Student Information

Name : Feyza Özlü

ID : e2580868

Answer 1

a)

For this part, we can use the formula in the textbook page-116 (Size of a Monte Carlo study). Since we don't have an estimator for p we can use this formula:

$$N \geq 0.25\left(\frac{z_{\alpha/2}}{\epsilon}\right)^2$$

We are given that $\alpha = 1 - 0.99 = 0.01$ and $\epsilon = 0.02$. And we can find the z value by looking at the textbook page-250 $z_{\alpha/2} = z_{0.01/2} = z_{0.005} = 2.576$.
If we substitute these values inside the equation:

$$\begin{aligned}
N &\geq 0.25\left(\frac{z_{\alpha/2}}{\epsilon}\right)^2 \\
N &\geq 0.25\left(\frac{2.576}{0.02}\right)^2 \\
N &\geq 4147.36
\end{aligned}$$

Hence we can say that, size of our Monte Carlo simulation should be 4148 so that with 0.99 probability, our answer should differ from the true value by no more than 0.02.

b)

1. Expected value for the weight of an automobile:

We are given the weight of each automobile is Gamma distributed with parameters $\alpha = 120$ and $\lambda = 0.1$. For a Gamma distribution Gamma(α, λ), the expected value is given by:

$$E(x) = \frac{\alpha}{\lambda} = \frac{120}{0.1} = 1200 \text{ kg}$$

2. Expected value for the weight of a truck:

We are given the weight of each truck is Gamma distributed with parameters $\alpha = 14$ and $\lambda = 0.001$. For a Gamma distribution Gamma(α, λ), the expected value is given by:

$$E(x) = \frac{\alpha}{\lambda} = \frac{14}{0.001} = 14000 \text{ kg}$$

3. Expected value for the total weights of all automobiles that pass over the bridge on a day: We are given that the number of automobiles per day follows a Poisson distribution with $\lambda = 60$. Let N_A be the number of automobiles. Then, the total weight of the automobiles is the sum of N_A Gamma distributed random variables. Therefore, the expected total weight is:

$$E(\text{total weights of automobiles}) = \lambda \cdot E(\text{weight of an automobile}) = 60 \cdot 1200 = 72000 \text{ kg}$$

4. Expected value for the total weights of all trucks that pass over the bridge on a day: We are given that the number of trucks per day follows a Poisson distribution with $\lambda = 12$. Let N_T be the number of trucks. Then, the total weight of the trucks is the sum of N_T Gamma distributed random variables. Therefore, the expected total weight is:

$$E(\text{total weights of trucks}) = \lambda \cdot E(\text{weight of a truck}) = 12 \cdot 14000 = 168000 \text{ kg}$$

Answer 2

For this question here is the Matlab code that does this simulation:

```
N = 4148;
lambdaA = 60;  % number of automobiles
lambdaT = 12;  % number of trucks

TotalWeight = zeros(N, 1); % a vector that keeps the total weight for each Monte Carlo r

for k = 1:N;
  % first generate the number of passed vehicles for each type from Poisson
  numA = 0;
  U = rand;
  F = exp(-lambdaA);
  while (U >= F)
      numA = numA + 1;
      F = F + exp(-lambdaA) * lambdaA^numA / gamma(numA + 1);
  end

  numT = 0;
  U = rand;
  F = exp(-lambdaT);
  while (U >= F)
      numT = numT + 1;
      F = F + exp(-lambdaT) * lambdaT^numT / gamma(numT + 1);
  end

  % calculate the total weight of automobiles
  totalWeightA = 0;
  for a = 1:numA
```

```
        totalWeightA = totalWeightA + sum(-1/0.1 * log(rand(120, 1)));
    end

    % calculate the total weight of trucks
    totalWeightT = 0;
    for a = 1:numT
        totalWeightT = totalWeightT + sum(-1/0.001 * log(rand(14, 1)));
    end

    % total weight of vehicles for this run
    weight(k) = totalWeightA + totalWeightT;
end;

p_est = mean(weight > 250000);
expectedWeight = mean(weight);
stdWeight = std(weight);

fprintf('Estimated probability = %f\n',p_est);
fprintf('Expected weight = %f\n',expectedWeight);
fprintf('Standard deviation = %f\n',stdWeight);
```

And when I run this code on Matlab I got these results:

```
fprintf('Estimated probability = %f\n',p_est);
fprintf('Expected weight = %f\n',expectedWeight);
fprintf('Standard deviation = %f\n',stdWeight);
Estimated probability = 0.414658
Expected weight = 241197.340460
Standard deviation = 50333.528366
>>
```

Hence we can say that the probability that the bridge would collapse is approximately 0.41, The total weight, X, of all the vehicles that pass over the bridge on a day is approximately 240000, and the Std(X) is approximately 50000.

According to the output of simulation, the standard deviation of X is estimated to be approximately 50000 kg. The accuracy of this estimator is determined by the size of the Monte Carlo simulation, (for our simulation it was N = 4148). We know that with a large sample size, the sample mean and sample standard deviation are good approximations of the real mean and real standard deviation. However, there can be always sampling errors. We can reduce these errors by increasing the number of simulations. For our simulation $N = 4148$, the standard deviation should provide a reasonably accurate estimate in the total weight of all vehicles crossing the bridge.

To decide on the new Poisson variable λ_T so that the probability of bridge's collapse is less than 0.1 we can run the above code on Matlab by decrementing the λ_T variable 1 by 1 until we get the probability of bridge's collapse is less than 0.1.

When we run the code with $\lambda_T = 9$ we will get these results:

$$\begin{aligned} \text{Estimated probability} &= 0.123192 \\ \text{Expected weight} &= 198719.901616 \\ \text{Standard deviation} &= 44378.719865 \end{aligned}$$

If we go one step further and run the code with $\lambda_T = 8$ we will get these results:

$$\begin{aligned} \text{Estimated probability} &= 0.070636 \\ \text{Expected weight} &= 184437.428505 \\ \text{Standard deviation} &= 41683.363656 \end{aligned}$$

Hence we can conclude that 8 would be the new Poisson variable λ_T so that the probability of bridge's collapse is less than 0.1. This is the highest possible integer limit.