# Mathematical Morphology

## Introduction

Mathematical morphology provides a set of operators to extract relevant image components for the representation and description of objects' shape. The morphological operators can also be used to implement pre and post-processing techniques, such as morphological filtering and thinning.

The IPT provides several functions for the implementation of morphological operators that can be used both in binary and monochromatic images; however, given the importance of morphological operators in the context of binary images, we will begin by presenting the IPT functions that implement the key concepts of the binary morphology.

### 1. Binary morphology

The dilation and erosion operations are fundamental in mathematical morphology. Dilation is an operation that makes (white) objects "grow" or "dilate" in a binary image, while erosion performs the reverse operation of "reduction" or "erosion" of the same objects. The specific form and extent, either for growth or reduction, are controlled by the shape and size of the structuring element.

The IPT function that implements the dilatation operator is *imdilate*, while erosion is implemented by the *imerode* function. The syntax of these two functions is:

g = imdilate(f, B)    and    g = imerode(f, B)

where f and g are binary images (input and output, respectively) and B is a matrix that specifies the structuring element to be used in the operation.

The structuring elements for binary image processing are also represented by binary matrices; in these matrices the value 0 means that the corresponding point does not belong to the structuring element, while a value of 1 indicates the opposite.

Although the arrays containing structuring elements can be set directly by the user, the IPT provides the *strel* function for creating structuring elements with a variety of shapes and sizes. The syntax of this function is:

se = strel(shape, parameters)

where shape is a *string* indicating the desired element shape, and parameters is a list of parameters that specify additional information, such as its size (see details in MATLAB Help). It should be in mind that the *strel* function does not create directly an array with the structuring element, having the advantage of producing decomposed structuring elements to reduce the processing time of morphological functions.

In applications requiring morphological processing, dilation and erosion operators are most often applied in combination, giving rise to other operators with different characteristics. Two of these combinations are the opening and closing operations (*opening* and *closing*). The *opening* operation consists of an erosion followed by a dilation and the *closing* operation is a dilation followed by an erosion.

These operations are implemented through the *imopen* (openness) and *imclose* (closing) functions. The syntax of these functions is:

g = imopen(f, B)    and    g = imclose(f, B)

where f, g and B have exactly the same meaning as indicated above for the *imdilate* and *imerode* functions.
The *bwmorph* IPT function enables the implementation of a large variety of binary morphological operators based on the combination of dilations, erosions and other neighborhood analysis operations. The syntax of this function is:

g = bwmorph(f, operation, n)

where f is the input binary image, operation is a *string* specifying the chosen operation and n is a positive integer that indicates the number of operation repetitions required to generate the output binary image g. Consult MATLAB Help to evaluate the available operations, their meaning and their parameter lists.

**2. Grayscale morphology**

All morphological operations discussed before (dilation, erosion, opening and closing) have a natural extension for grayscale images. The IPT functions that implement these operators are exactly the same (*imdilate, imerode, imopen, imclose*).
The main difference is that the structuring elements are arrays that can now hold values different from 1 in points belonging to the elements, thus introducing the possibility of performing operations with *non-flat structuring elements.*
As in binary images, the *strel* function can be used for constructing structuring elements. The creation of *flat structuring elements* is performed exactly as in binary morphology; for non-flat structuring elements creation two arrays are required as input arguments of the *strel* function, one consisting only of 0's and 1's to indicate the domain of the structuring element and the other with the values (weights) to be allocated to each domain element.
As with the binary image operators, the dilation, erosion, opening and closing operations can be combined to obtain a variety of effects. Some of the combinations most frequently used in image processing applications are presented in the following exercises.

**Work proposal**

**1.** (Pen and paper) Consider the portion of an image shown below. Compute the result of a 1) morphological dilation; 2) morphological erosion; 3) morphological opening and 4) morphological closing for the highlighted points. Use a 3×3 structuring element. Hint: you can use the intermediary steps of 3) and 4) to solve 1) and 2). If needed, pad the image's borders with zeros.

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | 1 | 22 | 29 | 9 | 1 | 1 |
| 9 | 21 | 21 | 25 | 22 | 5 | 3 |
| 7 | 23 | 22 | 3 | 5 | 6 | 7 |
| 8 | 26 | 21 | 2 | 1 | 25 | 9 |
| 10 | 23 | 24 | 20 | 2 | 29 | 10 |
| 11 | 2 | 25 | 21 | 26 | 25 | 5 |
| 5 | 3 | 5 | 21 | 25 | 0 | 1 |
| 4 | 5 | 9 | 10 | 11 | 13 | 6 |

**2.** Consider the “imp_digital_ruido.tif” binary image shown on the left in the figures below. As can be observed, the image contains noise that manifests itself in the form of small white fragments on dark background, as well as small dark holes in the white segments. Implement a processing sequence based on morphological operators to enable the removal of noise in order to obtain the image shown on the right.

 

**3.** The morphological gradient operator is defined as the difference between the dilation and erosion of a given image using the same structuring element. Determine the result of this operation in the “EDorsal(MRI).tif” image (you can use the IPT *imsubtract* function to perform subtraction).

**4.** The opening and closing morphological operations can be considered as smoothing operations, especially when used with flat structuring elements.

**A.** Implement a processing sequence based on opening and closing operators to obtain a smoothed version of the “histo_1.tif” image.

**B.** Repeat the problem in A using a sequence of opening-closing operators with increasing structuring elements of increasing size (ASF – *Alternating Sequential Filters*). Compare the result with the one obtained in A.

**5.** The images of the eye fundus are characterized by a gradual variation of intensity from the macular region to the periphery. The segmentation of the retina structures, in particular blood vessels, becomes easier if the background intensities are previously normalized. Since the intensity of the background is in general smaller than vessel intensity, we can use an opening operator to obtain an estimation of the background intensity, which can then be subtracted from the original image to obtain an image with an uniform background. Write a set of instructions to:

**A.** Read the “angio.tif” image.

**B.** Apply an opening operator using an appropriate structuring element (with an adequate shape and size) to estimate of the background image.

**C.** Subtract the background image from the original one and display the result.

**D.** The sequence of operations described in sections B and C form a well-known morphological operator, called “top-hat transformation”, available through the *imtophat* IPT function. Verify that the result of this operator is similar to that obtained in the preceding paragraph.

**6.** The dual operation of the “top-hat transformation” is the “bottom-hat transformation”, which consists in the difference between the closing of an image and the image itself, and is implemented through the IPT *imbothat* function. Each of these transformations lets you highlight the details of the highest or lowest image details present in the image, respectively (the size of the structuring element should be consistent with the size of the details to remove from the image). These two morphological transformations can also be combined to increase the image contrast.

**A.** Apply a “bottom-hat transformation” in the “angio.tif” image and observe the results.

**B.** Determine the image that results from the sum of the original image with the result of its "top-hat transformation" (you can use the *imadd* function to perform the sum). Comment on the obtained results.

**C.** Combine the two previous images through the difference between the image obtained in B and the image that resulted from the bottom-hat operator.

**7.** The image "blister.tif" was processed and binarized, resulting on the figure "blister_bin.png" shown in the middle. Starting from the binarized image, propose a method based on morphological operations that allows to obtain the vertical and horizontal separations, as shown in the figure on the right.