Energy efficient resource allocation problem formulation

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Chapter 1

Chapter 9 problems

1.1 Problem 21

1.1.1 Decision variables

Let $M \in \mathbb{N}^*$ be the number of users to allocate.
Let $N \in \mathbb{N}^*$ be the number of servers where to allocate the users.
The decision variables are:

- $(x_{ij})_{(i,j)\in[\![1,M]\!]\times[\![1,N]\!]}$: Binary variable denoting whether user i will be allocated to server j.

1.1.2 Essential data

The data that is useful to the green allocation consists of the usage in MegaBytes for each user, the energy consumption per MegaByte for each server, and the capacity in MegaBytes for each server. They are expressed as follows:

- $(U_i)_{i\in[\![1,M]\!]}$: Quantity describing how much MegaBytes user i needs.
- $(E_j)_{j\in[\![1,N]\!]}$: Quantity describing how much energy per MegaByte stored server j uses.
- $(C_j)_{j\in[\![1,N]\!]}$: Quantity describing how much total RAM capacity server j has.

1.1.3 Objective function

Let $U = \max\{U_i, i \in [\![1, M]\!]\}$ be the maximum usage among all users.
Let $E = \max\{E_j, j \in [\![1, N]\!]\}$ be the maximum energy consumption among all servers.
The objective is to maximize the gain of not allocating a user to the worst server[1] and the gain of not allocating the user that needs most space, as well as maximizing

[1] The server that has the maximum energy consumption.

the number of users allocated while minimizing the overall energy consumption along the way, which can be formulated as such:

$$\max_{(x_{ij})_{(i,j)\in[\![1,M]\!]\times[\![1,N]\!]}} z = \sum_{i=1}^{M}\sum_{j=1}^{N}(E - E_j + 1)(U - U_i + 1)x_{ij} \tag{1.1}$$

The "+1" in both terms means that allocating the user with most usage to the worst server is still better that not allocating him at all, since the objective is also to maximize the number of users allocated.

1.1.4 Constraints

The constraints are such that a user can be allocated to at most 1 server, and that the overall usage of the users allocated to a given server can not exceed the capacity of that server which can both be written as such:

$$\forall i \in [\![1, M]\!], \sum_{j=1}^{N} x_{ij} \leq 1 \ (User\ allocation\ limit) \tag{1.2}$$

$$\forall j \in [\![1, N]\!], \sum_{i=1}^{M} U_i x_{ij} \leq C_j \ (Server\ capacity\ limit) \tag{1.3}$$

The variables also need to be binary so:

$$\forall (i,j) \in [\![1, M]\!] \times [\![1, N]\!], x_{ij} \in \{0, 1\} \tag{1.4}$$

1.1.5 Problem formulation

The whole problem would be:

$$\begin{aligned}
&\max_{(x_{ij})_{(i,j)\in[\![1,M]\!]\times[\![1,N]\!]}} z = \sum_{i=1}^{M}\sum_{j=1}^{N}(E - E_j + 1)(U - U_i + 1)x_{ij} \\
&\quad subject\ to: \\
&\qquad \forall i \in [\![1, M]\!], \sum_{j=1}^{N} x_{ij} \leq 1 \\
&\qquad \forall j \in [\![1, N]\!], \sum_{i=1}^{M} U_i x_{ij} \leq C_j \\
&\qquad \forall (i,j) \in [\![1, M]\!] \times [\![1, N]\!], x_{ij} \in \{0, 1\}
\end{aligned} \tag{1.5}$$