Random seed

```
rng(01778525)
```

Import Data

```
train_data = readtable("train1778525.csv")
```

train_data = 500×6 table

	l100	mass	time100	displacement	type	colour
1	3.6000	997	8.6000	4.2000	'diesel'	'other'
2	4.8000	2050	10.6000	3.1000	'diesel'	'other'
3	4.6000	1265	9.7000	3.3000	'diesel'	'other'
4	4.3000	1578	9.5000	2.9000	'diesel'	'other'
5	4.9000	2343	11.8000	1.7000	'diesel'	'other'
6	3.7000	1306	9.5000	3.2000	'diesel'	'white'
7	5.1000	1934	10.4000	2.8000	'diesel'	'other'
8	3.9000	2051	10.2000	2.6000	'diesel'	'other'
9	2.6000	960	12.6000	1.6000	'diesel'	'other'
10	3.5000	934	11	1.8000	'diesel'	'white'
11	4.3000	1046	10.4000	2.2000	'diesel'	'other'
12	4.1000	1741	8.7000	2.7000	'diesel'	'other'
13	5.6000	1466	5.9000	6.9000	'diesel'	'other'
14	3.6000	1202	10.2000	2.4000	'diesel'	'white'
15	5.1000	1788	7.7000	4.7000	'diesel'	'white'
16	3.5000	1095	10.1000	3.4000	'diesel'	'white'
17	3.6000	1118	11.3000	1.7000	'diesel'	'other'
18	4	1967	9.7000	2.4000	'diesel'	'other'
19	3.1000	1695	11.4000	2.2000	'diesel'	'other'
20	4.1000	1782	10.2000	3.3000	'diesel'	'other'
21	4.2000	901	7.6000	4	'diesel'	'other'
22	3.8000	1591	12.5000	1.6000	'diesel'	'other'
23	4.2000	2137	11.1000	2	'diesel'	'other'
24	3.9000	1111	10.6000	2.5000	'diesel'	'other'
25	3.3000	1332	11.1000	2	'diesel'	'other'
26	4	1067	8.7000	3.2000	'diesel'	'other'

	l100	mass	time100	displacement	type	colour
27	3.6000	1077	11	2.5000	'diesel'	'other'
28	3.1000	984	11	2.1000	'diesel'	'other'
29	4.4000	917	7.4000	4.8000	'diesel'	'other'
30	5	2205	9	2.9000	'diesel'	'white'
31	3.4000	1256	10.7000	2.7000	'diesel'	'other'
32	4	1116	10.8000	2.7000	'diesel'	'other'
33	3.9000	1415	12.4000	2.1000	'diesel'	'other'
34	3.8000	1026	9.1000	3.3000	'diesel'	'white'
35	4.7000	1794	9.7000	2.4000	'diesel'	'other'
36	3.5000	1709	10.9000	2.4000	'diesel'	'white'
37	3.4000	1084	9.5000	2.7000	'diesel'	'other'
38	4.7000	1426	9.9000	3.4000	'diesel'	'white'
39	3.4000	964	12.3000	1.8000	'diesel'	'other'
40	4.6000	1454	9.3000	3.2000	'diesel'	'other'
41	3.2000	1106	8.3000	4.3000	'diesel'	'other'
42	3.9000	1792	10.7000	1.9000	'diesel'	'white'
43	4.9000	1986	9.7000	2.2000	'diesel'	'other'
44	3	927	10.4000	2.7000	'diesel'	'white'
45	3.9000	947	7.9000	3.9000	'diesel'	'white'
46	4.8000	1940	9.7000	2.5000	'diesel'	'other'
47	3.6000	1645	10.3000	2.4000	'diesel'	'other'
48	4.2000	1964	8	3.2000	'diesel'	'other'
49	3.3000	1015	11.3000	1.7000	'diesel'	'white'
50	5.1000	2078	9.3000	3.4000	'diesel'	'other'
51	4.2000	1246	7.9000	3.7000	'diesel'	'white'
52	5.6000	2359	7.8000	5.4000	'diesel'	'other'
53	4.1000	1185	10.2000	2.9000	'diesel'	'white'
54	3.6000	1113	10.4000	3	'diesel'	'other'
55	4.2000	2094	7.7000	3.9000	'diesel'	'white'
56	3.3000	976	8.8000	3.8000	'diesel'	'other'
57	3.8000	1065	12	2	'diesel'	'other'
58	5	2081	11	2.1000	'diesel'	'other'
59	3.6000	1256	9.6000	3.1000	'diesel'	'other'

	l100	mass	time100	displacement	type	colour
60	4.8000	1965	11.6000	2	'diesel'	'other'
61	6.4000	2216	8.9000	3.6000	'diesel'	'other'
62	4.4000	1037	9.3000	4.2000	'diesel'	'other'
63	3.4000	1303	11.5000	2.3000	'diesel'	'other'
64	4.5000	2364	12.2000	1.4000	'diesel'	'white'
65	3.7000	1000	10.6000	2.5000	'diesel'	'other'
66	3.5000	938	9.1000	3.2000	'diesel'	'white'
67	2.9000	1202	11.6000	2.5000	'diesel'	'white'
68	3.9000	1563	10.8000	2.3000	'diesel'	'other'
69	4.2000	1481	9.7000	2.4000	'diesel'	'other'
70	3.6000	1080	10.9000	2.6000	'diesel'	'other'
71	4.3000	1121	10.5000	2.3000	'diesel'	'other'
72	4.7000	1759	8.9000	3	'diesel'	'other'
73	4.8000	1864	9.1000	2.9000	'diesel'	'other'
74	3.9000	905	8.3000	4.5000	'diesel'	'white'
75	4.5000	1707	9	3.2000	'diesel'	'other'
76	5	1575	9.2000	3.8000	'diesel'	'other'
77	4.1000	1703	9.1000	3.7000	'diesel'	'white'
78	3.5000	1308	8.8000	3.7000	'diesel'	'white'
79	4.9000	1301	7.5000	5.8000	'diesel'	'white'
80	5.8000	2134	10.1000	2.8000	'diesel'	'white'
81	4.6000	1099	9	3.4000	'diesel'	'other'
82	4.7000	1615	8.8000	3.8000	'diesel'	'other'
83	4.2000	1840	9.7000	2.7000	'diesel'	'other'
84	5.2000	1141	7.1000	4.1000	'diesel'	'other'
85	3.9000	1073	10.1000	3	'diesel'	'other'
86	3.5000	1171	10.2000	2.7000	'diesel'	'white'
87	4.9000	1311	8.4000	4.4000	'diesel'	'other'
88	4.2000	1633	11.1000	2.2000	'diesel'	'other'
89	4.8000	929	9.7000	3.1000	'diesel'	'other'
90	4.1000	1589	11.2000	1.8000	'diesel'	'other'
91	3.1000	1208	10.3000	2.4000	'diesel'	'other'
92	3.6000	934	10	2.1000	'diesel'	'white'

	l100	mass	time100	displacement	type	colour
93	4.4000	1029	10.6000	2.2000	'diesel'	'other'
94	3.7000	1484	9.3000	3.1000	'diesel'	'white'
95	3.9000	1409	11.4000	1.8000	'diesel'	'white'
96	4.4000	2039	10.9000	2.2000	'diesel'	'other'
97	3.4000	1303	10.9000	2.8000	'diesel'	'white'
98	4.3000	1630	9.1000	3.4000	'diesel'	'other'
99	4.2000	1055	7.1000	4.1000	'diesel'	'other'
100	5	1728	9.9000	2.6000	'diesel'	'other'

⋮

```
test_data = readtable("test1778525.csv")
```

test_data = 500×6 table

	l100	mass	time100	displacement	type	colour
1	3.4000	2218	11.9000	2	'diesel'	'white'
2	3.8000	1977	10.8000	2.1000	'diesel'	'other'
3	4.6000	983	10.8000	2.1000	'diesel'	'other'
4	4	2310	9.9000	2	'diesel'	'white'
5	3.4000	1233	12	1.9000	'diesel'	'other'
6	4.1000	1004	10.1000	3.1000	'diesel'	'other'
7	3.7000	921	10.6000	2.1000	'diesel'	'white'
8	3.7000	1678	9.2000	3.8000	'diesel'	'other'
9	3.1000	1377	11.3000	2.2000	'diesel'	'other'
10	4.1000	2083	12	1.6000	'diesel'	'other'
11	3.7000	1054	10.1000	2.4000	'diesel'	'white'
12	3.3000	1217	11.7000	2.3000	'diesel'	'white'
13	3.7000	1940	8.9000	3.1000	'diesel'	'other'
14	4.9000	1899	9.1000	3.7000	'diesel'	'other'
15	6.2000	2152	9.2000	3.1000	'diesel'	'white'
16	4	1042	11	2.1000	'diesel'	'other'
17	4.4000	2050	9.2000	3	'diesel'	'other'
18	4.7000	2137	8.7000	4	'diesel'	'other'
19	4.1000	2393	11.1000	3	'diesel'	'other'
20	3.9000	1421	10.8000	2.1000	'diesel'	'other'
21	4.2000	1261	9.1000	3.7000	'diesel'	'white'

	l100	mass	time100	displacement	type	colour
22	5.7000	1999	9.1000	2.3000	'diesel'	'other'
23	5.2000	2272	7.8000	4.4000	'diesel'	'white'
24	4.4000	1678	6.6000	4.7000	'diesel'	'white'
25	3.8000	1064	8.9000	3.8000	'diesel'	'other'
26	4.4000	1917	7.2000	3.6000	'diesel'	'white'
27	3.9000	1634	9.4000	2.6000	'diesel'	'other'
28	3.7000	1044	10.6000	2.2000	'diesel'	'white'
29	3.4000	921	8.8000	3.3000	'diesel'	'white'
30	4.7000	2316	8.7000	3.7000	'diesel'	'white'
31	4.2000	2118	9.3000	2.9000	'diesel'	'white'
32	3.6000	1265	11.2000	2	'diesel'	'white'
33	4.6000	1431	8.5000	3.1000	'diesel'	'other'
34	4.8000	2412	9.3000	3	'diesel'	'white'
35	4.1000	1959	10.2000	2.7000	'diesel'	'other'
36	4.2000	1113	9.4000	2.4000	'diesel'	'other'
37	2.8000	1011	13.2000	1.6000	'diesel'	'other'
38	4.7000	1407	10.7000	2.8000	'diesel'	'white'
39	3.9000	1290	9.6000	3.1000	'diesel'	'other'
40	3.8000	1757	10.7000	2	'diesel'	'white'
41	3.5000	1106	10.1000	2	'diesel'	'other'
42	4.4000	1994	9.6000	2.3000	'diesel'	'white'
43	4.8000	1900	7.5000	5.5000	'diesel'	'white'
44	3.9000	956	9.8000	3	'diesel'	'other'
45	3.9000	1622	7.9000	4.2000	'diesel'	'other'
46	4.4000	2125	10.1000	2.1000	'diesel'	'other'
47	3.7000	950	10.5000	2.6000	'diesel'	'other'
48	4	2380	13	2	'diesel'	'other'
49	4.2000	1537	10.6000	2.3000	'diesel'	'other'
50	3.5000	1206	9.9000	2.8000	'diesel'	'other'
51	3.1000	1092	11.5000	2.1000	'diesel'	'white'
52	4.8000	1165	10	2.8000	'diesel'	'white'
53	3.1000	1334	12	1.5000	'diesel'	'other'
54	3.6000	1300	8.7000	3.9000	'diesel'	'other'

	l100	mass	time100	displacement	type	colour
55	4.3000	2297	11.1000	2.1000	'diesel'	'other'
56	4.9000	1280	8.9000	4	'diesel'	'other'
57	4.8000	1420	7.1000	5.6000	'diesel'	'other'
58	3.3000	1244	12.1000	2	'diesel'	'other'
59	4	2051	11.7000	1.8000	'diesel'	'other'
60	3.9000	1439	10.3000	2.8000	'diesel'	'other'
61	4	1831	9.3000	3.6000	'diesel'	'other'
62	3.2000	1009	12.4000	2.1000	'diesel'	'other'
63	3.3000	1437	10	2.4000	'diesel'	'other'
64	3.2000	1082	10.3000	2.9000	'diesel'	'other'
65	4.2000	2087	9.9000	3	'diesel'	'other'
66	3.4000	1201	9.9000	3.3000	'diesel'	'other'
67	3.8000	1617	11.2000	1.7000	'diesel'	'other'
68	3.6000	1241	10.4000	2.4000	'diesel'	'other'
69	4.3000	938	7.4000	5.1000	'diesel'	'other'
70	5.4000	2306	7.9000	3.8000	'diesel'	'other'
71	3.8000	1480	9.8000	3.3000	'diesel'	'other'
72	4.6000	2349	9.6000	2.9000	'diesel'	'other'
73	3	1103	12.1000	2	'diesel'	'white'
74	3.1000	911	10.4000	2.8000	'diesel'	'other'
75	3.4000	960	10.3000	3.3000	'diesel'	'white'
76	3.7000	910	10.4000	2.8000	'diesel'	'other'
77	4.7000	1634	9.1000	3.6000	'diesel'	'other'
78	3.4000	1290	10.6000	2.5000	'diesel'	'white'
79	3.1000	924	9.3000	2.8000	'diesel'	'other'
80	4	1294	9.1000	4	'diesel'	'other'
81	3.2000	1673	10.6000	2.3000	'diesel'	'white'
82	3.5000	1607	11.4000	1.8000	'diesel'	'white'
83	4	1174	8.6000	3.8000	'diesel'	'other'
84	4.3000	981	9.2000	4	'diesel'	'other'
85	3.7000	1278	12.3000	1.6000	'diesel'	'other'
86	3.7000	1049	11	1.5000	'diesel'	'other'
87	4	1139	10.4000	2	'diesel'	'white'

	l100	mass	time100	displacement	type	colour
88	5	2427	13.2000	1.3000	'diesel'	'other'
89	4.4000	1240	10	2.2000	'diesel'	'other'
90	4	1133	9	3.3000	'diesel'	'other'
91	4.1000	1507	10.2000	3.4000	'diesel'	'other'
92	3.5000	1194	11.3000	2.6000	'diesel'	'white'
93	3.9000	1007	11.4000	2.6000	'diesel'	'other'
94	3.3000	1469	10.9000	2.2000	'diesel'	'other'
95	3.2000	1770	13	1.4000	'diesel'	'other'
96	3.5000	1395	10.9000	2.2000	'diesel'	'other'
97	4.2000	1115	10.1000	2.6000	'diesel'	'other'
98	4.5000	1719	8	5	'diesel'	'other'
99	3.3000	1154	11	2.1000	'diesel'	'other'
100	3.3000	1549	10.3000	3.1000	'diesel'	'other'

⋮

Store training data in arrays

```
y = table2array(train_data(:,1));    % Fuel consumption
mass = table2array(train_data(:,2));
time = table2array(train_data(:,3)); % Acceleration time
disp = table2array(train_data(:,4)); % disp
fuel = table2array(train_data(:,5)); % Engine fuel type
color = table2array(train_data(:,6));
```

Question 1

A) Summary statistics

```
y_mean = mean(y)
```

```
y_mean = 4.9220
```

```
y_median = median(y)
```

```
y_median = 4.4000
```

```
y_std = std(y)
```

```
y_std = 1.5292
```

```
y_iqr = iqr(y)
```

```
y_iqr = 1.6000
```

Present in table

```
headers = ["Mean"; "Median"; "Standard deviation"; "Interquartile Range"];
y_stats = table(y_mean, y_median, y_std, y_iqr)
```

```
y_stats = 1×4 table
```

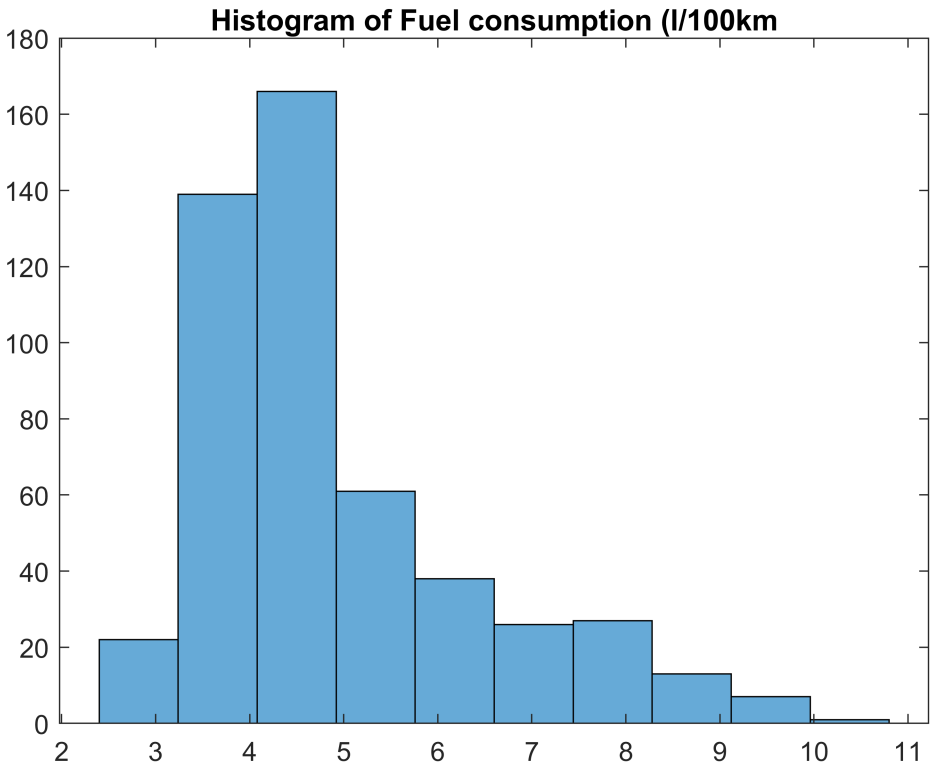
	y_mean	y_median	y_std	y_iqr
1	4.9220	4.4000	1.5292	1.6000

Plot histogram

```
y_hist = histogram(y,10)
```

```
y_hist =
  Histogram with properties:

            Data: [500×1 double]
          Values: [22 139 166 61 38 26 27 13 7 1]
         NumBins: 10
        BinEdges: [2.4000 3.2400 4.0800 4.9200 5.7600 6.6000 7.4400 8.2800 9.1200 9.9600 10.8000]
        BinWidth: 0.8400
       BinLimits: [2.4000 10.8000]
   Normalization: 'count'
       FaceColor: 'auto'
       EdgeColor: [0 0 0]

  Show all properties
```

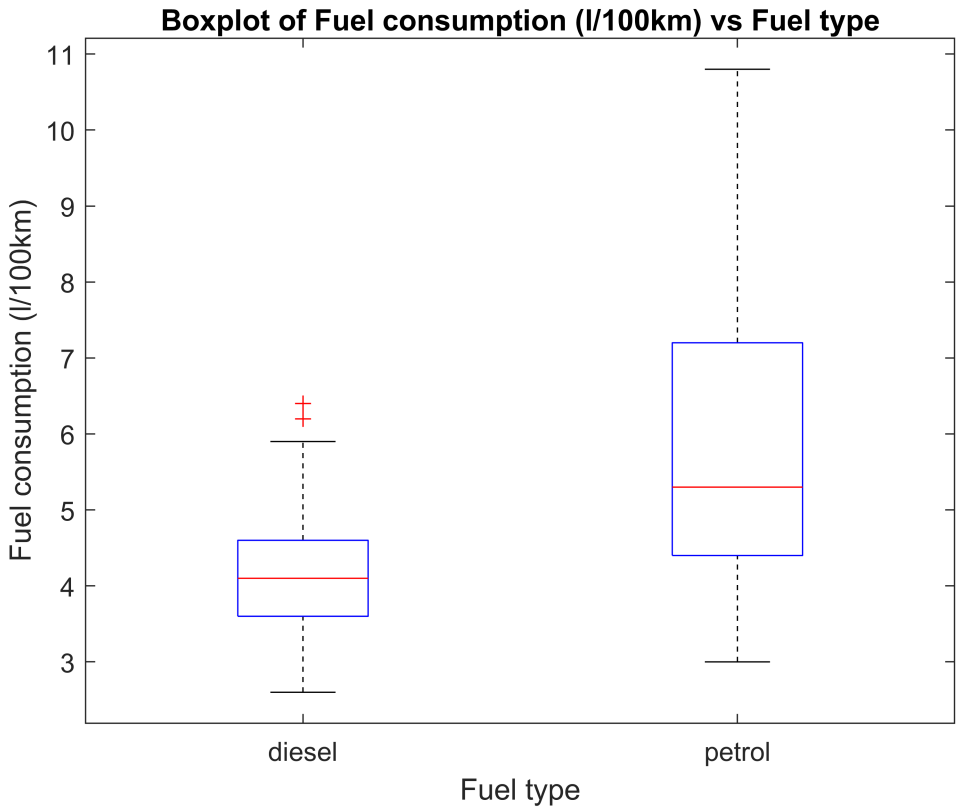
```
title("Histogram of Fuel consumption (l/100km")
```



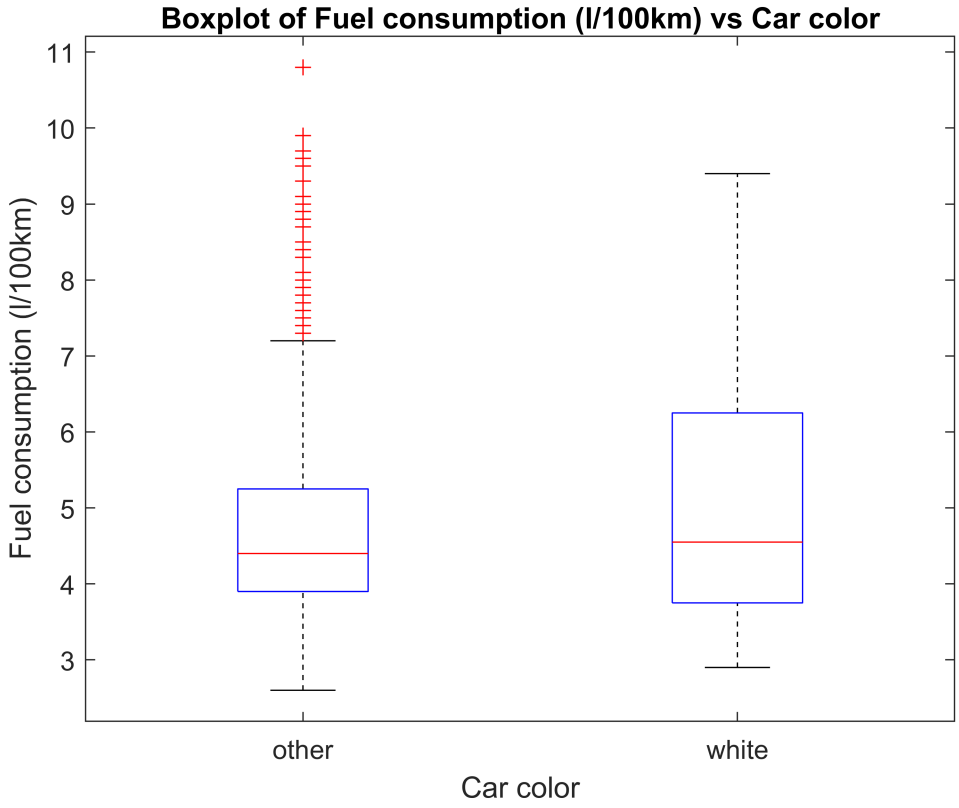
The data is not symmetric so it makes more sense to use the median to describe the data rather than the mean

B) i) Boxplots for categorical variables

```
boxplot(y,fuel)
title("Boxplot of Fuel consumption (l/100km) vs Fuel type")
xlabel("Fuel type")
ylabel("Fuel consumption (l/100km)")
```




```
boxplot(y,color)
title("Boxplot of Fuel consumption (l/100km) vs Car color")
xlabel("Car color")
ylabel("Fuel consumption (l/100km)")
```



The first boxplot makes clear that fuel type matters greatly for fuel efficiency. Diesel Cars tend to consume less fuel than petrol cars. So they should be included in our model

The second boxplot for color shows similar median values, but the cars not colored white show great variability and many observations fall outside of the boxplot range. Intuitively we know that a cars color should not matter for fuel consumption. The skewed distribution of the non-white cars is most likely explained by other parameters.

Before proceeding I'll recode catergorial variables so diesel = 1, petrol = 2

Encode categorical variable fuel

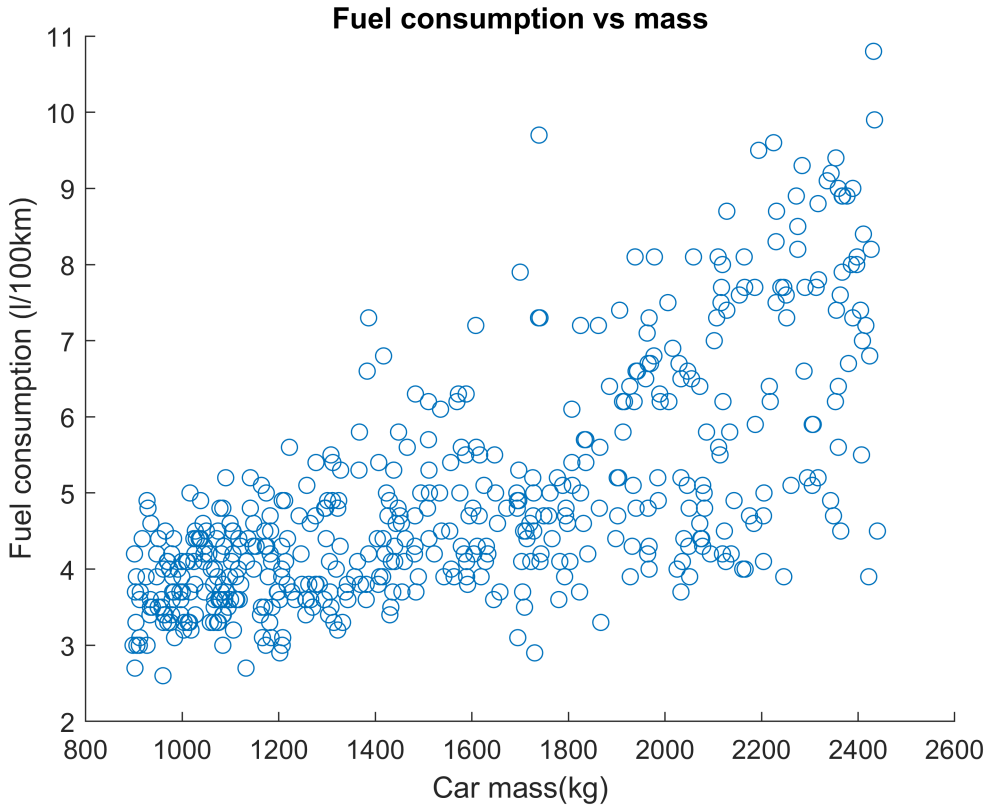
```
fuel = grp2idx(fuel)
```

```
fuel = 500×1
     1
     1
     1
     1
     1
     1
     1
     1
     1
     1
     ⋮
```

B)ii) Scatterplots of Continuous variables + correlation matrices

Mass

```
scatter(mass,y)
xlabel("Car mass(kg)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs mass")
```



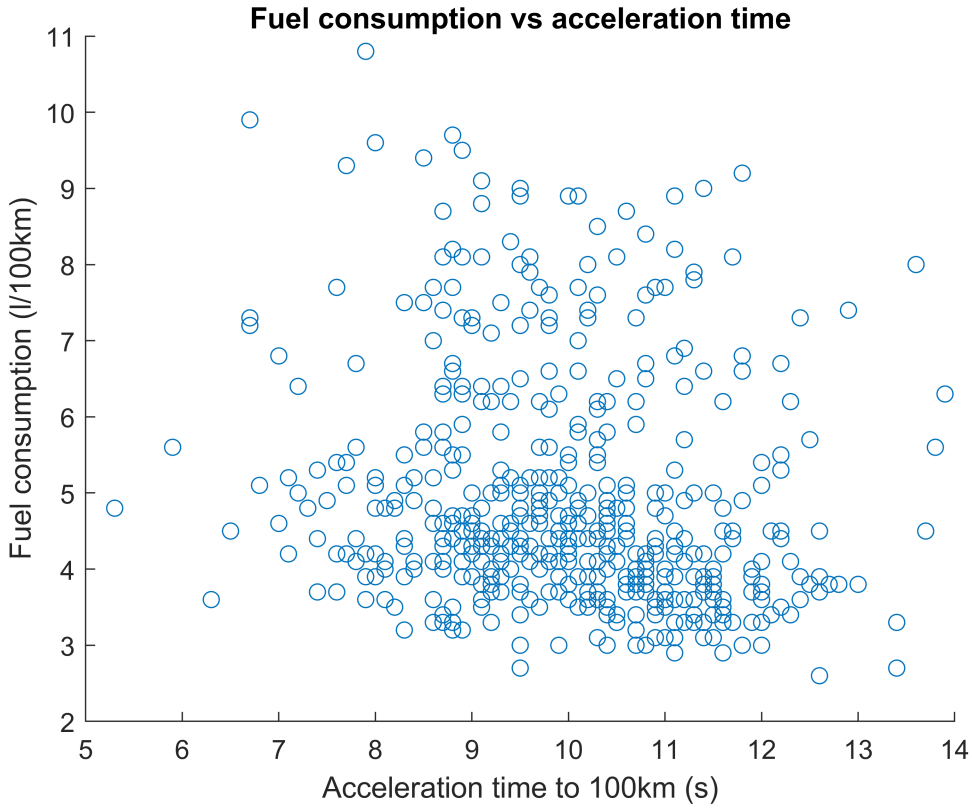

```
corrcoef(mass,y)
```

```
ans = 2×2
    1.0000    0.6986
    0.6986    1.0000
```

There is evidently a relationship between fuel consumption and mass, as would be expected. The relationship appears to be roughly linear, so we can use it in our model without any transformations. And the correlation coyicient is fairly high.

Acceleration time

```
scatter(time,y)
xlabel("Acceleration time to 100km (s)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs acceleration time")
```

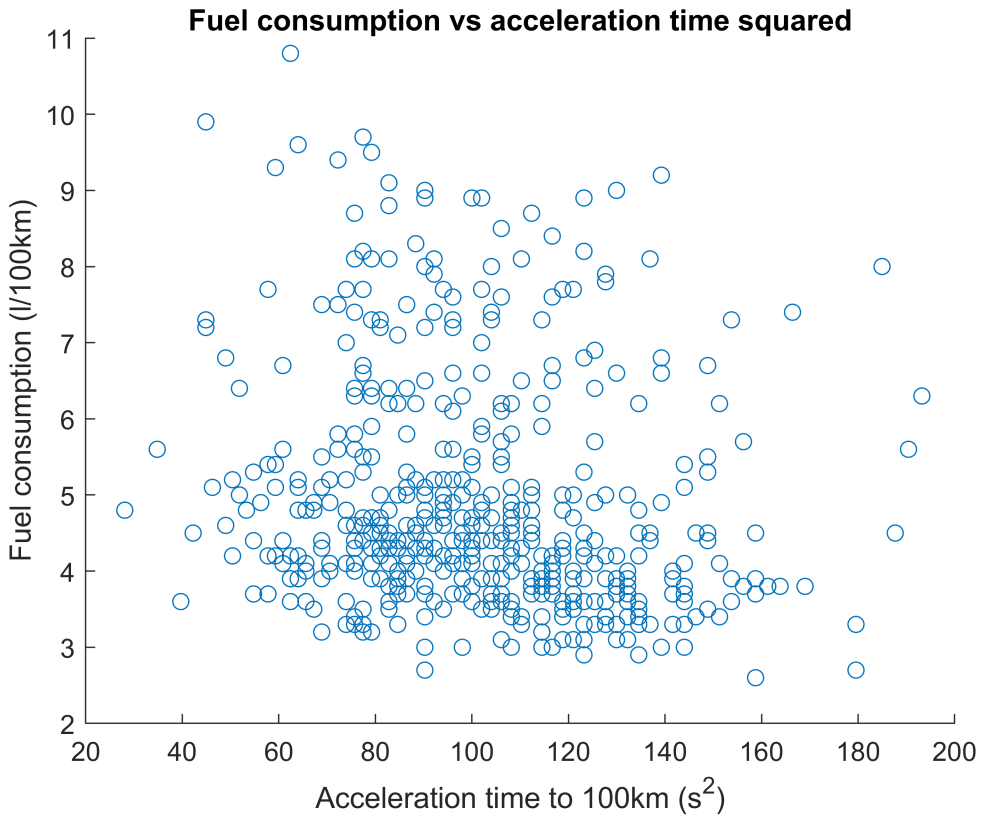



```
corrcoef(time,y)
```

```
ans = 2×2
    1.0000   -0.1780
   -0.1780    1.0000
```

We cant obviously see a linear relationship, so I will test some transformations

```
scatter(time.*time,y)
xlabel("Acceleration time to 100km (s^2)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs acceleration time squared")
```

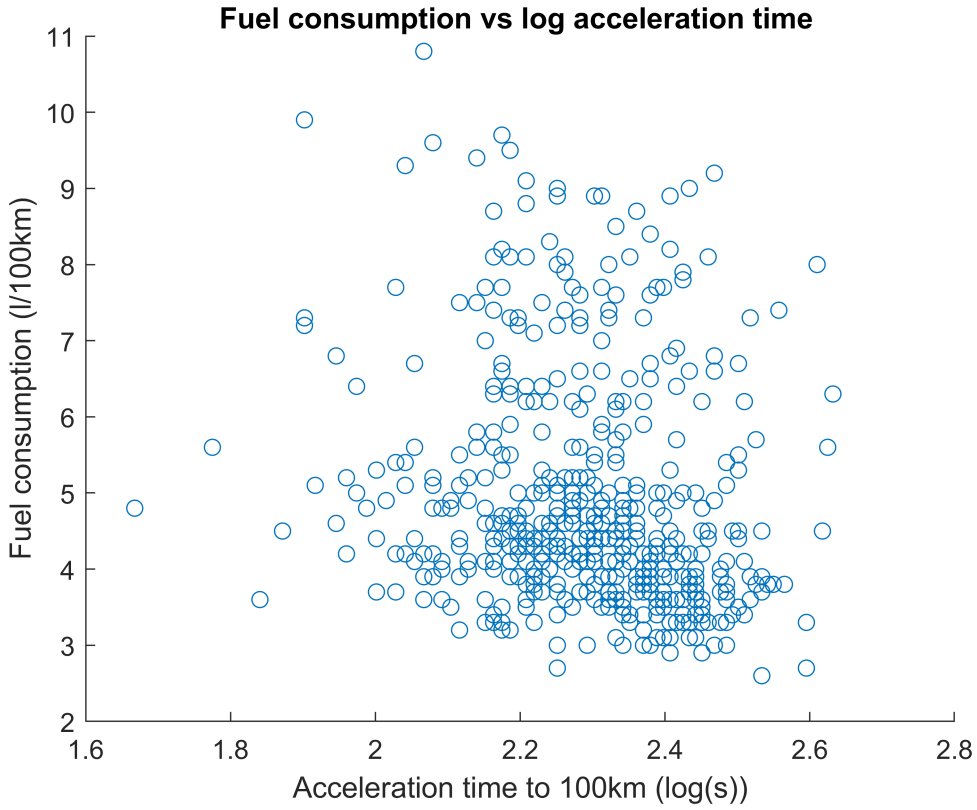



```
corrcoef(time.*time,y)
```

```
ans = 2×2
    1.0000   -0.1752
   -0.1752    1.0000
```

This does not appear to give a good relationship either

```
scatter(log(time),y)
xlabel("Acceleration time to 100km (log(s))")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs log acceleration time ")
```




```
corrcoef(log(time),y)
```

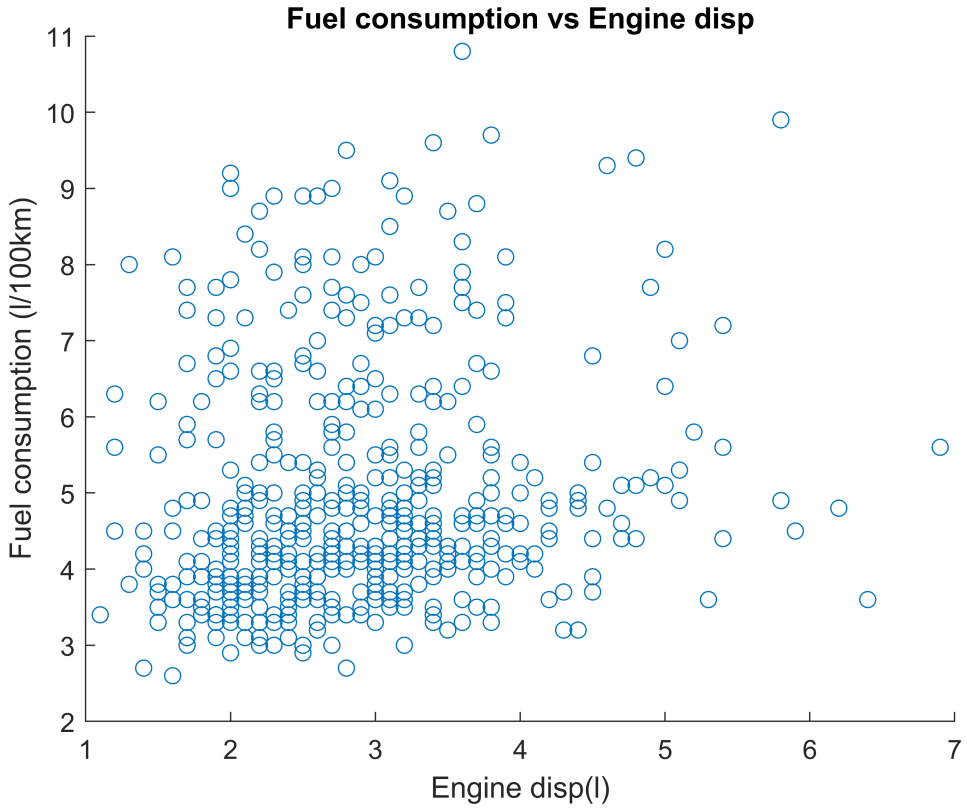
```
ans = 2×2
    1.0000   -0.1777
   -0.1777    1.0000
```

Overall fuel consumption tends to have a slighlty negative correlation with acceleration time. Perhaps the acceleration time can be coupled with another variable, say engine disp, to create a stronger predictor.

Overall the time to reach 100km/h does not appear to be a very useful predictor, because cars with a given acceleration show widely carying fuel consumption patterns. But I will still test its effect on performance in my models.

Engine displacement

```
scatter(disp,y)
xlabel("Engine disp(l)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs Engine disp")
```




```
corrcoef(disp,y)
```

```
ans = 2×2
    1.0000    0.1640
    0.1640    1.0000
```

There seems to be a correlation with engine disp, even if the data is very dispersed. I will therefore make use of it with my model

Q2) Modelling Simple linear regression with just mass

Start from simple models and work my way up by adding complexity

Fit a linear regression for fuel consumption vs mass

```
% Fit model and predict
model = fitlm(mass,y)
```

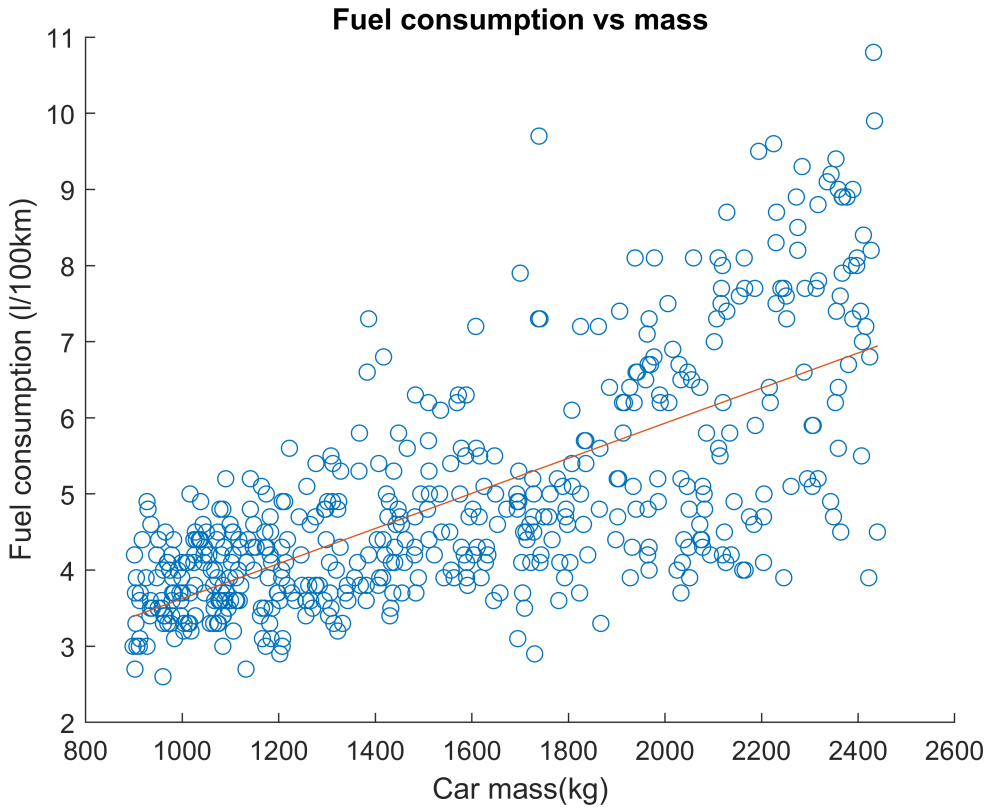
```
model =
Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate        SE         tStat       pValue
                   ________    __________    ______    __________

    (Intercept)       1.3215       0.17237    7.6668    9.3149e-14
    x1             0.0023041    0.00010576    21.786    2.0893e-74
```

```
Number of observations: 500, Error degrees of freedom: 498
Root Mean Squared Error: 1.1
R-squared: 0.488,  Adjusted R-Squared: 0.487
F-statistic vs. constant model: 475, p-value = 2.09e-74
```

```
pred = model.predict(mass)
```

```
pred = 500×1
    3.6188
    6.0450
    4.2363
    4.9575
    6.7201
    4.3307
    5.7777
    6.0473
    3.5335
    3.4736
      ⋮
```

```
% Plot
scatter(mass,y)
hold on
plot(mass,pred)
xlabel("Car mass(kg)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs mass")
hold off
```



Upon visual inspection the linear model with mass appears roughly in line with the data. However the model has an R2 value of only 0.48, meaning the change in mass can only explain 48% of the variability of the model, which means the linkage is weak. The root mean squared error of 1.1 is also quite significant, especially considering the interquartile range is 1.6.

In conclusion we need to make our model more complicated by adding more regression variables, while keeping mass in because it does explain some of the variablity.

B) Evaluate Performance Metrics

R^2 = 0.48

```
rsquared = model.Rsquared
```

```
rsquared = struct with fields:
    Ordinary: 0.4880
    Adjusted: 0.4870
```

This gives us an indication of what % of the varibility in y can be explained by changes in x. As R <0.5 there appears to be a weak linkage with just mass itself

MSE=1.1997

```
mse = model.MSE
```

```
mse = 1.1997
```

This tells us the average of the squares of error, by the predicions made by the regresison model. An MSE of roughly 1.2, and a RMSE of 1.1 suggests the model isn't performing very well, because our data for fuel yiciency is mostly in the single digits, with an iqr of 1.6 and mean and median of 4.92 and 4.4. A 1.1 RMSE error is quite signficiant given these summary statistics

AIC=1512

```
aic = model.ModelCriterion
```

```
aic = struct with fields:
     AIC: 1.5120e+03
    AICc: 1.5120e+03
     BIC: 1.5204e+03
    CAIC: 1.5224e+03
```

The Akaike information criterion is a mathematical method that evaluates how well a model fits the training data. AIC takes into account the number of independent varaibles used to build the model, and the maximum likelihood estimate of the model, so it allows us to pick the model taht explains the greatest amount of variation with the fewest independent varaible. Lower AIC scores are better

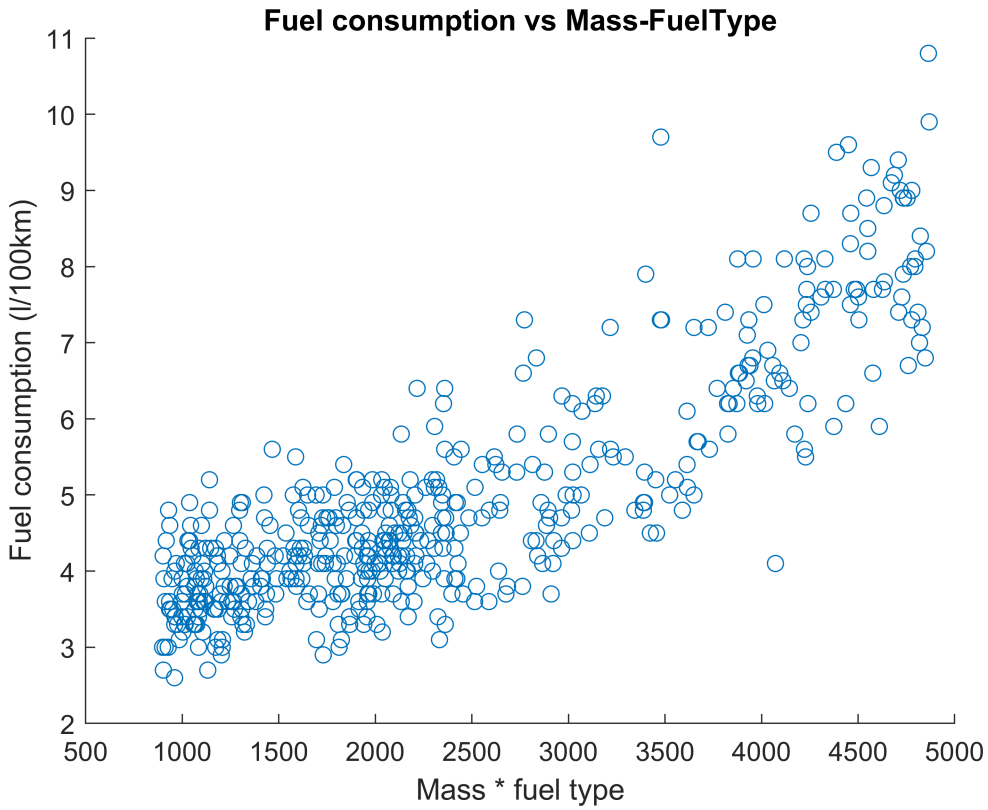
Overall, we should aim to maximise R2 and minimise MSE, but consider the change in aic as well when comparing models with similar R2 and MSE scores.

C) Test different models

- **First plot some scatter plots of possible interaction terms I can use**

Mass * fuel

```
scatter(mass.*fuel,y)
xlabel("Mass * fuel type")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs Mass-FuelType")
```



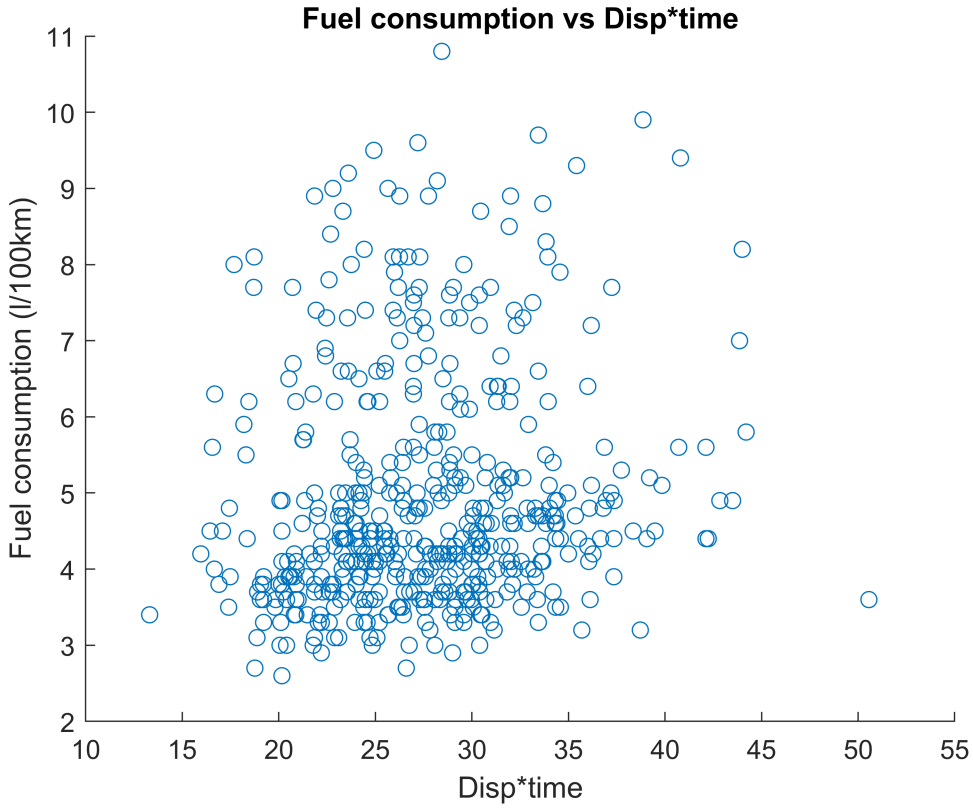

```
corrcoef(mass.*fuel,y)
```

```
ans = 2×2
    1.0000    0.8522
    0.8522    1.0000
```

This interaction term shows very strong correlation with fuel consumption

disp * time

```
scatter(disp.*time,y)
xlabel("Disp*time")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs Disp*time")
```



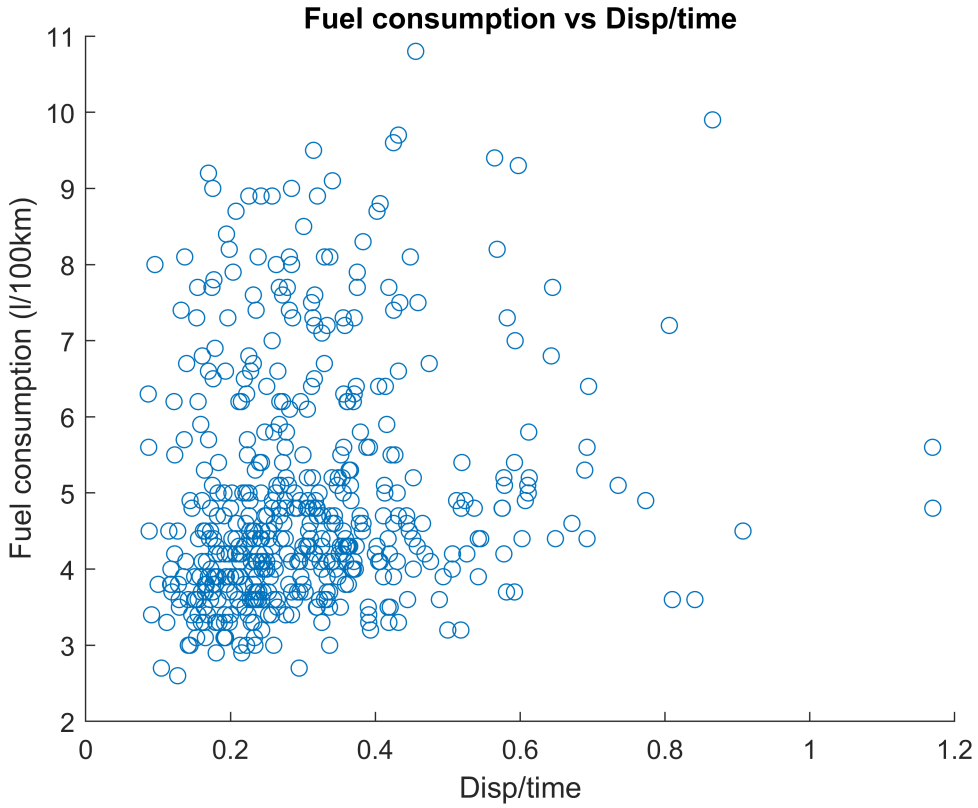

```
corrcoef(disp.*time,y)
```

```
ans = 2×2
    1.0000    0.1312
    0.1312    1.0000
```

Very weak correlation

disp/time

```
scatter(disp./time,y)
xlabel("Disp/time")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs Disp/time")
```



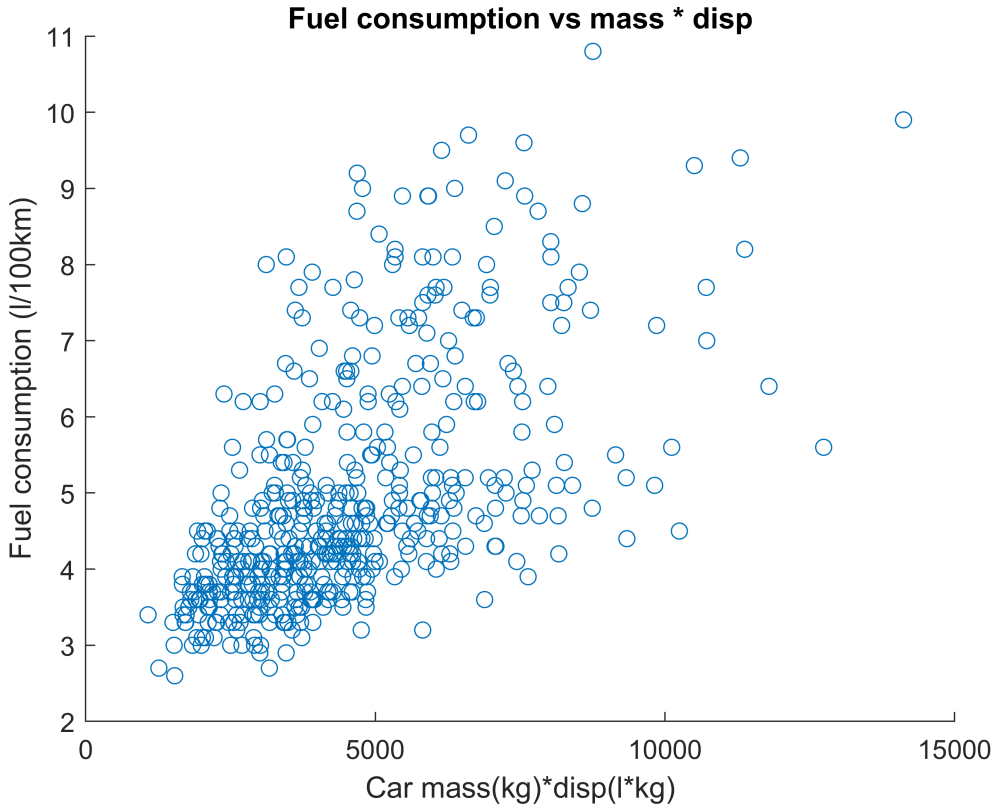

```
corrcoef(disp./time,y)
```

```
ans = 2×2
    1.0000    0.1643
    0.1643    1.0000
```

This shows a slightly higher correlation than disp*time100 so I'll prefer this one over that.

Mass*disp

```
scatter(mass.*disp,y)
xlabel("Car mass(kg)*disp(l*kg)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs mass * disp")
```



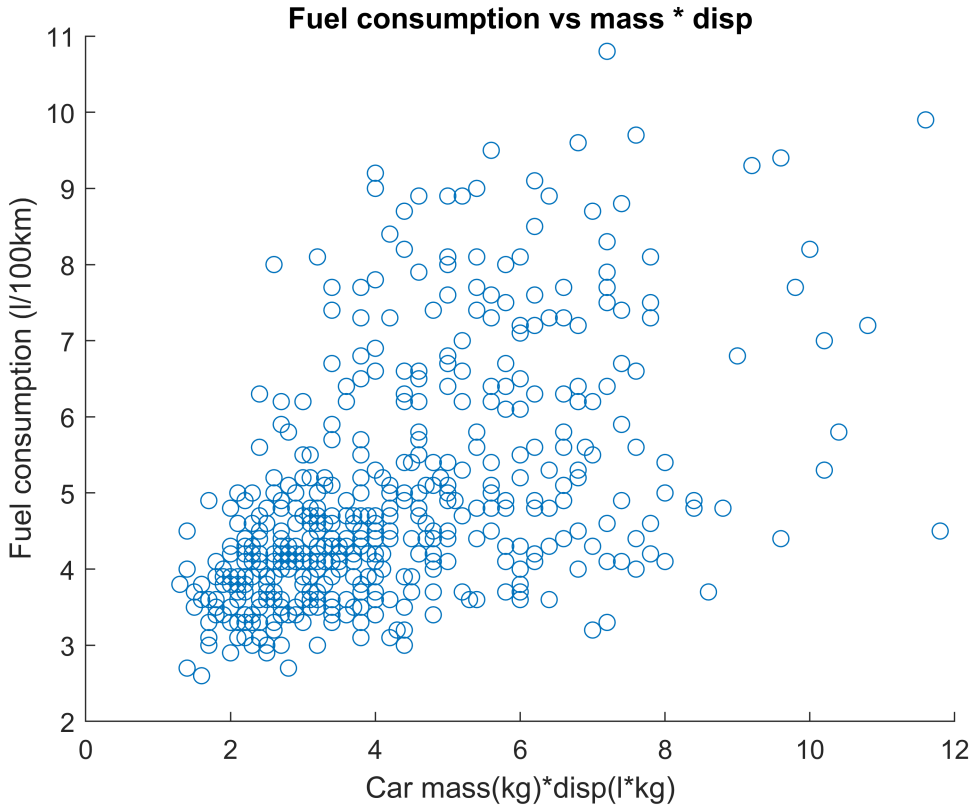

```
corrcoef(mass.*disp,y)
```

```
ans = 2×2
    1.0000    0.5863
    0.5863    1.0000
```

A visible correlation, though we'll see if it adds any useful info if we already have mass and disp in the model

Fuel*disp

```
scatter(fuel.*disp,y)
xlabel("Car mass(kg)*disp(l*kg)")
ylabel("Fuel consumption (l/100km)")
title("Fuel consumption vs mass * disp")
```




```
corrcoef(mass.*disp,y)
```

```
ans = 2×2
    1.0000    0.5863
    0.5863    1.0000
```

I could go on to try countless other combinations and parametric forms but the above seem good enough to work with.

Create table to populate with summary stats for the models I wish to test

```
sz = [10 5];
varTypes = ["double","string", "double","double","double"];
varNames = ["Model#", "Terms", "R^2", "MSE", "AIC"];
model_summaries = table('Size',sz,'VariableTypes',varTypes,'VariableNames',varNames)
```

```
model_summaries = 10×5 table
```

	Model#	Terms	R^2	MSE	AIC
1	0	<missing>	0	0	0
2	0	<missing>	0	0	0
3	0	<missing>	0	0	0

	Model#	Terms	R^2	MSE	AIC
4	0	<missing>	0	0	0
5	0	<missing>	0	0	0
6	0	<missing>	0	0	0
7	0	<missing>	0	0	0
8	0	<missing>	0	0	0
9	0	<missing>	0	0	0
10	0	<missing>	0	0	0

For all below models I first create the x dataset, fit the model, plot it, then add summary stats to the comparison table.

Model 1) Mass + fuel

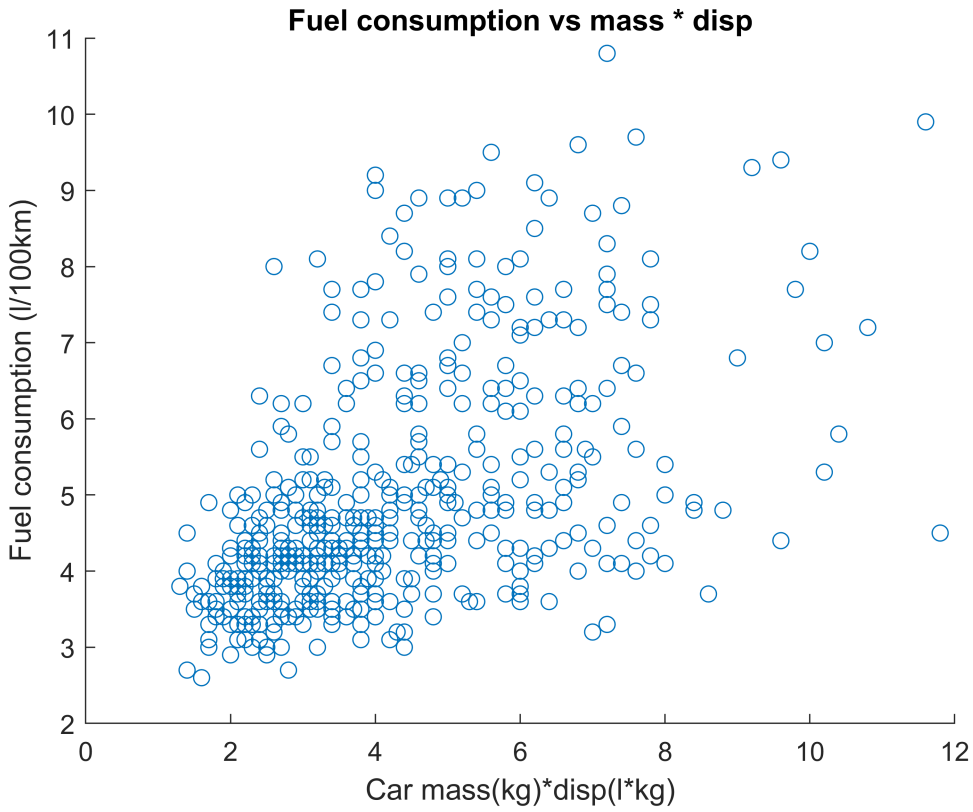
```
n = 1
```

```
n = 1
```

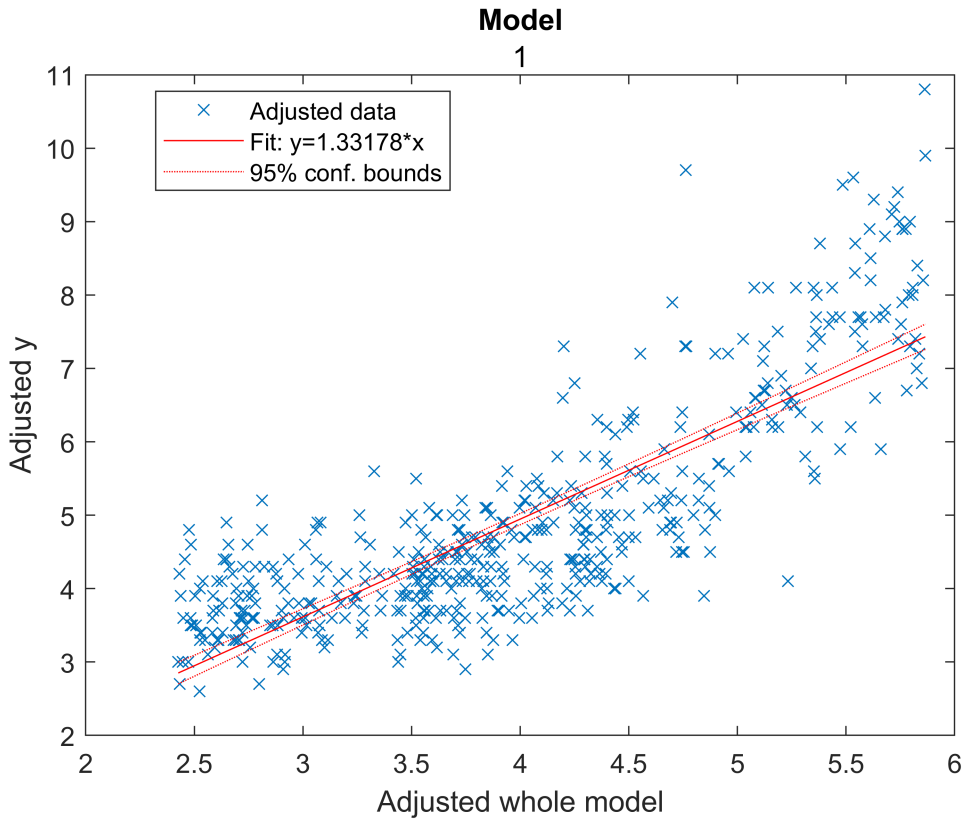
```
x = [mass,fuel];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                   ________    __________    _______    __________

    (Intercept)     -0.38046        0.1705    -2.2314      0.026098
    x1             0.0021149    8.5127e-05     24.844    3.5594e-89
    x2                1.3318      0.078852      16.89    6.5676e-51


Number of observations: 500, Error degrees of freedom: 497
Root Mean Squared Error: 0.874
R-squared: 0.675,  Adjusted R-Squared: 0.673
F-statistic vs. constant model: 515, p-value = 6.36e-122
```

```
hold off
```




```
plot(model)
title("Model",n)
```



```
model_summaries(n,:) = {n, "mass,fuel",model.Rsquared.Ordinary, model.MSE, model.ModelCriterion
```

Shows quite a bit of improvement from using just mass

Model 2) Mass, engine disp, and fuel type

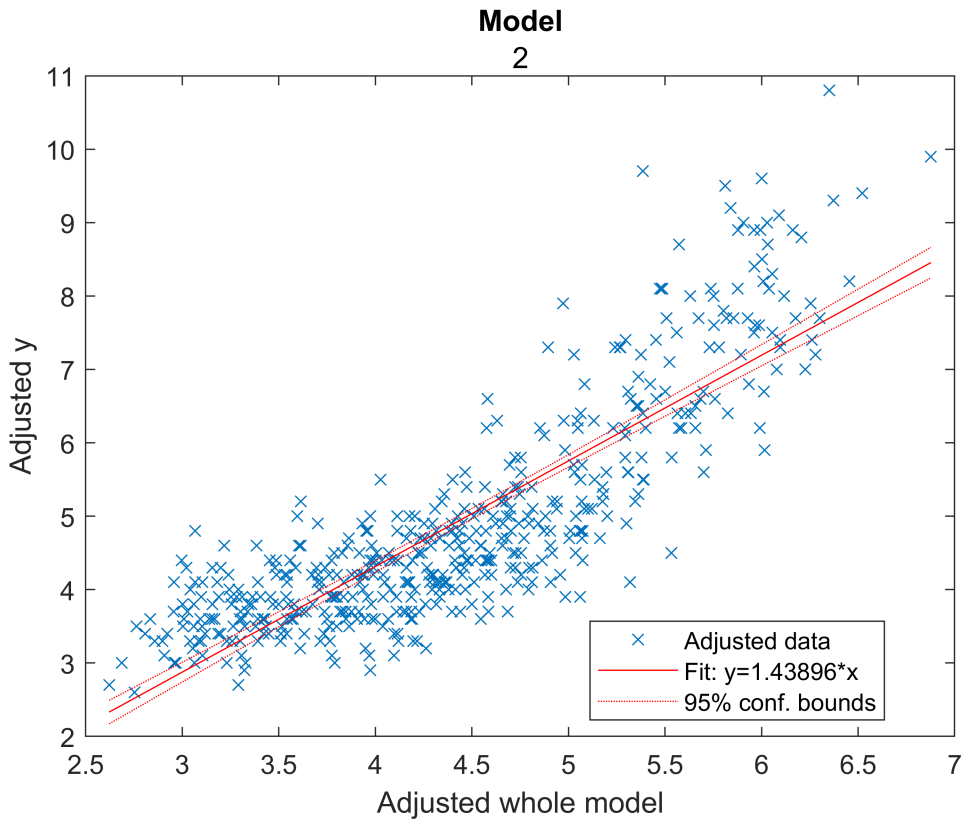
```
n = 2
```

```
n = 2
```

```
x = [mass,disp, fuel];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                   ________    __________    _______    __________

    (Intercept)      -1.4409       0.20175    -7.1419    3.2995e-12
    x1             0.0021006    7.9551e-05     26.405      1.32e-96
    x2               0.34271      0.040012     8.5651     1.369e-16
    x3                1.3975       0.07407     18.868    2.9677e-60


Number of observations: 500, Error degrees of freedom: 496
Root Mean Squared Error: 0.816
R-squared: 0.717,  Adjusted R-Squared: 0.715
F-statistic vs. constant model: 418, p-value = 2.34e-135
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel",model.Rsquared.Ordinary, model.MSE, model.ModelCrit
```

Noticeable improvement over Model 1

Model 3) Mass, engine disp, fuel, and time100

```
n = 3
```

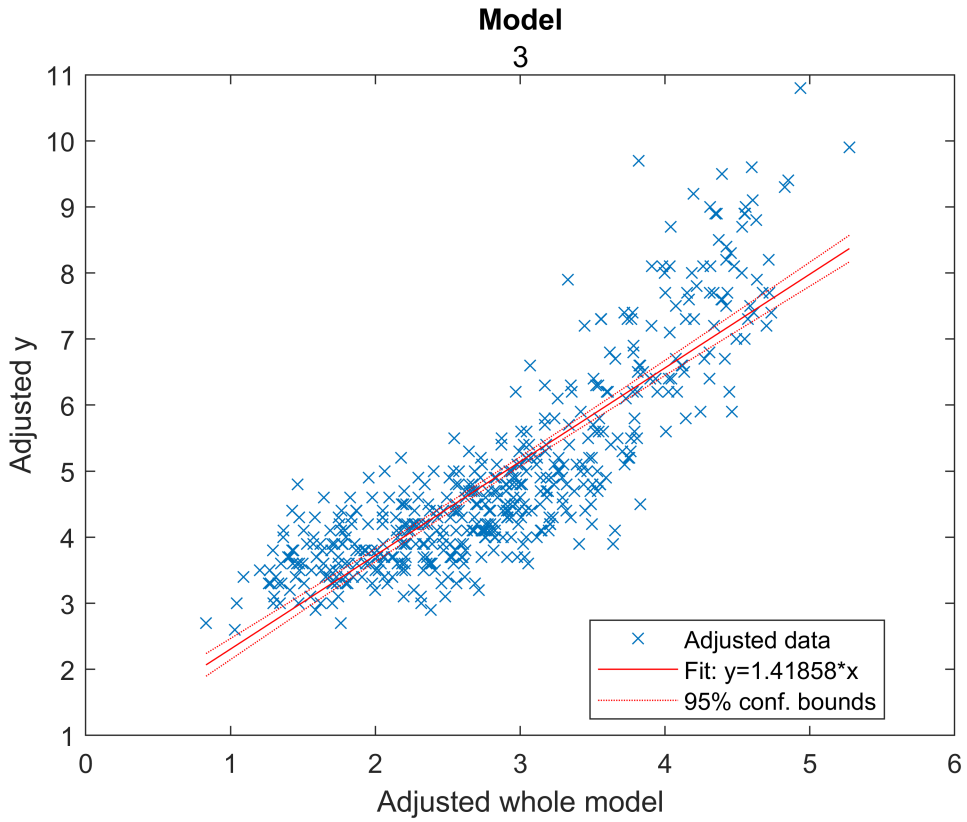
```
n = 3
```

```
x = [mass, disp, fuel,time];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                   Estimate        SE         tStat       pValue
                   ________    __________    _______    __________

    (Intercept)      0.88901       0.73379     1.2115       0.22627
    x1             0.0020982    7.8773e-05     26.636    1.2176e-97
    x2               0.12279      0.077532     1.5838       0.11389
    x3                1.4029       0.07336     19.123    1.8965e-61
    x4              -0.17085      0.051775    -3.2998     0.0010373


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.808
```

```
R-squared: 0.723,  Adjusted R-Squared: 0.72
F-statistic vs. constant model: 323, p-value = 2.4e-136
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,time",model.Rsquared.Ordinary, model.MSE, model.Mode
```

Again we observe an improvement in all metrics, although somewhat slightly

Model 4) Mass, engine disp, fuel, and mass*fuel

```
n = 4
```

```
n = 4
```

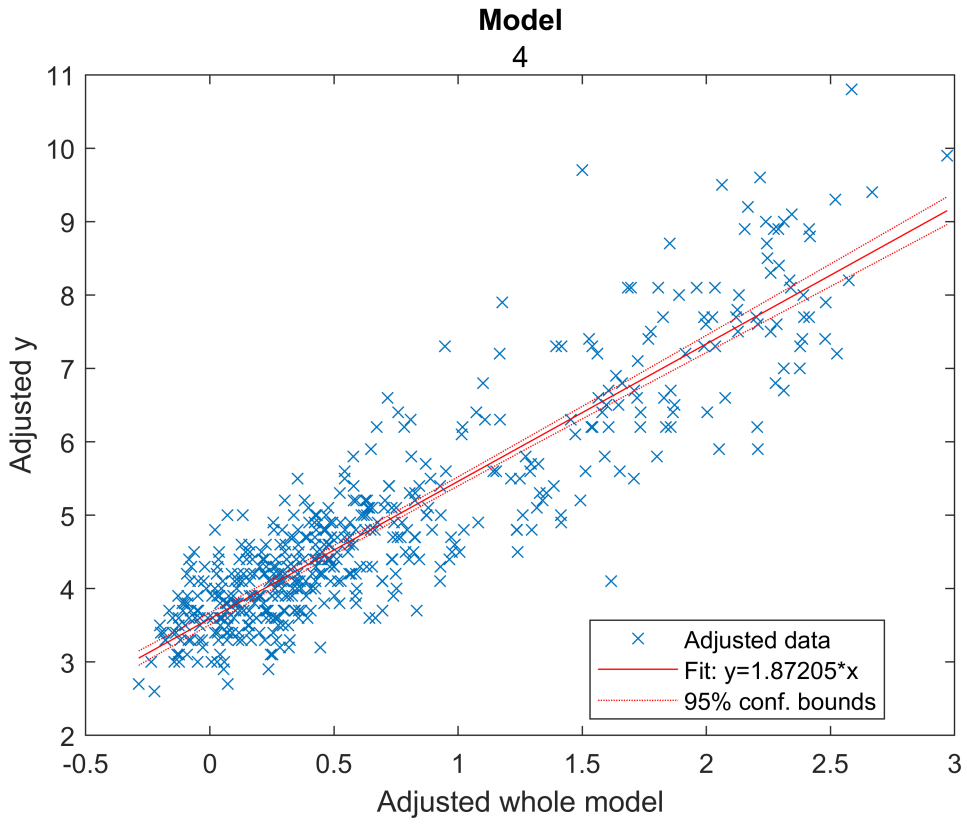
```
mass_fuel_inter = mass.*fuel;
x = [mass, disp, fuel ,mass_fuel_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                   Estimate        SE        tStat       pValue
                   ________    _________    _______    _________
```

```
(Intercept)        3.5894       0.35489      10.114     5.5041e-22
x1              -0.0011346    0.00021249    -5.3398     1.4203e-07
x2                 0.32393      0.032549     9.9521      2.156e-21
x3                 -1.8438       0.21154    -8.7161     4.3625e-17
x4                0.002081    0.00013019     15.984     1.1333e-46
```

```
Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.664
R-squared: 0.813,  Adjusted R-Squared: 0.812
F-statistic vs. constant model: 538, p-value = 1.09e-178
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,mass*fuel",model.Rsquared.Ordinary, model.MSE, model
```

We can observe a very significant improvement in all metrics now. I'll now test it with the time parameter to see if that adds improvement

Model 5) Mass, engine disp, fuel,time, and mass*fuel

```
n = 5
```

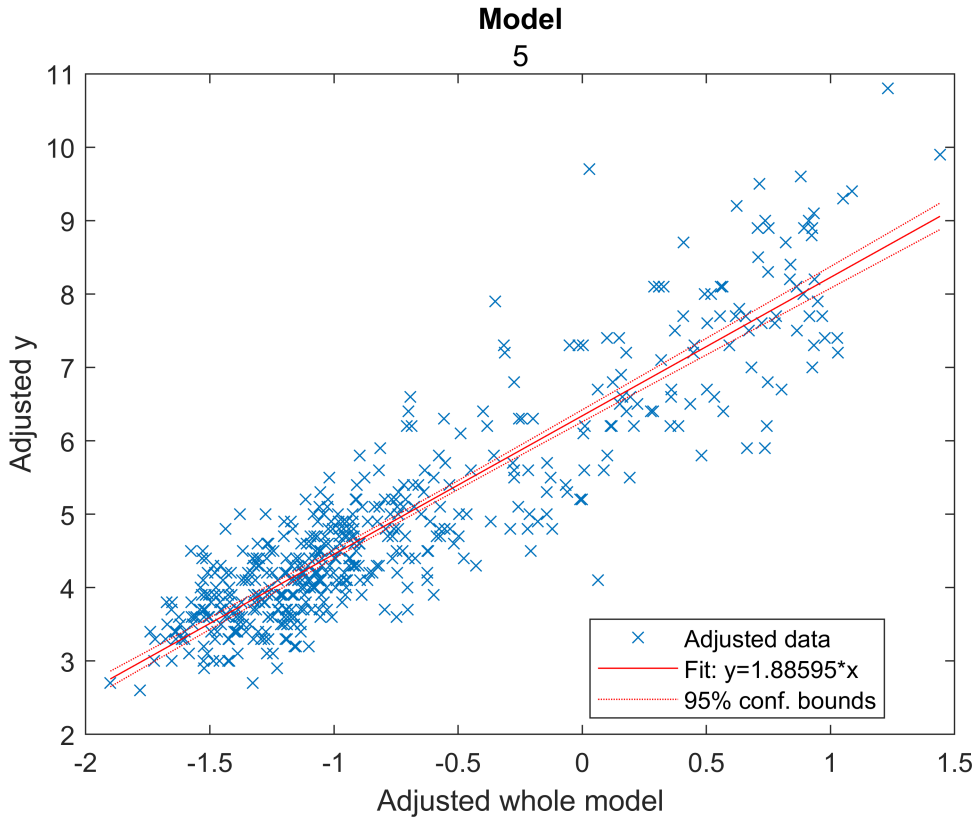
```
n = 5
```

```
mass_fuel_inter = mass.*fuel;
```

```
x = [mass, disp, fuel,time, mass_fuel_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                   Estimate         SE          tStat       pValue
                   ________    __________    _______    __________

    (Intercept)       6.3406       0.67603     9.3791    2.4031e-19
    x1             -0.001174    0.00020818    -5.6391    2.8791e-08
    x2              0.069416        0.0624     1.1124       0.26649
    x3               -1.8743       0.20719    -9.0462    3.3918e-18
    x4              -0.19757      0.041646    -4.7439    2.7477e-06
    x5             0.0021045    0.00012755       16.5    4.9118e-49


Number of observations: 500, Error degrees of freedom: 494
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 454, p-value = 4.55e-182
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,time,mass*fuel",model.Rsquared.Ordinary, model.MSE,
```

We see a slight improvement now as well. Let's try adding some more interaction terms to this model

Model 6) Mass, engine disp, fuel, time, mass*fuel, fuel*disp

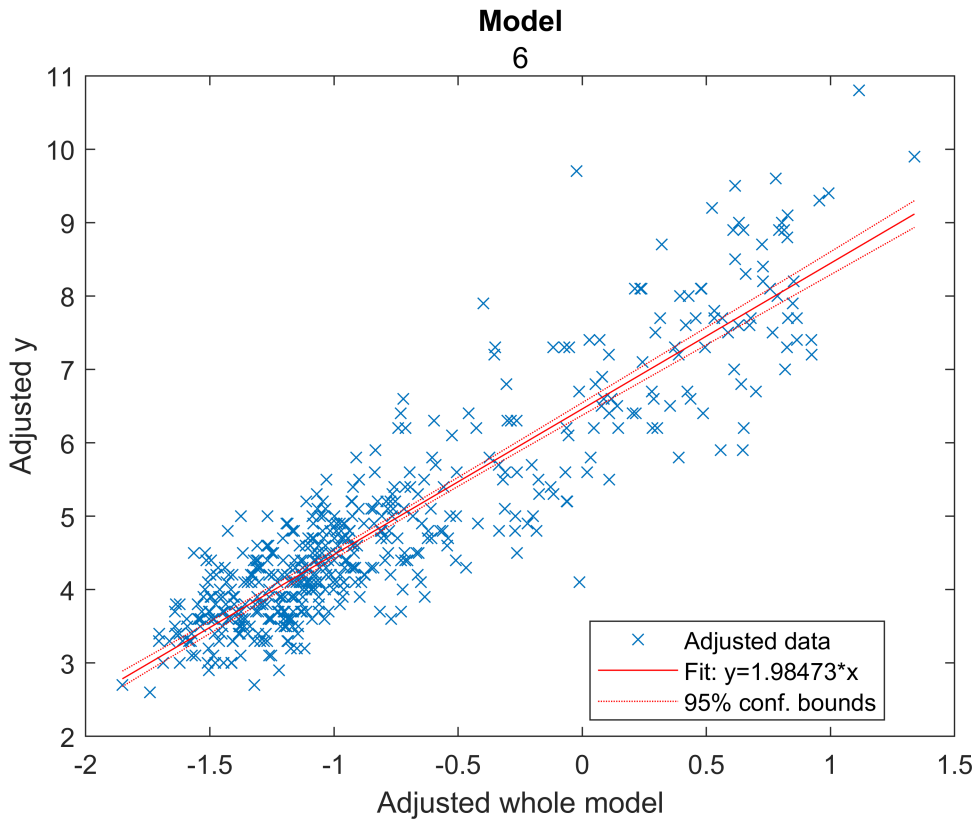
```
n = 6
```

```
n = 6
```

```
mass_fuel_inter = mass.*fuel;
fuel_disp_inter = fuel.*disp;
x = [mass, disp, fuel,time, mass_fuel_inter,fuel_disp_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:
                    Estimate         SE         tStat       pValue
                   __________    __________    _______    __________

    (Intercept)       6.4611       0.70992     9.1012    2.2145e-18
    x1             -0.0011727    0.00020834    -5.6288    3.0487e-08
    x2              0.019801       0.10838    0.18271        0.8551
    x3               -1.9747       0.27403    -7.2059    2.1753e-12
    x4              -0.19546      0.041845    -4.6709    3.8723e-06
    x5             0.0021028    0.00012767      16.47    7.0331e-49
    x6              0.036029      0.064323    0.56013       0.57565


Number of observations: 500, Error degrees of freedom: 493
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 378, p-value = 8.73e-181
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,time,mass*fuel,fuel*disp",model.Rsquared.Ordinary, r
```

Accuracy is barely changed from model5, and there is a small uptick in AIC. Likely too complicated of a model here.

Model 7) Mass, engine disp, fuel, time, mass*fuel, disp/time

```
n = 7
```

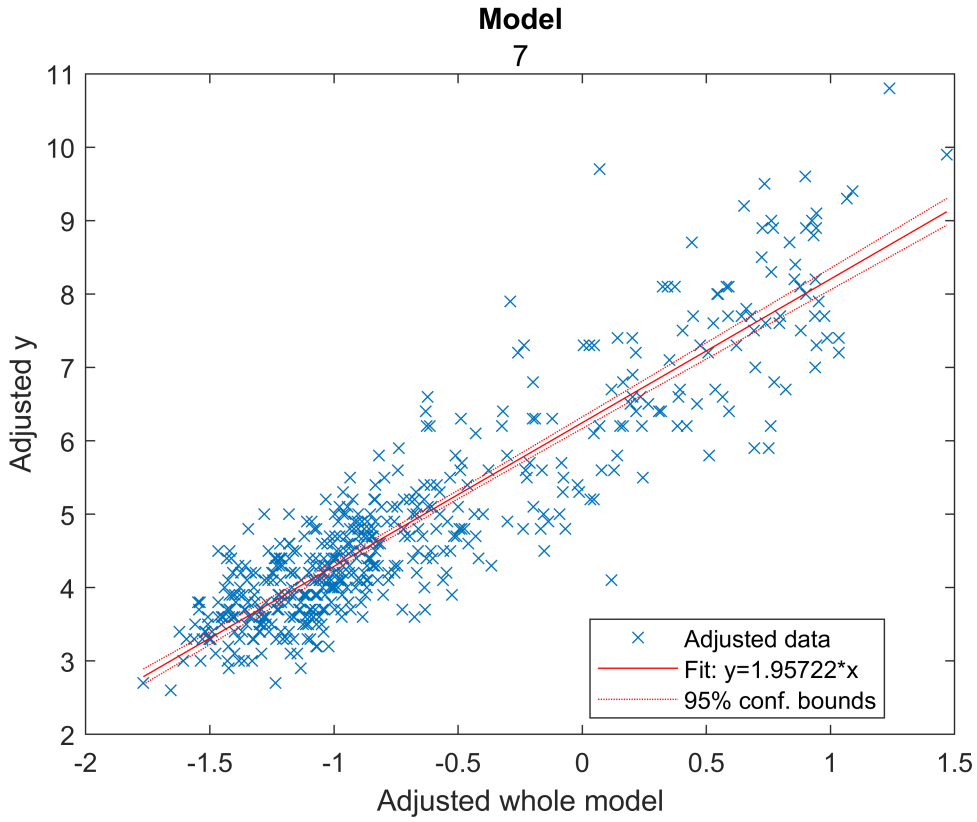
```
n = 7
```

```
mass_fuel_inter = mass.*fuel;
disp_time_inter = disp./time;
x = [mass, disp, fuel,time, mass_fuel_inter,disp_time_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5 + x6

Estimated Coefficients:
                   Estimate        SE          tStat        pValue
                   ________    _________    _________    _________

    (Intercept)       6.2465      0.69519       8.9854    5.4927e-18
    x1             -0.0011651    0.00020887    -5.5781    4.0158e-08
    x2              -0.002423       0.13743    -0.017631       0.98594
```

```
x3               -1.8677       0.20763       -8.9956     5.0748e-18
x4              -0.18584      0.046215       -4.0212     6.6965e-05
x5             0.0021002    0.00012784        16.428     1.0938e-48
x6               0.55476       0.94538       0.58681         0.5576


Number of observations: 500, Error degrees of freedom: 493
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 378, p-value = 8.6e-181
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,time,mass*fuel,disp/time",model.Rsquared.Ordinary, r
```

Pretty much the same metrics as model 6.

It seems making the model more complicated doesnt help much so let's try cutting down.

Model 8) Mass, engine disp, fuel, mass*fuel, disp/time

```
n = 8
```

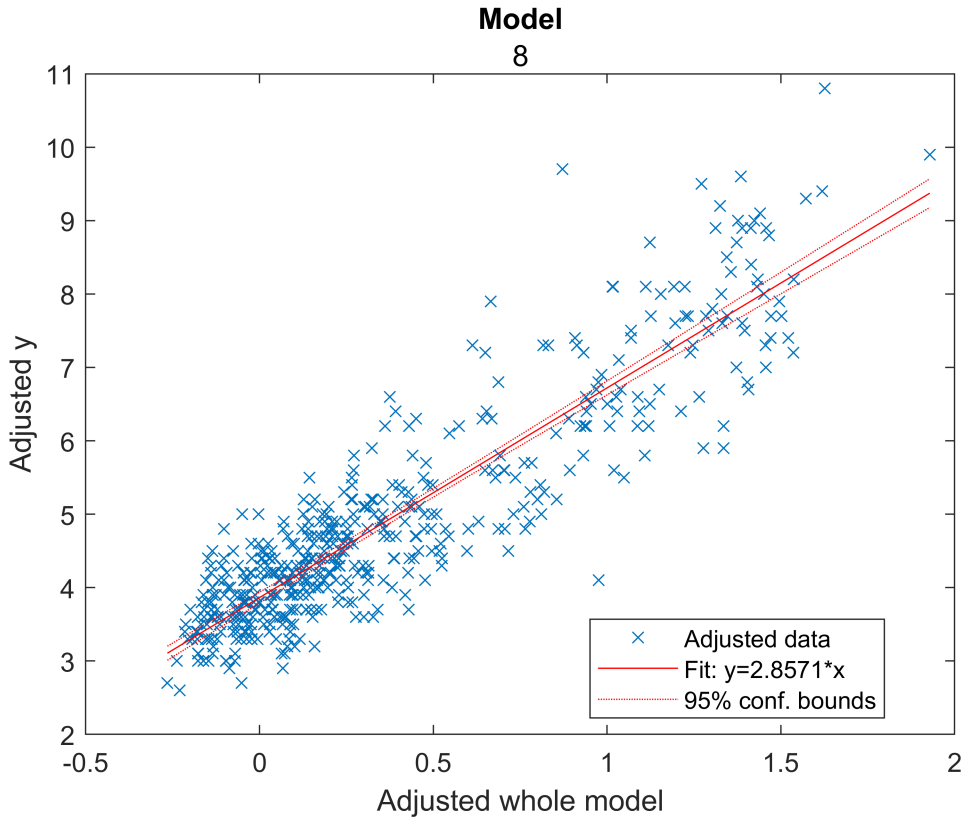
```
n = 8
```

```
mass_fuel_inter = mass.*fuel;
disp_time_inter = disp./time;
```

```
x = [mass, disp, fuel, mass_fuel_inter,disp_time_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                    Estimate        SE          tStat       pValue
                   __________    __________    ________    __________

    (Intercept)       3.8638        0.36911      10.468    2.6985e-23
    x1             -0.0011087    0.00021157    -5.2402    2.3799e-07
    x2              -0.020565       0.13945    -0.14748       0.88281
    x3                 -1.825       0.21052    -8.6692    6.2756e-17
    x4              0.0020696    0.00012956     15.974    1.3066e-46
    x5                 2.1982       0.86547     2.5399      0.011395


Number of observations: 500, Error degrees of freedom: 494
Root Mean Squared Error: 0.66
R-squared: 0.815,  Adjusted R-Squared: 0.814
F-statistic vs. constant model: 437, p-value = 1.1e-178
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,disp,fuel,mass*fuel,disp/time",model.Rsquared.Ordinary, model.
```

Comparing with model#5, it doesn't seem disp/time is a better substitute then time, as all metrics show degradation.

Model 9) Engine disp, time, mass*fuel

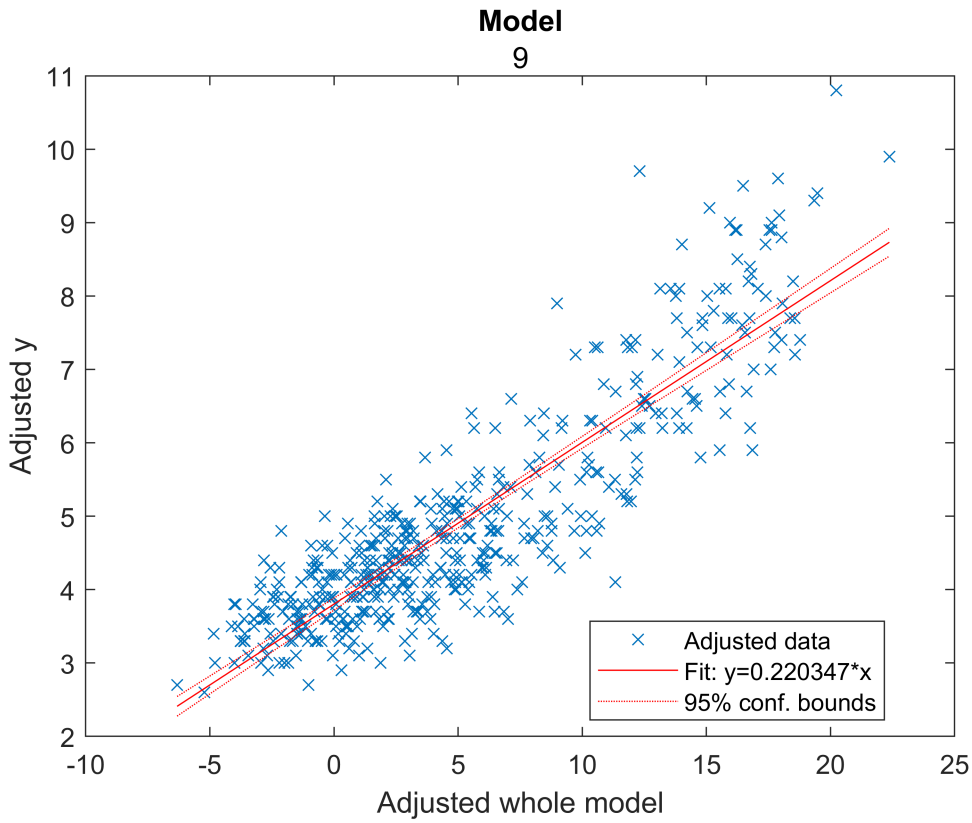
```
n = 9
```

```
n = 9
```

```
mass_fuel_inter = mass.*fuel;
x = [disp, time, mass_fuel_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                   ________    __________    _______    ___________

    (Intercept)       3.8027       0.64219     5.9215     5.9652e-09
    x1               0.10715       0.06906     1.5515        0.12141
    x2              -0.19254      0.046136    -4.1733     3.5458e-05
    x3             0.0011499    2.8059e-05     40.982    2.3559e-161


Number of observations: 500, Error degrees of freedom: 496
Root Mean Squared Error: 0.72
R-squared: 0.779,  Adjusted R-Squared: 0.778
F-statistic vs. constant model: 584, p-value = 2.76e-162
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "disp,time,mass*fuel",model.Rsquared.Ordinary, model.MSE, model.Mode
```

Comparing with model#5, it seems that mass*fuel term is not a substitute for mass and fuel separately. Accuracies decrease and AIC increases

Model 10) Mass,fuel, disp,time, + 5 interaction terms generated by matlab

Try something really complex

```
n = 10
```

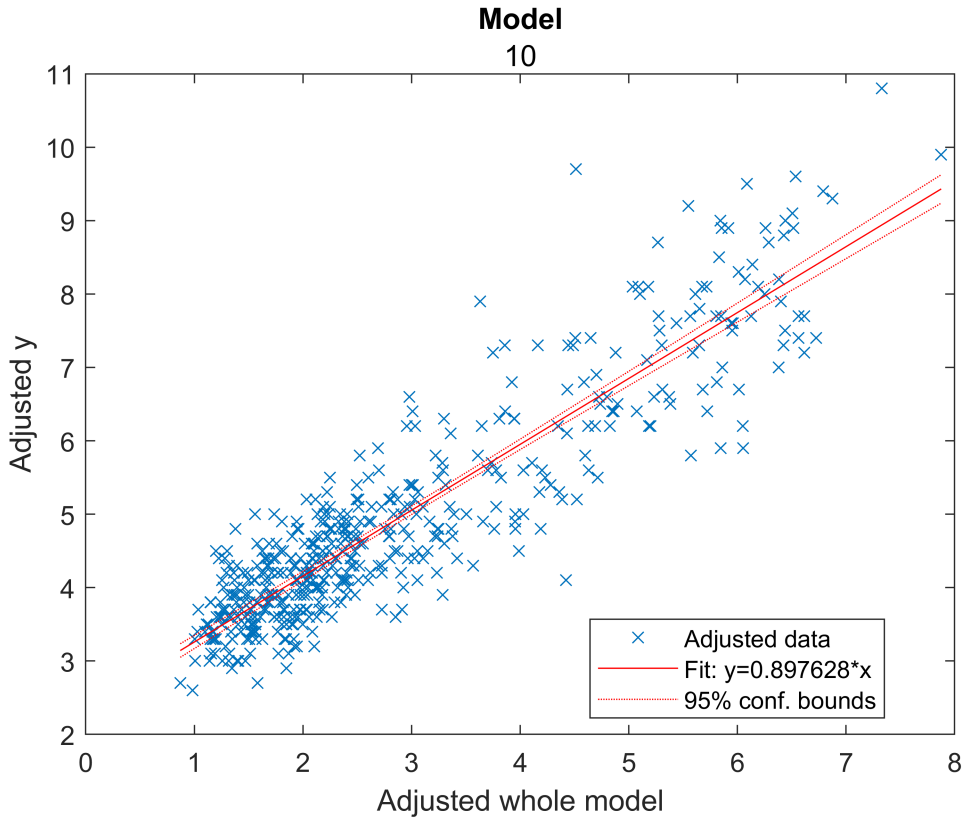
```
n = 10
```

```
x = [mass, fuel, disp,time];
model = fitlm(x,y,"interactions")
```

```
model =
Linear regression model:
    y ~ 1 + x1*x2 + x1*x3 + x1*x4 + x2*x3 + x2*x4 + x3*x4

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                   ________    _________    ________    __________

    (Intercept)       2.3613        2.542      0.9289        0.3534
    x1             0.00012595    0.0013196    0.095443         0.924
    x2               -0.77876       1.2058    -0.64583       0.51869
    x3                0.39814      0.30263      1.3156       0.18893
    x4                0.15624       0.1866     0.83733       0.40282
    x1:x2           0.0021236    0.00012848      16.529    4.4529e-49
```

```
x1:x3          -3.5794e-05    0.00013697    -0.26132       0.79396
x1:x4          -0.00012263    9.4416e-05     -1.2989        0.1946
x2:x3            -0.092879       0.12847    -0.72297       0.47004
x2:x4            -0.086319      0.086015     -1.0035        0.3161
x3:x4            -0.016305      0.018971    -0.85948        0.3905


Number of observations: 500, Error degrees of freedom: 489
Root Mean Squared Error: 0.648
R-squared: 0.824,  Adjusted R-Squared: 0.82
F-statistic vs. constant model: 229, p-value = 2.91e-177
```

```
plot(model)
title("Model",n)
```




```
model_summaries(n,:) = {n, "mass,fuel,diso,time + 5 interactions",model.Rsquared.Ordinary, mode
```

This is the most accurate model we got, but is really complex, and its AIC is higher than 5

Inspect Model Summaries

```
model_summaries
```

```
model_summaries = 10×5 table
```

	Model#	Terms	R^2	MSE	AIC
1	1	"mass,fuel"	0.6747	0.7637	1.2872e+03
2	2	"mass,disp,fuel"	0.7166	0.6667	1.2202e+03

	Model#	Terms	R^2	MSE	AIC
3	3	"mass,disp,fuel,time"	0.7227	0.6536	1.2113e+03
4	4	"mass,disp,fuel,mass*fuel"	0.8131	0.4406	1.0141e+03
5	5	"mass,disp,fuel,time,mass*fuel"	0.8212	0.4223	993.8367
6	6	"mass,disp,fuel,time,mass*fuel,fuel*disp"	0.8213	0.4228	995.5186
7	7	"mass,disp,fuel,time,mass*fuel,disp/time"	0.8214	0.4228	995.4876
8	8	"mass,disp,fuel,mass*fuel,disp/time"	0.8155	0.4358	1.0096e+03
9	9	"disp,time,mass*fuel"	0.7793	0.5191	1.0951e+03
10	10	"mass,fuel,diso,time + 5 interactions"	0.8239	0.4203	996.4042

Decide on best model

.Model 5 outperforms 1,2,3,4, in accuracy and has higher AIC, meaning its not overly complex.

Models 6 and 7 pretty much have the same accuracy in terms of R2 and MSE as Model5, but have an added interaction term that drives up the AIC. So they are not preferred over #5

Model 8 is not as accurate as 5, and is no less complex, as it substitutes time with an interaction term involving time.

Model 9 is simpler yet noticably less accurate then #5, and its AIC shows the simplification does not compensate.

Model 10 is the most accurate of them all. It has slightly higher R2 and lower MSE than #5, but its AIC is higher, meaning its overcomplicated given its accuracy. It will likely overfit the data

So I will go with Model#5: Mass,Fuel,Time,Disp,Mass*Fuel

Q3) A)Calculate residuals

Redo model

```
mass_fuel_inter = mass.*fuel
```

```
mass_fuel_inter = 500×1
         997
        2050
        1265
        1578
        2343
        1306
        1934
        2051
         960
         934
       ⋮
```

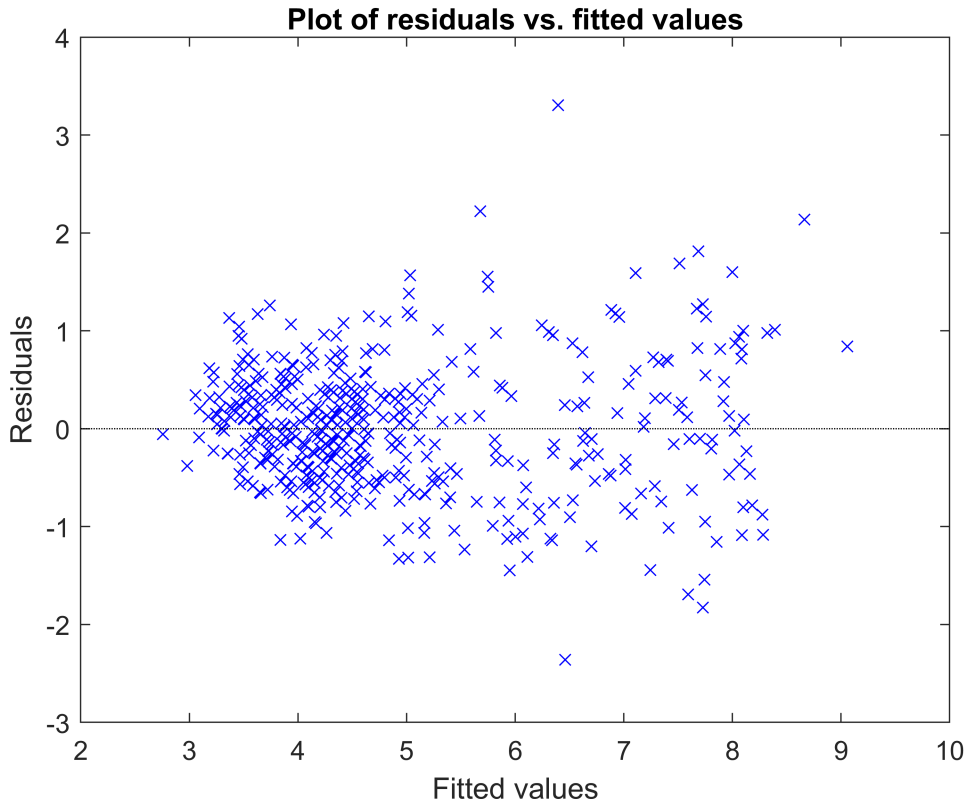
```
x = [mass, disp, fuel,time, mass_fuel_inter];
model = fitlm(x,y)
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                   ________    __________    _______    __________

    (Intercept)       6.3406       0.67603     9.3791    2.4031e-19
    x1             -0.001174    0.00020818    -5.6391    2.8791e-08
    x2              0.069416        0.0624     1.1124       0.26649
    x3               -1.8743       0.20719    -9.0462    3.3918e-18
    x4              -0.19757      0.041646    -4.7439    2.7477e-06
    x5             0.0021045    0.00012755       16.5    4.9118e-49


Number of observations: 500, Error degrees of freedom: 494
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 454, p-value = 4.55e-182
```

Plot residuals vs fitted values

Here I am checking if there is a noticable pattern between the errors(residuals) and the fitted data. Perhaps the model performs poorly on a given range of outputs.

```
plotResiduals(model,"fitted")
```



Upon inspection, it seems that the model is performing best on fuel consumption values between 3 and 5. For higher fuel consumption figures the residuals seem, on average, to be much larger, and are also dispersed more widely. This suggests the model is more accurate in predicting lower fuel consumptions than higher ones.

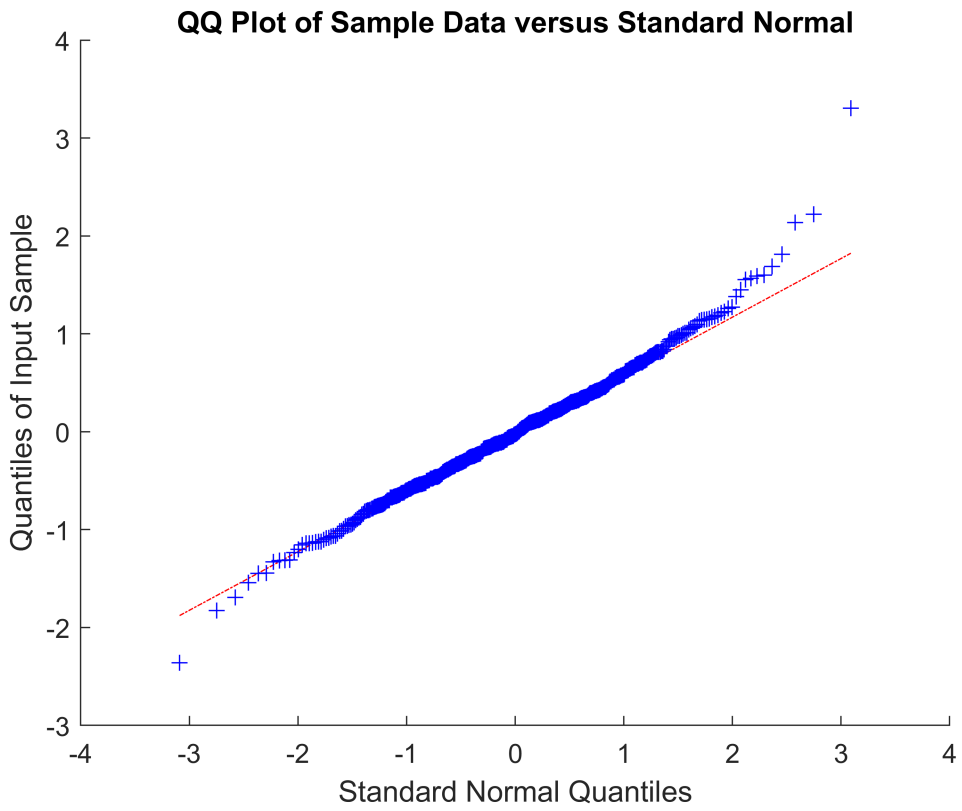
QQ Plot of Residuals

Here I am trying to check if the errors are indeed normally distributed. I will take the raw residuals

```
residuals = table2array(model.Residuals(:,1))
```

```
residuals = 500×1
   -0.3865
    0.3052
    0.6439
    0.0409
    0.4668
   -0.3268
    0.6944
   -0.6401
   -0.3813
    0.2129
      ⋮
```

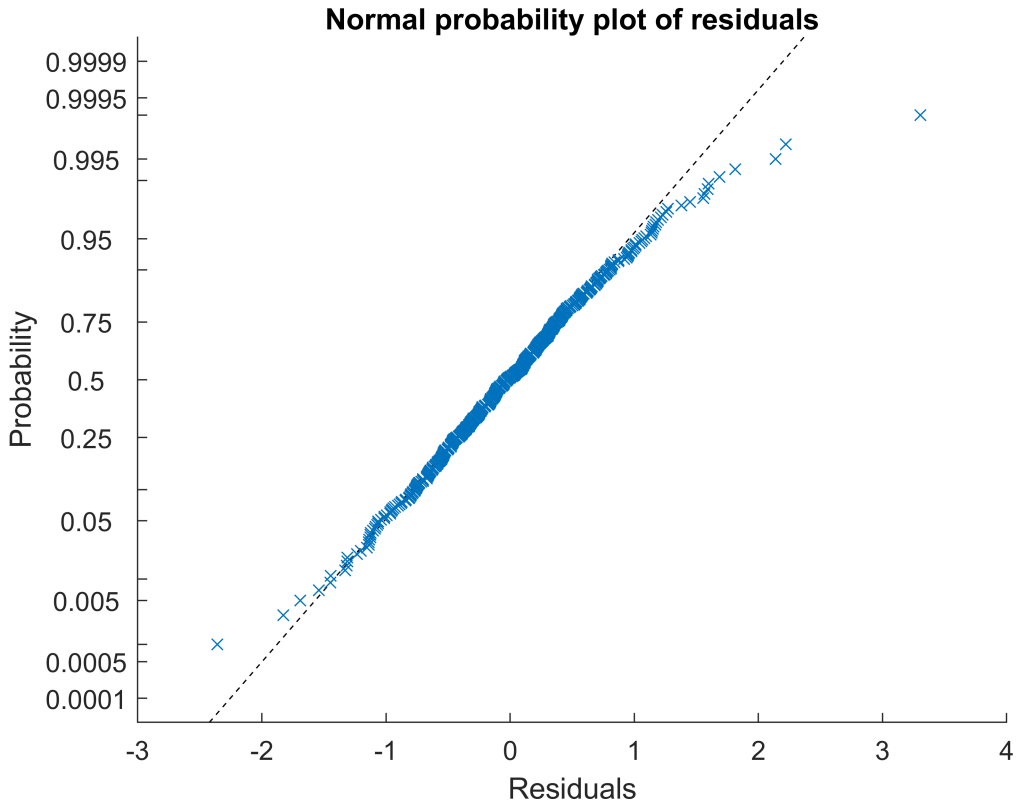
```
qqplot(residuals)
```



The QQ plot shows that the empirical quantiles approximate the true quantiles quite well up to the last segment of the data, i.e. the higher fuel consumption figures. So the residuals seem to be normally distributed overall, but again this QQplot confirms our previous suspicion about inaccuracies in predicting high fuel consumptions. There also seems to be deviations from a normal distribution while predicting low fuel consumptions

Theoretical and Empriical CDF

```
plotResiduals(model,"probability")
```



Plots of theoretical and empirical cdf (blue dots) serve to corraborate our conclusions from the qqplot.

B)Interpret the regression coefficients and suggest improvements

```
"X1= mass, X2= disp X3= fuel X4=time X5= mass*fuel"
```

```
ans =
"X1= mass, X2= disp X3= fuel X4=time X5= mass*fuel"
```

```
model
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
                   Estimate        SE          tStat       pValue
                   ________    __________    _______    __________

    (Intercept)       6.3406       0.67603     9.3791    2.4031e-19
    x1             -0.001174    0.00020818    -5.6391    2.8791e-08
    x2              0.069416        0.0624     1.1124       0.26649
    x3               -1.8743       0.20719    -9.0462    3.3918e-18
    x4               -0.19757      0.041646    -4.7439    2.7477e-06
    x5             0.0021045    0.00012755       16.5    4.9118e-49
```

```
Number of observations: 500, Error degrees of freedom: 494
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 454, p-value = 4.55e-182
```

Intercept

The intercept, 6.34, gives us the mean of the output when all the predictory variables we use are zero. This is sort of meaningless now here as a car with zero valued parameters couldn't really exist. Intuitively we could expect this intercept to be zero. We could force the intercept to be zero but this could make the model more inaccurate

```
model_int = fitlm(x,y,"Intercept",false)
```

```
model_int =
Linear regression model:
    y ~ x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
           Estimate          SE         tStat       pValue
          ___________    __________    _______    __________

    x1    -0.00018544    0.00019466    -0.9526       0.34126
    x2        0.54953      0.038693     14.202    1.2073e-38
    x3       -0.90473       0.19469    -4.6469    4.3258e-06
    x4        0.13751      0.023207     5.9255    5.8378e-09
    x5      0.0015198    0.00012066     12.596    9.4195e-32


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.705
```

```
model_int
```

```
model_int =
Linear regression model:
    y ~ x1 + x2 + x3 + x4 + x5

Estimated Coefficients:
           Estimate          SE         tStat       pValue
          ___________    __________    _______    __________

    x1    -0.00018544    0.00019466    -0.9526       0.34126
    x2        0.54953      0.038693     14.202    1.2073e-38
    x3       -0.90473       0.19469    -4.6469    4.3258e-06
    x4        0.13751      0.023207     5.9255    5.8378e-09
    x5      0.0015198    0.00012066     12.596    9.4195e-32


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.705
```

```
model_int.ModelCriterion.AIC
```

```
ans = 1.0738e+03
```

```
model_int.Rsquared
```

```
ans = struct with fields:
```

```
    Ordinary: 0.7894
    Adjusted: 0.7877
```

This is less accurate than our model and there is realistically a lower limit on car parameters (i.e. mass and engine size can't just e zero) so we can ignore this.

Betas

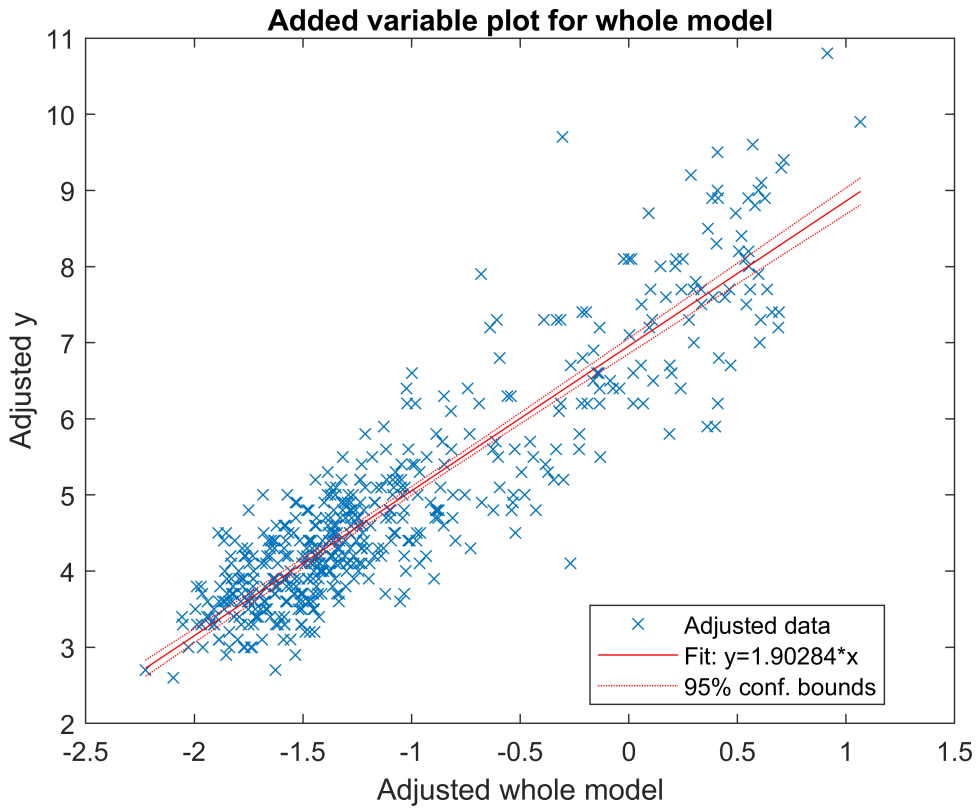
All the betas except beta/x2 have p values much smaller than 0.05, meaning they are very unlikely to be zero. X2's p value is 0.26649, so it may not be statistically significant to consider it as being yective.

All the other terms seem to be very signficiant, so I will try running the model without the disp term present.

```
mass_fuel_inter = mass.*fuel;
x_new = [mass, fuel,time, mass_fuel_inter];
model_new = fitlm(x_new,y)
```

```
model_new =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                    Estimate         SE         tStat       pValue
                   __________    __________    _______    __________

    (Intercept)        6.9575        0.3867     17.992    4.6632e-56
    x1             -0.0011852    0.00020799    -5.6984    2.0783e-08
    x2                 -1.888       0.20687    -9.1262    1.7963e-18
    x3                -0.2374      0.021271     -11.16    5.9471e-26
    x4              0.0021118    0.00012741     16.576    2.0899e-49


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 567, p-value = 3.33e-183
```

```
plot(model_new)
```




```
model_new.ModelCriterion.AIC
```

```
ans = 993.0877
```

```
model_new.Rsquared
```

```
ans = struct with fields:
    Ordinary: 0.8208
    Adjusted: 0.8193
```

```
model_new.MSE
```

```
ans = 0.4225
```

This yields a model that is slightly less accurate than model#5, but also has a sligthly lower AIC.

I will go with this model given that it is less complex then our previous model, and the high p value for x2 means I can not reject the null hypothesis that the yect of disp in this model is null with statistical confidence

```
x = x_new
```

```
x = 500×4
10^3 ×
    0.9970    0.0010    0.0086    0.9970
    2.0500    0.0010    0.0106    2.0500
    1.2650    0.0010    0.0097    1.2650
    1.5780    0.0010    0.0095    1.5780
    2.3430    0.0010    0.0118    2.3430
    1.3060    0.0010    0.0095    1.3060
    1.9340    0.0010    0.0104    1.9340
    2.0510    0.0010    0.0102    2.0510
```

```
    0.9600    0.0010    0.0126    0.9600
    0.9340    0.0010    0.0110    0.9340
     ⋮
```

```
model = model_new
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                   Estimate         SE         tStat       pValue
                  __________    __________    _______    __________

    (Intercept)       6.9575        0.3867     17.992    4.6632e-56
    x1             -0.0011852    0.00020799    -5.6984    2.0783e-08
    x2                -1.888       0.20687    -9.1262    1.7963e-18
    x3                -0.2374      0.021271     -11.16    5.9471e-26
    x4             0.0021118    0.00012741     16.576    2.0899e-49


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 567, p-value = 3.33e-183
```

Now we can say with confidence, thanks to the low pValues, that the regression parameters contribute meaningfully to our predictions.

Interpretation of coyicients

From our previous analysis of scatter plots and model 3 in particular we would expect coyicients x1 and x2 to be positive, so that cars with more mass, and cars burning petrol rather than diesel, consume more fuel. However the coyicient x4 for our interaction term, captures this relationship. It is, as expected, positive. X1 and X2 are counterintuitively negative. The positive x4 and negative x1 x2 are yective in fine tuning the model. The different models demonstrated that mass*fuel is a useful additional info and is a very good predictor. This might be because the yect of running a more economical diesel engine is more pronounced when the car is even heavier, and vice versa.

X3 is predictably negative, so that cars with higher acceleration times, i.e. slower accelerations, consume less fuel.

Q4) Clustering

Perform cluster analysis to identify two distinct groups of cars

```
% Perform K means
[idx,C] = kmeans(x,2);
% Use Logical indexing to separate data by category
x_idx = [idx,x]
```

```
x_idx = 500×5
10^3 ×
    0.0010    0.9970    0.0010    0.0086    0.9970
    0.0010    2.0500    0.0010    0.0106    2.0500
```

```
    0.0010    1.2650    0.0010    0.0097    1.2650
    0.0010    1.5780    0.0010    0.0095    1.5780
    0.0010    2.3430    0.0010    0.0118    2.3430
    0.0010    1.3060    0.0010    0.0095    1.3060
    0.0010    1.9340    0.0010    0.0104    1.9340
    0.0010    2.0510    0.0010    0.0102    2.0510
    0.0010    0.9600    0.0010    0.0126    0.9600
    0.0010    0.9340    0.0010    0.0110    0.9340
      ⋮
```

```
cat_1 = x_idx(:,1) < 2
```

```
cat_1 = 500×1 logical array
   1
   1
   1
   1
   1
   1
   1
   1
   1
   1
     ⋮
```

```
cat_2 = x_idx(:,1) > 1
```

```
cat_2 = 500×1 logical array
   0
   0
   0
   0
   0
   0
   0
   0
   0
   0
     ⋮
```

```
x_1 = x(cat_1,:);
x_2 = x(cat_2,:);
```

```
% Further indexing two separate each category by fuel type (will be useful)
% in below plot) - 1 is diesel, 2 is petrol
% Create logical indices
idx_1d =x_1(:,2) < 2
```

```
idx_1d = 362×1 logical array
   1
   1
   1
   1
   1
   1
   1
   1
   1
```

```
   1
     ⋮
```

```
idx_1p = x_1(:,2) > 1
```

```
idx_1p = 362×1 logical array
   0
   0
   0
   0
   0
   0
   0
   0
   0
   0
     ⋮
```

```
idx_2d = x_2(:,2) < 2
```

```
idx_2d = 138×1 logical array
   0
   0
   0
   0
   0
   0
   0
   0
   0
   0
     ⋮
```

```
idx_2p =x_2(:,2) > 1
```

```
idx_2p = 138×1 logical array
   1
   1
   1
   1
   1
   1
   1
   1
   1
   1
     ⋮
```

```
% Separate datasets
x_1_diesel = x_1(idx_1d,:)
```

```
x_1_diesel = 250×4
10^3 ×
    0.9970    0.0010    0.0086    0.9970
    2.0500    0.0010    0.0106    2.0500
    1.2650    0.0010    0.0097    1.2650
    1.5780    0.0010    0.0095    1.5780
    2.3430    0.0010    0.0118    2.3430
```

```
    1.3060    0.0010    0.0095    1.3060
    1.9340    0.0010    0.0104    1.9340
    2.0510    0.0010    0.0102    2.0510
    0.9600    0.0010    0.0126    0.9600
    0.9340    0.0010    0.0110    0.9340
     ⋮
```

```
x_1_petrol = x_1(idx_1p,:)
```

```
x_1_petrol = 112×4
10^3 ×
    1.2070    0.0020    0.0098    2.4140
    1.3190    0.0020    0.0092    2.6380
    1.2770    0.0020    0.0103    2.5540
    1.2760    0.0020    0.0104    2.5520
    0.9830    0.0020    0.0115    1.9660
    1.1830    0.0020    0.0103    2.3660
    1.3080    0.0020    0.0103    2.6160
    0.9950    0.0020    0.0099    1.9900
    1.1480    0.0020    0.0092    2.2960
    0.9700    0.0020    0.0086    1.9400
     ⋮
```

```
x_2_diesel = x_2(idx_2d,:) % No diesel cars in category 2
```

```
x_2_diesel =

  0×4 empty double matrix
```

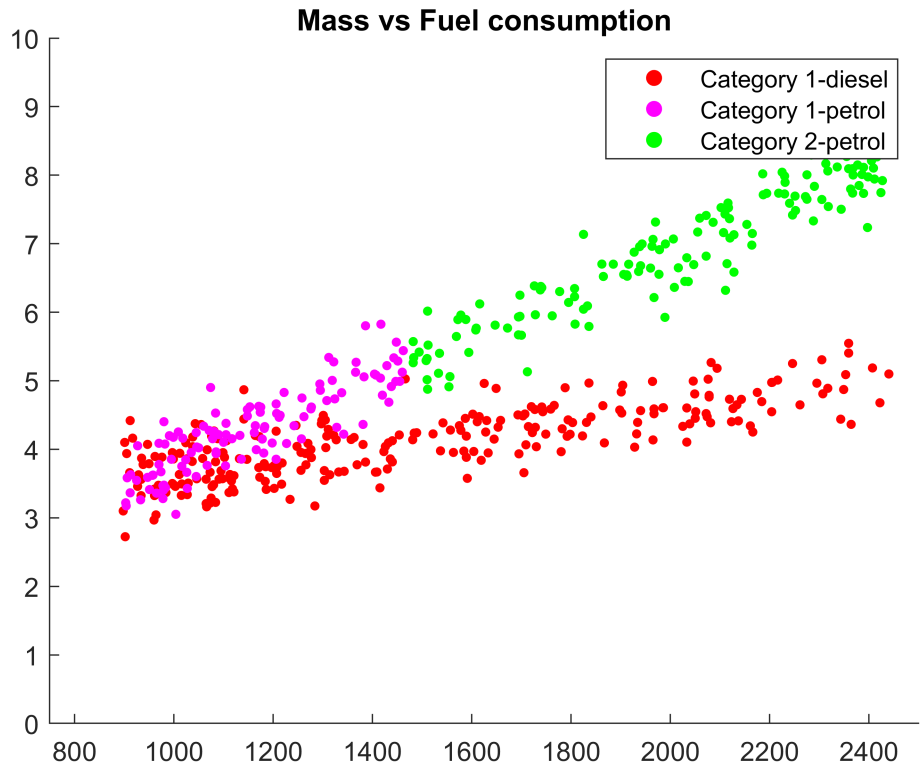
```
x_2_petrol = x_2(idx_2p,:)
```

```
x_2_petrol = 138×4
10^3 ×
    2.3770    0.0020    0.0095    4.7540
    1.5880    0.0020    0.0089    3.1760
    2.3540    0.0020    0.0085    4.7080
    1.7410    0.0020    0.0089    3.4820
    1.6720    0.0020    0.0105    3.3440
    2.1940    0.0020    0.0089    4.3880
    2.3970    0.0020    0.0136    4.7940
    2.2510    0.0020    0.0098    4.5020
    1.6940    0.0020    0.0101    3.3880
    2.1870    0.0020    0.0089    4.3740
     ⋮
```

Plot the different categories on cars in a mass v fuel consumption plot, with fuel types indicated

```
% Get the fuel consumptions for each category
y1_diesel = model.predict(x_1_diesel);
y1_petrol = model.predict(x_1_petrol);
y2_diesel = model.predict(x_2_diesel);
y2_petrol = model.predict(x_2_petrol);
% Plot
scatter(x_1_diesel(:,1), y1_diesel, 12, "red","filled")
hold on
scatter(x_1_petrol(:,1), y1_petrol, 12, "magenta","filled")
```

```
scatter(x_2_petrol(:,1), y2_petrol, 12, "green","filled")

legend("Category 1-diesel","Category 1-petrol", "Category 2-petrol")
title("Mass vs Fuel consumption")
axis([750,2500,0,10])
hold off
```



The clusters observed in the above plots point to an interesting observation. Notice that all diesel cars are in cluster 1, along with petrol cars with low fuel consumptions, up to about 5l/100km.

This suggests that petrol and diesel cars should have separate regression models, as their best fit lines clearly seem to be different.

Q5) Predictions on test set

Prepare data

```
test_data
```

test_data = 500×6 table

	l100	mass	time100	displacement	type	colour
1	3.4000	2218	11.9000	2	'diesel'	'white'
2	3.8000	1977	10.8000	2.1000	'diesel'	'other'
3	4.6000	983	10.8000	2.1000	'diesel'	'other'
4	4	2310	9.9000	2	'diesel'	'white'

	l100	mass	time100	displacement	type	colour
5	3.4000	1233	12	1.9000	'diesel'	'other'
6	4.1000	1004	10.1000	3.1000	'diesel'	'other'
7	3.7000	921	10.6000	2.1000	'diesel'	'white'
8	3.7000	1678	9.2000	3.8000	'diesel'	'other'
9	3.1000	1377	11.3000	2.2000	'diesel'	'other'
10	4.1000	2083	12	1.6000	'diesel'	'other'
11	3.7000	1054	10.1000	2.4000	'diesel'	'white'
12	3.3000	1217	11.7000	2.3000	'diesel'	'white'
13	3.7000	1940	8.9000	3.1000	'diesel'	'other'
14	4.9000	1899	9.1000	3.7000	'diesel'	'other'
15	6.2000	2152	9.2000	3.1000	'diesel'	'white'
16	4	1042	11	2.1000	'diesel'	'other'
17	4.4000	2050	9.2000	3	'diesel'	'other'
18	4.7000	2137	8.7000	4	'diesel'	'other'
19	4.1000	2393	11.1000	3	'diesel'	'other'
20	3.9000	1421	10.8000	2.1000	'diesel'	'other'
21	4.2000	1261	9.1000	3.7000	'diesel'	'white'
22	5.7000	1999	9.1000	2.3000	'diesel'	'other'
23	5.2000	2272	7.8000	4.4000	'diesel'	'white'
24	4.4000	1678	6.6000	4.7000	'diesel'	'white'
25	3.8000	1064	8.9000	3.8000	'diesel'	'other'
26	4.4000	1917	7.2000	3.6000	'diesel'	'white'
27	3.9000	1634	9.4000	2.6000	'diesel'	'other'
28	3.7000	1044	10.6000	2.2000	'diesel'	'white'
29	3.4000	921	8.8000	3.3000	'diesel'	'white'
30	4.7000	2316	8.7000	3.7000	'diesel'	'white'
31	4.2000	2118	9.3000	2.9000	'diesel'	'white'
32	3.6000	1265	11.2000	2	'diesel'	'white'
33	4.6000	1431	8.5000	3.1000	'diesel'	'other'
34	4.8000	2412	9.3000	3	'diesel'	'white'
35	4.1000	1959	10.2000	2.7000	'diesel'	'other'
36	4.2000	1113	9.4000	2.4000	'diesel'	'other'
37	2.8000	1011	13.2000	1.6000	'diesel'	'other'

	l100	mass	time100	displacement	type	colour
38	4.7000	1407	10.7000	2.8000	'diesel'	'white'
39	3.9000	1290	9.6000	3.1000	'diesel'	'other'
40	3.8000	1757	10.7000	2	'diesel'	'white'
41	3.5000	1106	10.1000	2	'diesel'	'other'
42	4.4000	1994	9.6000	2.3000	'diesel'	'white'
43	4.8000	1900	7.5000	5.5000	'diesel'	'white'
44	3.9000	956	9.8000	3	'diesel'	'other'
45	3.9000	1622	7.9000	4.2000	'diesel'	'other'
46	4.4000	2125	10.1000	2.1000	'diesel'	'other'
47	3.7000	950	10.5000	2.6000	'diesel'	'other'
48	4	2380	13	2	'diesel'	'other'
49	4.2000	1537	10.6000	2.3000	'diesel'	'other'
50	3.5000	1206	9.9000	2.8000	'diesel'	'other'
51	3.1000	1092	11.5000	2.1000	'diesel'	'white'
52	4.8000	1165	10	2.8000	'diesel'	'white'
53	3.1000	1334	12	1.5000	'diesel'	'other'
54	3.6000	1300	8.7000	3.9000	'diesel'	'other'
55	4.3000	2297	11.1000	2.1000	'diesel'	'other'
56	4.9000	1280	8.9000	4	'diesel'	'other'
57	4.8000	1420	7.1000	5.6000	'diesel'	'other'
58	3.3000	1244	12.1000	2	'diesel'	'other'
59	4	2051	11.7000	1.8000	'diesel'	'other'
60	3.9000	1439	10.3000	2.8000	'diesel'	'other'
61	4	1831	9.3000	3.6000	'diesel'	'other'
62	3.2000	1009	12.4000	2.1000	'diesel'	'other'
63	3.3000	1437	10	2.4000	'diesel'	'other'
64	3.2000	1082	10.3000	2.9000	'diesel'	'other'
65	4.2000	2087	9.9000	3	'diesel'	'other'
66	3.4000	1201	9.9000	3.3000	'diesel'	'other'
67	3.8000	1617	11.2000	1.7000	'diesel'	'other'
68	3.6000	1241	10.4000	2.4000	'diesel'	'other'
69	4.3000	938	7.4000	5.1000	'diesel'	'other'
70	5.4000	2306	7.9000	3.8000	'diesel'	'other'

	l100	mass	time100	displacement	type	colour
71	3.8000	1480	9.8000	3.3000	'diesel'	'other'
72	4.6000	2349	9.6000	2.9000	'diesel'	'other'
73	3	1103	12.1000	2	'diesel'	'white'
74	3.1000	911	10.4000	2.8000	'diesel'	'other'
75	3.4000	960	10.3000	3.3000	'diesel'	'white'
76	3.7000	910	10.4000	2.8000	'diesel'	'other'
77	4.7000	1634	9.1000	3.6000	'diesel'	'other'
78	3.4000	1290	10.6000	2.5000	'diesel'	'white'
79	3.1000	924	9.3000	2.8000	'diesel'	'other'
80	4	1294	9.1000	4	'diesel'	'other'
81	3.2000	1673	10.6000	2.3000	'diesel'	'white'
82	3.5000	1607	11.4000	1.8000	'diesel'	'white'
83	4	1174	8.6000	3.8000	'diesel'	'other'
84	4.3000	981	9.2000	4	'diesel'	'other'
85	3.7000	1278	12.3000	1.6000	'diesel'	'other'
86	3.7000	1049	11	1.5000	'diesel'	'other'
87	4	1139	10.4000	2	'diesel'	'white'
88	5	2427	13.2000	1.3000	'diesel'	'other'
89	4.4000	1240	10	2.2000	'diesel'	'other'
90	4	1133	9	3.3000	'diesel'	'other'
91	4.1000	1507	10.2000	3.4000	'diesel'	'other'
92	3.5000	1194	11.3000	2.6000	'diesel'	'white'
93	3.9000	1007	11.4000	2.6000	'diesel'	'other'
94	3.3000	1469	10.9000	2.2000	'diesel'	'other'
95	3.2000	1770	13	1.4000	'diesel'	'other'
96	3.5000	1395	10.9000	2.2000	'diesel'	'other'
97	4.2000	1115	10.1000	2.6000	'diesel'	'other'
98	4.5000	1719	8	5	'diesel'	'other'
99	3.3000	1154	11	2.1000	'diesel'	'other'
100	3.3000	1549	10.3000	3.1000	'diesel'	'other'

⋮

```
y_test = table2array(test_data(:,1));
% Get all x terms
x1 = table2array(test_data(:,2)); % Mass
```

```
x2 = table2array(test_data(:,5)); % Fuel
c = categorical({'petrol','diesel'})
```

```
c = 1×2 categorical
petrol       diesel
```

```
x2 = grp2idx(x2); % Encoded fuel
x3 = table2array(test_data(:,3)); % Time
x4 = x1.*x2 %mass *fuel
```

```
x4 = 500×1
        2218
        1977
         983
        2310
        1233
        1004
         921
        1678
        1377
        2083
      ⋮
```

```
% Overall x matrix
x_test = [x1,x2,x3,x4]
```

```
x_test = 500×4
10^3 ×
    2.2180    0.0010    0.0119    2.2180
    1.9770    0.0010    0.0108    1.9770
    0.9830    0.0010    0.0108    0.9830
    2.3100    0.0010    0.0099    2.3100
    1.2330    0.0010    0.0120    1.2330
    1.0040    0.0010    0.0101    1.0040
    0.9210    0.0010    0.0106    0.9210
    1.6780    0.0010    0.0092    1.6780
    1.3770    0.0010    0.0113    1.3770
    2.0830    0.0010    0.0120    2.0830
     ⋮
```

```
% Summary stats for output
y_mean = mean(y_test)
```

```
y_mean = 4.7372
```

```
y_median = median(y_test)
```

```
y_median = 4.4000
```

```
y_std = std(y_test)
```

```
y_std = 1.3232
```

```
y_iqr = iqr(y_test)
```

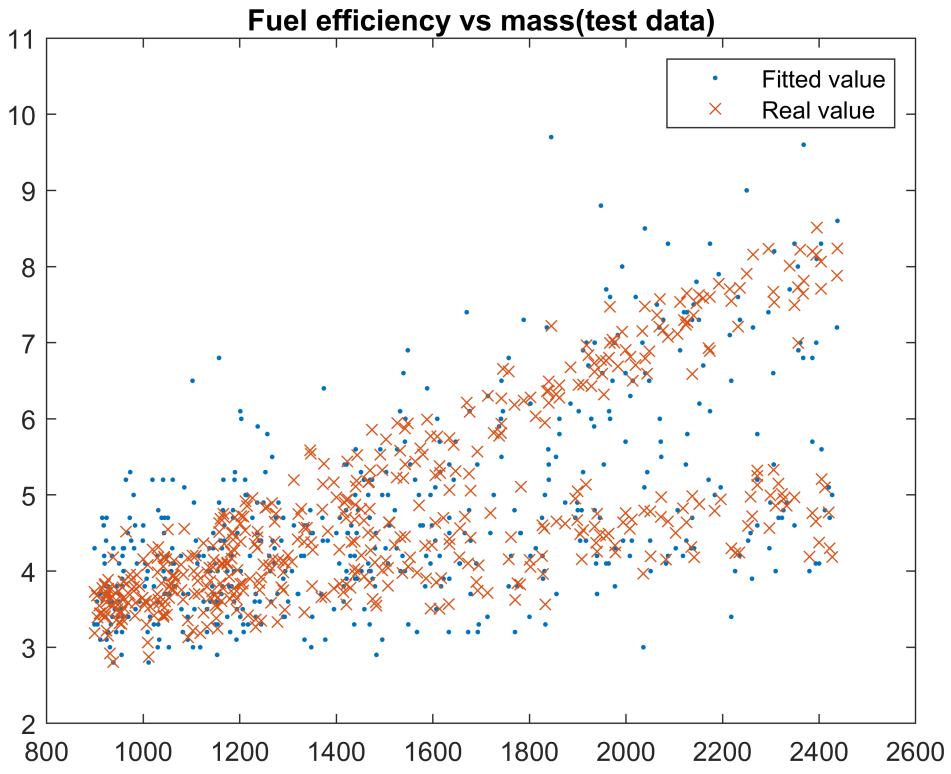
```
y_iqr = 1.5000
```

Fit model and plot

```
model
```

```
model =
Linear regression model:
    y ~ 1 + x1 + x2 + x3 + x4

Estimated Coefficients:
                   Estimate        SE          tStat       pValue
                   ________    __________    _______    __________

    (Intercept)       6.9575        0.3867     17.992    4.6632e-56
    x1             -0.0011852    0.00020799    -5.6984    2.0783e-08
    x2                -1.888        0.20687    -9.1262    1.7963e-18
    x3                -0.2374       0.021271    -11.16    5.9471e-26
    x4              0.0021118    0.00012741     16.576    2.0899e-49


Number of observations: 500, Error degrees of freedom: 495
Root Mean Squared Error: 0.65
R-squared: 0.821,  Adjusted R-Squared: 0.819
F-statistic vs. constant model: 567, p-value = 3.33e-183
```

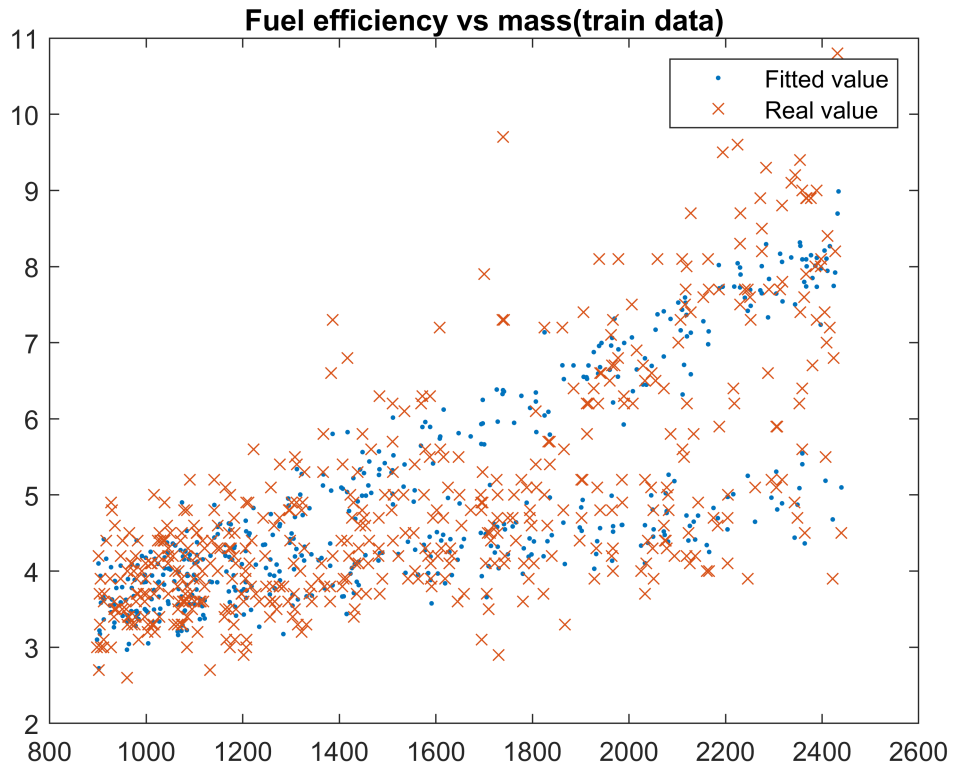
```
y_pred = predict(model,x_test);
plot(x1,y_test,".",x1,y_pred,"x")
legend("Fitted value", "Real value")
title("Fuel efficiency vs mass(test data)")
```



For comparison we can plot this identical plot for the training data

```
y_pred_test = predict(model,x);
```

```
plot(mass,y_pred_test,".",mass,y,"x")
legend("Fitted value", "Real value")
title("Fuel efficiency vs mass(train data)")
```



The model seems to perform similarly based on the plots

Calculate metrics

```
mse_test = mean((y_test - y_pred).^2)
```

```
mse_test = 0.4103
```

```
mse_train = model.MSE
```

```
mse_train = 0.4225
```

The MSE is slightly lower for our test data predictions. This is a good sign that the model is not overfitting.

The model seems simple enough and not in need of regularization.