STAT 248: Time Series

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Contents

1 Lecture 1

2020-01-21

Didn't attend.

2 Lecture 2

2020-01-22

Last time:

- Want to model $y_i \in \mathbb{R}^K$ for $i \in \mathbb{N}$
- Introduced Vector autoregressive model (VAR)

VAR(p)

$$\boldsymbol{y}_t = \boldsymbol{\nu} + \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{A}_2\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p\boldsymbol{y}_{t-p} + \underbrace{\boldsymbol{z}_t}_{\text{white noise}} \tag{1}$$

AR(p)

$$y_t = \nu + \alpha_1 y_{t-1} + \alpha_2 y_{t-1} + \cdots + \alpha_p y_{t-p} + z_t \tag{2}$$

Definition 1

$\{y_t\}_{t\in\mathbb{Z}}$ is (weak, wide-sense, second-order) ***stationary*** if $(y_{t_i})_{i=1}^k$ and $(y_{t_i-h})_{i=1}^k$ have the same mean and covariances for any $\{t_i\}_{i=1}^k$ and h.

Equivalently:

- $\mathbb{E}y_t$ is independent of t
- $\mathrm{Cov}(\boldsymbol{y}_t)$ doese not depend on t
- $\mathrm{Cov}(\boldsymbol{y}_t, \boldsymbol{y}_{t-h})$ only depends on h

Definition 2

Strong stationarity means $(\boldsymbol{y}_{t_i})_{i=1}^k$ and $(\boldsymbol{y}_{t_i-h})_{i=1}^k$ are equal in distribution.

Definition 3 (*Cross-covariance/correlation function*)

Suppose $\{\boldsymbol{y}_t, t \in \mathbb{Z}\}$ is stationary. The ***cross-covariance function***

$$\Gamma_{\boldsymbol{y}}(h) = \mathrm{Cov}(\boldsymbol{y}_t, \boldsymbol{y}_{t-h}) \tag{3}$$

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It is a $K \times K$ matrix, not generally symmetric since $\Gamma_y(h)^\top = \Gamma_y(-h)$
The ***cross-correlation function***

$$(R_y(h))_{i,j} = \mathrm{Corr}(y_{i,t}, y_{j,t-h}) = \frac{\Gamma_y(h)_{ij}}{\sqrt{\Gamma_y(0)_{ii}\Gamma_y(0)_{jj}}} \tag{4}$$

As matrices, let $D = \mathrm{diag}(\sqrt{\mathrm{var}\, y_{1t}}, \sqrt{\mathrm{var}\, y_{2t}}, \ldots)$. Then $R_y(h) = D^{-1}\Gamma_y(h)D^{-1}$.

Question: For what $\boldsymbol{\nu}$, $\boldsymbol{A}_i$ is the VAR(p) model stationary?

Let us first look at the AR(1) case:

$$y_t = \nu + \alpha y_{t-1} + z_t$$

It turns out: stationary solution exists iff $|\alpha| = 1$.