MA3740 R01- Statistical Programming and Analysis

Final Project

Substances' Effect on Candle Burning Rate

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Introduction

Candles have long charmed us with their soft glow, acting as a useful source of illumination and relaxation. But have you ever wondered why some candles burn faster than others? Even if some call it a myth, the mystery behind the varying burning rates among candles remains. With the goal of revealing the interactions between candles and their burning rates, our team, Data Explorers, is setting out to investigate these burning rate behaviors carefully.

The motivation behind this experiment is an eagerness to prove a belief about a candle's burning rate. People believe that adding salt, baking soda, and essential oils to candles makes them last longer. Some also suggest freezing candles before lighting them to decrease the burning rate. Scientific research states that sodium chloride compounds slow the melting and dripping processes. So we want to explore whether common substances, often suggested by individuals, can influence the rate at which candles burn.

The taper candle, a popular and widely used candle by people, is our experiment's target population. We took a sample of 5 candles and recorded the differences in the rate of burning between the different additives (baking soda, salt, and essential oils) with regular and frozen candles every fifteen minutes.

Our primary aim in this study is to address a fundamental statistical question: Do variations in additives, such as salt, essential oil, and baking soda, exert discernible influences on candle burning rates? Through meticulous data collection and rigorous statistical analysis, we endeavor to decipher the statistical intricacies that govern these burning behaviors, unveiling insights into the impact of different treatment groups on the burning rate of a candle.

As we embark on this empirical journey to decode the statistical patterns influencing candle burning rates, join us, the Data Explorers, in illuminating the path toward a comprehensive understanding of the interaction between time, additives, and the mesmerizing dance of candle flames.

Methods

Sampling Unit and Study Design:
The sampling unit in our study comprises individual taper candles, serving as the primary unit upon which measurements were conducted. This study is structured as a controlled experiment, aiming to investigate the impact of additives like salt, baking soda, essential oils, and freezing on candle burning rates.

Sampling Process/Scheme and Data Attainment:
We made sure the candles were the same size, shape, and composition by purchasing them from an established supplier. These candles were then categorized into different groups according to their treatment: Regular (untreated), Frozen, Salt-treated, Baking Soda-treated, and Essential Oil-treated, ensuring consistent environments for statistical reliability across experimental conditions. To ensure unbiased representation within each treatment group, we utilized a systematic random sampling method. Candles in each treatment category were assigned numeric identifiers, and random samples were selected using a random number generator. This approach was employed to mitigate any selection bias and guarantee that every candle had an equal opportunity to be selected for measurement.

Variables and Measurements:
During the experiment, we focused on observing and measuring two variables: time and candle length. We meticulously tracked time intervals at 15-minute increments using a digital timer. The primary variable of interest, candle length, was measured using a precision ruler, noting down the length of each candle to the nearest 0.1 centimeter at specified time intervals.

Controlled Experiment Approach:
In this controlled study, we systematically introduced specific quantities of additives (salt, baking soda, and essential oil) to individual candle groups. We ensured that the amount of each additive remained consistent within its respective treatment group to ensure uniformity and eliminate possible confounding variables.

Extraneous Variables and Control Measures:
To address potential influences on candle burning rates, we considered variables such as ambient temperature, air circulation, and candle positioning. To mitigate their impact, the experiment occurred in a controlled setting at a consistent room temperature (±76°F) and with minimal airflow. Moreover, candles were uniformly spaced to ensure even exposure to environmental conditions.

Results

We are experimenting with the effects of substances on the candle's burning rate so we used the candle's length as a response variable, whereas the independent variable pertains to treatment Types: untreated (Regular), frozen, salt-treated, and baking soda-treated candles. Our CSV file contains three columns namely time - indicating the time interval of 15 minutes (measured in minutes), type - indicating the type of treatment done on the candle (baking soda, regular, salt, essential oil, and frozen), length - length of the candle measured every 15 minutes after being lit.

We'll use a one-way ANOVA to see if treatment types affect candle-burning rates. Significant differences in mean lengths will reveal how substances alter burning rates. Additionally, post-hoc tests may be employed to ascertain specific treatment group differences if the ANOVA yields significant outcomes, aiding in a more nuanced understanding of treatment effects on candle burning rates.

Let's start our analysis by checking the boxplots of candle length across different treatment types to provide a visual summary of the key features of the distribution, including measures of central tendency, spread, and the presence of outliers if any.

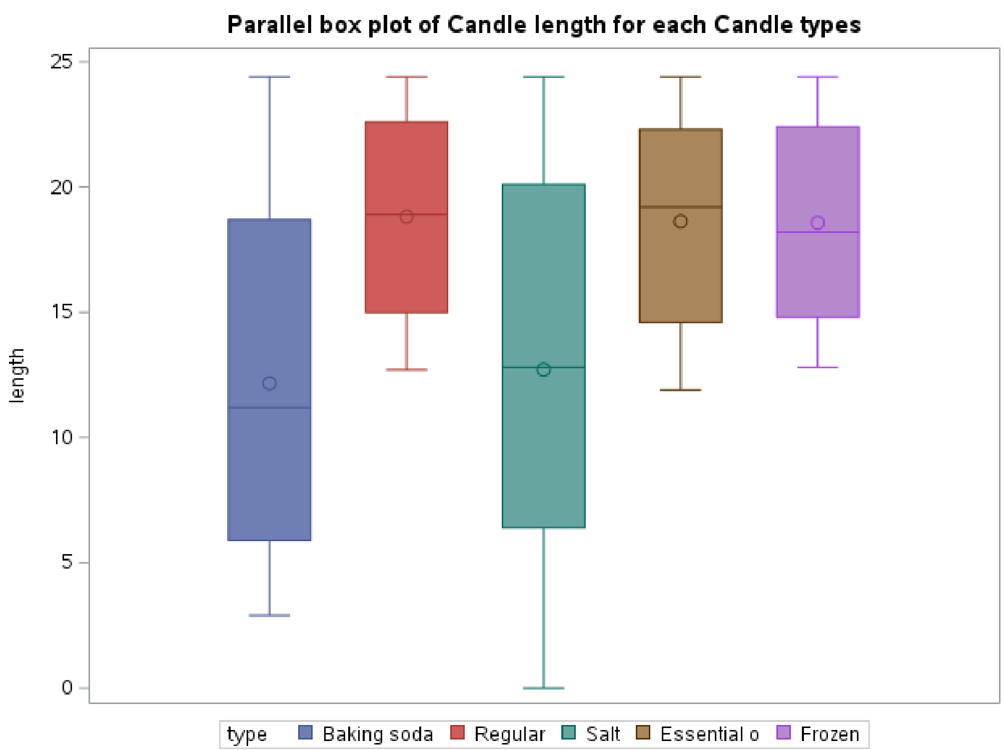

Fig 1: parallel boxplot of candle length for each group

As we see from the above parallel boxplots (Fig 1) there is a spread similarity among the three treatment types namely Regular, essential oil, and frozen. On the other hand, the other two treatment types (baking soda and salt) possess similar spread. The Baking soda group exhibits a positively skewed distribution, implying a tendency toward longer lengths or increased variability in higher lengths. Conversely, Salt, Essential oil, and Regular groups display slightly negatively skewed distributions, indicating a potential tendency towards shorter lengths or greater variability in lower lengths. The Frozen group demonstrates a nearly symmetric distribution with a very slight positive skewness, suggesting a minimal deviation towards longer lengths.

Additionally, let's create a table summarizing numerical statistics sample size, the mean, median, and standard deviation for the whole data and specifically for each Treatment Type.

The MEANS Procedure

Analysis Variable : length			
N	Mean	Median	Std Dev
55	16.1781818	17.0000000	6.3831285

Table 1: Summary Statistics of candle length for the whole group

The MEANS Procedure

Analysis Variable : length					
type	N Obs	N	Mean	Median	Std Dev
Baking soda	11	11	12.1636364	11.2000000	7.1606247
Essential o	11	11	18.6272727	19.2000000	4.2628842
Frozen	11	11	18.5727273	18.2000000	4.0323916
Regular	11	11	18.8181818	18.9000000	4.0609896
Salt	11	11	12.7090909	12.8000000	8.1414316

Table 2: Summary Statistics for Each Group

The summary statistics (Table 2) for the 'length' variable categorized by 'type' show consistent counts of 11 observations across all candle types (Baking soda, Essential oil, Frozen, Regular, Salt), indicating a balanced experimental design. Essential oil, Frozen, and Regular candles display similar mean and median lengths, while Baking soda and Salt candles have notably shorter mean and median lengths. Additionally, Baking soda and Salt candles exhibit higher variability in length, indicated by their larger standard deviations compared to the lower variability observed in Essential oil, Frozen, and Regular candles.

Overall, Essential oil, Frozen, and Regular candles demonstrate a tendency towards longer lengths with less variability, while Baking soda and Salt candles tend to have shorter lengths and higher length variability.

Analysis of One-Way ANOVA:

Before starting our hypothesis testing analysis we need to decide the significance level for our experiment which is alpha. Alpha represents the probability of rejecting a true null hypothesis or making a Type I error (false positive). Considering the fact that a higher significance level can help increase the chances of detecting real effects, we decided to use alpha=0.1 in our hypothesis testing.

So we want to check if there is a mean difference in candle length between the treatment groups, Our null hypothesis states that there is no significant difference in the mean of candle length between the treatment groups. On the other hand, our alternative hypothesis states that there is a significant mean difference in the candle length between at least one pair of the treatment groups.

While conducting a One-Way ANOVA, we obtain a table that includes the sum of squares, mean square, F-statistic value, and p-value.

The ANOVA Procedure

Dependent Variable: length

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	515.379273	128.844818	3.82	0.0087
Error	50	1684.814545	33.696291		
Corrected Total	54	2200.193818			

R-Square	Coeff Var	Root MSE	length Mean
0.234243	35.88074	5.804851	16.17818

Source	DF	Anova SS	Mean Square	F Value	Pr > F
type	4	515.3792727	128.8448182	3.82	0.0087

Table 3: ANOVA table

Before starting the analysis of One-Way ANOVA let's make sure that our sample will not violate all the assumptions of One-Way ANOVA. One-Way Anova has the following assumptions:

- Random samples are collected from their respective populations
- Samples are collected independently of one another
- Response variables all come from a normal distribution
- The population variances are all equal

We mentioned in the methods section that we sampled our data randomly and independently so the first two assumptions (Random samples and sample independence) are met.

Assessment of Normality Assumptions:

For checking the normality assumption we are going to use the quantile-quantile (Q-Q) plot of the residuals. This graphical representation compares the distribution of residuals against the expected normal distribution. We will also perform the Shapiro-Wilk test on the residuals.

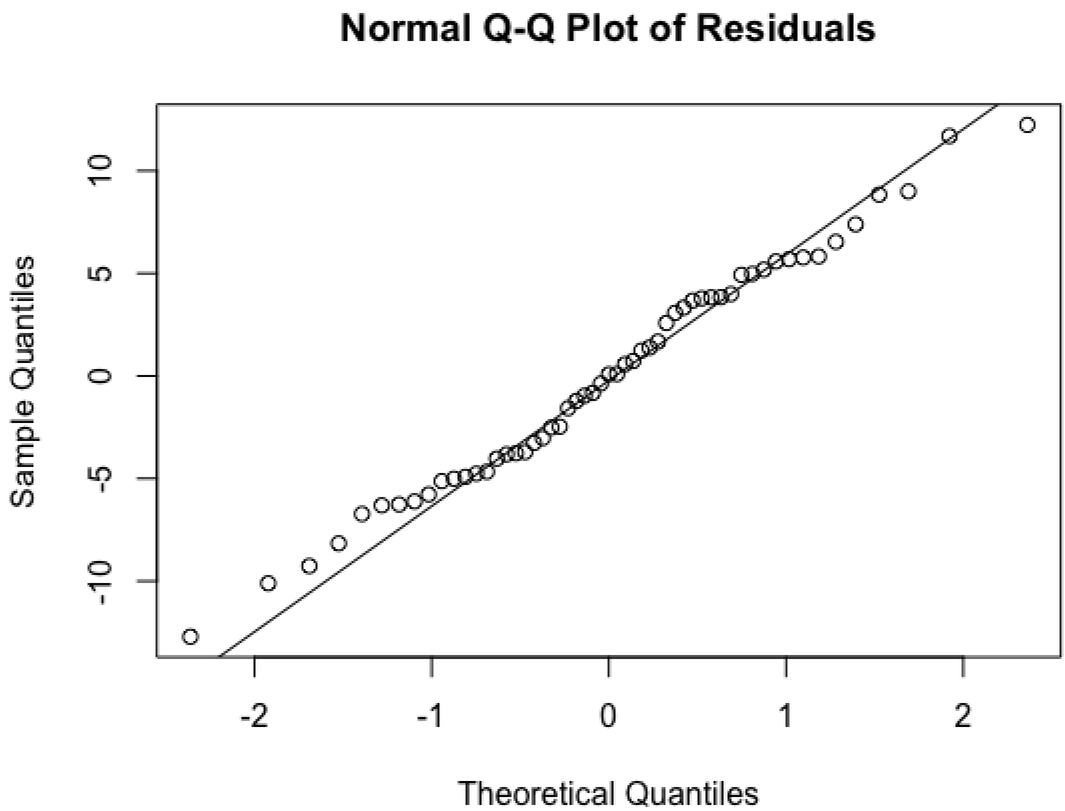

Fig 2: Q-Q Plot of Residuals

Observing the Q-Q plot (Fig 2), the residuals largely align along the diagonal line, suggesting a relatively close adherence to the pattern expected under normality. This pattern implies that the residuals are approximately normally distributed. The Shapiro-Wilk test further substantiates this observation, as its p-value of 0.8056 supports the notion of normality within the residuals, confirming that they follow a distribution similar to the normal curve.

Assessment of homoscedasticity Assumptions:
We will plot residuals against fitted values to find out about the homogeneity of our sample.

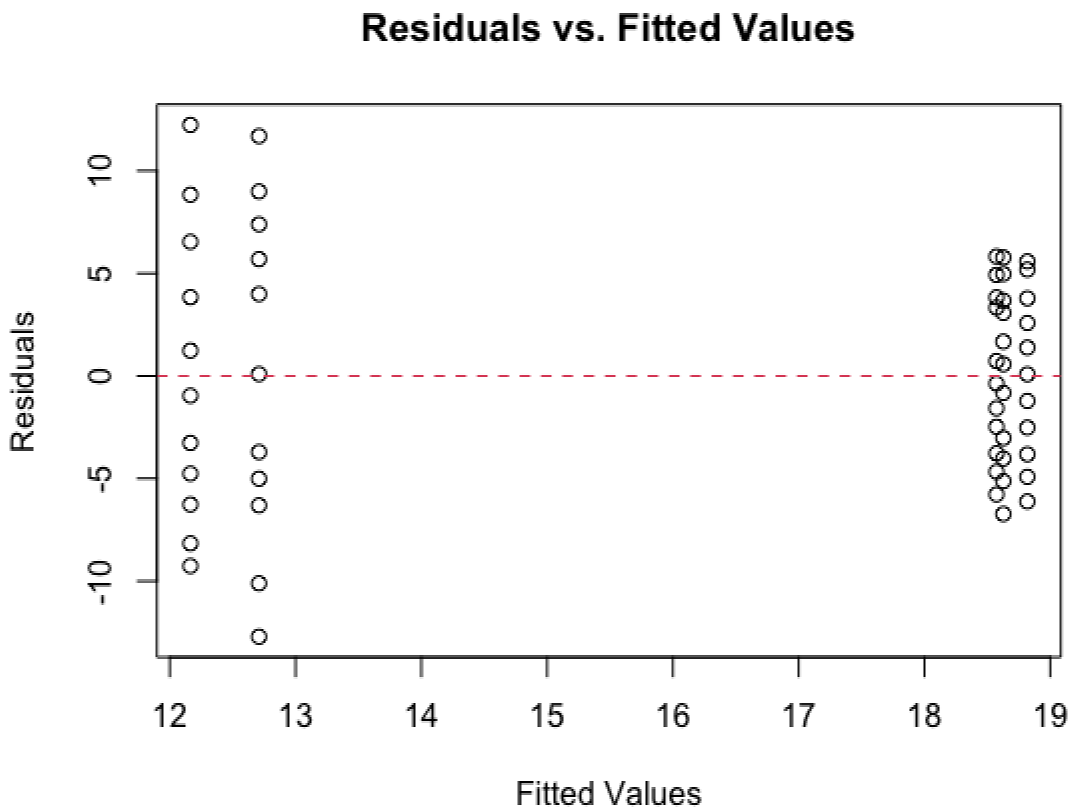

Fig 3: Residuals against Fitted values

Examining the residuals against Fitted values (Fig 3) we can see a clear pattern emerged, indicating heteroscedasticity. So our assumption of homoscedasticity is violated.

Our initial analysis involved a one-way analysis of means to assess differences in candle lengths among various types (baking soda, regular, salt, essential oil, and frozen). However, upon examining the residual plot depicting the relationship between residuals and fitted values, a clear pattern emerged, indicating heteroscedasticity. This violates the assumption of homogeneity of variances across groups, casting doubt on the reliability of the standard one-way ANOVA results. To address this issue, we conducted Welch's ANOVA, a robust alternative that does not presume equal variances among the groups.

Welch's ANOVA for length			
Source	DF	F Value	Pr > F
type	4.0000	2.91	0.0423
Error	24.6348		

Table 4: Welch's ANOVA table

The obtained p-value from Welch's ANOVA (Table 4) was determined to be 0.04, which falls below our predetermined alpha level of 0.1. Consequently, we reject the null hypothesis, indicating a statistically significant difference in the mean lengths of candles among the various treatment groups. The choice of Welch's ANOVA was made to accommodate the violation of homogeneity of variances while still rigorously assessing potential differences in means among the candle types.

Analysis of Post-Hoc Comparison:

Given that there was sufficient evidence to conclude that there is a significant mean difference in Candle Length among the Treatment Types, we can perform further analysis to find out which treatment groups were significantly different than the others. And that's when post-hoc comparison comes in; Let's perform Tukey's post-hoc comparison and find out which treatment groups were significantly different from others.

```
  Tukey multiple comparisons of means
    90% family-wise confidence level

Fit: aov(formula = length ~ type, data = candles)

$type
                                diff         lwr        upr     p adj
Essential oil-Baking soda  6.46363636   0.1997616 12.7275111 0.0835441
Frozen-Baking soda         6.40909091   0.1452161 12.6729657 0.0877959
Regular-Baking soda        6.65454545   0.3906707 12.9184202 0.0699982
Salt-Baking soda           0.54545455  -5.7184202  6.8093293 0.9994577
Frozen-Essential oil      -0.05454545  -6.3184202  6.2093293 0.9999999
Regular-Essential oil      0.19090909  -6.0729657  6.4547839 0.9999917
Salt-Essential oil        -5.91818182 -12.1820566  0.3456930 0.1346822
Regular-Frozen             0.24545455  -6.0184202  6.5093293 0.9999773
Salt-Frozen               -5.86363636 -12.1275111  0.4002384 0.1409343
Salt-Regular              -6.10909091 -12.3729657  0.1547839 0.1145061
```

Table 5: Tukey's HSD Confidence Intervals and P-Values

At a 90% confidence level, the results obtained from Tukey's HSD test following Welch's ANOVA analysis on the 'length' variable among various 'type' groups of candles suggest potential differences in mean lengths between the Baking soda group and the Essential oil, frozen and Regular groups, with p-values below 0.10 for 'Essential oil- baking soda', 'frozen - baking soda', and 'Regular - baking soda' comparisons. Conversely, comparisons involving other groups display p-values above 0.10, indicating no statistically significant distinctions in mean lengths.

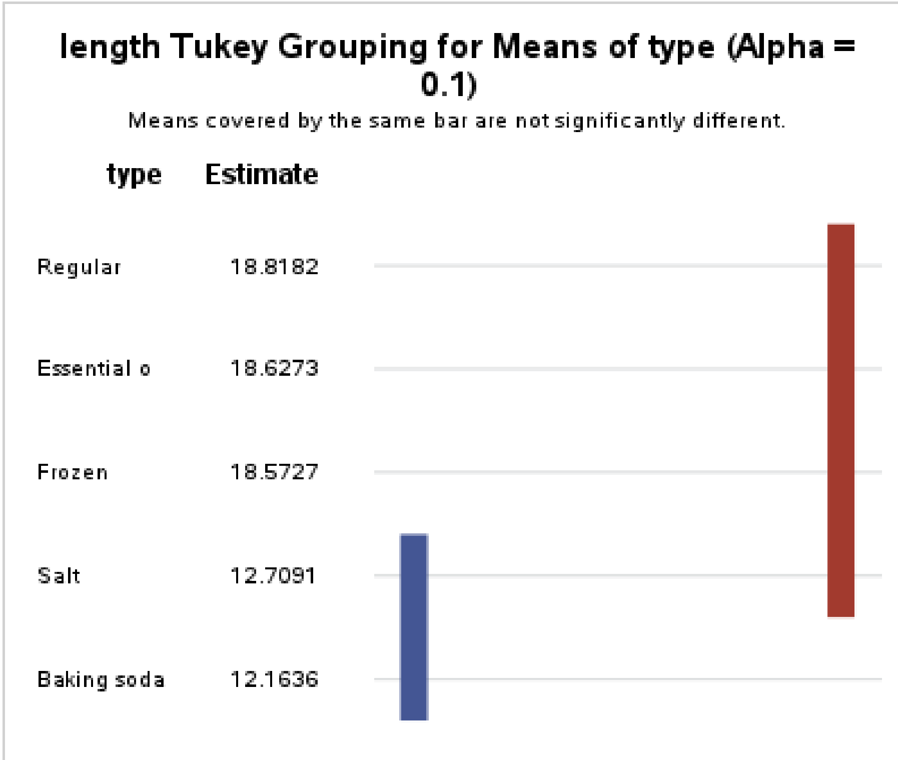

Fig 4: Tukey grouping chart

The Tukey grouping chart(Fig 4) also shows that the treatment groups Regular, essential oil, and frozen are covered by the same bar indicating they are not significantly different. On the other hand, treatment groups salt and baking soda are covered by the same bar showing they are not significantly different.

Conclusion

In summary, in our comprehensive analysis investigating the influence of different additives and freezing treatments on candle burning rates, intriguing insights surfaced, contradicting certain commonly held beliefs. Our study encompassed various candle types: those augmented with salt, baking soda, and essential oils, and subjected to freezing conditions. Contrary to popular belief, our findings revealed that candles enriched with sodium chloride (salt) and baking soda exhibited accelerated burning rates, contrary to the anticipated effect of elongating burn times. Moreover, despite claims that freezing candles diminishes their burning rates, our experiment did not support this notion. The results challenged prevailing assumptions, showcasing that the addition of sodium chloride compounds, specifically in salt and baking soda-infused candles, did not contribute to a slower burning process. This contradiction to established beliefs opens avenues for further investigation and prompts a reassessment of the traditional assumptions surrounding candle additives and freezing treatments. The scientific community may benefit from exploring the intricacies of these counterintuitive outcomes to enhance our understanding of candle-burning dynamics and potentially refine existing theories in this domain.