The long-run innovation risk component

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Abstract

A persistent component in productivity growth has been shown to be related to a persistent component in consumption, which has significant implications for asset prices. This paper studies a measure of R&D intensity defined as the stationary deviations in R&D investment from the equilibrium level in a semi-endogenous growth model. This measure results being the error correction term of the cointegration between R&D expenditures and the productivity level. The empirical counterpart strongly forecasts productivity growth and proves having a persistence that matches well previous evidence on the productivity growth *long-run risk component*. These findings support the identification of a long-run *innovation* risk component and all of the long-run risk framework. This claim is further verified by testing the main financial implication: stocks' cash flows sensitivities to this measure are indeed proven being associated with a significant cross-sectional risk premium.

Keywords: Asset Pricing, Long-run risk, Innovation, Cointegration

JEL Codes: E32, E44, G12, O30

1 Introduction

To reconcile consumption-based asset pricing theory with the data, especially with respect to the Equity Premium Puzzle posed by Mehra and Prescott (1985), Bansal and Yaron (2004) focused on a 'small' but persistent component of consumption growth, named the 'long-run risk' (LRR) component. This process can add little variance to consumption growth despite heavily impacting the whole consumption path, so, when paired with preferences where welfare is sensitive to uncertainty in expectations of consumption ahead, as those of Epstein and Zin (1989), it ends up being a huge source of risk. The presence of this risk has proven useful in studying various macro-financial phenomena,[1] but the technical difficulties in its empirical identification makes the validation of the whole mechanism challenging, thus attracting significant criticism.[2] Given the vastness of the literature that grew on the

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[1] For example, exchange rates dynamics as in Colacito and Croce (2011), climate change pricing as in Bansal, Ochoa, et al. (2016), term structures as in Ai et al. (2018), or oil dynamics as in Ready (2018).

[2] Most notably, Beeler and Campbell (2012) and Epstein, Farhi, et al. (2014).

LRR concept, it is important to provide evidence that supports its establishment: I do so by theoretically defining a LRR component in the economy that is specifically related to innovation and whose identification is empirically feasible, and by showing its fulfilment of theoretical predictions both on the macroeconomic and financial sides. Specifically, this paper focuses on a *semi*-endogenous growth model which predicts aggregate R&D and TFP level to be approximately cointegrated and the corresponding error correction term (ECT), i.e. how much they deviate from each other relative to equilibrium, to forecast TFP growth and constitute a priced risk factor in the cross-section of assets. This ECT is then what I define as R&D intensity measure and test on US data, where indeed it results being stationary and yet highly persistent as well as displaying a strong forecasting power with respect to TFP growth and being associated to a significant risk premium.

Previously, existence and relevance of long-run risks have been corroborated by directly overcoming the statistical difficulties in its identification, as done for example by Ortu et al. (2013) and Schorfheide et al. (2018), or by backing up the economic soundness of the concept, which translates to framing its origination in a richer structural model that can provide additional implications to test. Following the latter approach, Kaltenbrunner and Lochstoer (2010) first showed in general equilibrium how the long-run risk component can arise in consumption growth with standard productivity dynamics. Then, Croce (2014) went a step further, providing both empirical and theoretical evidence for a long-run consumption risk component being potentially originated in the persistence of the productivity growth process. This was additionally supported by Ortu et al. (2013), who found high correlation between the components with half-life within eight and sixteen years of consumption and Total Factor Productivity (TFP) growth rates. Moving one further step upstream, Kung and Schmid (2015) acknowledged the well-established role of Research and Development (R&D) expenditure in spurring productivity growth and showed how consumption and productivity long-run risk components could be driven by an endogenous and persistent aggregate R&D investment intensity – defined as R&D investment scaled by the stock of intangible capital. This paper builds on this point, circling around a definition of R&D intensity that is slightly different from that of Kung and Schmid (2015), but finds greater empirical support, thus providing further support to the long-run productivity risk asset pricing framework.

The cointegration relationship between R&D and ideas' stock has already been the subject of different studies comparing fully and semi-endogenous growth models, with the first examples being Ha and Howitt (2007) and Bottazzi and Peri (2007), and more recent examples in Herzer (2022) and Kruse-Andersen (2023). While not directly addressing the distinction, this study provides support to the semi-endogenous framework by testing it on the new ground of financial markets. More broadly, cointegration in macroeconomic variables has already been exploited widely to price assets, with notable examples in Lettau and Ludvigson (2001) and Melone (2021), but to my knowledge this is the first study to relate aggregate R&D and asset pricing.

Concerning the macroeconomic modelling, I employ a 'lab-equipment' R&D growth model, for example as studied by Sedgley and Elmslie (2013), where the allocation concerns goods

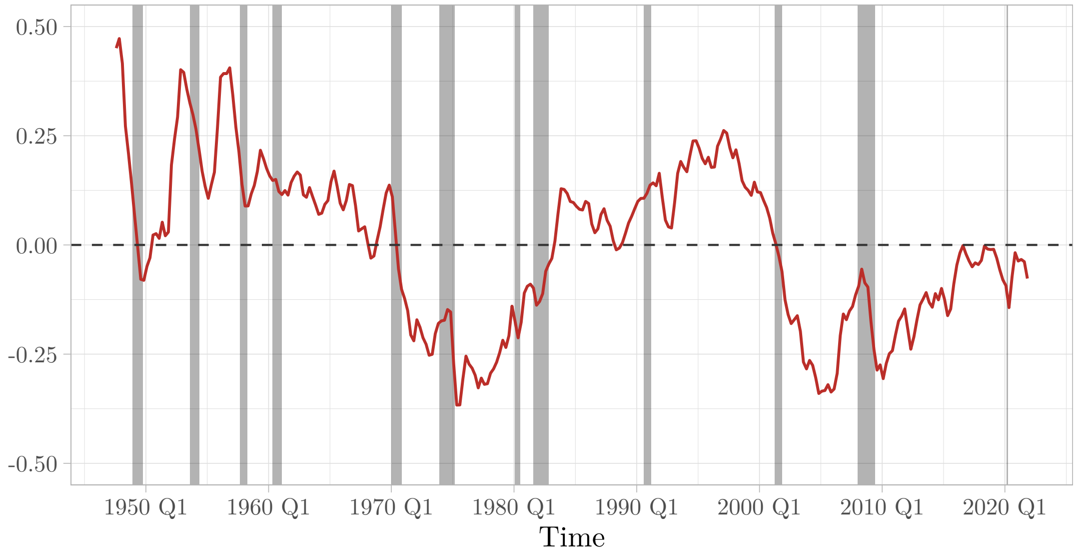

Figure 1: R&D excess intensity, see section 3 for details. Shaded areas mark NBER recessions.

rather than work, as it is in the more traditional cases à la Romer (1990). Further, leveraging the theoretical result from Kung and Schmid (2015) that R&D intensity persistence can emerge in general equilibrium, here the allocation is simply determined with a rule of thumb that ensures a balanced growth path, as in Jones (2022).

On the financial side of the analysis, the most important contribution of the paper is providing empirical evidence that persistent swings in R&D activities is a priced risk in the markets. This evidence comes from cross-sectional pricing tests pioneered by Bansal, Dittmar, and Lundblad (2005), and developed by Bansal, Dittmar, and Kiku (2009) among others. Like Bansal, Dittmar, and Lundblad (2005), in this paper I focus on the risk-premium related to cash-flow sensitivities rather than returns sensitivities, as discount rates may be impacted by many more factors. In this regard, I obviously depart from them by focusing on sensitivities to the estimated R&D intensity, which ends up showing a stronger risk premium than those to consumption growth. I also contribute by considering a wider set of test asset portfolios: in particular, I include portfolios sorted on firm-specific R&D. This is interesting because this sorting leads to the greatest dispersion in cash-flow growth rates across portfolios and it is a dimension likely relevant for heterogeneity in sensitivities to aggregate R&D. Indeed, a clear pattern emerge: cash-flows of more R&D intensive firms prove being much more positively sensitive to aggregate R&D intensity, which is in line with both them showing higher excess returns and spillover effects being positive, as previously shown by Jiang et al. (2016). This further supports the idea that firms investing in R&D gain more from other firms investing more too, so that their cash-flows are more sensitive to aggregate R&D investment.

From the methodological point of view, as often done in the recent macro-finance literature, I estimate the cointegration relationship employing the Dynamic OLS methology as illustrated in Phillips and Loretan (1991), Saikkonen (1991), and Stock and Watson (1993). There are technical reasons related to the timing of variables to do so, as will be clearer later. For the cross-sectional pricing test, a traditional Fama and Macbeth (1973) is employed.

The rest is structured as follows: in section 2 I outline the emergence of a R&D intensity measure in endogenous growth models and its asset pricing role; in section 3 I show the results of the estimation of this measure and proceed illustrating its proprieties, forecasting power with respect to TFP and links with markup and funding conditions, which are known to interact with R&D investment; in section 4 I carry out the cross-sectional pricing test; in section 5 I conclude.

2 The R&D component of long-run productivity risk

2.1 Background

The starting point of this study is the law of motion of the aggregate intangible capital stock N in Kung and Schmid (2015), which, using their notation is

$$N_{t+1} = (1-\phi)N_t + \chi \left(\frac{S_t}{N_t}\right)^{\eta} N_t, \tag{1}$$

where N_t can be interpreted as a measure of patented ideas and S_t of (R&D) expenditure. ϕ controls ideas' obsolescence rate, while η captures both the duplication effects in innovative efforts and spillover effects from past innovations. In their economy, the intangible capital contributes to equilibrium final goods production via technology

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \qquad \text{where} \qquad Z_t \equiv \bar{A}(e^{a_t} N_t)^{1-\alpha}, \tag{2}$$

with K_t being the physical capital, L_t the amount of labour, Z_t the standard Solow residual, and α the capital share. a_t is assumed to be a stationary process with innovations ε_t. When a_t is highly persistent and the TFP growth rate is small, the latter can be simply approximated as

$$\Delta \ln Z_{t+1} \approx (1-\alpha)\left[\chi \left(\frac{S_t}{N_t}\right)^{\eta} - \phi\right] + (1-\alpha)\varepsilon_{t+1}. \tag{3}$$

This formulation highlights the crucial role of the ratio S_t/N_t, defined as 'R&D intensity': as its dynamics drive conditional expectations of TFP growth, any persistent movement in it translates into a source of long-run productivity risk in the sense of Croce (2014).

Kung and Schmid (2015) show theoretically that innovation efforts endogenously driven by the fluctuations of profitability level set by the exogenous persistent process a_t spontaneously lead to persistence in growth prospects, rationalizing jointly macroeconomic and asset prices dynamics. To support the idea that R&D intensity identifies a long-run risk component, they study its empirical counterpart formed as the raw ratio of US annual private R&D expenditure from the National Science Foundation, measuring S_t, over the US Bureau of Labor Statistics' estimated R&D stock series, representing intangible capital N_t. This measure of R&D

intensity proves indeed being highly persistent and co-moving at low frequencies with the price-dividend ratio as well as forecasting the growth rates of consumption, GDP and TFP.

However, this approach has a few potential shortcomings. Indeed, their R&D intensity measure, as in the natural logarithm of S_t/N_t, is *extremely* persistent, with a point estimate of the yearly first autocorrelation equal to 0.987 and a standard error of 0.005.[3] It should be noted that the R&D stock series has been updated by the Bureau of Labor Statistics with respect to the one used in their paper and now covers a slightly different time period. Anyway, the 95% confidence interval, which spans from 0.977 to 0.998, highlights two potential issues with the use of this measure: the upper bound, being so close to 1, shows that the measure could well be non-stationary and the lower bound shows that this process identifies a long-run risk component in the economy that is unlikely to be related to productivity. Sample non-stationarity is not critical to the validity of the measure and the theory it is used to support: from a statistical point of view, R&D intensity is expected to be persistent and the more persistent a process is, the harder it becomes to assess its stationarity in finite samples, so the generating process could still be stationary; then, even if R&D intensity really was non-stationary the key mechanism studied by Kung and Schmid (2015) could still hold, at the price of a more complex model. Nonetheless, non stationarity of R&D intensity undermines the regressions in which it is employed, as any results would essentially be spurious. Unfortunately, the Augmented Dickey Fuller test with trend delivers for this series a statistic for the unit-root coefficient of -2.21, which is well above the 10% critical value of -3.15, thereby suggesting that the series is highly likely non-stationary. Furthermore, even with enough evidence backing its stationarity, another concerning issue is that this series' first autocorrelation is at least 0.977, which implies a half-life of shocks over 30 years for an AR(1) process. This suits the long-run risk component in consumption calibrated by Bansal and Yaron (2004), but is way more persistent than the component that Ortu et al. (2013) find consumption and productivity to share more strongly in the data, which has half-life between eight and sixteen years, corresponding to a maximum autocorrelation of 0.957 if modelled as a yearly AR(1). Therefore, all in all, this measure could well identify a long-run risk source in the economy, but the empirical evidence for it effectively identifying the long-run productivity risk originated in the R&D investment is fragile.

There are a few details that may drive this measure away from its aim. First, the measure of intangible capital stock used, which in this case is the stock of R&D. This might open a wedge between the model and the data because intangible capital, as shown in (1), is formed in a very different way than simple accumulation and depreciation of R&D expenses – R&D investments unlikely have constant marginal returns, considering duplication and spillover effects. Another related issue is that the production of ideas is likely not to rely solely on domestic R&D expenditure and stock of ideas anyway. This makes it difficult to rely on any measure of intangible capital stock for an empirical analysis because it would require to account for all spillover sources relevant to the formation of new patented ideas and make

[3]The standard error is obtained using the Delta method and 1-step GMM estimates of the fundamental moments.

Table 1: statistics of R&D intensity measure from Kung and Schmid (2015). In the first column, S is yearly R&D expenditure from the National Science Foundation and N is the R&D stock from Bureau of Labor Statistics, spanning 1963 to 2020; in the second and third column, S is quarterly real R&D expenditure from Bureau of Economic Analysis and Z is the quarterly utilization-adjusted TFP from Fernald (2012), spanning from 1947 Q1 to 2021 Q4. ADF u.r. stat is the statistic of the unit root coefficient in an Augmented Dickey-Fuller test with a time trend. AC(1) is the first autocorrelation, estimated as cross-correlation with the lagged value via 1-step GMM, and in parenthesis there are the HAC standard error recovered via Delta-method.

	$(\ln S_t - \ln N_t)$	$(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$	
α	$-$	0.35	0.3
ADF u.r. stat	-2.55	-2.11	-2.09
AC(1)	0.989	0.999	1.000
	(0.006)	(0.000)	(0.000)
Num. obs.	57	299	299

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

strong assumptions on the functional form to combine them. A way to bypass this issue could be to utilize directly the variable that the very concept of ideas' stock was born to drive and explain: Total Factor Productivity in the form of Solow residual. This quantity was born in the data and requires little structure to be identified.

Combining (2) and (3), with the assumption that the first moment of $\ln(S/N)$ exists and the log-ratio does not deviate too much from it, TFP growth can be fairly approximated in terms of S and Z as

$$\mathbb{E}_t\left[\Delta \ln Z_{t+1}\right] \approx \gamma_0 + \gamma_1 a_t + \gamma_1 \left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right). \tag{4}$$

Here, conditional expectations of ΔZ_{t+1} are impacted by R&D investment via $(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$, which contains the R&D fluctuations that moves TFP growth expectations around; a_t enters the equation mechanically following the substitution of N_t with $\bar{A}^{\frac{-1}{1-\alpha}} e^{-a_t} Z^{\frac{1}{1-\alpha}}$ and can be thought as merely compensating its implicit presence in the 'modified R&D intensity' term. On one side, this formulation is empirically convenient because it provides a way to measure TFP growth expectations involving variables that have more obvious empirical counterparts than (3) – a_t is not directly observable but can be represented by a combination of processes that are able to forecast TFP growth. Note that a similar process could be undertaken to express TFP growth in terms of other observables too, but it would require involving much more theoretical structure than just TFP definition and ideas' law of motion. Moreover, this formulation will prove being empirically successful, while, for example, similar expressions involving GDP and labor will not work as well. On the other side, however, this formulation makes it harder to identify the source of fluctuations in growth prospect: even if R&D intensity S/N and expected TFP growth were actually constant, $(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$ would still fluctuate, even persistently, because of a_t in Z_t. This could be amended recovering a_t from the forecasting regression and filtering it out of $(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$, but this would need γ_1 to be estimated consistently, which is difficult given the high non stationary behaviour shown by $(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$: this can be observed looking at table 1 where it is reported the unit root coefficient statistic from the ADF test performed on the series of $(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$ built using quarterly US R&D expenditure from the Bureau of Economic Analysis and US utility-adjusted TFP estimated by Fernald (2012),[4] for two different values of α.

A further point of departure of the theory from reality might be represented by the strong scale effects in the model. As highlighted by Bloom et al. (2020), there is wide evidence for decreasing research productivity in the data, so this is likely a realistic feature that is necessary for a model to be applied empirically. To take this into account, in the next section I formulate a model based on the key dynamics of Kung and Schmid (2015) reframed in a semi-endogenous framework, where I directly model R&D expenditure with a stochastic thumb rule, which allows to address R&D intensity dynamics more explicitly.

2.2 A simple data-friendly model

Consider a discrete-time model where goods Y are produced using only labour L and intangible capital I with technology

$$Y_t = e^{a_t} I_t^{\xi} L_t, \tag{5}$$

characterized by the degree of increasing returns ξ. a_t is a stationary exogenous process with innovations ε_t^a to keep track of possible external factors affecting the dynamics of output level. As common in the 'lab-equipment' literature, final goods can be employed in consumption C and R&D expenditures S. R&D investment implicitly employs both capital and labor, and produces new intangible capital with a schedule that embodies the insight of the semi-endogenous growth theory that ideas get harder to find. Specifically, the law of motion of the intangible capital stock is

$$I_{t+1} = (1-\phi) I_t + {S_t}^{\eta} {I_t}^{\Psi}. \tag{6}$$

Here, duplication effects are controlled, via η, independently from spillover and how-harder-finding-ideas-gets, set by Ψ; ϕ controls ideas' obsolescence rate. This formulation nests both fully endogenous models, which are described by setting $\Psi = 1$, and Kung and Schmid (2015), which requires $\Psi = 1-\eta$. Finally, as labour grows at an exogenous rate g_L and is entirely devoted to final goods production, R&D expenditure is the only economic allocation to be made in this set-up.

The Solow residual, or productivity, Z_t here is equal to $e^{a_t} I_t^{\xi}$ so, performing approximations similar to those that led to (4), its growth rate can be written as

$$\Delta \ln Z_{t+1} \approx \gamma_0 + \gamma_1 (\ln S_t - \psi \ln I_t) + \varepsilon_{t+1}^a. \tag{7}$$

[4] More details on the data are provided in the next section.

where $\psi = \frac{1-\Psi}{\eta}$, which is also the key difference with (3). In a way, the addition of this parameter makes obtaining an empirical measure of R&D intensity, now intended as S_t/I_t^{ψ}, more difficult because it demands the calibration of an additional coefficient on whose value there is limited evidence from the empirical literature. On the other hand, it provides the realism needed to fit the data and the wide support for a stationary $\Delta \ln Z_t$ provides the ground to directly estimate ψ, studying the cointegration relationship between S and I.

To set the level of S_t I will assume a simple stochastic thumb rule that ensures balanced growth. Specifically,

$$S_t = e^{\bar{r}+\tilde{r}_t}\ I_t^{\psi} \qquad , \tag{8}$$

where $\tilde{r}_t$ is stationary, meaning that S_t is a stochastic proportion of I_t^{ψ} fluctuating around the fixed value $e^{\bar{r}+\mathrm{Var}[\tilde{r}]/2}$. The steady state in this economy is on a balanced growth path where output, consumption and R&D investment grow at rate $g = \frac{\psi}{\psi-\xi} g_L$ while TFP grows at rate $g_{\mathrm{TFP}} = \frac{\xi}{\psi-\xi} g_L$. S_t could be actually set in a different manner, but this way of approximating and representing it allows to directly address the fluctuations in conditional expectations of TFP growth, which, depending only on how S_t and N_t^{ψ} relate to each other, are completely described in one process, $\tilde{r}_t$. Following this, I will refer to $\tilde{r}_t$ as 'excess R&D intensity', or 'excess innovative efforts', as in excess of the long-run equilibrium level.

This rule of thumb directly implies cointegration between $\ln S_t$ and $\ln I_t$, with $\tilde{r}_t$ being the error correction term, and implies a neat expression for TFP growth:

$$\Delta \ln Z_{t+1} \approx \mu + \gamma \cdot \tilde{r}_t + \varepsilon^a_{t+1}. \tag{9}$$

This formulation traces very closely the productivity process used in Croce (2014), illustrating the mapping between the typical productivity long-run component x_t and the excess R&D intensity $\tilde{r}$, which has the potential to be a 'long-run *innovation* risk component'. The only missing piece for the two specifications to be completely equivalent concerns the substantial persistence in x_t dynamics, which leaves open the issue of whether $\tilde{r}$ is persistent enough to identify it. I answer this empirically, in the following section.

The thumb rule can naturally be expressed in terms of TFP instead of intangible capital:

$$\ln S_t - \frac{\psi}{\xi} \ln Z_t = \bar{r} + \underbrace{\tilde{r}_t - \frac{\psi}{\xi} a_t}_{\hat{r}_t}, \tag{10}$$

meaning that the residual from the regression of $\ln S_t$ on $\ln Z_t$, $\hat{r}_t$, actually includes the level of any process that affect productivity levels other than intangible capital. In terms of $\hat{r}_t$, TFP growth is then:[5]

$$\Delta \ln Z_{t+1} \approx \mu + \gamma \frac{\psi}{\xi} a_t + \gamma \cdot \hat{r}_t + \varepsilon^a_{t+1}. \tag{11}$$

[5]Full derivation in Appendix 6.

Note that the coefficient of $\tilde{r}$ and $\hat{r}$ is the same.

2.3 The pricing of long-run innovation risk

The key object of study in asset pricing is the Stochastic Discount Factor (SDF) m_t. This is a stochastic process that tracks the growth in marginal utility of investors in a market, thus reflecting the shocks to the economy state variables that are relevant to them. In a typical long-run risk models this takes the form

$$m_{t+1} = \bar{m}_t - b_x \varepsilon_{x,t+1} - b_s \varepsilon_{s,t+1} \tag{12}$$

with $\bar{m}_t$ being its expectations conditional on previous-period information, and $\varepsilon_{x,t+1}$ and $\varepsilon_{s,t+1}$ being innovations that affect consumption marginal utility persistently and transiently with loadings b_x and b_s, respectively.

The SDF plays a relevant role in studying assets price dynamics because, in perfect markets, assets' expected returns in excess of the risk-free return are determined by the level of return innovations exposure to it, as the following holds:

$$\mathbb{E}_t \left[R^i_{t+1}\right] - R^f_t = -R^f_t \cdot \mathrm{Cov}_t \left[m_{t+1}, R^i_{t+1}\right]. \tag{13}$$

I complement these conditions with the following factor structure for returns,

$$R^i_{t+1} = \bar{R}^i_t + \beta^i_x \varepsilon_{x,t+1} + \beta^i_s \varepsilon_{s,t+1} + e^i_{t+1}, \tag{14}$$

where β^i_j is the sensitivity of the return i to shocks of the variable j. A potential driver of heterogeneity in sensitivities to the long-run innovation risk is represented by the firm-specific R&D intensity: evidence from Jiang et al. (2016) shows that R&D spillovers do get priced in financial markets, so it is sensible to hypothesize that fluctuations in the aggregate R&D investment leads to different return dynamics depending on the externality a firm can enjoy. This will be explored in the empirical analysis by looking at the sensitivity distribution over firm-specific-R&D-sorted stocks portfolios, but, as more mechanisms could play a role, there is ground for further investigation. Combining (12), (13) and (14) yields the main reduced-form pricing equation,

$$\mathbb{E}_t \left[R^i_{t+1}\right] - R^f_t = \lambda_x \beta^i_x + \lambda_s \beta^i_s, \tag{15}$$

with λ_j being the so called 'risk premium' associated with risk factor j.

The main take-away of these long-run risk models is that the heavy lifter in explaining stocks risk premium is, by orders of magnitude, the risk of exposure to the factor affecting the marginal utility in a persistent manner, i.e. β^i_x. So, for the sake of presentation clarity, I will proceed focusing on the cross-sectional pricing equation

$$\mathbb{E}_t \left[R^i_{t+1}\right] - R^f_t = \lambda_x \beta^i_x \tag{16}$$

instead. The long-run risk factor in the seminal paper by Bansal and Yaron (2004) is a persistent component of consumption directly; in Croce (2014) this is a persistent component in productivity growth instead; Kung and Schmid (2015) imputes this last one to persistency in R&D investment, which, following the framework of the previous section, should be embedded in $\tilde{r}_t$. I provide evidence of the whole chain, i.e. that persistence in R&D investment plays a relevant role in asset prices determination via productivity growth, I investigate the significance of $\tilde{r}$ as a risk factor in a cross-sectional pricing test. In other words, I set $x_t = \tilde{r}_t$, and study $\lambda_r \beta_r^i$. This is obviously a joint test of the theory and of the empirical identification of $\tilde{r}$, which I additionally address before performing the financial analysis.

Next, consider the decomposition shown in Campbell (1996), where returns innovations can be approximated as the sum of news to cash-flow growth rates and to discount rates:

$$\ln R_{t+1}^i - \mathbb{E}_t \left[\ln R_{t+1}^i\right] = \delta_{D,t+1}^i - \delta_{R,t+1}^i \qquad \text{where}$$

$$\delta_{D,t+1}^i = \{\mathbb{E}_{t+1} - \mathbb{E}_t\} \left[\sum_{j=0}^{\infty} \kappa^j \Delta \ln D_{i,t+j}\right] \quad \text{and} \quad \delta_{R,t+1}^i = \{\mathbb{E}_{t+1} - \mathbb{E}_t\} \left[\sum_{j=1}^{\infty} \kappa^j \ln R_{t+j}^i\right]. \tag{17}$$

Then, as

$$\beta_r^i = \frac{\text{Cov}\left[R_t^i, \tilde{r}_t\right]}{\text{Var}\left[\tilde{r}_t\right]} \approx \frac{\text{Cov}\left[\delta_{D,t}^i, \tilde{r}_t\right]}{\text{Var}\left[\tilde{r}_t\right]} - \frac{\text{Cov}\left[\delta_{R,t}^i, \tilde{r}_t\right]}{\text{Var}\left[\tilde{r}_t\right]} = \beta_{r,D}^i - \beta_{r,R}^i, \tag{18}$$

the cross-sectional pricing equation can be expressed as

$$\mathbb{E}_t \left[R_{t+1}^i\right] - R_t^f = \lambda_r \beta_{r,D}^i - \lambda_r \beta_{r,R}^i. \tag{19}$$

Following Bansal, Dittmar, and Lundblad (2005), being the long-run risk premium a phenomenon that mainly concerns assets fundamentals, rather than conditional discount rates, I focus on the cash-flows' exposure $\beta_{r,D}$ to aggregate R&D intensity, as key dimension of risk to explain excess returns.

3 Empirical long-run excess R&D intensity

The long-run innovation risk is embodied in $\tilde{r}_t$, which is the key object of the analysis. From the theoretical assumption of the allocation rule in (8) or the simple combination of assuming the ideas production function in (6) and the empirical observation of a stationary TFP growth, this could be directly estimated as the error correction term of the cointegration relationship between $\ln S_t$ and $\ln I_t$. I explore this by regressing a measure of ideas stock on a measure of R&D expenditure. The former is built in a spirit similar to Bottazzi and Peri (2007), i.e. recursively adding new patents, from the quarterly series from USPTO, to a depreciated value of past patents stock. The depreciation rate is assumed to be 0.15, a value that is in line with most of the literature, while higher than the one used by Bottazzi and Peri (2007) and lower than the one advocated by Li and Hall (2016). Different values leads to similar conclusions, so they are not shown. The empirical measure of R&D that I employ here, and through out the rest of analysis, is the quarterly private R&D expenditures series expressed in chained 2012 US Dollar prices provided by the Bureau of Economic Analysis in the National Income and Product Accounts tables.[6]

The results of a DOLS estimation of the cointegrating relationship are shown in the first two columns of Table 2, and they are not encouraging: the cointegration coefficient β_S is statistically insignificant with the addition of a time trend and the error correction terms are never stationary. This was to be expected to some extent, as patents are widely considered to be not a good measure of successful innovation, see for example Reeb and Zhao (2020) and Herzer (2022). Therefore, the analysis will focus instead on the error correction term $\hat{r}_t$ of the empirical counterpart of (10),

$$\ln S_t = b_0 + b_1 \ln Z_t + \hat{r}_t. \tag{20}$$

It should be remarked that $\hat{r}_t$, which equals $\tilde{r}_t - \frac{\psi}{\xi} a_t$, does not directly identify the R&D intensity $\tilde{r}$, which is the persistent component in TFP growth conditional expectations. In fact even with fixed $\tilde{r}$, one could still observe fluctuations in $\hat{r}$, with these being due to external factors acting on the level of TFP but not on the growth rate expectations at all. Nonetheless, assuming that a_t is spanned by some available factors $\mathbf{f}_t$, one could still measure the impact of $\tilde{r}$ on expected TFP growth rates, i.e. 'γ' in (9) and (11), by estimating k_r with

$$\Delta Z_{t+1} = k_0 + \mathbf{k}_f' \, \mathbf{f}_t + k_r \hat{r}_t + u_{t+1}. \tag{21}$$

In principle this also allows to explicitly recover $\tilde{r}_t$ by exploiting its definition, $\tilde{r}_t = \hat{r}_t + \frac{\psi}{\xi} a_t$:[7]

$$\tilde{r}_t = \hat{r}_t + \frac{\mathbf{k}_f' \, \mathbf{f}_t}{k_r}. \tag{22}$$

However, as will be shown later, it turns out that $\hat{r}$ is likely to identify $\tilde{r}$ already, for the purposes that are relevant to this project at least. I will now first go through the estimation of $\hat{r}$ and then proceed illustrating its case to be a close approximation of $\tilde{r}$.

Table 2: cointegration results. HAC Standard Errors in parenthesis, computed as advised by Lazarus et al. (2018). BIC values refer to the estimation of the same specifications on a sample where the first 32 observations were trimmed to allow for a fair comparison in model selection. AC(1) is the first autocorrelation estimated as cross-correlation with the lagged value, via 1-step GMM, whose HAC Standard Errors, below in parenthesis, are obtained via Delta-method.

	$\ln I$		$\ln Z$		$\ln Z$ (unadj.)
	(1)	(2)	(1)	(2)	(3)
β_S	0.09^{***}	0.06	0.20^{***}	0.18^{***}	
	(0.01)	(0.11)	(0.01)	(0.03)	
$\beta_{S,T}$					0.17^{***}
					(0.02)
β_{tt}		0.0004		0.0003	0.0005
		(0.0019)		(0.0004)	(0.0003)
J	0	0	0	0	0
K	11	11	14	15	11
BIC	−1087.9	−1088.5	−1095.3	−1092.0	
	$\tilde{r}_t$		$\hat{r}_t$		
Num. obs.	171	171	283	282	283
SD	11.8%	16.72%	18.2%	21.5%	18.8%
ADF u.r. stat	−3.09	−2.85	-3.89^{**}	-3.82^{**}	-3.44^{**}
AC(1)	0.986	0.987	0.979	0.982	0.964
	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

3.1 Estimation of $\hat{r}$

I estimate parameters of (20) via DOLS to avoid imposing any structure on the short-term dynamics. This implies estimating

$$\ln Z_t = \beta_0 + \beta_S \ln S_t + \beta_{tt} t + \sum_{i=-J}^{K} \beta_{\Delta i} \Delta S_{t-i} + u_t. \tag{23}$$

Terms are later re-arranged to form $\hat{r}_t = \ln S_t - \frac{1}{\beta_S} \ln Z_t + \frac{\beta_0}{\beta_S} + \frac{\beta_{tt}}{\beta_S} t$. The TFP series employed to measure Z is the quarterly utilization-adjusted series by Fernald (2012), which, paired with the R&D series, cover from 1947 to 2021. Since R&D is a flow variable that measures expenditures all along the quarter ending at time t, which in principle is continuously chosen by the agents between $t-1$ and t, while TFP level is a stock variable, to match the timing of economy state and economic choices at best, the main specification will refer to Z_t as the interpolated value of TFP between $t-1$ and t. At the same time, ΔZ_{t+1} will simply be the difference between utility-adjusted TFP at time $t+1$ and time t, to make TFP movements

[6] The real series is obtained deflating the nominal R&D series Y006RC of table 5.3.5 by the deflator series Y006RG of table 5.3.4.

[7] Another way is to add the same factors to the cointegration estimation and directly obtain $\tilde{r}$. However this option seems less sensible because a large number of potential regressors leads to poor estimation accuracy and the estimation of an impractical number of regressions to perform a formal model selection.

completely subsequent to any R&D expenditure between time $t-1$ and time t. This peculiar timing structure is a further motivation to estimate the cointegrating parameters using DOLS instead of estimating a full Vector Error Correction Model. As a robustness check, the results from the estimation using the raw TFP series from Fernald (2012) (no utilization adjustment and no timing-adjustment) are also reported in the last column of Table 2, where also a broader measure of R&D is employed - private plus government R&D expenditure. The results are extremely similar, with a cross correlation with the $\hat{r}$ of the specifications marked in Table 2 as (1) and (3) of 0.97%.

The formulation in (23) nests all the specifications tested. As advised by Choi and Kurozumi (2012), numbers of leads and lags are selected independently, i.e. J needs not be equal to K, and the selection is based on the Bayes Information Criterion (BIC). Specifically, leads of ΔS_t turn out to never be significant, so I focus on specifications with $J = 0$ and compare BIC values of the models estimated on a trimmed sample that allows fair comparisons up to $K = 32$ (8 years). Leads of ΔS_t never being significant also motivates keeping Z on the left-hand side: with this formulation all the first-differences of the regressor are lags, making the estimation based on the most recent observations of Z and S levels; vice versa, leaving S as a dependent variable would otherwise make the estimation performed on a dataset without the most recent observations in levels, lost to the missing leads of ΔZ_t. Table 2 shows the estimation results of the best performing specification with and without a time trend.

β_S and unit root ADF statistic of the error correction term are found to be significant, supporting the cointegration of S and Z. The time trend existence, on the other hand, finds little support. Therefore, the preferred specification, which will be employed in the rest of the paper, is the one used in column '$\ln Z(1)$' of Table 2. For this series, the first autocorrelation is 0.979, which fits in the range expected from an AR(1) long-run productivity risk component per Ortu et al. (2013) results – between 0.979 and 0.989. Correlation among the ECTs of the different specification can be seen in appendix at Table 8.

3.2 Forecast the TFP growth

The key property of $\tilde{r}$ is that it is supposed to drive conditional expectations of TFP growth, therefore it should display a strong forecasting ability. Employing $\hat{r}$, this can be tested estimating the regression of equation (21), where factors $\mathbf{f}$ are added as controls to capture the exogenous factor a_t hidden in $\hat{r}$, which can bias the estimates. Specifically, note that in the extreme case in which no controls are considered at all, and one is to estimate univariate regression of future TFP growth on $\hat{r}$, the OLS-estimated slope is obviously expected to be biased.[8] Specifically,

$$\hat{k}_r = k_r \frac{\sigma^2_{\tilde{r}}}{\sigma^2_{\tilde{r}}(1 - \frac{\psi}{\xi} d) + \left(\frac{\psi}{\xi}\right)^2 \sigma^2_a}, \tag{24}$$

[8] Full derivation in Appendix 8.

Table 3: TFP growth forecast regression results. TFP growth is the utilization-adjusted TFP growth from Fernald (2012); controls in (BS) specification are the predictive factors used in Bansal and Shaliastovich (2013) plus market integrated volatility, as in Ai et al. (2018); controls in (LN) specification are the factors computed in Ludvigson and Ng (2009)

	(BS)	(LN)	(uv)
(Intercept)	0.0034^{***}	0.0030^{***}	0.0031^{***}
	(0.0007)	(0.0005)	(0.0005)
$\hat{r}_t$	0.0121^{***}	0.0113^{***}	0.0123^{***}
	(0.0031)	(0.0025)	(0.0026)
p.v. (F_{controls})	21.2%	16.0%	–
R^2	9.18%	8.81%	6.90%
Adj. R^2	7.27%	4.88%	6.58%
Num. obs.	292	243	292

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

where d is the slope coefficient of the auxiliary regression of a_t on $\tilde{r}$, expected to be positive from Kung and Schmid (2015) and left unspecified in section 2.2. Correlation of a_t with TFP growth on the other hand is assumed to be close to 0, following theory in assuming a persistent a_t. It can be seen that $\hat{k}_r$ gets inflated for $\frac{\psi}{\xi}\sigma_a < \rho_{a,\tilde{r}}\sigma_{\tilde{r}}$, i.e. depending on the degree to which variations in $-\frac{\psi}{\xi}a_t$ go to 'compensate' variations of $\tilde{r}$, compressing the volatility of $\hat{r}$ without affecting its covariance with TFP growth rates.

To avoid this, I consider the sets of factors already employed for the same purpose in Ai et al. (2018): the main one is composed by price-dividend ratio, 3-month Treasury-bill yield, 3- and 5-year Treasury bond yields, and integrated volatility of the CRSP stock market index; a back-up one is formed by the 9 factors studied in Ludvigson and Ng (2009). The first set is preferred because it is available for a significantly longer timespan, starting in 1941 against the other one starting in 1970.

The results are in table 3. The most relevant facts are, first, that $\hat{k}_r$ is extremely significant with both sets of controls, and well in the confidence interval of the univariate estimate; second, that the control factors coefficients are jointly insignificant. These results can be interpreted in different ways: (1) both sets of factors are simply poor controls of a_t; (2) a_t is not there; (3) a_t is not really persistent. To see why persistency of a_t is relevant here note that the forecasting coefficient of a_t shown in equation (11) is a fair approximation only if ρ_a is close to 1, otherwise it reads $\rho_a - 1 + \frac{\psi}{\xi}k_1$: this could well be 0 even with a_t being very much alive. However, note that this, considering the previous estimate $\frac{\psi}{\xi}k_1 \approx 6\%$, would imply a quarterly $\rho_a = 0.94$ – a process with a half-life shorter than 3 years. In this case, it is true that $\hat{r}_t$ would not be strictly identifying $\tilde{r}_t$ because it would be determined by both $\tilde{r}_t$ and a_t, but the all of the low-frequency fluctuations in $\hat{r}$, which is what this paper mostly concerns about, would be generated by $\tilde{r}$. Therefore, even in this case, $\hat{r}$ would correctly identify the persistent component originated in R&D intensity for our purposes. For the possibility of both sets of factors being poor controls there are not trivial solutions, other than testing even more sets.

Table 4: estimates of the $\hat{r}$ regression from the VAR. In parenthesis, estimates' standard errors; 'max |roots|' is the maximum eigenvalue of the companion matrix estimated. Sample from 1970 Q2 to 2017 Q4.

	$\hat{r}$	ΔMark-Up	ΔI.C.R.
Lag: 1	1.520^{***}	-0.058	0.146
	(0.072)	(0.131)	(0.259)
Lag: 2	-0.756^{***}	0.008	0.341
	(0.121)	(0.137)	(0.258)
Lag: 3	0.213^{***}	-0.183	0.654^{**}
	(0.071)	(0.131)	(0.258)
T	R^2	$p(F)$	max |roots|
188	0.978	0	0.976

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

3.3 Investigating fluctuations determinants

While proving causal relations are beyond the scope of this paper, it is informative to outline the dynamic relation of R&D excess intensity with other macroeconomic variables. Specifically, R&D investment in Kung and Schmid (2015) is driven by markup level, but it is well known that financial constraints play a role too in the R&D investments dynamics, see for example Brown et al. (2012) and Li (2011). This might matter for both the macroeconomic 'origin' of the long-run innovation risk itself as well as for the determination of assets' sensitivities to this risk. To explore the dynamic relation between R&D intensity, mark-up and funding conditions, I estimate a VAR with endogenous variables being $\hat{r}$, the first principal component of the 5 measures of mark-up from Nekarda and Ramey (2020), which predicts 89% of the series' variance, and the intermediary capital ratio from He et al. (2017). Both the mark-up and the intermediaries funding conditions series result being non-stationary, with the ADF test unit-root statistics of -2.63 and -2.38 respectively; for this reasons I employ their first differences. The number of lags fixed for the VAR is 3, chosen by minimizing the AIC over a sample that allowed for a fair comparison up to 10 quarters. In Table 4 I report the results of the $\hat{r}$ regression.

The inverse root value indicates that the VAR is not too far from an explosive behaviour, but it is still stationary. What is most impressive from these results is that while mark-up does not show any predictive power with respect to R&D excess intensity, intermediaries' capital ratio does, with a highly significant coefficient when lagged thrice. While this is not conclusive, it is suggestive of a role for aggregate funding conditions on R&D and the long-run risk, which calls for deeper research.

4 Cross-sectional risk premium

The key asset pricing implication of swings in R&D intensity generating persistent fluctuations in expected growth rates of the economy, is that asset returns covarying more with R&D

intensity should be regarded as riskier and be held for a higher compensation, i.e. a higher expected excess return. Following Bansal, Dittmar, and Lundblad (2005), this hypothesis is tested in the cross-section of US stocks, by forming portfolios based on stocks sorts that give rise to a documented spread in average excess returns and testing whether the differences in sensitivities of these portfolios' cash flows to aggregate R&D intensity are related to the differences in excess returns in a manner consistent with theory.

4.1 Test assets

Following Bansal, Dittmar, and Lundblad (2005), the set of test assets considered here are all stocks portfolios, 10 based on size sorting, 10 on Book/Market equity sorting, 10 on past-year return sorting and 5 on firm-specific R&D intensity. The R&D-sorted portfolios are less than the other sortings to keep a level of diversification inside the portfolio that is homogeneous with the others, considering the severe under-reporting of R&D expenditures which Koh and Reeb (2015) reports being 42% between 1980 and 2006.

Cash-flows growth rates of each portfolio is computed as in Bansal, Dittmar, and Lundblad (2005). A measure h_t of capital gain is built for each stock and then summed up with those of the other stocks proportionally to the respective portfolio weight, obtaining a portfolio capital gain series $h_{p,t}$. From this series the current value of a dollar invested at the beginning of the series is computed as $V_{p,t+1} = h_{p,t+1}V_t$, where V_t is naturally initialized setting $V_{p,0} = 1$. The measure of cash-flows obtained with such strategy is then $D_{p,t+1} = y_{p,t+1}V_{p,t}$ where $y_{p,t+1}$ is the portfolio dividend-yield, obtained exploiting $R_{p,t} = h_{p,t} + y_{p,t}$. h_t is computed adjusting CRSP ex-dividend returns `RETX` for share repurchases as follows:

$$h_t = \left(\frac{P_{t+1}}{P_t}\right) \cdot \min\left[\left(\frac{n_{t+1}}{n_t}\right), 1\right]. \tag{25}$$

Essentially, capital gains are less than proportional to price appreciation when there is a reduction in (equivalent) shares outstanding, which is likely related to share repurchases, a form of payout not accounted for in dividends records. Then, quarterly dividends series are obtained by simply summing monthly values up and deflating them by the implicit price deflator of nondurable and services consumption shown in Hansen et al. (2005). As the quarterly series still show strong seasonalities, quarterly values are de-seasoned by applying a 4-quarter rolling mean. The series of cash-flows growth rates are then obtained taking the first difference of the log-series of de-seasoned real quarterly dividends.

Monthly stock data is from CRSP, starting at the beginning of 1926 and stopping and the end of 2021. Yearly accounting data is from Compustat Fundamentals dataset, starting in 1950 and ending in 2021. All the monthly returns are compounded to obtain a quarterly figure and then deflated with the same deflator used for dividends. The construction of the portfolios closely follows Bansal, Dittmar, and Lundblad (2005) for comparison purposes; detailed procedure descriptions follow while the main statistics of the formed portfolios' returns and cash-flow growth rates are in table 5.

Table 5: Test asset portfolios returns and cash-flows growth: quarterly summary statistics. All series are from 1947 Q2 to 2022 Q1, a part from the R&D portfolios, which start from 1975 Q1.

Portfolio	Returns Mean	Returns SD	CF growth Mean	CF growth SD
size.01	0.06569	0.18418	0.02767	0.17561
size.02	0.03768	0.15135	0.01470	0.15258
size.03	0.03366	0.14015	0.01166	0.15642
size.04	0.03014	0.13445	0.00962	0.16473
size.05	0.02812	0.13136	0.00491	0.14426
size.06	0.02720	0.11971	0.01022	0.14199
size.07	0.02575	0.11947	0.01026	0.12372
size.08	0.02432	0.11418	0.00699	0.14256
size.09	0.02213	0.10717	0.00659	0.15094
size.10	0.01758	0.09801	0.00241	0.09691
bm.01	0.02476	0.10114	0.02050	0.28804
bm.02	0.02337	0.09135	0.01872	0.25541
bm.03	0.02499	0.08891	0.01845	0.23877
bm.04	0.02297	0.08454	0.01540	0.26606
bm.05	0.02271	0.10876	0.00682	0.14774
bm.06	0.02234	0.10538	0.00496	0.13942
bm.07	0.02118	0.10769	0.00411	0.14106
bm.08	0.02991	0.09674	0.01757	0.22745
bm.09	0.02741	0.11662	0.00933	0.20893
bm.10	0.03312	0.12231	0.01144	0.19516
mom.01	0.01498	0.21583	-0.01330	0.22345
mom.02	0.01176	0.12973	-0.00812	0.16180
mom.03	0.01468	0.11705	-0.00452	0.15323
mom.04	0.01739	0.10650	-0.00042	0.20205
mom.05	0.01824	0.09819	0.00122	0.15589
mom.06	0.01622	0.09966	0.00043	0.15794
mom.07	0.01882	0.09758	0.00126	0.16452
mom.08	0.02378	0.09693	0.00536	0.17665
mom.09	0.02605	0.10352	0.00245	0.26211
mom.10	0.03639	0.12087	-0.00819	0.29443
rd.01	0.02895	0.10367	0.00954	0.15829
rd.02	0.02464	0.08621	0.00528	0.12731
rd.03	0.02935	0.09370	0.01006	0.17154
rd.04	0.03991	0.11387	0.01552	0.16616
rd.05	0.06591	0.19221	0.03406	0.20777

Size-sorted portfolios All firms covered by CRSP are assigned to deciles based on their market capitalization at the end of June of each year relative to NYSE breakpoints. Weights are assigned based on the market capitalization relative to the total capitalization of the portfolio and are re-assigned at the end of every June. Both returns and cash-flows growth display a remarkable reduction for greater-size portfolios, which is in line the usual Small-minus-Big returns spread and cash-flows patterns observed in Bansal, Dittmar, and Lundblad (2005).

B/M-sorted portfolios All firms covered by both CRSP and Compustat are assigned to deciles based on their book to market ratio and NYSE breakpoints. Portfolios are value-weighted and formed at the end of every June, where for year t the book-to-market ratio is based on book equity of fiscal year $t-1$ and market capitalization at the end of calendar year $t-1$. Both portfolio returns and cash-flows growth rates show an increasing pattern with the B/M ratio, in line with previous evidence on the value premium and Bansal, Dittmar, and Lundblad (2005).

Momentum portfolios This set of portfolios employs stocks traded on NYSE or AMEX markets only. The assignment of a stock to a decile portfolio is determined at each end-of-quarter month t and is based the rank of the respective stock compound return from the beginning of month $t-12$ to the end of month $t-1$. These portfolios too are value-weighted. In line with previous evidence both returns and cash-flows increase with momentum, with the exception of the cash-flows growth of the most positive momentum portfolio.

R&D-sorted portfolios Firm-specific R&D intensity has been known to be associated to dispersion in excess returns since Chan et al. (2001). I specifically include these portfolios to provide further evidence that can be relevant in the study of the effects of R&D efforts aggregation. If spillover effects are stronger than fishing-out effects, then one would expect more R&D intensive firms to gain more when the whole economy invests more in R&D and the innovation LRR is higher, which leads to sensitivity heterogeneity along the R&D dimension. To enter these portfolios a stock has to be: of ordinary or common type; traded on either NYSE, AMEX, or NASDAQ; not being of a firm working in the utility or financial sectors; have at least one record of R&D expenditure. Similarly to book-market-ratio sorting, at the end of each June each firm is ranked depending on its own R&D intensity, measured by the ratio of R&D expenditure in the previous fiscal year over market capitalization at the end of the previous calendar year. Then, stocks are value weighted. The data highlights higher returns and higher cash-flows growth for higher firm-specific R&D intensity.

4.2 Time-series sensitivities

As in Bansal, Dittmar, and Lundblad (2005), $\theta_{p,x}$, the sensitivity of portfolio p to a risk factor – the long-run risk component in variable x, is estimated with the following regression:

$$\Delta \ln D_{p,t} = \theta_{p,x} \left(\frac{1}{L} \sum_{l=1}^{L} x_{t-l} \right) + v_{p,t}. \tag{26}$$

Both dependent and independent variables are demeaned before estimation. Estimating the coefficient over the rolling mean of the process x_t has the purpose of filtering persistent components of the regressor that should have a long-lasting impact on cash-flows growth. Indeed, the coefficient is asymptotically equivalent to the one estimated in the regression

$$\frac{1}{L} \sum_{l=1}^{L} \Delta \ln D_{p,t+l} = \theta_{p,x} x_t + v'_{p,t}. \tag{27}$$

Table 6: Test assets cash-flows sensitivity to long-run risk components. From 1975 Q1 to 2022 Q1.

Portfolio	β_C	β_Z	$\beta_{\hat{r}}$	$\beta_{\tilde{r}}$
size.01	1.865	6.873	12.360	9.570
size.02	0.773	7.586	8.042	8.557
size.03	-0.072	1.919	1.013	-7.730
size.04	0.655	3.010	2.808	-3.880
size.05	1.231	2.638	2.087	7.114
size.06	1.104	0.541	0.025	0.411
size.07	1.087	3.675	2.443	-9.770
size.08	1.443	-0.924	1.416	-6.097
size.09	1.011	1.257	-3.608	-16.928
size.10	0.190	-0.688	-0.348	-7.804
bm.01	0.631	7.751	-3.374	-8.638
bm.02	0.902	6.230	-0.130	-4.021
bm.03	1.677	7.184	0.152	-2.351
bm.04	1.105	7.258	-2.307	-3.631
bm.05	0.954	2.592	2.052	-1.085
bm.06	0.144	0.354	-1.179	-4.207
bm.07	-0.166	2.900	-0.417	-12.966
bm.08	0.226	6.503	-2.226	-1.546
bm.09	0.129	0.334	0.536	-11.556
bm.10	0.437	-0.834	1.842	-0.085
mom.01	-0.770	-4.243	-0.953	2.946
mom.02	1.496	-2.634	-0.821	-8.086
mom.03	1.305	-2.806	-3.117	-0.962
mom.04	-0.418	-2.404	0.893	-8.338
mom.05	1.971	-2.639	-2.557	-6.691
mom.06	0.488	-3.783	1.353	-6.335
mom.07	-0.125	0.069	3.677	-8.790
mom.08	-0.204	-4.465	1.471	2.005
mom.09	2.283	-3.785	1.352	2.935
mom.10	1.099	-2.887	-2.676	-20.877
rd.01	0.366	-0.605	-1.988	2.864
rd.02	-1.151	3.810	-1.506	-8.603
rd.03	-1.452	2.862	-2.036	-16.581
rd.04	-0.307	7.867	-0.220	-6.877
rd.05	-0.779	4.454	7.927	11.295

with an inferential advantage in small samples, as illustrated by Hodrick (1992). The long-run risk components studied here are those contained in consumption growth, productivity growth and R&D intensity, i.e. $x \in \{\Delta \ln C,\ \Delta \ln Z,\ \hat{r},\ \tilde{r}\}$, where I also include $\tilde{r}$, the series based on ideas proxied by patents, for robustness. K is fixed to 12, i.e. 3 years, in the main analysis, but results are not significantly different for reasonable changes. Results over the period where all the portfolios are available, i.e. from 1975 to 2022, are shown in table 6.

It can be noted that sensitivities to persistent movements in consumption show a pattern for size and BM portfolios, but not quite as much for momentum and R&D portfolios. Long-run *productivity* risk component produce much starker patterns across all sortings and the long-run *innovation* risk component too. Even more interestingly, the sensitivities to

Table 7: cross-sectional risk premia estimated following Fama and Macbeth (1973). t-statistics are HAC, computed as advised by Lazarus et al. (2018), and corrected for error-in-variable following Shanken (1992). From 1947 Q2 to 2022 Q1.

	C	Z	$\hat{r}$	$\tilde{r}$
intcpt (%)	1.920***	1.621***	1.730***	2.329***
t-stats	(3.899)	(3.225)	(3.625)	(4.450)
lambda (%)	0.015	0.196***	0.315***	0.096***
t-stats	(0.083)	(3.100)	(3.619)	(3.619)
R^2 (%)	0.01	29.12	55.71	24.93

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

R&D intensity increases with firm-specific R&D intensity, meaning that cash-flows of firms investing more in R&D grow more when the whole economy is investing relatively more too. This could support the thesis empirically studied by Jiang et al. (2016) that firms gain from higher R&D investment of peers, here on a economy-wide scale, but changes in payout policies would have to be controlled for in a more formal setting to validate such claim.

4.3 Cross-sectional risk premium

Following Fama and Macbeth (1973), risk premia are estimated with a second-step where each period the returns are regressed on a constant and the risk measure – the cash-flows sensitivities.

Estimates are shown in table 7. The most surprising result is that the premium associated to long-run consumption risk is far from significant. In estimations over different time periods and not shown here, for example exploiting the series from the beginning of its availability in 1947 and ignoring R&D portfolios, it becomes stronger, but over this sample has no grip. On the other hand, results strongly support the existence of a premium for long-run productivity risk, both directly and through the innovation channel, i.e. related to sensitivities of cash-flows to R&D excess intensity. In both cases the premium is significantly different from 0 and the cross-sectional R^2 is remarkable for a single non-traded factor. This is further supported by the premium associated to sensitivity to the R&D intensity measure based on patents being significant too. These results suggest that persistence of innovation originated in R&Dis indeed priced as expected by the long-run risk framework.

5 Conclusion

Persistent fluctuations in consumption are theorized to heavily impact investors welfare and how they price financial assets. These swings have also been shown to be originated in persistent swings in productivity, which has, itself, proven to be strictly related to R&D investments in the economy. This paper defines a relevant and empirically-feasible measure of R&D investment, then estimates it and displays its adherence to theoretical predictions. Specifically, deviations of R&D investment from an equilibrium proportion of TFP level, labelled ‘long-run *innovation* risk component’, prove being persistent, predict productivity growth rates and are associated to a significant risk premium in the cross section for assets whose cash-flows are more sensitive to them. This provides further support to the existence of a long-run risk component and the relevance of the long-run risk framework.

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6 R&D-TFP cointegration

Appendix 6.A In Kung and Schmid (2015)

Using their notation, the starting conditions are:

$$Z_t = \bar{A}(e^{a_t} N_t)^{1-\alpha} \tag{28}$$

$$\frac{N_{t+1}}{N_t} = 1 - \phi + \chi \left(\frac{S_t}{N_t}\right)^{\eta}. \tag{29}$$

Then, the intangible capital growth rate is

$$\Delta \ln N_{t+1} \approx \chi \left(\frac{S_t}{N_t}\right)^{\eta} - \phi \tag{30}$$

$$= \chi \exp\left\{\eta\left(\ln S_t - \ln N_t\right)\right\} - \phi \tag{31}$$

$$= \chi \exp\left\{\eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right\} e^{\eta\bar{r}} - \phi \tag{32}$$

$$\approx \chi e^{\eta\bar{r}}\left\{1 + \eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right\} - \phi \tag{33}$$

$$= \chi e^{\eta\bar{r}}(1 - \eta\bar{r}) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \ln N_t\right) \tag{34}$$

$$= a_N + b_N\left(\ln S_t - \ln N_t\right), \tag{35}$$

and the TFP growth rate, in terms of intangible capital is[9]

$$\frac{Z_{t+1}}{Z_t} = e^{(1-\alpha)(a_{t+1}-a_t)}\left(\frac{N_{t+1}}{N_t}\right)^{(1-\alpha)} \tag{36}$$

$$\Delta \ln Z_{t+1} = (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{N_t}\right)^{\eta}\right] \tag{37}$$

$$\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(\frac{S_t}{N_t}\right)^{\eta} - \phi\right] \tag{38}$$

$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi e^{\eta(\ln S_t - \ln N_t) - \eta\bar{r}} e^{\eta\bar{r}} - \phi\right] \tag{39}$$

$$\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(1 + \eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right) e^{\eta\bar{r}} - \phi\right] \tag{40}$$

$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi e^{\eta\bar{r}}(1 - \eta\bar{r}) - \phi + \chi\eta e^{\eta\bar{r}}\left(\ln S_t - \ln N_t\right)\right]. \tag{41}$$

Expressing this in terms of TFP level, from Equation 37,

$$\Delta \ln Z_{t+1} = (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}\bar{A}^{\frac{1}{\alpha-1}}e^{-a_t}}\right)^{\eta}\right] \tag{42}$$

$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}}\bar{A}^{\frac{1}{1-\alpha}}e^{a_t}\right)^{\eta}\right] \tag{43}$$

$$\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}}\bar{A}^{\frac{1}{1-\alpha}}e^{a_t}\right)^{\eta} - \phi\right] \tag{44}$$

$$\begin{aligned} &= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi\cdot\exp\left(\eta(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t) - \eta\bar{r}\right)e^{\eta\bar{r}} - \phi\right]\end{aligned} \tag{45}$$

$$\begin{aligned} &\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi\left(1 + \eta(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t) - \eta\bar{r}\right)e^{\eta\bar{r}} - \phi\right]\end{aligned} \tag{46}$$

$$\begin{aligned} &= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi e^{\eta\bar{r}}(1-\eta\bar{r}) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t\right)\right]\end{aligned} \tag{47}$$

[9]In this simple formulation the presence of a deterministic trend would surely deteriorate the accuracy of the last approximation but would not necessarily invalidate it, depending on its magnitude. Anyway, as shown in the following analysis, the presence of a time trend is statistically rejected.

$$= (1-\alpha)((\rho - 1 + \chi e^{\eta\bar{r}}\eta)a_t + \varepsilon_{t+1}) + \tag{48}$$

$$+ (1-\alpha)\left[\chi e^{\eta\bar{r}}\left(1 - \eta\bar{r} + \frac{\eta \ \ln \bar{A}}{1-\alpha}\right) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right)\right]$$

$$= (1-\alpha)((\rho - 1 + \chi e^{\eta\bar{r}}\eta)a_t + \varepsilon_{t+1}) + \tag{49}$$

$$+ (1-\alpha)\left[\chi e^{\eta\bar{r}}\eta\left(\frac{1}{\eta} - \bar{r} + \frac{\ln \bar{A}}{1-\alpha}\right) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right)\right]$$

$$= a_Z + b_Z a_t + c_Z \varepsilon_{t+1} + d_Z\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right). \tag{50}$$

Appendix 6.B In my model

The conditions needed for derivation of (7):

$$Z_T \equiv e^{a_t} I_t^{\xi} \tag{51}$$

$$I_{t+1} = (1-\phi)I_t + S_t^{\eta} {I_t}^{\Psi}. \tag{52}$$

Consider the following basic manipulations,

$$\frac{I_{t+1}}{I_t} = 1 - \phi + \left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} \tag{53}$$

$$\Delta \ln I_{t+1} \approx \left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} - \phi \tag{54}$$

$$\ln Z_{t+1} = a_t + \xi \ln I_t \tag{55}$$

$$\Delta \ln Z_{t+1} = (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \Delta \ln I_{t+1} \tag{56}$$

$$\approx (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi\left[\left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} - \phi\right] \tag{57}$$

$$= (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi\left[\exp\left\{\eta(\ln S_t - \psi \ln I_t)\right\} - \phi\right] \tag{58}$$

$$\approx (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi\left[1 + \eta(\ln S_t - \psi \ln I_t) - \phi\right] \tag{59}$$

$$= (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi\left[1 - \phi\right] + \xi\eta(\ln S_t - \psi \ln I_t). \tag{60}$$

Then, assuming $\rho^a \approx 1$,

$$\Delta \ln Z_{t+1} = \gamma_0 + \gamma_1(\ln S_t - \psi \ln I_t) + \varepsilon^a_{t+1}. \tag{61}$$

7 Half-lives

The half-life of the AR(1) process of interest is between 8 and 16 years,

$$\rho_Y^{N_Y} = 0.5 \quad \Rightarrow \quad \frac{\ln(0.5)}{\ln \rho_Y} = N_Y \in [8, 16]. \tag{62}$$

The coefficient ρ_Y such that this is true can range between

$$0.5^{1/8} = 0.9170 < \rho_Y < 0.9576 = 0.5^{1/16}. \tag{63}$$

Quarterly,

$$\rho_Q^{N_Q} = 0.5 \quad \Rightarrow \quad \frac{\ln(0.5)}{\ln \rho_Q} = N_Q \in [32, 64]. \tag{64}$$

So, the AR(1) coefficient can take values

$$0.5^{1/32} = 0.9786 < \rho_Q < 0.9892 = 0.5^{1/64}. \tag{65}$$

8 Forecast regression bias

In case of omitted controls for a_t, the regression reads:

$$\Delta \ln Z_{t+1} = \beta_0 + \beta_{\hat{r}} \hat{r}_t + \hat{w}_{t+1} \tag{66}$$

Then, $\beta_{\hat{r}}$ is estimated as

$$\beta_{\hat{r}} = \frac{\mathrm{Cov}\left[\Delta \ln Z_{t+1}, \tilde{r}_t\right] - \frac{\psi}{\xi}\mathrm{Cov}\left[\Delta \ln Z_{t+1}, a_t\right]}{\mathrm{Var}\left[\tilde{r}\right] + \left(\frac{\psi}{\xi}\right)^2 \mathrm{Var}\left[a_t\right] - \frac{\psi}{\xi}\mathrm{Cov}\left[\tilde{r}_t, a_t\right]} \tag{67}$$

$$= \frac{\mathrm{Cov}\left[\Delta \ln Z_{t+1}, \tilde{r}_t\right]}{\mathrm{Var}\left[\tilde{r}\right]} \frac{1 - \frac{\psi}{\xi}\mathrm{Cov}\left[\Delta \ln Z_{t+1}, a_t\right]/\mathrm{Cov}\left[\Delta \ln Z_{t+1}, \tilde{r}_t\right]}{1 + \left(\frac{\psi}{\xi}\right)^2 \mathrm{Var}\left[a_t\right]/\mathrm{Var}\left[\tilde{r}\right] - \frac{\psi}{\xi}\mathrm{Cov}\left[\tilde{r}_t, a_t\right]/\mathrm{Var}\left[\tilde{r}\right]} \tag{68}$$

$$= \beta_{\tilde{r}} \frac{1 - \frac{\psi}{\xi}\mathrm{Cov}\left[\Delta \ln Z_{t+1}, a_t\right]/\mathrm{Cov}\left[\Delta \ln Z_{t+1}, \tilde{r}_t\right]}{1 + \left(\frac{\psi}{\xi}\right)^2 \mathrm{Var}\left[a_t\right]/\mathrm{Var}\left[\tilde{r}\right] - \frac{\psi}{\xi}\mathrm{Cov}\left[\tilde{r}_t, a_t\right]/\mathrm{Var}\left[\tilde{r}\right]}. \tag{69}$$

Assuming a_t is extremely persistent,[10] $\mathrm{Cov}\left[\Delta \ln Z_{t+1}, a_t\right] \approx 0$. Further, if one assumes that the relation between $\tilde{r}$ and a can be specified as $a_t = d \cdot \tilde{r}_t + w_t$, where w_t are shocks uncorrelated to $\tilde{r}$ and d is expected from theory to be positive,

$$\beta_{\hat{r}} = \beta_{\tilde{r}} \frac{1}{1 + \left(\frac{\psi}{\xi}\right)^2 \mathrm{Var}\left[a_t\right]/\mathrm{Var}\left[\tilde{r}\right] - \frac{\psi}{\xi} d}. \tag{70}$$

So the proportional bias in $\beta_{\hat{r}}$ with respect to $\beta_{\tilde{r}}$ is

$$\frac{\beta_{\hat{r}} - \beta_{\tilde{r}}}{\beta_{\tilde{r}}} = \frac{d - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{r}}}\right)^2}{\frac{\xi}{\psi} + \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{r}}}\right)^2 - d}, \tag{71}$$

[10] In case it is not, bias in the forecasting regression coefficient would be more easily positive, but concerns for pricing implications about using $\hat{r}$ instead of $\tilde{r}$ would alleviate significantly, since the source of persistency of $\hat{r}$ would be more likely $\tilde{r}$ then a_t.

which is positive only in the case

$$0 < d - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{r}}}\right)^2 < \frac{\xi}{\psi} \tag{72}$$

or, considering the OLS estimator of d:

$$0 < \rho_{a,\tilde{r}} - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{r}}}\right) < \frac{\xi}{\psi}\left(\frac{\sigma_{\tilde{r}}}{\sigma_a}\right). \tag{73}$$

9 Additional tables and graphs

Correlations among the R&D intensity measures are in Table 8.

Table 8: correlation among specifications of the ECTs. 't.t.' stands for 'time trend'.

	$\tilde{r}(1)$	$\tilde{r}(2)$	$\hat{r}(1)$	$\hat{r}(2)$	$\hat{r}(1b)$	$\hat{r}(2b)$
$\tilde{r}(1)$	1.000	0.989	0.798	0.817	0.718	0.663
$\tilde{r}(2)$	0.989	1.000	0.765	0.822	0.719	0.675
$\hat{r}(1)$	0.798	0.765	1.000	0.960	0.763	0.723
$\hat{r}(2)$	0.817	0.822	0.960	1.000	0.784	0.761
$\hat{r}(unadj.)$	0.718	0.719	0.763	0.784	1.000	0.969
$\hat{r}(unadj. + t.t.)$	0.663	0.675	0.723	0.761	0.969	1.000