CENG216 Numerical Analysis
Assignment 3

Solution 1. Table :

Time of Day	t	Temperature
12 mid.	0	18
3 am	$\frac{1}{8}$	16
6 am	$\frac{2}{8}$	22
9 am	$\frac{3}{8}$	25
12 noon	$\frac{4}{8}$	26
3 pm	$\frac{5}{8}$	26
6 pm	$\frac{6}{8}$	23
9 pm	$\frac{7}{8}$	19

Model :

$$y = c_1 + c_2cos(2\pi t) + c_3sin(2\pi t)$$

Substituting the data into the model :

$$y = c_1 + c_2cos(2\pi(0)) + c_3sin(2\pi(0)) = 18$$
$$y = c_1 + c_2cos(2\pi(\frac{1}{8})) + c_3sin(2\pi(\frac{1}{8})) = 16$$
$$y = c_1 + c_2cos(2\pi(\frac{2}{8})) + c_3sin(2\pi(\frac{2}{8})) = 22$$
$$y = c_1 + c_2cos(2\pi(\frac{3}{8})) + c_3sin(2\pi(\frac{3}{8})) = 25$$
$$y = c_1 + c_2cos(2\pi(\frac{4}{8})) + c_3sin(2\pi(\frac{4}{8})) = 26$$
$$y = c_1 + c_2cos(2\pi(\frac{5}{8})) + c_3sin(2\pi(\frac{5}{8})) = 26$$
$$y = c_1 + c_2cos(2\pi(\frac{6}{8})) + c_3sin(2\pi(\frac{6}{8})) = 23$$
$$y = c_1 + c_2cos(2\pi(\frac{7}{8})) + c_3sin(2\pi(\frac{7}{8})) = 19$$

The corresponding inconsistent matrix equation is

$$Ax = b$$

$$A = \begin{bmatrix} 1 & cos(0) & sin(0) \\ 1 & cos(\pi/4) & sin(\pi/4) \\ 1 & cos(2\pi/4) & sin(2\pi/4) \\ 1 & cos(3\pi/4) & sin(3\pi/4) \\ 1 & cos(4\pi/4) & sin(4\pi/4) \\ 1 & cos(5\pi/4) & sin(5\pi/4) \\ 1 & cos(6\pi/4) & sin(6\pi/4) \\ 1 & cos(7\pi/4) & sin(7\pi/4) \end{bmatrix} \quad b = \begin{bmatrix} 18 \\ 16 \\ 22 \\ 25 \\ 26 \\ 26 \\ 23 \\ 19 \end{bmatrix} \quad and \quad A^T.A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The normal equations $A^T Ac = A^T b$ are

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 175 \\ -19.3137 \\ -3.8284 \end{bmatrix} c_1 = 21,875 \quad c_2 = -4,8284 \quad c_3 = -0,9571$$

Therefore new b is

$$b = \begin{bmatrix} 17,0465 \\ 17,7840 \\ 20,9179 \\ 24,6124 \\ 26,7034 \\ 25,9659 \\ 22,8321 \\ 19,1375 \end{bmatrix}$$

After the calculations,

$$RMSE = \frac{\sqrt{SE}}{\sqrt{m}} = \frac{2,4404}{2,8284} = 0,8628$$

Solution 2. We know that with Gauss-Newton Method we solve

To minimize

$$r_1(x)^2 + r_2(x)^2 + ... + r_m(x)^2$$

Set x_0 initial vector, for $k = 0, 1, 2, ...$

$$\begin{aligned} A &= Dr(x^k) \\ A^T Av^k &= -A^T r(x^k) \\ x^{k+1} &= x^k + v^k \end{aligned}$$

To prove that $Ax = b$ converges in one iteration to the solution of the normal equation we should show that

$$r = b - Ax$$

for all linear equations

$$Dr = -A$$

Therefore given a A_1 matrix $Dr = -A_1$ substituting this into the method converts the equation to a normal equation which we used to solve for linear equations.

$$\begin{aligned} A &= -A_1 \\ (-A_1^T)(-A_1)v^k &= -(-A_1^T)r(x^k) \\ A_1^T(A_1)v_k &= A_1^T r(x^k) \end{aligned}$$

we know that $r = b - Ax$ so continuing with

$$\begin{aligned} A_1^T A_1 v^k &= A_1^T(b - A_1 x) \\ A_1^T A_1 v^k &= A_1^T b - A_1^T A_1 x^k \\ A_1^T A_1(v^k + x^k) &= A_1^T b \end{aligned}$$

if we iterate one time for x^0 initial vector, equation become

$$\begin{aligned} A_1^T A_1(v^0 + x^0) &= A_1^T b \\ x_1 &= v^0 + k^0 \\ A_1^T A_1 x^1 &= A_1^T b \end{aligned}$$

At the end only one iteration we obtain normal equation for the linear system.

Solution 3. Jacobian matrix needed to apply the Gauss-Newton method to the model fitting problem $y = c_1 x^{c_2}$ with $(x_1, y_1), (x_2, y_2), (x_3, y_3)$.

We know that Jacobian is

$$J_F(c_1, c_2) = \begin{bmatrix} \frac{\partial y_1}{\partial c_1} & \frac{\partial y_1}{\partial c_2} \\ \frac{\partial y_2}{\partial c_1} & \frac{\partial y_2}{\partial c_2} \\ \frac{\partial y_3}{\partial c_1} & \frac{\partial y_3}{\partial c_2} \end{bmatrix}$$

so our J for our problem is

$$J_F(c_1, c_2) = \begin{bmatrix} x_1^{c_2} & c_1 x_1^{c_2} \ln x_1 \\ x_2^{c_2} & c_1 x_2^{c_2} \ln x_2 \\ x_3^{c_3} & c_1 x_3^{c_2} \ln x_3 \end{bmatrix}$$