# The Long-run Innovation Risk Component

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## LONG-RUN Innovation RISK Component

Worries about long-run prospects: persistence is key...

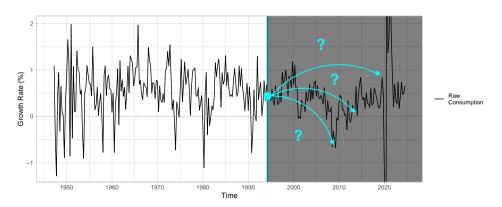


Figure 1: Consumption is US expenditures in services and non-durable from BEA;

## Long-Run Innovation Risk Component

...what persistence?

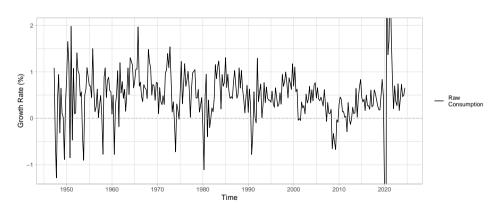


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## Long-Run Innovation Risk COMPONENT

A persistent part, predictable at long horizons (eg  $\sim$ 12y)

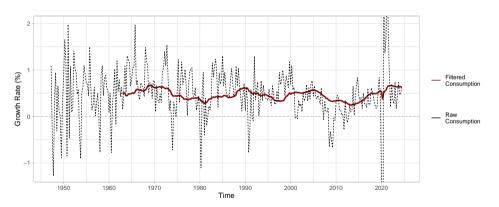


Figure 1: Consumption is US expenditures in services and non-durable from BEA; Ortu et al. (2013) decomposition.

The filtered series are the  $6^{\mbox{th}}$  component of

## Long-Run INNOVATION Risk Component

#### Consumption tracks Total Factor Productivity (TFP) in the medium-long term

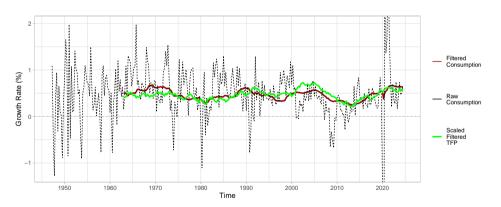


Figure 1: Consumption is US expenditures in services and non-durable from BEA; TFP\* is the utilization-adjusted TFP from Fernald (2012). The filtered series are the 6<sup>th</sup> component of Ortu et al. (2013) decomposition. Cross-correlation between Consumption and TFP is 0.62;

#### LONG-RUN INNOVATION RISK COMPONENT

#### Innovation key driver of productivity growth

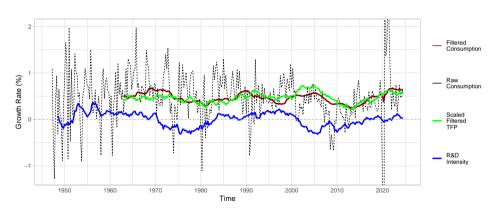


Figure 1: Consumption is US expenditures in services and non-durable from BEA; "TFP" is the utilization-adjusted TFP from Fernald (2012). The filtered series are the 6<sup>th</sup> component of Ortu et al. (2013) decomposition. Cross-correlation between Consumption and TFP is 0.62; with (an arbitrary 1y lag) R&D is 0.60 and 0.53, respectively.

Key element: Aggregate Research and Development (R&D) investment intensity

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R&D intensity process identifies the persistent component of TFP

- stationary but highly persistent
- forecasts TFP and consumption growth

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- stationary but highly persistent
- forecasts TFP and consumption growth
- · associated to a positive risk premium in cross-section of US stocks

(Innovation) LRR framework Bansal and Yaron (2004); Croce (2014); Kung and Schmid (2015); ...

Contribution: supports the framework by direct empirical validation of predictions

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Contribution: studies dynamics of R&D intensity and tests framework on financial markets

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Contribution: studies dynamics of R&D intensity and tests framework on financial markets

Macroeconomic risk factors Lettau and Ludvigson (2001); Bansal et al. (2005); Melone (2021); ...

Contribution: first risk factor related to aggregate R&D

## **Theoretical framework**

### Key ingredients to define R&D intensity

TFP driven by an exogenous process and ideas

$$Z_{t} = e^{a_{t}} \cdot I_{t}^{\xi} \tag{1}$$

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Ideas' dynamics embody semi- AND fully-endogenous production schedule (Jones (1999))

$$I_{t} = (1 - \phi)I_{t-1} + \chi \cdot S_{t-1}^{\eta} I_{t-1}^{\psi} Q_{t-1}^{-\omega}$$
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» R&D key determinant of productivity growth

$$\Delta \ln Z_{t+1} \approx \gamma_0 + \gamma_1 \left( \ln S_t - \frac{1-\psi}{\eta} \ln I_t - \frac{\omega}{\eta} \ln Q_t \right) + \Delta a_t$$
 (3)

» Convenient definition of R&D intensity

$$s_{t} := \ln S_{t} - \frac{1 - \psi}{\eta} \ln I_{t} - \frac{\omega}{\eta} \ln Q_{t}$$
 (4)

Assuming a stationary s<sub>t</sub>

$$\tilde{s}_t = s_t - \bar{s}$$

It drives conditional expectations' fluctuations

$$\mathsf{E}_{\mathsf{t}}\left[\Delta \ln \mathsf{Z}_{\mathsf{t}+1}\right] \approx \mu + \gamma_1 \cdot \tilde{\mathsf{s}}_{\mathsf{t}} + \Delta \mathsf{a}_{\mathsf{t}} \tag{5}$$

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Persistence in R&D makes shocks affect economy for longer

$$\{E_{t+1} - E_t\} \Big(\sum_{j=0}^{\infty} \Delta \ln Z_{t+1+j}\Big)$$
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$$\{E_{t+1} - E_t\} \left( \sum_{j=0}^{\infty} \Delta \ln Z_{t+1+j} \right) = \frac{\rho_s}{1 - \rho_s} \varepsilon_{t+1}^s$$
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Competitive markets:

$$\mathsf{E}_{\mathsf{t}}\left[\mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] - \mathsf{R}_{\mathsf{t}}^{\mathsf{f}} = -\mathsf{R}_{\mathsf{t}}^{\mathsf{f}} \cdot \mathsf{Cov}_{\mathsf{t}}\left[\mathsf{m}_{\mathsf{t}+1}, \mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] \tag{7}$$

Epstein and Zin (1989) preferences (EIS = 1, risk aversion set by  $\theta$ ):

$$m_{t+1} - E_t [m_{t+1}] = -\Delta c_{t+1} - (\theta - 1) \cdot \{E_{t+1} - E_t\} \sum_{j=1}^{\infty} \Delta c_{t+j}$$
 (8)

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Result, testable pricing equation

$$\mathsf{E}_{\mathsf{t}}\left[\mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] - \mathsf{R}_{\mathsf{t}}^{\mathsf{f}} = \lambda_{\mathsf{c}} \; \beta_{\mathsf{c}}^{\mathsf{i}} \; + \; \lambda_{\mathsf{x}} \beta_{\mathsf{x}}^{\mathsf{i}} \tag{10}$$

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» Does  $s_t$  identify a LRR  $x_t$ ?

## The empirical R&D intensity

### Fully-endogenous R&D intensity is non-stationary and too persistent

Table 1: statistics of Kung and Schmid (2015) R&D intensity measure. In the first column original data sources are used; S is yearly R&D expenditure from the National Science Foundation and I is the R&D stock from BLS, spanning 1963 to 2020. Data of second and third columns span 1947 Q1 to 2021 Q4, sources follow.

	$(\ln S_t - \ln I_t)$	$(\ln S_t - \frac{1}{\xi} \ln Z_t)$	
	$\tilde{s}_t$	ŝt	
1 — ξ,	_	0.35	0.3
ADF u.r. stat AC(1)	-2.55 0.989 (0.006)	-2.11 0.999 (0.000)	-2.09 1.000 (0.000)
Num. obs.	57	299	299
***p < 0.01, **p < 0.05, *p < 0.1			

• Non-stationary  $\tilde{s}$  or  $\hat{s}$  can produce spurious forecasting results

lacktriangle  $\Delta$ TFP stationarity

- $\cdot$  Half-life of shocks > 30 years, but innovation component in consumption < 16 years
- » Previous fully-endogenous-based evidence is unreliable

$$\tilde{s}_t = \ln S_t - \frac{1-\psi}{\eta} \ln I_t - \frac{\omega}{\eta} \ln Q_t - \bar{s}$$

I follow the literature:

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$$Q_t = L_t^\kappa \qquad 0 < \kappa < 1$$

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Ideas stock is challenging to identify empirically! TFP more robust

$$\ln I_t = \frac{1}{\xi} \ln Z_t - \frac{1}{\xi} \alpha_t$$

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External factor assumed to be spanned by a set of macro factors f

$$\alpha_t = b^\prime f_t$$

### From the model to the data

$$\tilde{s}_t = \ln S_t - \frac{1-\psi}{\eta\xi} (\ln Z_t - b'f_t) - \frac{\omega\kappa}{\eta} \ln L_t - \bar{s}$$

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Real US R&D expenditure from Bureau of Economic Analysis, quarterly series, spanning 1947 Q1 to 2021 Q4.
 Baseline series: Y006RC, table 5.3.5, deflated by Y006RG, table 5.3.4.

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US TFP estimated by Fernald (2012), quarterly series, spanning 1947 Q2 to 2021 Q4.
 Baseline series: utilization-adjusted series, using capital values without R&D capital.

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Employment Level from Bureau of Labor Statistics, monthly series, spanning 1948-01 to 2024-11.
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- 9 factors from Ludvigson and Ng (2009), monthly series, spanning 1960-03 to 2024-06.

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Real US Consumption from Bureau of Economic Analysis, quarterly series, spanning 1947 Q1 to 2023 Q4.
 Baseline series: non-durable goods (A797RX) plus services (A796RX).

$$\ln Z_t = \alpha_0 + \alpha_S \ln S_t + \alpha_L \ln L_t + \alpha_f' f_t + \varepsilon_t^Z \tag{11}$$

### **DOLS instead of VECM**

· stock-vs-flow variables timing issue

$$\ln Z_t = \alpha_0 + \alpha_S \ln S_t + \alpha_L \ln L_t + \alpha_f' f_t + \varepsilon_t^Z + \sum_{i \in \{S, L, f_1\}}$$
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► More details

### Z as the dependent variable

- use of most recent LEVELS observations  $\,(\hat{Lg}_i>>\hat{Ld}_i\quad\forall i)\,$ 

Table 2: Standard Errors in parenthesis, computed as in Mendes (2011). AC(1) is the coefficient of an AR(1) model fit.

In Z:	Adj TFP	Raw TFP	Adj T	FP
In L:	Tot. e	mp.	Nonfarm emp.	Tot. emp.
f:		BS		LN
$\alpha_{\rm S}$	0.233***	0.269***	0.217***	0.227***
	(0.022)	(0.020)	(0.021)	(0.029)
max lag	20	32	20	10
lags n.	8	19	8	6
max lead	0	4	0	0
leads n.	0	1	0	0
$\alpha_{\rm L}$	-0.098***	-0.261***	-0.046***	-0.085***
	(0.013)	(0.012)	(0.013)	(0.018)
max lag	0	1	0	0
lags n.	0	1	0	0
leads n.	0	0	0	0
tt	F	F	F	F
tt <sup>2</sup>	F	F	F	F
I(1) controls	0	0	0	0
I(0) controls	3	5	3	4
Num. obs.	262	262	262	245
		ŝ	t	
SD	0.149	0.128	0.162	0.139
ADF u.r. stat	-2.51**	-2.66***	-2.36**	-2.23**
KPSS p.v.	0.1+	0.1 +	0.09	0.1 +
AC(1)	0.961	0.954	0.960	0.962
. ,	(0.015)	(0.017)	(0.015)	(0.016)
*** 0 01 **	- 2 25 * 2 1			

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Cross-correlations

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lags n.	0	1	0	0	
leads n.	0	0	0	0	
tt	F	F	F	F	
tt <sup>2</sup>	F	F	F	F	
I(1) controls	0	0	0	0	
I(0) controls	3	5	3	4	
Num. obs.	262	262	262	245	
		ĩ	t		
SD ADF u.r. stat KPSS p.v. AC(1)	0.149 -2.51** 0.1+ 0.961 (0.015)	0.128 -2.66*** 0.1+ 0.954 (0.017)	0.162 -2.36** 0.09 0.960 (0.015)	0.139 -2.23** 0.1+ 0.962 (0.016)	
***p < 0.01, **p	< 0.05,*p < 0.1				

Cross-correlation

Table 2: Standard Errors in parenthesis, computed as in Mendes (2011). AC(1) is the coefficient of an AR(1) model fit.

In Z :	Adj TFP Raw TFP		Adj T	FP	
In L :	Tot. e	mp.	Nonfarm emp.	Tot. emp.	
f:		BS		LN	
$\alpha_{\mathrm{S}}$	0.233***	0.269***	0.217***	0.227***	
	(0.022)	(0.020)	(0.021)	(0.029)	
max lag	20	32	20	10	
lags n.	8	19	8	6	
max lead	0	4	0	0	
leads n.	0	1	0	0	
$\alpha_{\rm L}$	-0.098***	-0.261***	-0.046***	-0.085***	
	(0.013)	(0.012)	(0.013)	(0.018)	
max lag	0	1	0	0	
lags n.	0	1	0	0	
leads n.	0	0	0	0	
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Cross-correlation

# The long-run innovation risk component

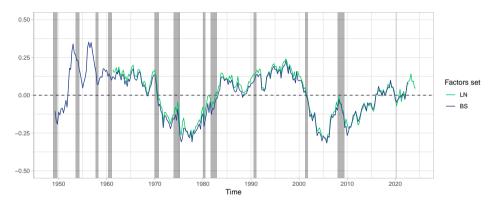


Figure 2: Shaded areas mark NBER recessions. Cross-correlation: 0.990.

# **TFP predictability**

$$\mathsf{E}_{\mathsf{t}}\left[\Delta \ln \mathsf{Z}_{\mathsf{t}+1}\right] = \mu + \gamma_1 \cdot \tilde{\mathsf{s}}_{\mathsf{t}} + \gamma_g' \mathsf{g}_{\mathsf{t}} \tag{12}$$

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Table 3: TFP growth forecast regression results. TFP growth is the utilization-adjusted TFP growth from Fernald (2012); controls in (BS) specification are the predictive factors used in Bansal and Shaliastovich (2013) plus market integrated volatility, as in Ai et al. (2018); controls in (LN) specification are the factors computed in Ludvigson and Ng (2009)

In Z :	Adj T	FP	Raw TFP	Adj T	FP
In L:		Tot. emp.		Nonfarm emp.	Tot. emp.
f:		BS			LN
$\tilde{s}_t$	0.193*** [4.74]	0.171*** [3.91]	0.160** [2.46]	0.194*** [4.80]	0.158*** [3.89]
ARMA	(1,0)	(1,0)	(1,2)	(0,1)	(0,1)
Controls set	BS	LN	BS	BS	LN
p.v. (F <sub>controls</sub> )	0.00%	0.00%	14.46%	0.21%	0.00%
p.v. (LR <sub>controls</sub> )	0.00%	0.00%	8.36%	0.07%	0.00%
$R^2$	10.6%	11.9%	5.7%	10.6%	11.6%
Num. obs.	294	251	294	294	252

<sup>\*\*\*</sup>p < 0.01; \*\*p < 0.05; \*p < 0.1

1 SD shock to  $\tilde{s}_t \approx +0.18\%$  in TFP growth, quarterly

# TFP predictability

$$\mathsf{E}_{\mathsf{t}}\left[\Delta \ln \mathsf{Z}_{\mathsf{t}+1}\right] = \mu + \gamma_{\mathsf{1}} \cdot \tilde{\mathsf{s}}_{\mathsf{t}} + \gamma_{\mathsf{g}}' \mathsf{g}_{\mathsf{t}} \tag{12}$$

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ARMA	(1,0)	(1,0)	(1,2)	(0,1)	(0,1)
Controls set	BS	LN	BS	BS	LN
p.v. (F <sub>controls</sub> )	0.00%	0.00%	14.46%	0.21%	0.00%
p.v. (LR <sub>controls</sub> )	0.00%	0.00%	8.36%	0.07%	0.00%
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Num. obs.	294	251	294	294	252

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# **Consumption predictability**

$$\mathsf{E}_{\mathsf{t}}\left[\Delta \ln \mathsf{C}_{\mathsf{t}+\mathsf{j}}\right] = \pi_{\mathsf{0}} + \pi_{\mathsf{s},\mathsf{j}} \tilde{\mathsf{s}}_{\mathsf{t}} + \pi'_{\mathsf{g},\mathsf{j}} \mathsf{g}_{\mathsf{t}} \tag{13}$$

# **Consumption predictability**

$$E_{t} \left[ \Delta \ln C_{t+j} \right] = \pi_{0} + \pi_{s,j} \tilde{s}_{t} + \pi'_{q,j} g_{t}$$
(13)

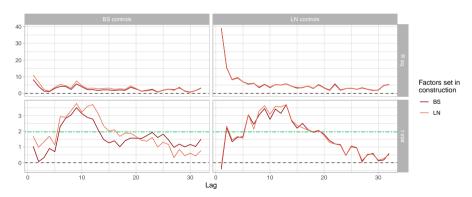


Figure 3: 'BS' stands for controls used in Bansal and Shaliastovich (2013), starting in 1948 Q1; 'LNG' in Ludvigson and Ng (2009), starting in 1960 Q2.

The premium of long-run innovation risk

#### **Financial Data**

#### Returns

- US stocks from Center for Research in Security Prices (CRSP), monthly series, spanning 1926-12 to 2021-12.
  - Quarterly real returns (deflated as in Hansen et al. (2005)), monthly compounding, adjusted for delistings as in Bali et al. (2016)
  - Quarterly cash-flow growth rates, obtained following Bansal et al. (2005) and Hansen et al. (2005)
     Details
- US stocks factors from Global Factor Data (Jensen et al. (2021)), monthly series, 1926-01 to 2024-11. Used series: 118 out of 153, spanning 1951 Q4 to 2023 Q4.

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### **Accounting data**

US firms accounting data from Compustat, yearly series, spanning 1975 Q1 to 2021 Q4.
 Used series: book equity, R&D expenditures.

$$\mathsf{E}_{\mathsf{t}}\left[\mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] - \mathsf{R}_{\mathsf{t}}^{\mathsf{f}} = \lambda_{\mathsf{c}} \; \beta_{\mathsf{c}}^{\mathsf{i}} + \lambda_{\mathsf{x}} \; \beta_{\mathsf{x}}^{\mathsf{i}} \tag{14}$$

Following Bansal et al. (2005)

$$\mathsf{E}_{\mathsf{t}}\left[\mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] - \mathsf{R}_{\mathsf{t}}^{\mathsf{f}} = \lambda_{\mathsf{c}} \; \beta_{\mathsf{c}}^{\mathsf{i}} + \lambda_{\mathsf{x}} \; \beta_{\mathsf{x}}^{\mathsf{i}} \tag{14}$$

Following Bansal et al. (2005)

- Focus on long-run risk only  $\beta_x^{i}$ 

Exploiting Campbell (1996)

$$\text{In } R_{t+1}^i - E_t \left[ \text{In } R_{t+1}^i \right] = \{ E_{t+1} - E_t \} \left[ \sum_{j=0}^{\infty} \kappa^j \Delta \ln D_{\mathfrak{t}, \mathfrak{t}+j} \right] - \{ E_{t+1} - E_t \} \left[ \sum_{j=1}^{\infty} \kappa^j \ln R_{t+j}^i \right]$$

 $\beta^i$ s can be decomposed:

$$\mathsf{E}_{\mathsf{t}}\left[\mathsf{R}_{\mathsf{t}+1}^{\mathsf{i}}\right] - \mathsf{R}_{\mathsf{t}}^{\mathsf{f}} = \lambda_{\mathsf{c}} \; \beta_{\mathsf{c}}^{\mathsf{i}} + \lambda_{\mathsf{x}}(\beta_{\mathsf{x},\mathsf{D}}^{\mathsf{i}} - \beta_{\mathsf{x},\mathsf{R}}^{\mathsf{i}}) \tag{14}$$

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- Focus on long-run risk only  $\beta_x^{\,i}$ 

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$$\text{ln}\,R_{t+1}^i - E_t\left[\text{ln}\,R_{t+1}^i\right] = \{E_{t+1} - E_t\} \bigg[\sum_{j=0}^\infty \kappa^j \Delta \,\text{ln}\,D_{t,t+j}\bigg] - \{E_{t+1} - E_t\} \bigg[\sum_{j=1}^\infty \kappa^j \,\text{ln}\,R_{t+j}^i\bigg]$$

 $\beta^i$ s can be decomposed:

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- Focus on long-run risk only  $\beta_x^{\,i}$
- Focus on cash-flows exposure to long-run risk

Table 4: Test asset portfolios returns and cash-flows growth: quarterly summary statistics. First series are from 1947 Q2 to 2022 Q1, the R&D portfolios start from 1975 Q1.

Portfolio	Returns Mean	Returns SD	CF growth Mean	CF growth SD
size.01	0.06569	0.18418	0.02767	0.17561
size.02	0.03768	0.15135	0.01470	0.15258
size.09	0.02213	0.10717	0.00659	0.15094
size.10	0.01758	0.09801	0.00241	0.09691
bm.01	0.02476	0.10114	0.02050	0.28804
bm.02	0.02337	0.09135	0.01872	0.25541
bm.09	0.02741	0.11662	0.00933	0.20893
bm.10	0.03312	0.12231	0.01144	0.19516
mom.01	0.01498	0.21583	-0.01330	0.22345
mom.02	0.01176	0.12973	-0.00812	0.16180
mom.09	0.02605	0.10352	0.00245	0.26211
mom.10	0.03639	0.12087	-0.00819	0.29443
rd.01	0.02895	0.10367	0.00954	0.15829
rd.02	0.02464	0.08621	0.00528	0.12731
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## Cash-flows sensitivity to long-run risks

$$\Delta \ln D_{i,t} = \beta_{x,D}^{i} \left( \frac{1}{L} \sum_{l=1}^{L} x_{t-l} \right) + \nu_{i,t}$$

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Table 5: Test assets cash-flows sensitivity to long-run risk components. L=16. From 1975 Q1 to 2022 Q1.

Portfolio	β <sub>C</sub>	$\beta_{Z\text{-raw}}$	$\beta_{Z-adj}$	$\beta_{\tilde{s}\text{-BS}}$	$\beta_{\bar{s}\text{-LN}}$
size.01	1.245	4.624	8.145	16.042	18.977
size.02	0.238	7.076	9.422	12.454	13.968
size.09	1.432	7.282	1.317	-3.427	-4.200
size.10	-0.415	4.014	-2.535	-0.894	-0.934
bm.01	-2.126	12.756	6.596	2.437	2.456
bm.02	-1.861	8.248	2.371	4.085	4.683
bm.09	3.041	9.280	5.844	1.936	1.713
bm.10	-0.276	4.359	-5.128	2.698	3.078
mom.01	-5.273	-1.836	-1.620	-3.097	-3.641
mom.02	3.279	5.482	-4.846	-1.532	-1.791
mom.09	9.293	12.116	3.322	1.253	1.023
mom.10	6.509	15.769	-1.913	-2.662	-5.490
rd.01	0.794	4.354	1.465	-3.177	-2.795
rd.02	-1.902	13.573	8.317	4.332	2.802
rd.04	0.125	15.837	12.275	7.127	5.621
rd.05	-1.294	20.648	11.143	15.662	14.056

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mom.10	6.509	15.769	-1.913	-2.662	-5.490
rd.01	0.794	4.354	1.465	-3.177	-2.795
rd.02	-1.902	13.573	8.317	4.332	2.802
rd.04	0.125	15.837	12.275	7.127	5.621
rd.05	-1.294	20.648	11.143	15.662	14.056

Innovative firms' cash-flows grow more when all the economy innovates more: spillovers?

## **Cross-sectional risk premium**

$$E[R^{i}] = \lambda_{0} + \lambda_{x} \cdot \beta_{x,D}^{i}$$
(16)

Table 6: cross-sectional risk premia estimated following Fama and Macbeth (1973). t-statistics are HAC, computed as advised by Lazarus et al. (2018). From 1975 Q1 to 2022 Q1.

	Cons.	Raw TFP	Util+R&D-adj. TFP	$\tilde{s}$ (BS)	ŝ (LN)
		Hor	rizon: 1 year		
λ <sub>0</sub> (%)	6.06***	5.76***	7.05***	7.71***	7.59***
t-stat	[3.11]	[2.89]	[3.89]	[4.18]	[4.13]
$\lambda_{\chi}$ (%)	0.08	0.12	0.24***	0.56***	0.51***
t-stat	[1.49]	[1.61]	[2.60]	[3.88]	[3.67]
MAPE (%)	0.83	0.84	0.86	0.81	0.82
R <sup>2</sup> (%)	1.5	2.5	4.6	31.5	27.8
		Hor	rizon: 8 year		
λ <sub>0</sub> (%)	6.59***	5.70***	7.18***	4.40**	4.62**
t-stat	[3.68]	[3.17]	[4.02]	[2.41]	[2.54]
λ <sub>x</sub> (%)	0.05	0.39***	0.46***	0.72***	0.64***
t-stat	[0.21]	[4.24]	[4.44]	[4.15]	[3.94]
MAPE (%)	0.85	0.72	0.69	0.54	0.57
R <sup>2</sup> (%)	-1.2	14.5	13.3	70.1	66.8

<sup>\*\*\*</sup>p < 0.01, \*\*p < 0.05, \*p < 0.

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## **Omitted-factors robust premium**

Giglio and Xiu (2021) provides a 3-pass procedure to better control for omitted factors.

It relies on Principal Components of a wide cross-section of test assets.

Table 7: risk premia estimation. Optimal number of components p∗ estimated as in Alessi et al. (2010).

	p*	р	Т
PCs n.	9	13	
Test assets R <sup>2</sup>	62.0%	83.4%	289
$\lambda_{\tilde{s} ext{-BS}}$	3.63	6.09*	285
$\lambda_{\tilde{s}\text{-LN}}$	[1.48] 7.14*** [2.94]	[1.82] 10.25*** [3.18]	251

 $<sup>^{***}</sup>p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$ 

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# **Delving deeper**

## On R&D fluctuations' origins

A step towards identification of R&D intensity structural shocks and stricter test of theory

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Markup and funding liquidity often assumed as drivers (Kung and Schmid (2015), Li (2011)):

**Table 8:** estimates of the  $\tilde{s}$  regression from the VAR. In brackets, estimates' t statistics; "max |roots|" is the maximum eigenvalue of the companion matrix estimated.  $\Delta$  mark-up is the 1st principal component of markup differenced series, while Intermediary Capital Ratio (ICR) is from He et al. (2017), sample from 1970 Q2 to 2017 Q4. Lags selected by AIC.

	ŝ	$\Delta$ Mark-Up	I.C.R.
Lag: 1	0.961*** [47.25]	-0.054** [-2.75]	-0.022 [-1.07]
Т	$R^2$	p(F)	max  roots
191	92.9%	0	0.946

<sup>\*\*\*</sup>p < 0.01; \*\*p < 0.05; \*p < 0.1

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Т	$R^2$	p(F)	max  roots
191	92.9%	0	0.946
***p < 0.0	1; **p < 0.05; *p <	0.1	

» more work needed

### **Further extensions**

- · Intra- and inter-sectorial R&D sensitivities
- R&D intensity and aggregate uncertainty:
  - possible strong non-linearities
  - · possible significant role of funding conditions

### **Conclusion**

- Evidence in support of Long-Run Risk framework
- R&D matters for investors and exposure to it is a significant risk measure

### **Conclusion**

- · Evidence in support of Long-Run Risk framework
- R&D matters for investors and exposure to it is a significant risk measure
- R&D fluctuations from 2<sup>nd</sup>-gen endogenous growth models easier to study
- R&D intensity is highly persistent and forecasts TFP and consumption growth
- R&D intensity is associated to a positive risk premium in financial markets

# The Long-run Innovation Risk Component

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# **Appendix**

## TFP growth rate approximation 1

Ideas growth rate from law of motion/production schedule

$$\frac{I_{t}}{I_{t-1}} = 1 - \phi + \chi \left( S_{t-1}^{\eta} I_{t-1}^{(-1+\psi)} Q_{t-1}^{-\omega} \right)$$
 (17)

$$\ln\left(\frac{I_t}{I_{t-1}}\right) \approx -\phi + \chi\left(S_{t-1}^{\eta} I_{t-1}^{(-1+\psi)} Q_{t-1}^{-\omega}\right)$$

$$(l_{t-1})$$

$$= -\phi + v \cdot \exp\left(n \ln S - v - (1 - ib) \ln I\right)$$

$$\ln 7_{+} = a_{+} + \xi \ln I_{+}$$

$$\ln Z_t = a_t + \xi \ln I_t$$

$$\ln Z_t = \alpha_t + \xi \ln I_t$$

$$\ln Z_t = a_t + \xi \ln I_t$$

$$\Delta \ln Z_t = \Delta \alpha_t + \xi \Delta \ln I_t$$

$$\cong \Delta a_{+} + \varepsilon \left[ -\phi + \gamma + \gamma \left( n \ln S_{+-1} - (1 - \psi) \ln I_{+-1} - \omega \ln O_{+-1} \right) \right]$$

$$_{-1}-\omega \ln Q_{t-1})$$

$$\approx -\phi + \chi + \chi \left( \eta \ln S_{t-1} - (1-\psi) \ln I_{t-1} - \omega \ln Q_{t-1} \right)$$

$$= -\phi + \chi \cdot \exp\{\eta \ln S_{t-1} - (1-\psi) \ln I_{t-1} - \omega \ln Q_{t-1}\}\$$

$$\Delta \ln I_{t} \approx -\varphi + \chi \cdot \text{exp} \left\{ \ln \left( S_{t-1}^{\eta} I_{t-1}^{(-1+\psi)} Q_{t-1}^{-\omega} \right) \right\}$$

$$\left(-\left(\frac{2n}{2}\right)^{2}-\left(\frac{1+4y}{2}\right)^{2}-\left(\frac{4y}{2}\right)^{2}\right)$$

$$(-1+\psi)$$
  $(-\psi)$ 

$$\mathbf{n} \, \mathbf{Q}_{t-1}$$

(18)

(19)

(20)

(25)

# TFP growth rate approximation 2

Assuming  $O_t = L_t^{\kappa}$  $\Delta \ln Z_t \cong \Delta \alpha_t + \xi(\chi - \phi) + \xi \chi \left( \eta \ln S_{t-1} - (1 - \psi) \ln I_{t-1} - \omega \kappa \ln L_{t-1} \right)$ (26)

Expressing in term of Z

$$\Delta \ln Z_{t} \approx \Delta a_{t} + \xi(\chi - \phi) + \xi \chi \left( \eta \ln S_{t-1} - \frac{1 - \psi}{\xi} (\ln Z_{t-1} - a_{t-1}) - \omega \kappa \ln L_{t-1} \right)$$
 (27)

Rearranging and assuming  $a_t = \rho_a a_{t-1} + \epsilon_t^a$ 

$$\Delta \ln Z_{t} \approx \xi(\chi - \varphi) + \xi \chi \left( \eta \ln S_{t-1} - \frac{1 - \psi}{\xi} \ln Z_{t-1} - \omega \kappa \ln L_{t-1} \right) + (\rho_{\alpha} - 1 + \chi(1 - \psi)) \alpha_{t-1} + \epsilon_{t}^{\alpha}$$
 (28)

Assuming 
$$(\ln S_t - \frac{1-\psi}{\eta} \ln I_t - \frac{\omega}{\eta} \ln Q_t) = \bar{s} + \tilde{s}_t \sim I(0)$$

$$\bar{z} + \tilde{z}$$
  $z = 1 - \psi$ 

$$\bar{s} + \tilde{s}_t = \eta \ln S_t - \frac{1 - \psi}{\xi} \ln Z_t + \frac{1 - \psi}{\xi} a_t - \omega \kappa \ln L_t$$

$$\ln Z_{t} = \frac{-s\xi}{1-\psi} + \frac{\eta\xi}{1-\psi} \ln S_{t} - \frac{\omega\kappa\xi}{1-\psi} \ln L_{t} + \alpha_{t} + \frac{-\xi}{1-\psi} \tilde{s}_{t}$$
(30)

(29)

(32)

$$\Delta \ln Z_{t+1} = \Delta a_t + \xi(\chi - \phi) + \xi \chi \eta \left(\bar{s} + \tilde{s}_t\right) \tag{31}$$

$$= (\xi(\chi - \phi) + \xi \chi \eta \bar{s}) + \xi \chi \eta \tilde{s}_{t} + \Delta a_{t}$$

## **TFP stationarity**

Table 9: data from Fernald (2012).

	Unadj. TFP	Adj. TFP			
ADF u.r. stat AC(1)	-10.81*** 0.194*** (0.057)	-12.38*** 0.050 (0.058)			
Num. obs.	299	299			
*** $p < 0.01, **p < 0.05, *p < 0.1$					

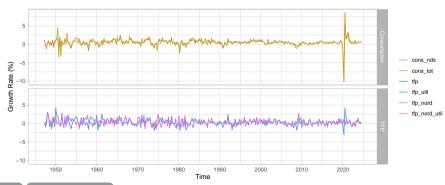


Figure 4: first differences of the macroeconomic series forecasted.

#### AdaLASSO DOLS

1. Start by setting standard maximum number of lags and leads (see Choi and Kurozumi (2012)):

$$Lg = Ld = 12 \times (T/100)^{1/4}$$

2. Perform 10-fold cross-validation to select optimal regularization parameter  $\lambda$  in

$$\min_{ \begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\alpha}_{f} \\ \boldsymbol{\delta} \end{pmatrix} } \sum_{t=1}^{T} \left\{ \ln Z_{t} - \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\alpha}_{f} \end{bmatrix}' \begin{bmatrix} \boldsymbol{l}_{\boldsymbol{n}} S_{t} \\ \boldsymbol{l}_{\boldsymbol{n}} L_{t} \\ \boldsymbol{f}_{t} \end{bmatrix} + \delta' \begin{bmatrix} \Delta S_{t-Lg} \\ \dots \\ \Delta S_{t+Ld} \\ \Delta L_{t-Lg} \\ \dots \\ \Delta (f_{1})_{t-Lg} \\ \dots \\ \Delta (f_{1})_{t+Ld} \end{bmatrix} \right\}^{2} \\ + \lambda \left( \sum_{i} w_{i} \cdot |\delta_{i}| + \sum_{j} w_{j} \cdot |(\alpha_{f})_{i}| \right)$$

- 3. Set  $w_i = |\delta_i|^{-0.9}$  and  $w_i = |(\alpha_f)_i|^{-0.9}$  (initial values set by a preliminary OLS or Ridge Regression)
- 4. Repeat steps 2 and 3 until convergence
- Replicate steps 2, 3 and 4, 999 times; select median model (i.e. the one associated to the median value of  $\lambda$ )
- 6. Repeat steps 2, 3, 4 and 5 by increasing/decreasing lags/leads by 4 (a year) if boundaries are hit/slack

## **Cross-correlations across specifications**

Table 10: correlation among specifications of the ECTs. Naming format: Z-variable, S-variable, Q-variable, factors set.

	nord_util.tot.bs	raw.tot.bs	nord_util.nonfarm.bs	nord_util.tot.ln
raw.tot.bs nord_util.nonfarm.bs nord_util.tot.ln	0.829 0.993 0.990	0.798 0.876	0.99	-



## Dividends growth rate computation

 $D_{\mathfrak{p},t+1} = y_{\mathfrak{p},t+1} V_{\mathfrak{p},t} \quad \text{where} \quad$ 

$$\boldsymbol{\cdot} \ V_{p,t+1} = h_{p,t+1} V_t \quad \text{with} \quad V_{p,0} = 1$$

$$\cdot y_{p,t} = R_{p,t} - h_{p,t}$$

All relies on  $h_{p,t}$ , which is the weighted sum of all portfolios stocks' RETX adjusted for share repurchases as

$$h_{t} = \left(\frac{P_{t+1}}{P_{t}}\right) \cdot \min\left[\left(\frac{n_{t+1}}{n_{t}}\right), 1\right] \tag{33}$$

◆ Back