5 Performance considerations

Performance criteria:

Performance	Noise
Disturbances	Robustness

- Increasing γ, we are unhappy with the oscillations of our parameters θ and therefore with the oscillations of $u(t)$.
- We have no clue what the adaptive closed loop will do between $t = 0$ and $t = \infty$ other than boundedness

5.1 Adaptation with a closed loop reference model

Now Deal with transient response

Idea Adaptation changes with signals

$$\dot{\theta} = -\operatorname{sgn}(\varepsilon)\, e\gamma\phi$$

where the value of e and γ are changeable.
$\Rightarrow$ we can alter the transient with γ (leads to oscillations), or we can change $e(t)$.

So far Open loop reference model (ORM)

$$\dot{x}_m^o(t) = a_m x_m^o(t) + k_m r(t) \tag{2}$$

Now Closed loop reference model (CRM)

$$\dot{x}_m^c(t) = a_m x_m^c(t) + k_m r(t) - l e^c(t) \tag{19}$$

ORM = CRM if $l = 0$.

"CRM is observer-like; M helps G by moving towards G and retreating to original position." Through the movement, the reference model now has a different behaviour ($M \to M'$!) and the plant P is trying to follow M'.

γ - learning effect
- decreasing γ helps P follow M',
- but the learning becomes slower

l - movement to P
- increasing l helps P follow M'

5.2 Stability proof

$$\dot{x}_m^c(t) = a_m x_m^c(t) + k_m r(t) - l e^c(t) \tag{19}$$

$$\dot{x}_p(t) = a_p x_p(t) + k_p u(t) \tag{1}$$

Input is

$$u = \begin{bmatrix} a(t) & k(t) \end{bmatrix} \begin{bmatrix} x_p(t) \\ r(t) \end{bmatrix} = \theta^T(t)\boldsymbol{\phi}(t)$$

$$\dot{x}_p(t) = a_m x_p(t) + k_m r(t) + k_p \theta^T(t)\boldsymbol{\phi}(t)$$

Tracking error

$$\begin{aligned} \dot{e}^c(t) &= \dot{x}_p(t) - \dot{x}_m^c(t) \\ &= (a_m + l)\, e^c(t) + k_p \theta^T(t)\boldsymbol{\phi}(t) \end{aligned}$$

Lyapunov-like function

$$\begin{aligned} V(e^c, \tilde{\theta}) &= \frac{1}{2}(e^c)^2 + \frac{1}{2}\Gamma^{-1}|k_p|\tilde{\theta}^T(t)\tilde{\theta}(t) \\ \dot{V} &= e^c\dot{e}^c + \Gamma^{-1}|k_p|\tilde{\theta}^T\dot{\tilde{\theta}} = \dots \\ &= (a_m + l)\,(e^c)^2 \\ &\quad + \underbrace{e^c k_p \tilde{\theta}\boldsymbol{\phi} + \Gamma^{-1}|k_p|\tilde{\theta}^T\dot{\tilde{\theta}}}_{\overset{!}{=}0} \\ \dot{V} &= (a_m + l)\,(e^c)^2 \leqslant 0, \qquad l < 0 \end{aligned}$$

Adaptive law

$$\dot{\theta} = -\Gamma \operatorname{sgn} k_p e^c \boldsymbol{\phi}$$

Proof as before. $e^c(t) \to 0$ for $t \to \infty$ [8].

Questions

- How do we show increased performance? (using $\|e^c(t)\|_{\mathcal{L}_2}$ as a performance criterion)
- How do we show that the oscillations decrease?

5.3 Analysing transient performance

Check the performance criterion $\mathcal{L}_2$-norm of e^c

$$\begin{aligned} \int_0^\infty \dot{V}(e^c, \theta)\,d\tau &= V(\infty) - V(0) \\ -|a_m + l| \int_0^\infty e^{c2}\,d\tau &= V(\infty) - V(0) \\ V(0) &= \underbrace{V(\infty)}_{\geqslant 0} + |a_m + l| \cdot \|e^c\|_2^2 \\ V(0) &\geqslant |a_m + l| \cdot \|e^c\|_2^2 \\ \|e^c\|_2 &\leqslant \sqrt{\frac{V(0)}{|a_m + l|}} \end{aligned}$$

$$\boxed{\|e^c\|_2^2 \leqslant \frac{1}{2}\frac{(e^c(0))^2 + \frac{|k_p|}{\gamma}\theta^T(0)\theta(0)}{|a_m + l|} \tag{20}}$$

[8] We assume here that $e^c(t) \to 0$ follows from $e^o(t) \to 0$. In actuality, though, $e^o(t)$ can't be proven for special functions. However, these cases are usually not relevant to engineering/industry. Therefore, strictly speaking, we can't actually assume that $e^c(t) \to 0$

Discussion

- Increasing γ reduces $\|e^c\|_{\mathcal{L}_2}$ depending on the parameter errors $\tilde{\theta}$
- Increasing the value of l reduces $\|e^c\|_{\mathcal{L}_2}$ also from $e^c(0)$

5.4 Analysing the signal oscillations

$\mathcal{L}_2$-norm of $\dot{k}$

$$
\begin{aligned}
\dot{k} &= -\gamma \operatorname{sgn} k_p e^c r(t) \\
\int_0^\infty |\dot{k}|^2 d\tau &= \gamma^2 \int_0^\infty (e^c)^2 r^2 d\tau \\
& \qquad \big(r(t) \leqslant \|r\|_{\mathcal{L}_\infty}\big) \\
&\leqslant \gamma^2 \|r\|^2_{\mathcal{L}_\infty} \int_0^\infty (e^c)^2 d\tau \\
&\leqslant \gamma^2 \|r\|^2_{\mathcal{L}_\infty} \|e^c\|^2_{\mathcal{L}_2}
\end{aligned}
$$

$$
\boxed{\|\dot{k}\|_2 \leqslant \gamma \|r\|_{\mathcal{L}_\infty} \sqrt{\frac{V(0)}{|a_m + l|}}}
$$

Increasing γ or reducing l causes $\dot{k}$ to decrease in magnitude.

$\mathcal{L}_2$-norm of $\dot{\theta}$

$$
\begin{aligned}
\dot{\theta} &= -\gamma \operatorname{sgn} k_p e^c x_p(t) \\
&= -\gamma \operatorname{sgn} k_p e^c \left(e^c + x_m(t)\right) \\
|\dot{\theta}|^2 &= \gamma^2 (e^c)^2 \left(e^c + x_m(t)\right)^2 \\
& \qquad (a+b)^2 \leqslant 2a^2 + 2b^2 \\
&\leqslant 2\gamma^2 (e^c)^2 \left[(e^c)^2 (e^c)^2 + x_m^2\right] \\
\int_0^\infty |\dot{\theta}|^2 d\tau &\leqslant 2\gamma^2 \left[\int_0^\infty (e^c)^2 (e^c)^2 d\tau + \int_0^\infty (e^c)^2 x_m^2 d\tau\right] \\
& \vdots
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
&\int_0^\infty |\dot{\theta}|^2 d\tau \\
&\leqslant 2\gamma^2 \frac{V(0)}{|a_m + l|} \left[V(0) \left(2 + \frac{l^2}{|a_m| \cdot |a_m + l|}\right) \right. \\
&\qquad \left. + 2 \|\dot{x}_m\|^2_{\mathcal{L}_\infty}\right]
\end{aligned}
}
$$

Discussion

- l reduces contribution of the ORM $\|\dot{x}_m(t)\|_{\mathcal{L}_\infty}$ on $\|\dot{\theta}\|_{\mathcal{L}_2}$
- l has no clear effect on the contributions of $V(0)$.
- γ always increases the oscillations, s. $\|\dot{\theta}\|_{\mathcal{L}_2}$

6 Output feedback adaptive control

Let $M(s)$ be a linear time-invariant, asymptotically stable reference model, with I/O $\{r(.), y_m(.)\}$. r is uniform, bounded, piecewise continuous function of time. The plant G is defined such that

$$G(s) = k_p \frac{Z_p(s)}{R_p(s)}$$

- Z_p, R_p are monic[9] polynomials of orders $n-1$ and n
- $G(s)$ is controllable, observable $\Leftrightarrow$ Z_p, R_p are coprime[10] polynomials

We know

(i) sign of high-frequency gain k_p
(ii) upper bound n on the order of $G(s)$
(iii) relative degree n^* [11]
(iv) zeroes of $Z_p(s)$ lie in $\mathbb{C}^-$
$\Rightarrow$ min. phase[12], no inverse response

Minimal phase systems "In minimal phase systems, we can predict the phase φ given the magnitude $|G| \Leftarrow$ only if G stable with no time delay".

e.g. a plane, or a forklift, are examples of non minimal phase systems (they have different inverse responses depending on where you measure, s. Figure 5)

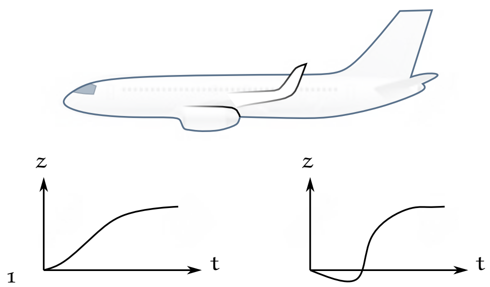

Figure 5: An ascending plane.

Solution in three parts

(i) k_p unknown
(ii) Z_p unknown
(iii) R_p unknown

In each case

a) "Matching conditions"
Show that $\exists$ parameters $\theta^* = \text{const.}$, such that the closed loop behaves exactly as the reference

b) Error dynamics Derive error model as

$$e = \frac{1}{k^*} M(s) \left[\tilde{\theta}^T \phi\right]$$

c) Use KY-lemma
d) Show that $e \to 0$ for $t \to \infty$ (Barbalat's lemma)

6.1 k_p unkown

Matching conditions

[9] monic: coefficient of the highest order is 1, e.g. $s^2 + 2s + 1$
[10] coprime: no cancellations, i.e. no common roots
[11] in this course, we only handle cases where $n^* = 1$
[12] there is no standard definition for minimal phase