BLG 202 E , Numerical Methods in CE

CRN:

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Homework - 1

1) a) $\sin(\phi) - \sin(\psi) = 2\cos\left(\frac{\phi+\psi}{2}\right).\sin\left(\frac{\phi-\psi}{2}\right)$

$$\left.\begin{array}{l}\phi = x_0 + h \\ \psi = x_0\end{array}\right\} \Rightarrow 2.\cos\left(\frac{2x_0+h}{2}\right).\sin\left(\frac{h}{2}\right)$$

b) $$f'(x) = \frac{f(x_0+h) - f(x_0)}{h}$$

$$f'(x) = \frac{\sin(x_0+h) - \sin(x_0)}{h} \Big\} \Rightarrow$$

I changed the numerator with $2.\cos\left(\frac{2x_0+h}{2}\right).\sin\left(\frac{h}{2}\right)$

$$f'(x) = \frac{2.\cos\left(x_0+\frac{h}{2}\right).\sin\left(\frac{h}{2}\right)}{h} \Big\} \Rightarrow$$

I used this function in the function named "funct" in the Python file part1.py.

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2) a) $x = \frac{a}{a^2 - b^2}$ $\quad y = \frac{b}{b^2 - a^2}$ (from 2.c)

- When $a \approx b$ denominator goes to 0.
- Also determinant of the matrix is going to be 0.

These conditions cause to numerical issues

b) $\begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ => I converted the matrix into equations.

$ax + by = 1$

$bx + ay = 0$

=> Sum of the equations =>

$ax + ay + bx + by = 1$

$a(x+y) + b(x+y) = 1$

$(x+y)(a+b) = 1$

$x + y = \frac{1}{a+b}$

c) • $x+y = \frac{1}{a+b}$ (2.b)

• $ax - ay + by - bx = 1$

$a(x-y) - b(x-y) = 1$

$(x-y)(a-b) = 1$

$x - y = \frac{1}{a-b}$

I found "x-y" to find x and y

$x+y = \frac{1}{a+b}$

$x-y = \frac{1}{a-b}$

$>$ $x = \frac{a}{a^2-b^2}$

$y = \frac{b}{b^2-a^2}$

If small changes in inputs make big changes in output, we can say it is ill-conditioned.

* If $a = 1$ $\quad x = -4999.75$

$b = 1.0001$ $\quad y = 5000.25$

$x+y = 0.5$

* If $a = 1.001$ $\quad x = 555.8054$

$b = 1.0001$ $\quad y = -555.3057$

$x+y = 0.499707 \approx 0.5$

Calculating x and y are ill conditioned, but x+y is not ill-conditioned. Because there is no significant difference between first x+y value and second x+y value.

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3) $x^y = e^{y \ln x}$, $x>0$

formula: $fl(\ln z) = (\ln z)(1+\epsilon)$ $\quad |\epsilon| \leq n$

$$fl(x^y) = e^{y \cdot \underbrace{fl(\ln x)}_{\Downarrow}} = e^{y\ln x + y\epsilon \ln x} = \underbrace{e^{y\ln x}}_{\Downarrow} \cdot e^{y\epsilon \ln x} = x^y \cdot e^{y\epsilon \ln x}$$

$fl(\ln x) = (\ln x)(1+\epsilon)$ $\qquad x^y$

$$fl(x^y) = x^y \cdot e^{y\epsilon \ln x}$$

Relative error: $\dfrac{|u-v|}{|u|} \Rightarrow \dfrac{|x^y - x^y \cdot e^{y\epsilon \ln x}|}{|x^y|} = \dfrac{\cancel{|x^y|}\,|1-e^{y\epsilon \ln x}|}{\cancel{|x^y|}}$

Relative error is unbounded $\qquad = \underbrace{|1-e^{y\epsilon \ln x}|}$

Relative error

That shows that elementary function such as sin, ln, and exponentiation do not have bounded relative error.