rangle$.

- A **line** through $p = (a, b, c)$ parallel to $\overrightarrow{v} = \langle v_x, v_y, v_z \rangle$ is $\overrightarrow{x}(t) = \overrightarrow{p} + t\overrightarrow{v}$
- **velocity** is characterized completely by $\overrightarrow{v}(t) = \overrightarrow{x}'(t) = \langle x'(t), y'(t), z'(t) \rangle$.
- The **speed** of an object along that line versus t is the length of v ($\|v\|$)
- Therefore, the speed of an object along line
$$\langle x(t), y(t), z(t) \rangle = \langle 0, 2, -3 \rangle + t\langle 1, -2, 2 \rangle$$
is
$$\sqrt{1^2 + (-2)^2 + 2^2} = 3$$
- Note that $\overrightarrow{v}$ need not be constant. The speed of
$$\overrightarrow{x}(t) = \overrightarrow{p} + 3\sin(2\pi t)\hat{u}, \|\hat{u}\| = 1$$
would then be
$$\|6\pi \cos(2\pi t)\hat{u}\| = |6\pi \cos(2\pi t)|$$
- **Acceleration** $a(t) = v'(t) = x''(t)$ is straightforward. Acceleration of
$$x(t) = \langle -1 + \cos(t), 1, \cos(t) \rangle = \langle -\cos(t), 0, -\cos(t) \rangle$$
- An example position vector for a planet of distance r from the sun could be $\langle r\cos(t), r\sin(t) \rangle$. The acceleration vector points in the opposite direction: $\langle -r\cos(t), -r\sin(t) \rangle$. In addition to being the analytical second derivative, consider that the *force* of gravity, (which, by $F = ma$ is proportional to acceleration) points towards the sun.
- A **helix** could be a 3D extension like $\langle r\cos(t), r\sin(t), b \cdot t \rangle$.