Spotting Graph Theory Problems in Spot It

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Abstract

TODO

1 The Game

2 The Graph Theorem

A deck of n Spot It cards with m symbols over s slots can be represented by a graph G on n nodes of degree s and edges of m unique colors, and for any color m_i, the edges of that color (and all adjacent nodes) form a complete subgraph of G_i of G.

Note: Self-loop objection.

3 The Core Question

For what choices of g and s can graphs be constructed that satisfy our constraints?

TODO

4 The Candidate Theorem

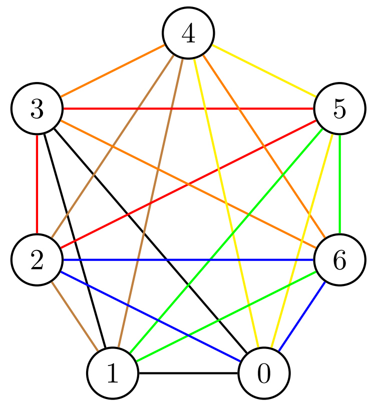

Figure 1: s=3, g=3, n=7, m=7

Suppose further that that every symbol s has exactly g cards containing it [1]. Then

- $n = (g-1)s + 1$,
- $\binom{g}{2} | \binom{n}{2}$, and
- $g | s(s-1)$.

TODO Proof

For example, a tiling of triangles ($g = 3$) means that either $s \equiv 0 \mod 3$ or $s \equiv 1 \mod 3$. Since $n = (g-1)s + 1 = 2s + 1$ or $n \equiv 1 \mod 2$, then $n = 2(3k) + 1 = 6k + 1$ or $n = 2(3k+1) + 1 = 6k + 3$, meaning $n \in \{1, 3\} \mod 6$.

5 Some examples with $g = 3$

6 Generating any $g = s - 1$

TODO: The Dave construction

7 Generating any $g = s$

TODO: The James construction

TODO Proof

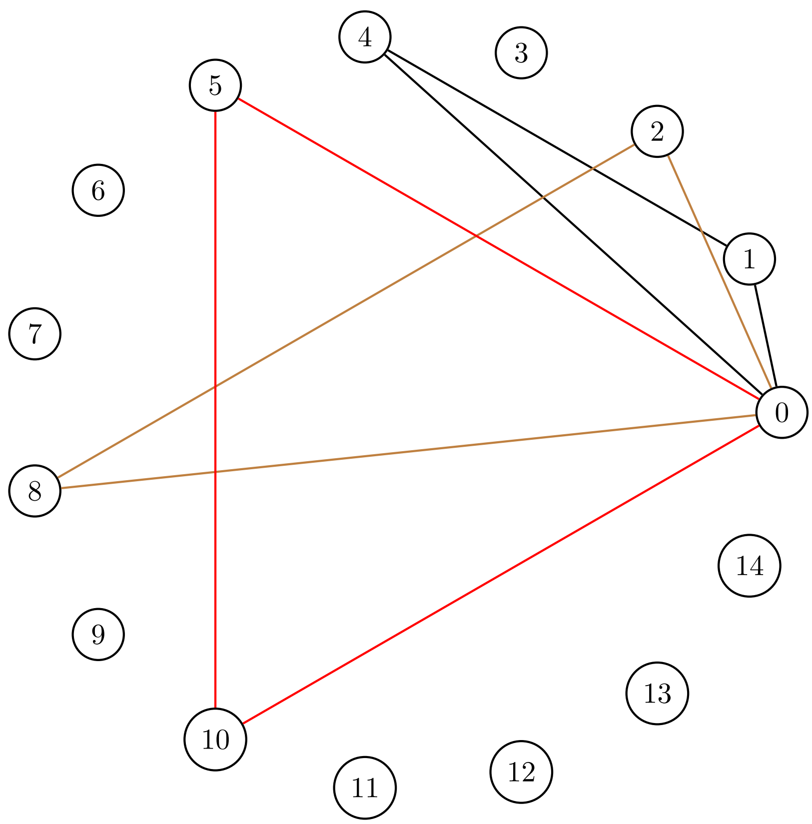

Figure 2: s=7, g=3, n=15, m=35, node 0 adjacencies

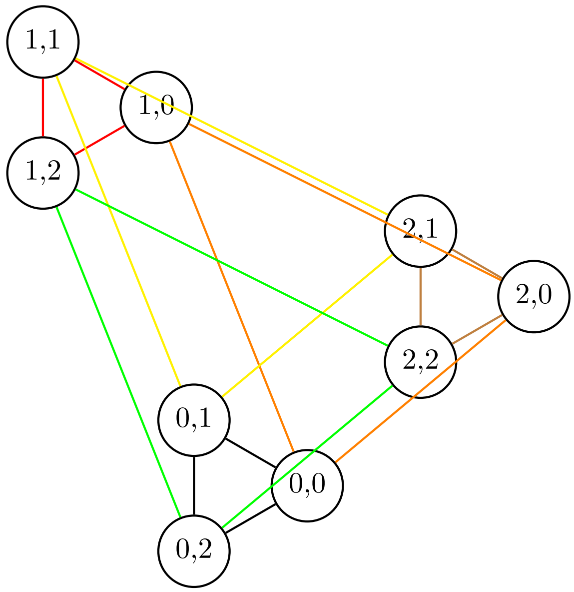

Figure 3: complete graphs with increment 0

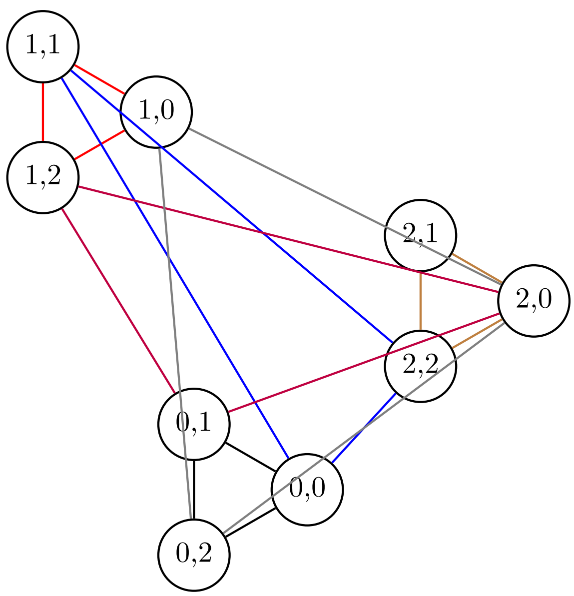

Figure 4: complete graphs with increment 1

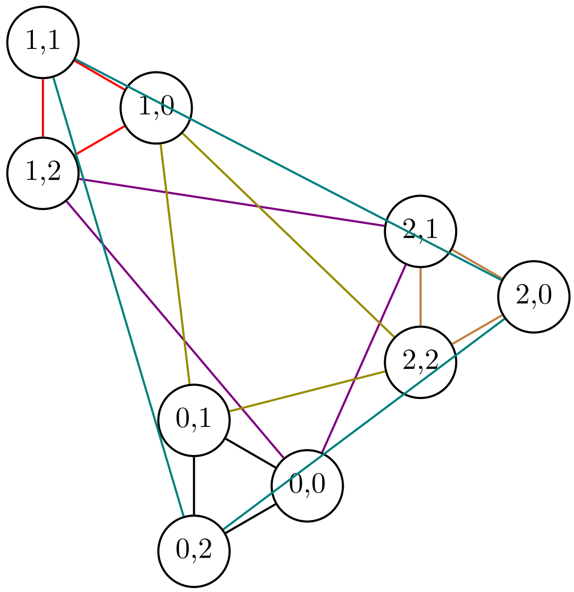

Figure 5: complete graphs with increment 2

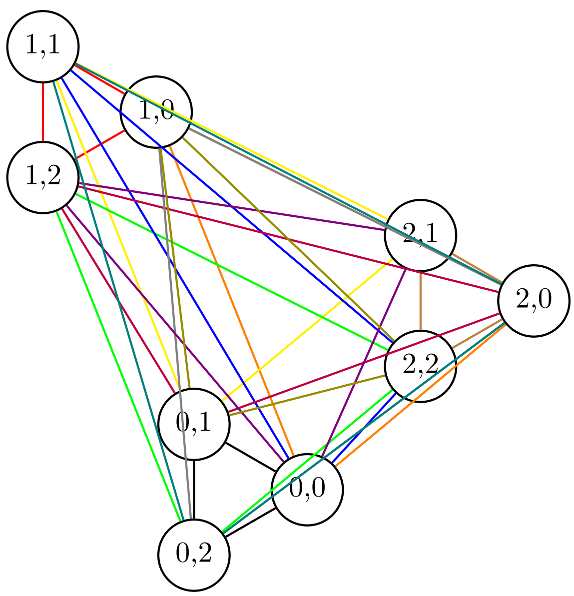

Figure 6: whole C_9: $s = 4, g = 3, n = 9, m = 12$

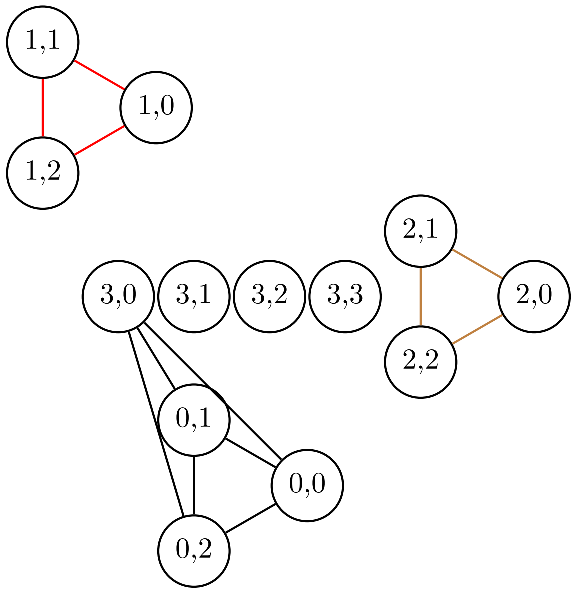

Figure 7: adding to original cliques

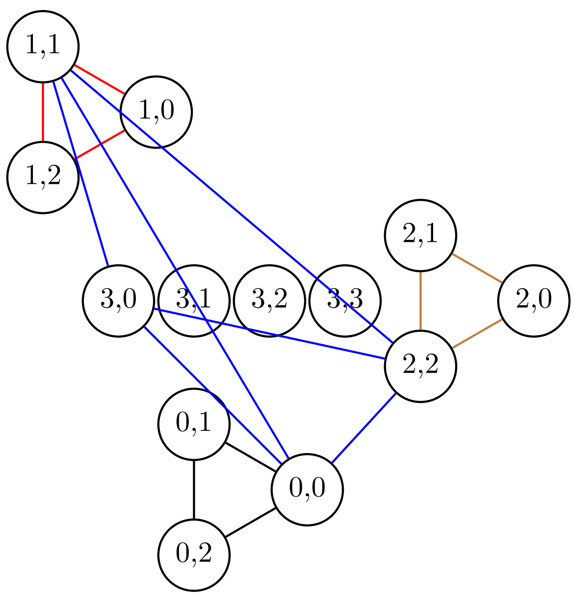

Figure 8: adding to increment cliques

8 Constructions with mixed g

8.1 Trivial

TODO

8.2 Chopping

TODO

8.3 Inception

TODO

9 The main question: Are all candidates viable?

TODO I actually don't have a proof of this yet, but I suspect this is the main question.

10 Further questions