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BLG 202E - HW2

(1)

$x_1 - x_2 + 3x_3 = 2$

$x_1 + x_2 = 4$

$3x_1 - 2x_2 + x_3 = 1$

Augmented matrix form of given equations

$$\left[\begin{array}{ccc:c} 1 & -1 & 3 & 2 \\ 1 & 1 & 0 & 4 \\ 3 & -2 & 1 & 1 \end{array}\right]$$

Some elementary row operations

$R_2 - R_1 \rightarrow R_2$: $\left[\begin{array}{ccc:c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 2 \\ 3 & -2 & 1 & 1 \end{array}\right]$

$R_3 - 3R_1 \rightarrow R_3$: $\left[\begin{array}{ccc:c} 1 & -1 & 3 & 2 \\ 0 & 2 & -3 & 2 \\ 0 & 1 & -8 & -5 \end{array}\right]$

$R_2/2 \rightarrow R_2$: $\left[\begin{array}{ccc:c} 1 & -1 & 3 & 2 \\ 0 & 1 & -3/2 & 1 \\ 0 & 1 & -8 & -5 \end{array}\right]$

$R_3 - R_2 \rightarrow R_2$ $\underbrace{\left[\begin{array}{ccc:c} 1 & -1 & 3 & 2 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & -13/2 & -6 \end{array}\right]}_{\text{resulting upper triangular matrix}}$

Backward Substitution

- $-\frac{13}{2}x_3 = -6$ $\qquad x_3 = \frac{12}{13}$
- $x_2 - \frac{3x_3}{2} = 1$ $\qquad x_2 = 1 + \frac{18}{13} = \frac{31}{13}$
- $x_1 - x_2 + 3x_3 = 2$ $\qquad x_1 = 2 + \frac{31}{13} - \frac{36}{13} = \frac{21}{13}$

$$X = \begin{bmatrix} 21/13 \\ 31/13 \\ 12/13 \end{bmatrix}$$

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(2) a) A permutation matrix makes a matrix change it's rows when multiplication happens between P and A matrix.

If we choose P as the replacements of the 3rd and 4th rows of the identity matrix, we get an upper triangular matrix that we need for LU decomposition.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \leftarrow P \text{ matrix}$$

$$P \cdot A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow U \text{ matrix}$$

L matrix is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ lower part of the diagonal is 0, because we did not use elementary operations to get U matrix

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & -1 \end{bmatrix}}_{U}$$

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2) b) $Ax = b$, We do not know the LU decomposition of A, bu we know PA's. To get A, we multiply PA with P^{-1} $(P^{-1}PA)$

$$PA = LU$$

$$\underbrace{P^{-1}.P.A}_{A} = P^{-1}LU \quad \Rightarrow \quad A = P^{-1}LU$$

$$Ax = b$$

$$P^{-1}LUx = b$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P^{-1} = P} \underbrace{\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x}}_{a} = \underbrace{\begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix}}_{b}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ 1 \\ -3 \end{bmatrix} \qquad \begin{aligned} a_1 &= 26 \\ a_2 &= 9 \\ a_3 &= -3 \\ a_4 &= 1 \end{aligned} \qquad a = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 5 & 6 & 7 & 8 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{b}}^{a} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \\ -3 \\ 1 \end{bmatrix} \qquad \begin{aligned} b_1 &= 26 \\ b_2 &= 9 \\ b_3 &= -3 \\ b_4 &= 1 \end{aligned}$$

$$\underbrace{\begin{bmatrix}5&6&7&8\\0&4&3&2\\0&0&-1&-2\\0&0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}}_{b} = \overbrace{\begin{bmatrix}26\\9\\-3\\1\end{bmatrix}}^{b}$$

$$5x_1 + 6x_2 + 7x_3 + 8x_4 = 26$$

$$4x_2 + 3x_3 + 2x_4 = 9$$

$$-x_3 - 2x_4 = -3$$

$$x_4 = 1$$

$$x_1 = 1$$

$$x_2 = 1$$

$$x_3 = 1$$

$$x_4 = 1$$

$$X = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

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(3) $A = \begin{bmatrix} -2 & 1 & 4 \\ 1 & 1 & 1 \\ 4 & 1 & -2 \end{bmatrix}$

n_1: $V_0 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ $A \cdot x_0 = \begin{bmatrix} -4 \\ 2 \\ 8 \end{bmatrix} \xrightarrow{\text{Normalize it}} \begin{bmatrix} -0.5 \\ 0.25 \\ 1 \end{bmatrix}$ get max : 8

Eigenvalue 1 : 8

n_2: $V_1 = \begin{bmatrix} -0.5 \\ 0.25 \\ 1 \end{bmatrix}$ $A \cdot x_1 = \begin{bmatrix} 5.25 \\ 0.75 \\ -3.75 \end{bmatrix} \xrightarrow{\text{Normalize it}} \begin{bmatrix} 1 \\ 0.142857 \\ -0.71428 \end{bmatrix}$ get max : 5.25

Eigenvalue2 : 5.25

It goes like that

```
Iteration 1
Eigenvalue 1: 8
Eigenvector 1: [-0.5   0.25  1.  ]

Iteration 2
Eigenvalue 1: 5.25
Eigenvector 1: [ 1.          0.14285714 -0.71428571]

Iteration 3
Eigenvalue 1: 5.571428571428572
Eigenvector 1: [-0.84615385  0.07692308  1.        ]

Iteration 4
Eigenvalue 1: 5.769230769230769
Eigenvector 1: [ 1.    0.04 -0.92]

Iteration 5
Eigenvalue 1: 5.88
Eigenvector 1: [-0.95918367  0.02040816  1.        ]
```

```
Iteration 1
Eigenvalue2: 4
Eigenvector2: [1. 1. 1.]

Iteration 2
Eigenvalue2: 3.0
Eigenvector2: [1. 1. 1.]

Iteration 3
Eigenvalue2: 3.0
Eigenvector2: [1. 1. 1.]

Iteration 4
Eigenvalue2: 3.0
Eigenvector2: [1. 1. 1.]

Iteration 5
Eigenvalue2: 3.0
Eigenvector2: [1. 1. 1.]
```

When $V_0 = [1,2,-1]^T$, the eigenvalues converges to 6.
Eigenvectors converges to the vector below in iteration 100

```
Iteration 97
Eigenvalue 1: 6.0
Eigenvector 1: [-1.  0.  1.]

Iteration 98
Eigenvalue 1: 6.0
Eigenvector 1: [ 1.  0. -1.]

Iteration 99
Eigenvalue 1: 6.0
Eigenvector 1: [-1.  0.  1.]

Iteration 100
Eigenvalue 1: 6.0
Eigenvector 1: [ 1.  0. -1.]
```

When $V_0 = [1,2,1]^T$, the eigenvalues converges to 6.
Eigenvectors converges to $V = [1,1,1]^T$.

```
Iteration 47
Eigenvalue 2: 3.0
Eigenvector 2: [1. 1. 1.]

Iteration 48
Eigenvalue 2: 3.0
Eigenvector 2: [1. 1. 1.]

Iteration 49
Eigenvalue 2: 3.0
Eigenvector 2: [1. 1. 1.]

Iteration 50
Eigenvalue 2: 3.0
Eigenvector 2: [1. 1. 1.]
```

They do not converts to same values because every matrix can have different eigenvalues and we can get them by using different initial vectors.

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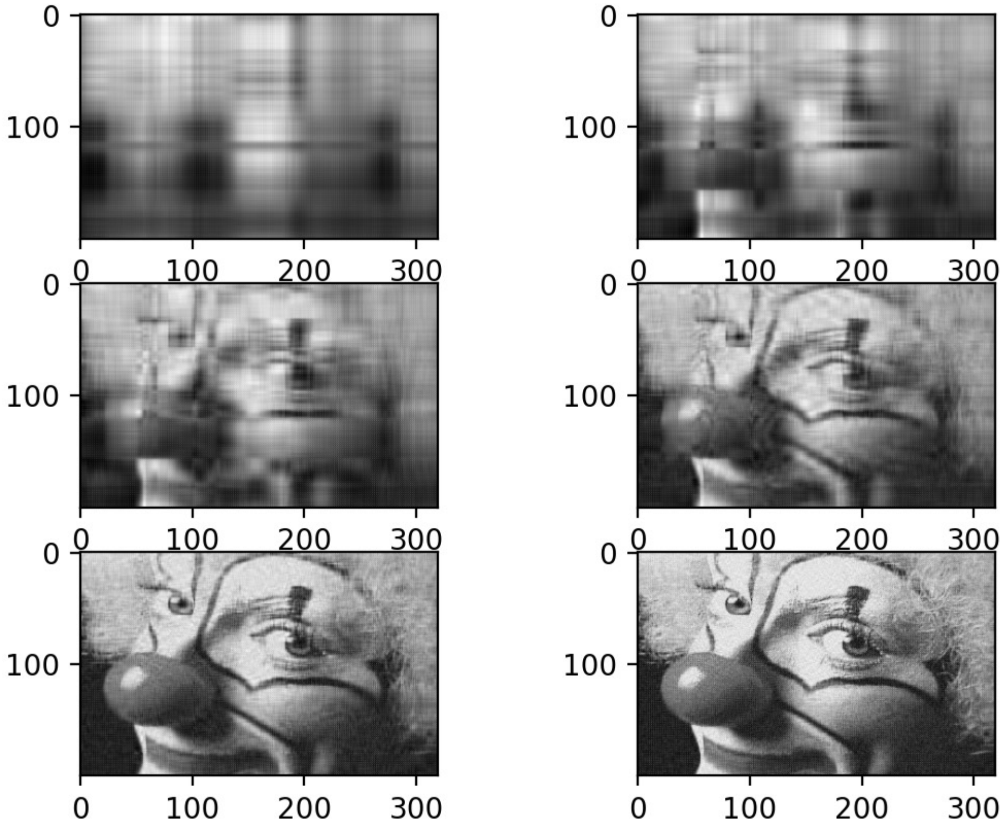
4 a)



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(4) b) The storage required is increase by r. If we want to more clear image we need more storage area.