1) $\lim_{x\to 0}\left[\lim_{y\to 0}\frac{x+y}{\sqrt{x^2+y^2}}\right]=\lim_{x\to 0}\left[\frac{x}{\sqrt{x^2}}\right]=\boxed{1}$

$\lim_{y\to 0}\left[\lim_{x\to 0}\frac{x+y}{\sqrt{x^2+y^2}}\right]=\lim_{y\to 0}\frac{y}{\sqrt{y^2}}=\boxed{1}$

Si $y=ax$; $x\to 0$; $ax\to 0$

$\lim_{x\to 0}\frac{x(a+1)}{\sqrt{x^2(1+a^2)}}=\lim_{x\to 0}\frac{a+1}{\sqrt{1+a^2}}$

depende de a

2) $z \begin{cases} x \begin{cases} u \\ t \end{cases} \\ y \begin{cases} u \\ t \end{cases} \end{cases}$

$\frac{\partial f}{\partial x}=2xy^2$; $\frac{\partial f}{\partial y}=2yx^2$; $\frac{\partial x}{\partial u}=1$; $\frac{\partial x}{\partial t}=\operatorname{sen}(t)$

$\frac{\partial y}{\partial u}=2\cos(2u)$; $\frac{\partial y}{\partial t}=-1$

$$\boxed{\frac{\partial f}{\partial u}=2xy^2\cdot 1+2yx^2\cdot 2\cos(2u)}\qquad \boxed{\frac{\partial f}{\partial t}=2xy^2\operatorname{sen}(t)+2yx^2(-1)}$$

3a) $F(x,y)=x^2+y^2-4$ $\longrightarrow$ Polinomica, Cont. y dif.

a) $Df: \mathbb{R}^2$; $Rf: [-4,\infty)$

b) $\nabla f=\langle 2x, 2y\rangle$

c) $\vec{u}=\langle 1-0, 0-1\rangle=\langle 1,-1\rangle$

$|\vec{u}|=\sqrt{2}$; $\vec{u}_u=\left\langle\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right\rangle$

$D_{\vec{u}}F=\nabla f(2,2)\cdot \vec{u}_u=2\sqrt{2}-2\sqrt{2}=\boxed{0}$

d) $\begin{cases} 2x=0 \\ 2y=0 \end{cases} \longrightarrow P(0,0)$ unico (f. Cont, Polinom. y dif)

$f_{xx}=2$; $f_{yy}=2$; $f_{xy}=0$

$D=\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix}=4$

$D(0,0)=4>0$ y $f_{xx}>0$ $\Rightarrow$ $\boxed{\text{mínimo Local}}$

4) $y'+2xy=x^3$

$P(x)=2x$, $Q(x)=x^3$

$e^{\int P(x)dx}=e^{\int 2x dx}=e^{x^2}$

$e^{-\int P(x)dx}=e^{-x^2}$

$\int Q(x)\, e^{\int P(x)dx}dx=\int x^3 e^{x^2}dx$

$u=x^2$

$du=2x\,dx$

$u\cdot du=2x^3dx$

$\frac{1}{2}\int u\cdot e^u du=$

$\frac{1}{2}e^u(u-1)=\frac{e^{x^2}}{2}(x^2-1)$

$$\boxed{y_g(x)=e^{-x^2}\left[c+\frac{e^{x^2}}{2}(x^2-1)\right]}$$

4) b) $12y'' - 5y' - 2y = 0$

$P(\lambda) = 12\lambda^2 - 5\lambda - 2$

$\Delta = 25 - 4(-2) \cdot 12 = 25 + 96$

$\Delta = 121$

$\lambda_1 = \dfrac{+5+11}{24} = \dfrac{16}{24} = 2/3$

$\lambda_2 = \dfrac{5-11}{24} = \dfrac{-6}{24} = -1/4$

$$\boxed{y_g(x) = c_1 e^{-\frac{1}{4}x} + c_2 e^{\frac{2}{3}x}}$$