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Computer Engineering #1

1- Express the following values in binary notation

a. $3\frac{1}{4}$ b. $5\frac{3}{4}$

Solution

a) $(3)_{10} = (\overset{2^1 2^0}{11})_2$

$(1/4)_{10} = (0,01)$ (2^0, 2^{-1}, 2^{-2})

$(11)_2 + (0,01)_2 = (11,01)_2$ ✓

b) $(5)_{10} = (101)_2$ ($2^2\,2^1\,2^0$)

$\left(\frac{3}{4}\right)_{10} = \left(\frac{1}{4} + \frac{1}{2}\right)_{10}$

$= (0,11)_2$ (2^0, 2^{-1}, 2^{-2})

$(101)_2 + (0,11)_2 = (101,11)_2$ ✓

2- Encode the following values using 8 bit floating point format

a. $-4,5$ b. $5/32$

a) $(4)_{10} + (1/2)_{10} = (100)_2 + (0,1)_2$

$(4,5)_{10} = (100,1)_2$ ($2^2\,2^1\,2^0\,2^{-1}$)

Exponent $= +3$

Excess notation $\Rightarrow (3)_{10} = 111$

Sign bit in excess notation (+)

$-(100,1)_2$

$\underbrace{1}_{\text{Sign bit }(-)}\underbrace{111}_{\text{exponent}}\underbrace{1001}_{\text{Mantissa}}$

✓

b) $\left(5/32\right)_{10} = \left(\frac{1}{32}\right)_{10} + \left(\frac{4}{32}\right)_{10}$

$= \left(\frac{1}{32}\right)_{10} + \left(\frac{1}{8}\right)_{10}$

$\left(\frac{1}{32}\right)_{10} + \left(\frac{1}{8}\right)_{10} = (0,\overset{\text{define exponent}}{\overbrace{00}}101)_2$

2^{-1} 2^{-2} 2^{-3} 2^{-4} 2^{-5}

Exponent $= -2$

Excess notation $= (+2)_{10} = 110$

$= (-2)_{10} = 010$

Sign bit excess notation (-)

Two's complement

$(0,00101)_2$

$\underset{\text{Sign bit}(-)}{0}\,\underbrace{010}_{\text{exponent}}\,\underbrace{1010}_{\text{mantissa}}$

3)

⇒ Lets consider some basic idea

* What is the complementation?

if we define equation;

$$8 = 6 + x$$

we can find $x = 2$, when we are doing this. We complement $x + 6 = 8$

⇒ Lets consider 1's complement and 2's complement theory

It is just sign bit, we don't consider when we are obtaining value. (+, -)

$(7)_{10} \Rightarrow 0111$

(+)

$2^0 \times 1$

$2^1 \times 1$

$2^2 \times 1$

$0111 \rightarrow 7$

$1000 \rightarrow$ 1's complement

$+ \quad 1$

$1001 \rightarrow$ 2's complement (-7)

(−) It is representation of (-7)

Lets focus on some special cases

$0111 \rightarrow 7$

$1000 \rightarrow$ 1's

$+ \quad 1$

2's ← 1001

Sign bit

$2^0 \times 1$

$2^1 \times 0$

$2^2 \times 0$

We know it is representation of (-7), but if we don't know this format. We can read this number 1 (Assume we know existing of sign bit)

finally;

$0111 \rightarrow 7$

$+ \, 1001 \rightarrow 1 \, (-7)$, $+ \, 8$

$\leftarrow 1{,}0000$, 0

$7 - 7 = 0$

$0110 \rightarrow 6$

$+ \, 1010 \rightarrow 2 \, (-6)$, $+ \, 8$

$1{,}0000$, 0

$6 - 6 = 0$

$0101 \rightarrow 5$

$+ \, 1011 \rightarrow 3 \, (-5)$, $+ \, 8$

$1{,}0000$, 0

$5 - 5 = 0$

$7 - 5 = 2$

$0111 \rightarrow 7$

$+ \, 1011 \rightarrow 3 \, (-5)$, $+ \, 10$

$1{,}0010$

2

$6 - 5 = 1$

$0110 \rightarrow 6$

$+ \, 1011 \rightarrow 3 \, (-5)$, $+ \, 9$

$1{,}0001$

1

As we can see, we have relations between result of base two and base ten.

$8 \div 8$ → 0

$10 \div 8$ → 2

$9 \div 8$ → 1

→ Let's consider $(5-7=-2)$ and see which result we obtain

$5-7=-2$

$$\begin{array}{r} 0101 \rightarrow 5 \\ +\ 1001 \rightarrow 1\ (-7) \\ \hline 1110 \end{array} \qquad \begin{array}{r} \\ +\ 1 \\ \hline 6 \end{array}$$

sign bit (−)

$\rightarrow 2^1 \times 1$

$\rightarrow 2^2 \times 1$ ①

② What is the representation of 2?

$(2)_{10} = 0010$

③ What is the representation of (-2)?

$$\begin{array}{r} 0010 \rightarrow 2 \\ 1101 \rightarrow 1\text{'s complement} \\ +\ \ \ \ 1 \\ \hline 1110 \rightarrow 2\text{'s complement } (-2) \end{array}$$

$\rightarrow 2^1 \times 1$

$\rightarrow 2^2 \times 1$) 6

→ As we can see in examples. We manipulate substraction by complementing negative number. Then we are able to obtain result. Sign bit help us to specify value positive or negative

Main number (8)

$5-3=\underline{2}$

$5+5=10 \mid 8$

2

$7-4=\underline{3}$

$7+4=11 \mid 8$

3

$6-1=\underline{5}$

$6+7=13 \mid 8$

5

$-5+2=\underline{-3}$

$+8 \downarrow$

$3+2=5$

$5=x+8$

$x=-3$

$-7+3=\underline{-4}$

$+8 \downarrow$

$1+3=4$

$4=x+8$

$x=-4$