The long-run innovation risk component

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Abstract

This paper provides evidence that aggregate Research and Development (R&D) intensity drives persistent fluctuations in productivity growth and that it embodies a risk priced in financial markets. The analysis relies on a definition of R&D intensity that is cast in a semi-endogenous growth model, which results in an empirically stationary process, contrary to the fully endogenous case. This allows to reliably document key statistical properties, such as its forecasting power of relevant macroeconomic variables and the significance of a cross-sectional risk premium associated to stocks' cash-flows sensitivities to it.

Keywords: Asset Pricing, Long-run risk, Innovation, Cointegration

JEL Codes: E32, E44, G12, O30

1 Introduction

To reconcile consumption-based asset pricing theory with the data, Bansal and Yaron (2004) focused on a 'small' but persistent component of consumption growth, named the 'long-run risk' (LRR) component. This process can add little variance to consumption growth despite heavily impacting the whole consumption path. Therefore, when coupled with preferences that are sensitive to uncertainty in future consumption expectations, as in Epstein and Zin (1989), it becomes a significant source of risk. Risks of this kind have proven useful in studying various macro-financial phenomena.[1] However, detecting LRR components empirically proves challenging, undermining the validation of mechanisms relying on them and drawing significant criticism towards the entire framework.[2] Given the extensive literature that has developed around the LRR concept, it is essential to provide evidence that supports its establishment: in this paper I contribute by directly documenting a LRR

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[1] For example, exchange rates dynamics as in Colacito and Croce (2011), climate change pricing as in Bansal, Ochoa, et al. (2021), term structures as in Ai et al. (2018), or oil dynamics as in Ready (2018).

[2] Most notably, Beeler and Campbell (2012) and Epstein, Farhi, et al. (2014).

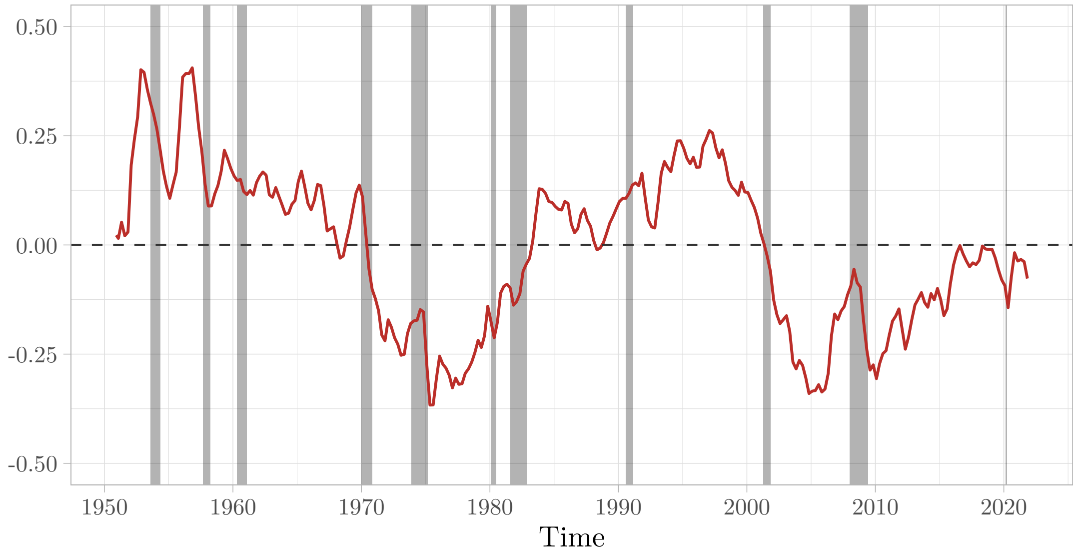

Figure 1: R&D excess intensity, see section 3 for details. Shaded areas mark NBER recessions.

component related to innovation efforts, plotted in figure 1. More specifically, I empirically show that aggregate Research and Development (R&D) investment intensity fulfills several theoretical requirements, by being highly persistent, forecasting consumption and Total Factor Productivity (TFP) growth, and being associated to a positive risk premium in financial markets.

The significance of low-frequency macroeconomic fluctuations, long-run risks, has been previously corroborated either by directly tackling the statistical difficulties in its detection, as done for example by Ortu et al. (2013), Dew-Becker and Giglio (2016) and Schorfheide et al. (2018), or by framing their origin in richer structural models, which provide additional implications to test. Following the latter approach, Kaltenbrunner and Lochstoer (2010) first showed in general equilibrium how the long-run risk component can arise in consumption growth with standard productivity dynamics. Then, Croce (2014) went a step further, providing both theoretical arguments and empirical evidence for a long-run consumption risk component being originated in the persistence of the productivity growth process. This found additional support in Ortu et al. (2013), which empirically found high correlation between the components of consumption and TFP growth rates with half-life within eight and sixteen years. Kung and Schmid (2015) moved one further step upstream, acknowledging the well-established role of R&D in spurring productivity growth and showing how a long-run risk component in consumption could ultimately be driven by an endogenous and persistent aggregate R&D investment intensity, both theoretically and empirically. The empirical evidence they provided to support this claim, however, relied on a measure of R&D intensity that, with updated data, shows undesirable statistical properties, most importantly an apparent non-stationarity. This paper improves on this, providing empirical evidence for a long-run risk originating in R&D efforts that is based on a more reliable R&D intensity measure. The crucial difference from Kung and Schmid (2015) is the definition of R&D

intensity, which in this paper stems from a semi-endogenous growth model rather than a fully endogenous one: in semi-endogenous models R&D and TFP level are approximately cointegrated and the associated Error Correction Term (ECT) reflects the fluctuations in R&D intensity, which drive conditional expectations of productivity growth. As the estimated ECT is stationary, empirical analysis relying on this process are less likely to produce spurious results. A further novelty of this paper with respect to Kung and Schmid (2015) consists of a cross-sectional asset pricing test. This returns a positive and significant risk premium associated to assets' exposure to it, as expected. Building on theoretical arguments put forward by previous research, this paper does not provide any theoretical result concerning why R&D intensity fluctuates as persistently as it does. However, the drivers of fluctuations in R&D are investigated descriptively, with aggregate funding liquidity forecasting R&D intensity more strongly than aggregate mark-up.

The definition of R&D intensity is illustrated exploiting a semi-endogenous 'lab-equipment' R&D growth framework, specifically.[3] This provides enough structure to interpret the cointegrating relation empirically emerging between R&D and TFP, allowing to identify the long-run innovation risk component in R&D fluctuations. From a methodological point of view, as often done in the recent macro-finance literature, the cointegrating relation is estimated with the Dynamic OLS methodology studied in Phillips and Loretan (1991), Saikkonen (1991), and Stock and Watson (1993). In the macroeconomic literature, cointegration methods have already been employed to study the relation between R&D and technological progress in different studies, such as Ha and Howitt (2007), Bottazzi and Peri (2007), and more recently Herzer (2022b) and Kruse-Andersen (2023). These papers are mostly concerned with the assessment of foreign spillovers and the comparison of fully- versus semi-endogenous growth models, with more recent evidence leaning towards semi-endogenous ones, as also backed by Bloom et al. (2020). This paper does not contribute directly to the debate on the modelling comparison, rather it leverages the empirical fit of semi-endogenous growth models to perform a more effective empirical analysis of the LRR mechanism, while testing endogenous growth theory on new grounds, namely that of financial markets. In the financial economics litetarure, cointegration in macroeconomic variables has been widely exploited to price assets, with notable examples in Lettau and Ludvigson (2001) and Melone (2021), but to my knowledge this is the first application to relate aggregate R&D and asset pricing in a cross-sectional analysis. The forecasting exercise of R&D with respect to consumption and TFP growth is carried out with local projections, as standard in the literature, while descriptive investigation of the R&D fluctuations' drivers is performed estimating a VAR system with the first differences of the intermediaries' equity ratio from He et al. (2017) and of the mark-up from Nekarda and Ramey (2020).

On the financial side of the analysis, the most important contribution of the paper is providing empirical evidence that persistent swings in R&D activities is a priced risk in equity markets. This evidence comes from cross-sectional pricing tests pioneered by Bansal, Dittmar,

[3] This class of models was introduced in Romer (1987), and is characterized by the use of units of the final output good to produce ideas, instead of using labor as in more traditional cases à la Romer (1990).

and Lundblad (2005), and developed by Bansal, Dittmar, and Kiku (2009) among others. Like Bansal, Dittmar, and Lundblad (2005), in this paper I focus on the risk premium related to cash-flow growth rates' sensitivities rather than returns' sensitivities because in LRR general equilibrium models the key determinant of risk premia is exposure of dividends to long-run risks, while discount rates may be impacted by many other elements. I obviously depart from Bansal, Dittmar, and Lundblad (2005) by focusing on sensitivities to the estimated R&D intensity rather than to consumption, which ends up showing a stronger risk premium than those related to consumption. I also differ in considering a wider set of test asset portfolios: in particular, I include portfolios sorted on firm-specific R&D. This is interesting because this sorting leads to the greatest dispersion in cash-flow growth rates across portfolios and it is a dimension likely relevant for heterogeneity in sensitivities to aggregate R&D. Indeed, a clear pattern emerge: cash-flows of more R&D intensive firms prove being much more positively sensitive to aggregate R&D intensity, meaning that cash-flows of more R&D-intensive firms grow more when other firms invest more in R&D too. This is in line with both R&D-intensive firms showing higher excess returns and spillover effects being stronger than the fishing-out effect, as previously shown by Jiang et al. (2016). For the cross-sectional pricing test, a traditional Fama and Macbeth (1973) is employed.

The rest is structured as follows: in section 2 I illustrate the theoretical framework, outlining the emergence of a long-run innovation risk component in a semi-endogenous growth models as well as its role in pricing assets; in section 3 I show the first empirical results from the estimation of the R&D intensity measure, and proceed illustrating its proprieties as well as its forecasting power with respect to TFP and consumption, in addition to investigating the relation with markup and funding conditions; in section 4 I carry out the cross-sectional pricing test; in section 5 I conclude.

2 Theoretical framework

2.1 A simple economy with ideas

Consider a discrete-time economy where the production of goods can be represented by a neo-classical production function, such as, without loss of generality,

$$Y_t = Z_t L_t \ , \tag{1}$$

where L_t is the labor employed in production and Z_t is the level of Total Factor Productivity (TFP). Then, following the 'lab-equipment' literature, further assume that such goods can be employed in consumption C_t and R&D expenditures S_t, i.e.

$$Y_t = C_t + S_t \ . \tag{2}$$

As in Kung and Schmid (2015), to focus on the role of innovation in the economy, in this paper TFP is modelled as determined by an exogenous process a_t and an intangible capital

I_t, as follows:

$$Z_t = e^{a_t} I_t^{\xi} \ , \tag{3}$$

with I_t consisting of the stock of ideas that defines the technological frontier of the economy, $\xi > 0$ determining the degree of increasing returns in the economy, and a_t capturing all of the other factors that can affect the productivity level, such as misallocation. The key assumption of this framework, from which the novelty of the results follow, concerns the shape of the law of motion of the intangible capital, which embodies the typical production schedule of new ideas in the semi-endogenous literature:[4]

$$I_t = (1-\phi) I_{t-1} + \chi \cdot S_{t-1}^{\eta} I_{t-1}^{\psi} \ , \tag{4}$$

with $\phi \in [0,1]$ capturing the probability of ideas becoming obsolete; $\chi > 0$ being a scale parameter; $\eta > 0$ capturing duplication effects; and ψ capturing the strenght of spillovers coming from the current stock of ideas, net of fishing-out effects. This formulation nests the law of motion of Kung and Schmid (2015), which is replicated by setting $\psi = 1 - \eta$, but offers greater flexibility, which results being crucial in an empirical application.

The model could be closed with laws of motion for L_t and S_t, and it would be able to describe an economy that endogenously grows on a Balanced Growth Path. Kung and Schmid (2015) solved a fully-endogenous model with fixed labor force that boils down to similar reduced form relations, obtaining an optimal policy rule for S_t, which results being persistent and makes the economy endogenously grow with low-frequency fluctuations. Matching such a complete theoretical description of the economy is out of the scope of this paper, which instead is focused on empirically assess the dynamics of R&D and the fit of the theoretical predictions associated to it.

2.2 The R&D component of productivity growth

Relying on the set-up described above, equations (3) and (4) provide sufficient structure to describe the relation between R&D investments and TFP growth, which also yields a meaningful definition of R&D intensity:

$$\Delta \ln Z_{t+1} \approx \gamma_0 + \gamma_1 \cdot s_t + \Delta a_{t+1}, \tag{5}$$

with $\gamma_0 = \xi(\chi - \phi)$, $\gamma_1 = \xi \chi \eta$, and R&D intensity s_t being defined as

$$s_t = \ln S_t - \frac{1-\psi}{\eta} \ln I_t \ . \tag{6}$$

In words, the growth rate of TFP can be approximated as a linear function of the first difference of the external factors a_{t+1} and of the R&D intensity level of previous period s_t, which is the key variable of this study. Interpreting (5) literally, a stationary TFP growth,

[4] For a broad review of the semi-endogenous growth literature see Jones (2005).

for which there is wide support, and the process describing exogenous factors not being integrated above order 2, as widely assumed, imply s_t being stationary too. This, in turn, would imply two things: (1) TFP growth fluctuations around its unconditional mean are driven by the extent to which R&D intensity exceeds its own unconditional value; (2) this "excess R&D intensity" can be empirically retrieved by studying the cointegration between S_t and I_t. More precisely, defining "excess R&D intensity" $\tilde{s}_t$ as

$$\tilde{s}_t = s_t - \bar{s} \ , \tag{7}$$

the cointegrating relationship between S_t and I_t can be written as

$$\ln S_t = \bar{s} + \frac{1-\psi}{\eta} \ln I_t + \tilde{s}_t \ . \tag{8}$$

Assuming for simplicity, as in Kung and Schmid (2015), that a_t is a highly persistent AR(1) process, meaning that

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \qquad \varepsilon_t^a \sim N(0, \sigma_a^2) \ ; \tag{9}$$

then it is easy to appreciate the crucial role of R&D intensity fluctuations in driving conditional expectations of TFP growth, which with ρ_a close to 1 is determined as

$$\mathbb{E}_t \left[\Delta \ln Z_{t+1}\right] \approx \mu + \gamma_1 \cdot \tilde{s}_t \ . \tag{10}$$

This formulation traces very closely the productivity process used in Croce (2014), illustrating the mapping between the typical productivity long-run component x_t and the excess R&D intensity $\tilde{s}_t$, which has the potential to be a 'long-run *innovation* risk component'. The only missing piece for the two specifications to be completely equivalent concerns the substantial persistence in x_t dynamics, which leaves open the issue of whether $\tilde{s}_t$ is persistent enough to identify it. Kung and Schmid (2015) have theoretically showed how persistence can emergence in an optimization problem, and I test this empirically in the following section. Further assuming an AR(1) structure for s_t,

$$\tilde{s}_t = \rho_s \tilde{s}_{t-1} + \varepsilon_t^s \qquad \varepsilon_t^s \sim N(0, \sigma_s^2) \ , \tag{11}$$

allows to display the key role of persistence in how R&D shocks impact the economy path:

$$\{\mathbb{E}_{t+1} - \mathbb{E}_t\} \Big(\sum_{j=0}^{\infty} \Delta \ln Z_{t+1+j} \Big) = \frac{\rho_s}{1-\rho_s} \varepsilon_{t+1}^s . \tag{12}$$

The greater is the persistence of R&D, reflected in a greater ρ_s, the more a shock to R&D moves the whole expected path of TFP ahead.

Previous studies, among which Herzer (2022a) and Kruse-Andersen (2023), have recognized the difficulty of empirically identifying I_t. The main solution adopted has been to study the key relations employing the TFP level instead of the ideas. This does not actually require the addition of any further assumptions and, plugging (3) in (5) returns the following relation:[5]

$$\Delta \ln Z_{t+1} \approx \mu + \gamma_1 \cdot \hat{s}_t + \left(\rho_a - 1 + \gamma_1 \frac{1-\psi}{\eta\xi} \right) a_t + \varepsilon^a_{t+1}, \tag{13}$$

where

$$\hat{s}_t = \ln S_t - \frac{1-\psi}{\eta\xi} \ln Z_t - \bar{s} \ . \tag{14}$$

Note that $\hat{s}$ is the ECT in the cointegration between $\ln S_t$ and $\ln Z_t$, which can be thought as a cycle-adjusted measure of R&D intensity $\tilde{s}_t - \frac{1-\psi}{\eta\xi} a_t$. However, $\hat{s}_t$ does not only capture fluctuations in R&D intensity, particular care will be needed in its employment instead of $\tilde{s}_t$. Anyway, further notice that the coefficient of $\tilde{s}$ and $\hat{s}$ in the TFP forecasting regression is the same.

2.3 The pricing of long-run innovation risk

In perfect markets, the expected return in excess of the risk-free return of any asset i is determined by the exposure of the return to the 'Stochastic Discount Factor' (SDF) M_t, as the following holds:

$$\mathbb{E}_t \left[R^i_{t+1} \right] - R^f_t = -R^f_t \cdot \mathrm{Cov}_t \left[M_{t+1}, R^i_{t+1} \right] \ . \tag{15}$$

It is then clear that the SDF plays a key role in studying assets prices. This is a stochastic process that tracks the growth in marginal utility of investors in a market, thus reflecting the shocks to the state variables that are relevant to them. As such, asset prices can, in turn, provide insights on investors' view on variables that are relevant outside the financial markets. In a typical long-run risk model, such as Bansal and Yaron (2004), the log-SDF takes the form

$$\ln M_{t+1} = \ln \bar{M}_t - b_x \varepsilon_{x,t+1} - b_c \varepsilon_{c,t+1} \ , \tag{16}$$

with $\ln \bar{M}_t$ being its expectations conditional on previous-period information, $\varepsilon_{c,t+1}$ being the shocks to contemporaneous consumption and $\varepsilon_{x,t+1}$ being shocks that affect consumption persistently, with loadings b_c and b_x, respectively. Then, R&D intensity enters the SDF due to the fact that long-run prospects of consumption are often related to prospects of TFP.[6] Indeed, Croce (2014) has established that shocks of the $\varepsilon_{x,t+1}$ kind can be originated in low-frequencies shocks to TFP growth, with Ortu et al. (2013) more precisely identifying consumption fluctuations with half-life between 8 and 16 years into TFP growth fluctuations

[5] Full derivation in Appendix 6.

[6] For example, consumption level choices in the theoretical model of Blanchard et al. (2013) is directly set to be equal to the long-run productivity expectations.

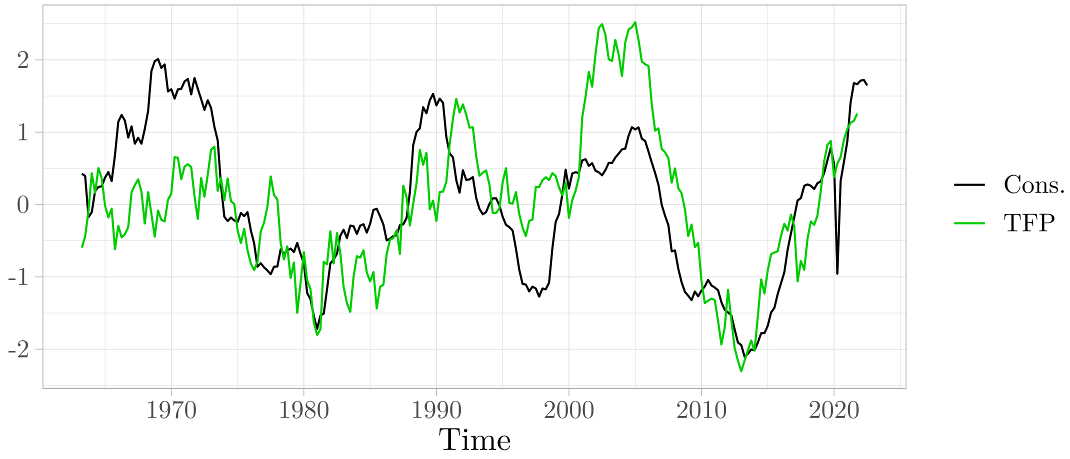

Figure 2: 'Cons.' is US non-durable consumption from FRED, 'TFP' is the utilization-adjusted TFP from Fernald (2012). The joint dataset covers from 1947 Q2 to 2021 Q4. The series plotted are the 6^{th} component of Ortu et al. (2013) filtering method. Cross-correlation between the two series is 0.60.

for US. The latter result is shown with updated data in Figure 2. Then, as illustrated in (12), the more persistent R&D is, the more it will impact TFP growth long-run expectations and, thus, the SDF. Kung and Schmid (2015) showed theoretically how it could be the case that R&D is persistent enough to result being a significant long-run risk, whose factor should be associated to a priced premium in the cross-section of assets.

Complementing these conditions with a standard assumption of factor structure for returns,

$$R^i_{t+1} = \bar{R}^i_t + \beta^i_x \varepsilon_{x,t+1} + \beta^i_s \varepsilon_{s,t+1} + e^i_{t+1}, \tag{17}$$

where β^i_j is the sensitivity of the return i to shocks of the variable j, yields the main reduced-form pricing equation,

$$\mathbb{E}_t \left[R^i_{t+1} \right] - R^f_t = \lambda_x \beta^i_x + \lambda_s \beta^i_s, \tag{18}$$

with λ_j being the so called 'risk premium' associated with risk factor j. This paper will exactly test the significance of a λ_x for x being $\tilde{s}$. This obviously represents a joint test of the theory and of the empirical identification of $\tilde{s}$, which I address before performing the financial analysis, with special emphasis on the persistency. A potential driver of heterogeneity in sensitivities to the long-run innovation risk is represented by the firm-specific R&D intensity: evidence from Jiang et al. (2016) shows that R&D spillovers do get priced in financial markets, so it is sensible to hypothesize that fluctuations in the aggregate R&D investment leads to different return dynamics depending on the externality a firm can enjoy. This will be explored in the empirical analysis by looking at the sensitivity distribution over firm-specific-R&D-sorted stocks portfolios, but, as more mechanisms could play a role, there is ground for further investigation.

Answering to the classical failure of short-term fluctuations in consumption to justify the stock market risk premium, the main take-away of the long-run risk models is that the heavy lifter in explaining stocks risk premium is, by orders of magnitude, the risk of exposure to the LRR factor, i.e. β_x^i. So, as Bansal, Dittmar, and Lundblad (2005), I will proceed focusing on the cross-sectional pricing equation

$$\mathbb{E}_t\left[R_{t+1}^i\right] - R_t^f = \lambda_x \beta_x^i \tag{19}$$

instead. Next, consider the decomposition shown in Campbell (1996), where returns innovations can be approximated as the sum of news to cash-flow growth rates and to discount rates:

$$\ln R_{t+1}^i - \mathbb{E}_t\left[\ln R_{t+1}^i\right] = \delta_{D,t+1}^i - \delta_{R,t+1}^i \qquad \text{where}$$

$$\delta_{D,t+1}^i = \{\mathbb{E}_{t+1} - \mathbb{E}_t\}\left[\sum_{j=0}^{\infty} \kappa^j \Delta \ln D_{i,t+j}\right] \quad \text{and} \quad \delta_{R,t+1}^i = \{\mathbb{E}_{t+1} - \mathbb{E}_t\}\left[\sum_{j=1}^{\infty} \kappa^j \ln R_{t+j}^i\right]. \tag{20}$$

Then, as

$$\beta_r^i = \frac{\text{Cov}\left[R_t^i, \tilde{s}_t\right]}{\text{Var}\left[\tilde{s}_t\right]} \approx \frac{\text{Cov}\left[\delta_{D,t}^i, \tilde{s}_t\right]}{\text{Var}\left[\tilde{s}_t\right]} - \frac{\text{Cov}\left[\delta_{R,t}^i, \tilde{s}_t\right]}{\text{Var}\left[\tilde{s}_t\right]} = \beta_{s,D}^i - \beta_{s,R}^i, \tag{21}$$

the cross-sectional pricing equation can be expressed as

$$\mathbb{E}_t\left[R_{t+1}^i\right] - R_t^f = \lambda_s \beta_{s,D}^i - \lambda_s \beta_{s,R}^i. \tag{22}$$

In the theoretical models that are under scrutiny here, assets' exposure to LRRs is grounded in fundamentals' sensitivities to LRRs, therefore, following Bansal, Dittmar, and Lundblad (2005), I focus on the cash-flows' exposure $\beta_{r,D}$ to aggregate R&D intensity, as key dimension of risk to explain excess returns, rather than conditional discount rates. The key pricing relation that I will test is then

$$\mathbb{E}_t\left[R_{t+1}^i\right] - R_t^f = \lambda_s \beta_{s,D}^i. \tag{23}$$

3 Empirical long-run excess R&D intensity

3.1 The fully-endogenous specification

Kung and Schmid (2015) focus on the specification that returns an R&D intensity in the simple form of the ratio S_t/I_t and form its empirical counterpart as the raw ratio of US annual private R&D expenditure from the National Science Foundation, measuring S_t, over the R&D stock series estimate by the US Bureau of Labor Statistics, representing intangible capital I_t. This measure of R&D intensity proves indeed being highly persistent and co-moving at low frequencies with the price-dividend ratio as well as forecasting the growth rates of consumption, GDP and TFP.

However, this approach has a few potential shortcomings. Indeed, their R&D intensity measure, as in the natural logarithm of S_t/I_t, is *extremely* persistent, with a point estimate of the yearly first autocorrelation equal to 0.987 and a standard error of 0.005.[7] These and other key statistics on the R&D intensity as defined in Kung and Schmid (2015) are in Table 1. It should be noted that the R&D stock series has been updated by the Bureau of Labor Statistics with respect to the one used in their paper and now covers a slightly different time period. Anyway, the 95% confidence interval, which spans from 0.977 to 0.998, highlights two potential issues with the use of this measure: the upper bound, being so close to 1, shows that the process could well be non-stationary while the lower bound is still so high that makes it unlikely for this process to identify a long-run risk component in the economy related to productivity. Sample non-stationarity is not critical to the validity of the measure and the theory it is used to support: from a statistical point of view, R&D intensity is expected to be persistent and the more persistent a process is, the harder it becomes to assess its stationarity in finite samples, so the generating process could still be stationary. Then, even if R&D intensity really was non-stationary the key mechanism studied by Kung and Schmid (2015) could still hold, at the price of a more complex model. Nonetheless, non stationarity of R&D intensity undermines the regressions in which it is employed, as any results could essentially be spurious. Unfortunately, the Augmented Dickey Fuller test with trend delivers for this series a statistic for the unit-root coefficient of -2.21, which is well above the 10% critical value of -3.15, thereby suggesting that the series is highly likely non-stationary. Furthermore, even with enough evidence backing its stationarity, another concerning issue is that this series' first autocorrelation is likely at least 0.977, which implies a half-life of shocks over 30 years for a yearly AR(1) process. This suits the long-run risk component in consumption calibrated by Bansal and Yaron (2004), but it is way more persistent than the component that Ortu et al. (2013) find consumption and productivity to share more strongly in the data, which has half-life between eight and sixteen years, corresponding to a maximum autocorrelation of 0.957 if modelled as a yearly AR(1). Therefore, all in all, this measure could well identify a long-run risk source in the economy, but the empirical evidence for it effectively identifying the long-run productivity risk originated in the R&D investment is fragile.

There are a few details that may drive this measure away from its aim. First, the measure of intangible capital stock used, which in this case is the stock of R&D. This might open a wedge between the model and the data because intangible capital, as shown in (4), is formed in a very different way than simple accumulation and depreciation of R&D expenses – R&D investments unlikely have constant marginal returns, considering duplication and spillover effects. Another related issue is that the production of ideas is likely not to rely solely on domestic R&D expenditure and stock of ideas anyway. This makes it difficult to rely on any measure of intangible capital stock for an empirical analysis because it would require to

[7]The standard error is obtained using the Delta method and 1-step GMM estimates of the fundamental moments.

Table 1: statistics of R&D intensity measure from Kung and Schmid (2015). In the first column, S is yearly R&D expenditure from the National Science Foundation and I is the R&D stock from Bureau of Labor Statistics, spanning 1963 to 2020; in the second and third column, S is quarterly real R&D expenditure from Bureau of Economic Analysis and Z is the quarterly utilization-adjusted TFP from Fernald (2012), spanning from 1947 Q1 to 2021 Q4. ADF u.r. stat is the statistic of the unit root coefficient in an Augmented Dickey-Fuller test with a time trend. AC(1) is the first autocorrelation, estimated as cross-correlation with the lagged value via 1-step GMM, and in parenthesis there are the HAC standard error recovered via Delta-method.

	$(\ln S_t - \ln I_t)$	$(\ln S_t - \frac{1}{1-\alpha}\ln Z_t)$	
α	–	0.35	0.3
ADF u.r. stat	-2.55	-2.11	-2.09
AC(1)	0.989	0.999	1.000
	(0.006)	(0.000)	(0.000)
Num. obs.	57	299	299

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

account for all spillover sources relevant to the formation of new patented ideas and make strong assumptions on the functional form to combine them. A way to bypass this issue could be to utilize directly the variable that the very concept of ideas' stock was born to drive and explain: Total Factor Productivity in the form of Solow residual. This quantity was born in the data and requires little structure to be identified. Table 1 also reports statistics for the $\hat{s}_t$ from (14) as specified in their paper, with similar conclusions to $\tilde{s}_t$. This measure was built using quarterly US R&D expenditure from the Bureau of Economic Analysis and US utility-adjusted TFP estimated by Fernald (2012).[8]

A further point of departure of the theory from reality might be represented by the strong scale effects in the model. As highlighted by Bloom et al. (2020), there is wide evidence for a decreasing research productivity in the data, so this is likely a realistic feature that is necessary for a model to be applied empirically. This is why in the previous section I showed a semi-endogenous framework and, in the following section, I proceed and compute the R&D intensity measure that is prescribed there.

3.2 A semi-endogenous specification

The long-run innovation risk is embodied in $\tilde{s}_t$, which is the key object of the analysis. This can be estimated as the error correction term of the cointegration relationship between $\ln S_t$ and $\ln I_t$. I explore this by regressing a measure of ideas stock on a measure of R&D expenditure. The former is built in a spirit similar to Bottazzi and Peri (2007), i.e. recursively adding new patents, from the quarterly series from USPTO, to a depreciated value of past patents stock. The depreciation rate is assumed to be 0.15, a value that is in line with most of the literature – higher than the one used by Bottazzi and Peri (2007) and lower than the one advocated by Li and Hall (2016). Different values leads to similar conclusions, so they

[8]More details on the data are provided in the next section.

are not shown. The empirical measure of R&D that I employ here, and through out the rest of analysis, is the quarterly private R&D expenditures series expressed in chained 2012 US Dollar prices provided by the Bureau of Economic Analysis in the National Income and Product Accounts tables.[9]

The results of a DOLS estimation of the cointegrating relationship are shown in the first two columns of Table 2, and they are not encouraging: the cointegration coefficient β_S is statistically insignificant with the addition of a time trend and the error correction terms are never stationary. This was to be expected to some extent, as patents are widely considered to be not a good measure of successful innovation – see for example Reeb and Zhao (2020) and Herzer (2022b). Therefore, the analysis will focus instead on the error correction term $\hat{s}_t$ from (14). The empirical model (slightly abusing notation) is then

$$\ln S_t = b_0 + b_1 \ln Z_t + \hat{s}_t. \tag{24}$$

It should be remarked that $\hat{s}_t$, which equals $\tilde{s}_t - \frac{1-\psi}{\eta\xi} a_t$, does not directly identify the R&D intensity $\tilde{s}$, which is the persistent component in TFP growth conditional expectations. In fact even with fixed $\tilde{s}$, one could still observe fluctuations in $\hat{s}$, with these being due to external factors acting on the level of TFP but not on the growth rate expectations at all. Nonetheless, assuming that a_t is spanned by some available factors $\mathbf{f}_t$, one could still measure the impact of $\tilde{s}$ on expected TFP growth rates, i.e. 'γ_1' in (13), by estimating k_s in

$$\Delta Z_{t+1} = k_0 + \mathbf{k}_f' \, \mathbf{f}_t + k_s \hat{s}_t + u_{t+1}. \tag{25}$$

In principle this also allows to explicitly recover $\tilde{s}_t$ by exploiting its definition, $\tilde{s}_t = \hat{s}_t + \frac{1-\psi}{\eta\xi} a_t$, with[10]

$$\tilde{s}_t = \hat{s}_t + \frac{\mathbf{k}_f' \, \mathbf{f}_t}{k_s}. \tag{26}$$

However, as will be shown later, it turns out that $\hat{s}$ is likely to identify $\tilde{s}$ already, for the purposes that are relevant to this project at least. I will now first go through the estimation of $\hat{s}$ and then proceed illustrating its case to be a close approximation of $\tilde{s}$.

3.3 Estimation of $\hat{s}$

I estimate parameters of (24) via DOLS to avoid imposing any structure on the short-term dynamics. This implies estimating

$$\ln Z_t = \beta_0 + \beta_S \ln S_t + \beta_{tt} t + \sum_{i=-J}^{K} \beta_{\Delta i} \Delta S_{t-i} + u_t. \tag{27}$$

[9] The real series is obtained deflating the nominal R&D series Y006RC of table 5.3.5 by the deflator series Y006RG of table 5.3.4.

[10] Another way is to add the same factors to the cointegration estimation and directly obtain $\tilde{s}$. However this option seems less sensible because a large number of potential regressors leads to poor estimation accuracy and the estimation of an impractical number of regressions to perform a formal model selection.

Table 2: cointegration results. HAC Standard Errors in parenthesis, computed as advised by Lazarus et al. (2018). BIC values refer to the estimation of the same specifications on a sample where the first 32 observations were trimmed to allow for a fair comparison in model selection. AC(1) is the first autocorrelation estimated as cross-correlation with the lagged value, via 1-step GMM, whose HAC Standard Errors, below in parenthesis, are obtained via Delta-method.

	$\ln I$		$\ln Z$		$\ln Z$ (unadj.)
	(1)	(2)	(1)	(2)	(3)
β_S	0.09^{***}	0.06	0.20^{***}	0.18^{***}	
	(0.01)	(0.11)	(0.01)	(0.03)	
$\beta_{S,T}$					0.17^{***}
					(0.02)
β_{tt}		0.0004		0.0003	0.0005
		(0.0019)		(0.0004)	(0.0003)
J	0	0	0	0	0
K	11	11	14	15	11
BIC	−1087.9	−1088.5	−1095.3	−1092.0	
	$\tilde{s}_t$		$\hat{s}_t$		
Num. obs.	171	171	283	282	283
SD	11.8%	16.72%	18.2%	21.5%	18.8%
ADF u.r. stat	−3.09	−2.85	-3.89^{**}	-3.82^{**}	-3.44^{**}
AC(1)	0.986	0.987	0.979	0.982	0.964
	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Terms are later re-arranged to form $\hat{s}_t = \ln S_t - \frac{1}{\beta_S} \ln Z_t + \frac{\beta_0}{\beta_S} + \frac{\beta_{tt}}{\beta_S} t$. The TFP series employed to measure Z is the quarterly utilization-adjusted series by Fernald (2012), which, paired with the R&D series, cover from 1947 to 2021. Since R&D is a flow variable that measures expenditures all along the quarter ending at time t, which in principle is continuously chosen by the agents between $t-1$ and t, while TFP level is a stock variable, to match the timing of economy state and economic choices at best, the main specification will refer to Z_t as the interpolated value of TFP between $t-1$ and t. At the same time, ΔZ_{t+1} will simply be the difference between utility-adjusted TFP at time $t+1$ and time t, to make TFP movements completely subsequent to any R&D expenditure between time $t-1$ and time t. This peculiar timing structure is a further motivation to estimate the cointegrating parameters using DOLS instead of estimating a full Vector Error Correction Model. As a robustness check, the results from the estimation using the raw TFP series from Fernald (2012) (no utilization adjustment and no timing-adjustment) are also reported in the last column of Table 2, where also a broader measure of R&D is employed - private plus government R&D expenditure. The results are extremely similar, with a cross correlation with the $\hat{s}$ of the specifications marked in Table 2 as (1) and (3) of 0.97%.

The formulation in (27) nests all the specifications tested. As advised by Choi and Kurozumi (2012), numbers of leads and lags are selected independently, i.e. J needs not be equal to K, and the selection is based on the Bayes Information Criterion (BIC). Specifically, leads of ΔS_t turn out to never be significant, so I focus on specifications with $J = 0$ and compare BIC values of the models estimated on a trimmed sample that allows fair comparisons up to $K = 32$ (8 years). Leads of ΔS_t never being significant also motivates keeping Z on the left-hand side: with this formulation all the first-differences of the regressor are lags, making the estimation based on the most recent observations of Z and S levels; vice versa, leaving S as a dependent variable would otherwise make the estimation performed on a dataset without the most recent observations in levels, lost to the missing leads of ΔZ_t. Table 2 shows the estimation results of the best performing specification with and without a time trend.

β_S and unit root ADF statistic of the error correction term are found to be significant, supporting the cointegration of S and Z. The time trend existence, on the other hand, finds little support. Therefore, the preferred specification, which will be employed in the rest of the paper, is the one used in column '$\ln Z(1)$' of Table 2. For this series, the first autocorrelation is 0.979, which fits in the range expected from an AR(1) long-run productivity risk component per Ortu et al. (2013) results – between 0.979 and 0.989. Correlation among the ECTs of the different specification can be seen in appendix at Table 8.

3.4 Forecast the TFP growth

The key property of $\tilde{s}$ is that it is supposed to drive conditional expectations of TFP growth, therefore it should display a strong forecasting ability. Employing $\hat{s}$, this can be tested estimating the regression of (25), where factors $\mathbf{f}$ are added as controls to capture the exogenous factor a_t hidden in $\hat{s}$, which can bias the estimates. Specifically, note that in the extreme case in which no controls are considered at all, and one is to estimate univariate regression of future TFP growth on $\hat{s}$, the OLS-estimated slope is obviously expected to be biased.[11] Specifically,

$$\hat{k}_s \xrightarrow{p} k_s \frac{\sigma^2_{\tilde{s}}}{\sigma^2_{\tilde{s}}(1 - \frac{\psi}{\xi}d) + \left(\frac{\psi}{\xi}\right)^2 \sigma^2_a}, \tag{28}$$

where d is the slope coefficient of the auxiliary regression of a_t on $\tilde{s}$, expected to be positive from Kung and Schmid (2015) and left unspecified in section 2.2. Correlation of a_t with TFP growth on the other hand is assumed to be close to 0, following theory in assuming a persistent a_t. It can be seen that $\hat{k}_s$ gets inflated for $\frac{\psi}{\xi}\sigma_a < \rho_{a,\tilde{s}}\sigma_{\tilde{s}}$, i.e. depending on the degree to which variations in $-\frac{\psi}{\xi}a_t$ go to 'compensate' variations of $\tilde{s}$, compressing the volatility of $\hat{s}$ without affecting its covariance with TFP growth rates.

To avoid this, I consider the sets of factors already employed for the same purpose in Ai et al. (2018): the main one is composed by price-dividend ratio, 3-month Treasury-bill yield, 3- and 5-year Treasury bond yields, and integrated volatility of the CRSP stock market index; a back-up one is formed by the 9 factors studied in Ludvigson and Ng (2009). The first set is preferred because it is available for a significantly longer timespan, starting in 1941 against the other one starting in 1970.

[11] Full derivation in Appendix 8.

Table 3: TFP growth forecast regression results. TFP growth is the utilization-adjusted TFP growth from Fernald (2012); controls in (BS) specification are the predictive factors used in Bansal and Shaliastovich (2013) plus market integrated volatility, as in Ai et al. (2018); controls in (LN) specification are the factors computed in Ludvigson and Ng (2009)

	(BS)	(LN)	(uv)
(Intercept)	0.0034^{***}	0.0030^{***}	0.0031^{***}
	(0.0007)	(0.0005)	(0.0005)
$\hat{r}_t$	0.0121^{***}	0.0113^{***}	0.0123^{***}
	(0.0031)	(0.0025)	(0.0026)
p.v. (F_{controls})	21.2%	16.0%	–
R^2	9.18%	8.81%	6.90%
Adj. R^2	7.27%	4.88%	6.58%
Num. obs.	292	243	292

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

The results are in table 3. The most relevant facts are, first, that $\hat{k}_s$ is extremely significant with both sets of controls, and well in the confidence interval of the univariate estimate; second, that the control factors coefficients are jointly insignificant. These results can be interpreted in different ways: (1) both sets of factors are simply poor controls of a_t; (2) a_t is not there; (3) a_t is not really persistent. To see why persistency of a_t is relevant here note that the forecasting coefficient of a_t shown in (13) is a fair approximation only if ρ_a is close to 1, otherwise it reads $\rho_a - 1 + \frac{1-\psi}{\eta\xi}k_1$: this could well be 0 even with a_t being very much alive. However, note that this, considering the previous estimate $\frac{1-\psi}{\eta\xi}k_1 \approx 6\%$, would imply a quarterly $\rho_a = 0.94$ – a process with a half-life shorter than 3 years. In this case, it is true that $\hat{s}_t$ would not be strictly identifying $\tilde{s}_t$ because it would be determined by both $\tilde{s}_t$ and a_t. Nonetheless, as $\hat{s}$ shows a high persistency, high-frequency components not related to $\tilde{s}$ cannot drive a significant part of its fluctuations. Therefore, for the purposes of this paper, $\hat{s}$ well identifies the persistent component originated in R&D intensity. For the possibility of both sets of factors being poor controls there are no trivial solutions, other than testing even more sets.

3.5 Forecast consumption growth

Figure 3 reports the t-statistics and the R^2 of the following consumption growth forecasting regression, for different values of j:

$$\Delta \ln C_{t+j} = b_0 + \mathbf{b}'\mathbf{f}_t + b_s\hat{s}_t + w_{t+j}. \tag{29}$$

$\hat{s}_t$ results being significant at 5% level in forecasting consumption growth for both control factors sets. The peak of forecasting power is observed with BS factors, which also allows the longest time coverage of the test and returns the best fit in terms of R^2, at approximately 2 years horizon.

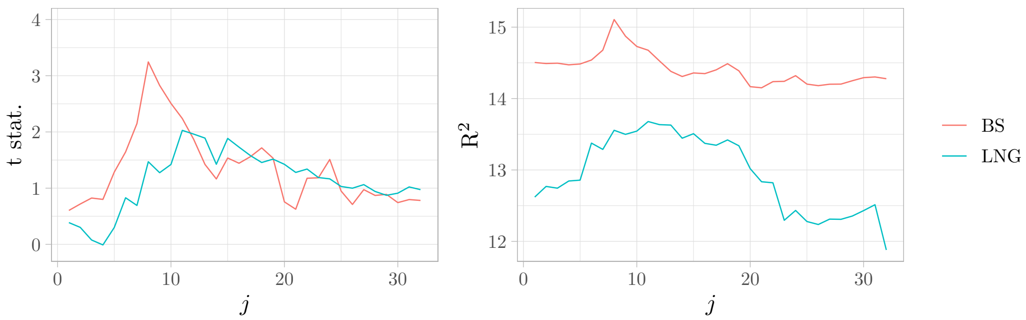

Figure 3: "BS" stands for controls used in Bansal and Shaliastovich (2013), starting in 1950 Q4; "LNG" in Ludvigson and Ng (2009), starting in 1960 Q2.

Table 4: estimates of the $\hat{s}$ regression from the VAR. In parenthesis, estimates' standard errors; 'max |roots|' is the maximum eigenvalue of the companion matrix estimated. Sample from 1970 Q2 to 2017 Q4.

	$\hat{s}$	ΔMark-Up	ΔI.C.R.
Lag: 1	1.520^{***}	-0.058	0.146
	(0.072)	(0.131)	(0.259)
Lag: 2	-0.756^{***}	0.008	0.341
	(0.121)	(0.137)	(0.258)
Lag: 3	0.213^{***}	-0.183	0.654^{**}
	(0.071)	(0.131)	(0.258)
T	R^2	$p(F)$	max |roots|
188	0.978	0	0.976

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

3.6 Investigating fluctuations determinants

While proving causal relations are beyond the scope of this paper, it is informative to outline the dynamic relation of R&D excess intensity with other macroeconomic variables. Specifically, R&D investment in Kung and Schmid (2015) is driven by markup level, but it is well known that financial constraints play a role too in the R&D investments dynamics, see for example Brown et al. (2012) and Li (2011). This might matter for both the macroeconomic 'origin' of the long-run innovation risk itself as well as for the determination of assets' sensitivities to this risk. To explore the dynamic relation between R&D intensity, mark-up and funding conditions, I estimate a VAR with endogenous variables being $\hat{s}$, the first principal component of the 5 measures of mark-up from Nekarda and Ramey (2020), which predicts 89% of the series' variance, and the intermediary capital ratio from He et al. (2017). Both the mark-up and the intermediaries funding conditions series result being non-stationary, with the ADF test unit-root statistics of -2.63 and -2.38 respectively; for this reasons I employ their first differences. The number of lags fixed for the VAR is 3, chosen by minimizing the AIC over a sample that allowed for a fair comparison up to 10 quarters. In Table 4 I report the results of the $\hat{s}$ regression.

The inverse root value indicates that the VAR is not too far from an explosive behaviour, but it is still stationary. What is most impressive from these results is that while mark-up does not show any predictive power with respect to R&D excess intensity, intermediaries' capital ratio does, with a highly significant coefficient when lagged thrice. While this is not conclusive, it is suggestive of a role for aggregate funding conditions on R&D and the long-run risk, which calls for deeper research.

4 Cross-sectional risk premium

The key asset pricing implication of swings in R&D intensity generating persistent fluctuations in expected growth rates of the economy, is that asset returns covarying more with R&D intensity should be regarded as riskier and be held for a higher compensation, i.e. a higher expected excess return. Following Bansal, Dittmar, and Lundblad (2005), this hypothesis is tested in the cross-section of US stocks, by forming portfolios based on stocks sorts that give rise to a documented spread in average excess returns and testing whether the differences in sensitivities of these portfolios' cash flows to aggregate R&D intensity are related to the differences in excess returns in a manner consistent with theory.

4.1 Test assets

Following Bansal, Dittmar, and Lundblad (2005), the set of test assets considered here are all stocks portfolios, 10 based on size sorting, 10 on Book/Market equity sorting, 10 on past-year return sorting and 5 on firm-specific R&D intensity. The R&D-sorted portfolios are less than the other sortings to keep a level of diversification inside the portfolio that is homogeneous with the others, considering the severe under-reporting of R&D expenditures which Koh and Reeb (2015) reports being 42% between 1980 and 2006.

Cash-flows growth rates of each portfolio is computed as in Bansal, Dittmar, and Lundblad (2005). A measure h_t of capital gain is built for each stock and then summed up with those of the other stocks proportionally to the respective portfolio weight, obtaining a portfolio capital gain series $h_{p,t}$. From this series the current value of a dollar invested at the beginning of the series is computed as $V_{p,t+1} = h_{p,t+1}V_t$, where V_t is naturally initialized setting $V_{p,0} = 1$. The measure of cash-flows obtained with such strategy is then $D_{p,t+1} = y_{p,t+1}V_{p,t}$ where $y_{p,t+1}$ is the portfolio dividend-yield, obtained exploiting $R_{p,t} = h_{p,t} + y_{p,t}$. h_t is computed adjusting CRSP ex-dividend returns `RETX` for share repurchases as follows:

$$h_t = \left(\frac{P_{t+1}}{P_t}\right) \cdot \min\left[\left(\frac{n_{t+1}}{n_t}\right), 1\right]. \tag{30}$$

Essentially, capital gains are less than proportional to price appreciation when there is a reduction in (equivalent) shares outstanding, which is likely related to share repurchases, a form of payout not accounted for in dividends records. Then, quarterly dividends series are obtained by simply summing monthly values up and deflating them by the implicit price deflator of nondurable and services consumption shown in Hansen et al. (2005). As the quarterly series still show strong seasonalities, quarterly values are de-seasoned by applying a 4-quarter rolling mean. The series of cash-flows growth rates are then obtained taking the first difference of the log-series of de-seasoned real quarterly dividends.

Monthly stock data is from CRSP, starting at the beginning of 1926 and stopping and the end of 2021. Yearly accounting data is from Compustat Fundamentals dataset, starting in 1950 and ending in 2021. All the monthly returns are compounded to obtain a quarterly figure and then deflated with the same deflator used for dividends. The construction of the portfolios closely follows Bansal, Dittmar, and Lundblad (2005) for comparison purposes; detailed procedure descriptions follow while the main statistics of the formed portfolios' returns and cash-flow growth rates are in table 5.

Size-sorted portfolios All firms covered by CRSP are assigned to deciles based on their market capitalization at the end of June of each year relative to NYSE breakpoints. Weights are assigned based on the market capitalization relative to the total capitalization of the portfolio and are re-assigned at the end of every June. Both returns and cash-flows growth display a remarkable reduction for greater-size portfolios, which is in line the usual Small-minus-Big returns spread and cash-flows patterns observed in Bansal, Dittmar, and Lundblad (2005).

B/M-sorted portfolios All firms covered by both CRSP and Compustat are assigned to deciles based on their book to market ratio and NYSE breakpoints. Portfolios are value-weighted and formed at the end of every June, where for year t the book-to-market ratio is based on book equity of fiscal year $t-1$ and market capitalization at the end of calendar year $t-1$. Both portfolio returns and cash-flows growth rates show an increasing pattern with the B/M ratio, in line with previous evidence on the value premium and Bansal, Dittmar, and Lundblad (2005).

Momentum portfolios This set of portfolios employs stocks traded on NYSE or AMEX markets only. The assignment of a stock to a decile portfolio is determined at each end-of-quarter month t and is based the rank of the respective stock compound return from the beginning of month $t-12$ to the end of month $t-1$. These portfolios too are value-weighted. In line with previous evidence both returns and cash-flows increase with momentum, with the exception of the cash-flows growth of the most positive momentum portfolio.

R&D-sorted portfolios Firm-specific R&D intensity has been known to be associated to dispersion in excess returns since Chan et al. (2001). I specifically include these portfolios to provide further evidence that can be relevant in the study of the effects of R&D efforts aggregation. If spillover effects are stronger than fishing-out effects, then one would expect more R&D intensive firms to gain more when the whole economy invests more in R&D and the innovation LRR is higher, which leads to sensitivity heterogeneity along the R&D dimension. To enter these portfolios a stock has to be: of ordinary or common type; traded

Table 5: Test asset portfolios returns and cash-flows growth: quarterly summary statistics. All series are from 1947 Q2 to 2022 Q1, a part from the R&D portfolios, which start from 1975 Q1.

Portfolio	Returns Mean	Returns SD	CF growth Mean	CF growth SD
size.01	0.06569	0.18418	0.02767	0.17561
size.02	0.03768	0.15135	0.01470	0.15258
size.03	0.03366	0.14015	0.01166	0.15642
size.04	0.03014	0.13445	0.00962	0.16473
size.05	0.02812	0.13136	0.00491	0.14426
size.06	0.02720	0.11971	0.01022	0.14199
size.07	0.02575	0.11947	0.01026	0.12372
size.08	0.02432	0.11418	0.00699	0.14256
size.09	0.02213	0.10717	0.00659	0.15094
size.10	0.01758	0.09801	0.00241	0.09691
bm.01	0.02476	0.10114	0.02050	0.28804
bm.02	0.02337	0.09135	0.01872	0.25541
bm.03	0.02499	0.08891	0.01845	0.23877
bm.04	0.02297	0.08454	0.01540	0.26606
bm.05	0.02271	0.10876	0.00682	0.14774
bm.06	0.02234	0.10538	0.00496	0.13942
bm.07	0.02118	0.10769	0.00411	0.14106
bm.08	0.02991	0.09674	0.01757	0.22745
bm.09	0.02741	0.11662	0.00933	0.20893
bm.10	0.03312	0.12231	0.01144	0.19516
mom.01	0.01498	0.21583	-0.01330	0.22345
mom.02	0.01176	0.12973	-0.00812	0.16180
mom.03	0.01468	0.11705	-0.00452	0.15323
mom.04	0.01739	0.10650	-0.00042	0.20205
mom.05	0.01824	0.09819	0.00122	0.15589
mom.06	0.01622	0.09966	0.00043	0.15794
mom.07	0.01882	0.09758	0.00126	0.16452
mom.08	0.02378	0.09693	0.00536	0.17665
mom.09	0.02605	0.10352	0.00245	0.26211
mom.10	0.03639	0.12087	-0.00819	0.29443
rd.01	0.02895	0.10367	0.00954	0.15829
rd.02	0.02464	0.08621	0.00528	0.12731
rd.03	0.02935	0.09370	0.01006	0.17154
rd.04	0.03991	0.11387	0.01552	0.16616
rd.05	0.06591	0.19221	0.03406	0.20777

on either NYSE, AMEX, or NASDAQ; not being of a firm working in the utility or financial sectors; have at least one record of R&D expenditure. Similarly to book-market-ratio sorting, at the end of each June each firm is ranked depending on its own R&D intensity, measured by the ratio of R&D expenditure in the previous fiscal year over market capitalization at the end of the previous calendar year. Then, stocks are value weighted. The data highlights higher returns and higher cash-flows growth for higher firm-specific R&D intensity.

Table 6: Test assets cash-flows sensitivity to long-run risk components. From 1975 Q1 to 2022 Q1.

Portfolio	β_C	β_Z	$\beta_{\hat{s}}$	$\beta_{\tilde{s}}$
size.01	1.865	6.873	12.360	9.570
size.02	0.773	7.586	8.042	8.557
size.03	-0.072	1.919	1.013	-7.730
size.04	0.655	3.010	2.808	-3.880
size.05	1.231	2.638	2.087	7.114
size.06	1.104	0.541	0.025	0.411
size.07	1.087	3.675	2.443	-9.770
size.08	1.443	-0.924	1.416	-6.097
size.09	1.011	1.257	-3.608	-16.928
size.10	0.190	-0.688	-0.348	-7.804
bm.01	0.631	7.751	-3.374	-8.638
bm.02	0.902	6.230	-0.130	-4.021
bm.03	1.677	7.184	0.152	-2.351
bm.04	1.105	7.258	-2.307	-3.631
bm.05	0.954	2.592	2.052	-1.085
bm.06	0.144	0.354	-1.179	-4.207
bm.07	-0.166	2.900	-0.417	-12.966
bm.08	0.226	6.503	-2.226	-1.546
bm.09	0.129	0.334	0.536	-11.556
bm.10	0.437	-0.834	1.842	-0.085
mom.01	-0.770	-4.243	-0.953	2.946
mom.02	1.496	-2.634	-0.821	-8.086
mom.03	1.305	-2.806	-3.117	-0.962
mom.04	-0.418	-2.404	0.893	-8.338
mom.05	1.971	-2.639	-2.557	-6.691
mom.06	0.488	-3.783	1.353	-6.335
mom.07	-0.125	0.069	3.677	-8.790
mom.08	-0.204	-4.465	1.471	2.005
mom.09	2.283	-3.785	1.352	2.935
mom.10	1.099	-2.887	-2.676	-20.877
rd.01	0.366	-0.605	-1.988	2.864
rd.02	-1.151	3.810	-1.506	-8.603
rd.03	-1.452	2.862	-2.036	-16.581
rd.04	-0.307	7.867	-0.220	-6.877
rd.05	-0.779	4.454	7.927	11.295

4.2 Time-series sensitivities

As in Bansal, Dittmar, and Lundblad (2005), $\theta_{p,x}$, the sensitivity of portfolio p to a risk factor – the long-run risk component in variable x, is estimated with the following regression:

$$\Delta \ln D_{p,t} = \theta_{p,x} \left(\frac{1}{L} \sum_{l=1}^{L} x_{t-l} \right) + v_{p,t}. \tag{31}$$

Both dependent and independent variables are demeaned before estimation. Estimating the coefficient over the rolling mean of the process x_t has the purpose of filtering persistent components of the regressor that should have a long-lasting impact on cash-flows growth.

Indeed, the coefficient is asymptotically equivalent to the one estimated in the regression

$$\frac{1}{L}\sum_{l=1}^{L}\Delta\ln D_{p,t+l} = \theta_{p,x}x_t + v'_{p,t}. \tag{32}$$

with an inferential advantage in small samples, as illustrated by Hodrick (1992). The long-run risk components studied here are those contained in consumption growth, productivity growth and R&D intensity, i.e. $x \in \{\Delta\ln C,\ \Delta\ln Z,\ \hat{s},\ \tilde{s}\}$, where I also include $\tilde{s}$, the series based on ideas proxied by patents, for robustness. K is fixed to 12, i.e. 3 years, in the main analysis, but results are not significantly different for reasonable changes. Results over the period where all the portfolios are available, i.e. from 1975 to 2022, are shown in table 6.

It can be noted that sensitivities to persistent movements in consumption show a pattern for size and BM portfolios, but not quite as much for momentum and R&D portfolios. Long-run *productivity* risk component produce much starker patterns across all sortings and the long-run *innovation* risk component too. Even more interestingly, the sensitivities to R&D intensity increases with firm-specific R&D intensity, meaning that cash-flows of firms investing more in R&D grow more when the whole economy is investing relatively more too. This could support the thesis empirically studied by Jiang et al. (2016) that firms gain from higher R&D investment of peers, here on a economy-wide scale, but changes in payout policies would have to be controlled for in a more formal setting to validate such claim.

4.3 Cross-sectional risk premium

Following Fama and Macbeth (1973), risk premia are estimated with a second-step where each period the returns are regressed on a constant and the risk measure – the cash-flows sensitivities. Estimates are shown in table 7.

The most surprising result is that the premium associated to long-run consumption risk is far from significant. This could be related to known measurement error in consumption series,[12] as well as the predominance of other factors in pricing R&D portfolios. Indeed, in estimations over different time periods not shown here, exploiting the series from the beginning of its availability in 1947 and ignoring R&D portfolios, it becomes stronger. The other results, on the other hand, strongly support the existence of a premium for long-run productivity risk, both directly and through the innovation channel, i.e. related to sensitivities of cash-flows to R&D excess intensity. In both cases the premium is significantly different from 0 and the cross-sectional R^2 is remarkable for a single non-traded factor. This is further supported by the premium associated to sensitivity to the R&D intensity measure based on patents being significant too. These results suggest that persistent innovation originated in R&D is indeed priced, as expected by the long-run risk framework.

[12] See, for example, Savov (2011).

Table 7: cross-sectional risk premia estimated following Fama and Macbeth (1973). t-statistics are HAC, computed as advised by Lazarus et al. (2018), and corrected for error-in-variable following Shanken (1992). From 1947 Q2 to 2022 Q1.

	C	Z	$\hat{s}$	$\tilde{s}$
λ_0 (%)	1.920***	1.621***	1.730***	2.329***
t-stat	(3.899)	(3.225)	(3.625)	(4.450)
λ_x (%)	0.015	0.196***	0.315***	0.096***
t-stat	(0.083)	(3.100)	(3.619)	(3.619)
R^2 (%)	0.01	29.12	55.71	24.93

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

5 Conclusion

Persistent fluctuations in consumption are theorized to heavily impact investors welfare and how they price financial assets. These swings have also been shown to be originated in persistent swings in productivity, which has, itself, proven to be strictly related to R&D investments in the economy. This paper defines a relevant and empirically-feasible measure of R&D investment intensity and its estimates adhere to theoretical predictions. Specifically, deviations of R&D investment from an equilibrium proportion of TFP level, labelled 'long-run *innovation* risk component', prove being persistent, predict productivity growth rates and are associated to a significant risk premium in the cross section for assets whose cash-flows are more sensitive to them. This provides further support to the existence of a long-run risk component and the relevance of the long-run risk framework.

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6 R&D-TFP cointegration

Appendix 6.A In Kung and Schmid (2015)

Using their notation, the starting conditions are:

$$Z_t = \bar{A}(e^{a_t} N_t)^{1-\alpha} \tag{33}$$

$$\frac{N_{t+1}}{N_t} = 1 - \phi + \chi \left(\frac{S_t}{N_t}\right)^{\eta}. \tag{34}$$

Then, the intangible capital growth rate is

$$\begin{aligned}
\Delta \ln N_{t+1} &\approx \chi \left(\frac{S_t}{N_t}\right)^{\eta} - \phi && (35)\\
&= \chi \exp\left\{\eta\left(\ln S_t - \ln N_t\right)\right\} - \phi && (36)\\
&= \chi \exp\left\{\eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right\} e^{\eta\bar{r}} - \phi && (37)\\
&\approx \chi e^{\eta\bar{r}} \left\{1 + \eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right\} - \phi && (38)\\
&= \chi e^{\eta\bar{r}}(1 - \eta\bar{r}) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \ln N_t\right) && (39)\\
&= a_N + b_N\left(\ln S_t - \ln N_t\right), && (40)
\end{aligned}$$

and the TFP growth rate, in terms of intangible capital is[13]

$$\begin{aligned}
\frac{Z_{t+1}}{Z_t} &= e^{(1-\alpha)(a_{t+1}-a_t)} \left(\frac{N_{t+1}}{N_t}\right)^{(1-\alpha)} && (41)\\
\Delta \ln Z_{t+1} &= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{N_t}\right)^{\eta}\right] && (42)\\
&\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(\frac{S_t}{N_t}\right)^{\eta} - \phi\right] && (43)
\end{aligned}$$

[13] In this simple formulation the presence of a deterministic trend would surely deteriorate the accuracy of the last approximation but would not necessarily invalidate it, depending on its magnitude. Anyway, as shown in the following analysis, the presence of a time trend is statistically rejected.

$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi e^{\eta(\ln S_t - \ln N_t) - \eta\bar{r}} e^{\eta\bar{r}} - \phi\right] \tag{44}$$
$$\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(1 + \eta\left(\ln S_t - \ln N_t\right) - \eta\bar{r}\right) e^{\eta\bar{r}} - \phi\right] \tag{45}$$
$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi e^{\eta\bar{r}}(1-\eta\bar{r}) - \phi + \chi\eta e^{\eta\bar{r}}\left(\ln S_t - \ln N_t\right)\right]. \tag{46}$$

Expressing this in terms of TFP level, from Equation 42,

$$\Delta \ln Z_{t+1} = (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}\bar{A}^{\frac{1}{\alpha-1}} e^{-a_t}}\right)^{\eta}\right] \tag{47}$$
$$= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\ln\left[1 - \phi + \chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}}\bar{A}^{\frac{1}{1-\alpha}} e^{a_t}\right)^{\eta}\right] \tag{48}$$
$$\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + (1-\alpha)\left[\chi\left(\frac{S_t}{Z_t^{\frac{1}{1-\alpha}}}\bar{A}^{\frac{1}{1-\alpha}} e^{a_t}\right)^{\eta} - \phi\right] \tag{49}$$
$$\begin{aligned}&= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi\cdot\exp\left(\eta(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t) - \eta\bar{r}\right)e^{\eta\bar{r}} - \phi\right]\end{aligned} \tag{50}$$
$$\begin{aligned}&\approx (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi\left(1 + \eta(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t) - \eta\bar{r}\right)e^{\eta\bar{r}} - \phi\right]\end{aligned} \tag{51}$$
$$\begin{aligned}&= (1-\alpha)((\rho-1)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi e^{\eta\bar{r}}(1-\eta\bar{r}) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t + \frac{\ln\bar{A}}{1-\alpha} + a_t\right)\right]\end{aligned} \tag{52}$$
$$\begin{aligned}&= (1-\alpha)((\rho-1+\chi e^{\eta\bar{r}}\eta)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi e^{\eta\bar{r}}\left(1 - \eta\bar{r} + \frac{\eta\,\ln\bar{A}}{1-\alpha}\right) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right)\right]\end{aligned} \tag{53}$$
$$\begin{aligned}&= (1-\alpha)((\rho-1+\chi e^{\eta\bar{r}}\eta)a_t + \varepsilon_{t+1}) + \\ &\quad + (1-\alpha)\left[\chi e^{\eta\bar{r}}\eta\left(\frac{1}{\eta} - \bar{r} + \frac{\ln\bar{A}}{1-\alpha}\right) - \phi + \chi e^{\eta\bar{r}}\eta\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right)\right]\end{aligned} \tag{54}$$
$$= a_Z + b_Z a_t + c_Z\varepsilon_{t+1} + d_Z\left(\ln S_t - \frac{1}{1-\alpha}\ln Z_t\right). \tag{55}$$

Appendix 6.B In my model

The conditions needed for derivation of (**??**):

$$Z_T \equiv e^{a_t} I_t^{\xi} \tag{56}$$
$$I_{t+1} = (1-\phi)I_t + S_t^{\eta} {I_t}^{\Psi}. \tag{57}$$

Consider the following basic manipulations,

$$\frac{I_{t+1}}{I_t} = 1 - \phi + \left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} \tag{58}$$

$$\Delta \ln I_{t+1} \approx \left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} - \phi \tag{59}$$

$$\ln Z_{t+1} = a_t + \xi \ln I_t \tag{60}$$

$$\begin{aligned} \Delta \ln Z_{t+1} &= (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \Delta \ln I_{t+1} &(61)\\ &\approx (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \left[\left(\frac{S_t}{{I_t}^{\psi}}\right)^{\eta} - \phi\right] &(62)\\ &= (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \left[\exp\left\{\eta(\ln S_t - \psi \ln I_t)\right\} - \phi\right] &(63)\\ &\approx (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \left[1 + \eta(\ln S_t - \psi \ln I_t) - \phi\right] &(64)\\ &= (\rho^a - 1)a_t + \varepsilon^a_{t+1} + \xi \left[1 - \phi\right] + \xi\eta(\ln S_t - \psi \ln I_t). &(65) \end{aligned}$$

Then, assuming $\rho^a \approx 1$,

$$\Delta \ln Z_{t+1} = \gamma_0 + \gamma_1(\ln S_t - \psi \ln I_t) + \varepsilon^a_{t+1}. \tag{66}$$

7 Half-lives

The half-life of the AR(1) process of interest is between 8 and 16 years,

$$\rho_Y^{N_Y} = 0.5 \quad \Rightarrow \quad \frac{\ln(0.5)}{\ln \rho_Y} = N_Y \in [8, 16]. \tag{67}$$

The coefficient ρ_Y such that this is true can range between

$$0.5^{1/8} = 0.9170 < \rho_Y < 0.9576 = 0.5^{1/16}. \tag{68}$$

Quarterly,

$$\rho_Q^{N_Q} = 0.5 \quad \Rightarrow \quad \frac{\ln(0.5)}{\ln \rho_Q} = N_Q \in [32, 64]. \tag{69}$$

So, the AR(1) coefficient can take values

$$0.5^{1/32} = 0.9786 < \rho_Q < 0.9892 = 0.5^{1/64}. \tag{70}$$

8 Forecast regression bias

In case of omitted controls for a_t, the regression reads:

$$\Delta \ln Z_{t+1} = \beta_0 + \beta_{\hat{s}} \hat{s}_t + \hat{w}_{t+1} \tag{71}$$

Then, $\beta_{\hat{s}}$ is estimated as

$$\beta_{\hat{s}} = \frac{\text{Cov}\left[\Delta \ln Z_{t+1}, \tilde{s}_t\right] - \frac{\psi}{\xi}\text{Cov}\left[\Delta \ln Z_{t+1}, a_t\right]}{\text{Var}\left[\tilde{s}\right] + \left(\frac{\psi}{\xi}\right)^2 \text{Var}\left[a_t\right] - \frac{\psi}{\xi}\text{Cov}\left[\tilde{s}_t, a_t\right]} \tag{72}$$

$$= \frac{\text{Cov}\left[\Delta \ln Z_{t+1}, \tilde{s}_t\right]}{\text{Var}\left[\tilde{s}\right]} \frac{1 - \frac{\psi}{\xi}\text{Cov}\left[\Delta \ln Z_{t+1}, a_t\right] / \text{Cov}\left[\Delta \ln Z_{t+1}, \tilde{s}_t\right]}{1 + \left(\frac{\psi}{\xi}\right)^2 \text{Var}\left[a_t\right] / \text{Var}\left[\tilde{s}\right] - \frac{\psi}{\xi}\text{Cov}\left[\tilde{s}_t, a_t\right] / \text{Var}\left[\tilde{s}\right]} \tag{73}$$

$$= \beta_{\tilde{s}} \frac{1 - \frac{\psi}{\xi}\text{Cov}\left[\Delta \ln Z_{t+1}, a_t\right] / \text{Cov}\left[\Delta \ln Z_{t+1}, \tilde{s}_t\right]}{1 + \left(\frac{\psi}{\xi}\right)^2 \text{Var}\left[a_t\right] / \text{Var}\left[\tilde{s}\right] - \frac{\psi}{\xi}\text{Cov}\left[\tilde{s}_t, a_t\right] / \text{Var}\left[\tilde{s}\right]}. \tag{74}$$

Assuming a_t is extremely persistent,[14] $\text{Cov}\left[\Delta \ln Z_{t+1}, a_t\right] \approx 0$. Further, if one assumes that the relation between $\tilde{s}$ and a can be specified as $a_t = d \cdot \tilde{s}_t + w_t$, where w_t are shocks uncorrelated to $\tilde{s}$ and d is expected from theory to be positive,

$$\beta_{\hat{s}} = \beta_{\tilde{s}} \frac{1}{1 + \left(\frac{\psi}{\xi}\right)^2 \text{Var}\left[a_t\right] / \text{Var}\left[\tilde{s}\right] - \frac{\psi}{\xi} d}. \tag{75}$$

So the proportional bias in $\beta_{\hat{s}}$ with respect to $\beta_{\tilde{s}}$ is

$$\frac{\beta_{\hat{s}} - \beta_{\tilde{s}}}{\beta_{\tilde{s}}} = \frac{d - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{s}}}\right)^2}{\frac{\xi}{\psi} + \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{s}}}\right)^2 - d}, \tag{76}$$

which is positive only in the case

$$0 < d - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{s}}}\right)^2 < \frac{\xi}{\psi} \tag{77}$$

or, considering the OLS estimator of d:

$$0 < \rho_{a,\tilde{s}} - \frac{\psi}{\xi}\left(\frac{\sigma_a}{\sigma_{\tilde{s}}}\right) < \frac{\xi}{\psi}\left(\frac{\sigma_{\tilde{s}}}{\sigma_a}\right). \tag{78}$$

9 Additional tables and graphs

Correlations among the R&D intensity measures are in Table 8.

[14] In case it is not, bias in the forecasting regression coefficient would be more easily positive, but concerns for pricing implications about using $\hat{s}$ instead of $\tilde{s}$ would alleviate significantly, since the source of persistency of $\hat{s}$ would be more likely $\tilde{s}$ then a_t.

Table 8: correlation among specifications of the ECTs. 't.t.' stands for 'time trend'.

	$\tilde{s}(1)$	$\tilde{s}(2)$	$\hat{s}(1)$	$\hat{s}(2)$	$\hat{s}(1b)$	$\hat{s}(2b)$
$\tilde{s}(1)$	1.000	0.989	0.798	0.817	0.718	0.663
$\tilde{s}(2)$	0.989	1.000	0.765	0.822	0.719	0.675
$\hat{s}(1)$	0.798	0.765	1.000	0.960	0.763	0.723
$\hat{s}(2)$	0.817	0.822	0.960	1.000	0.784	0.761
$\hat{s}(unadj.)$	0.718	0.719	0.763	0.784	1.000	0.969
$\hat{s}(unadj. + t.t.)$	0.663	0.675	0.723	0.761	0.969	1.000