

Question 1: a. $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$
 $5n^3 + 2n^2 + 3n \leq 10n^3$

$\therefore \text{~~no~~ } 5n^3 + 2n^2 + 3n = O(n^3)$
 $n_0 = 1, C = 10$

b. $\sqrt{7n^2 + 2n - 8} < \sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n}$

$\sqrt{7n^2 + 2n - 8} \leq \sqrt{9n^2}$

$\sqrt{7n^2 + 2n - 8} \leq 3n$

$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2}$

$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7} n$

$2n - 8 \geq 0$
 $2n \geq 8$
 $n \geq 4$

$\therefore \sqrt{7n^2 + 2n - 8} = \Theta(n) \quad (n_0 = 4, C_1 = \sqrt{7}, C_2 = 3)$

$$C. \quad d(n) \leq a \cdot f(n) \quad (n \geq n_0)$$

$$e(n) \leq b \cdot g(n) \quad (n \geq n_0)$$

$$d(n), e(n) \leq a \cdot b \cdot f(n) \cdot g(n)$$

$$C = ab$$

$$\therefore d(n), e(n) \leq C \cdot f(n) \cdot g(n)$$

$$\therefore d(n), e(n) = O(f(n) \cdot g(n)) \text{ where}$$

$$n_0 = n_1 \text{ or } n_0 = n_2,$$

$$C = ab$$

Question 2:

$$\text{example 1: } \Theta(n^2)$$

$$\text{example 2: } \Theta(n)$$

$$\text{example 3: } \Theta(n^2)$$

$$\text{example 4: } \Theta(n \log_2 n)$$