1.已知物体上某点的应力张量，求切面l=m=0.5上的全应力，正应力，剪应力。

$$\sigma_{ij} = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$l=m=0.5.\quad n=\frac{\sqrt{2}}{2}$

$$\begin{cases} S_x = \sigma_x l + \tau_{xy} m + \tau_{zx} n \\ S_y = \tau_{xy} l + \sigma_y m + \tau_{zy} n \\ S_z = \tau_{xz} l + \tau_{yz} m + \sigma_z n \end{cases} \Rightarrow \begin{cases} S_x = 1 \\ S_y = 1 \\ S_z = \frac{3\sqrt{2}}{2} \end{cases}$$

$$S^2 = S_x^2 + S_y^2 + S_z^2$$

$$S^2 = \frac{13}{2}$$

$$S = \pm\sqrt{\frac{13}{2}} = \pm\frac{\sqrt{26}}{2}$$

$$\sigma = S_x l + S_y m + S_z n = \frac{5}{2}$$

$$\tau = \sqrt{S^2 - \sigma^2} = \pm\sqrt{\frac{13}{2} - \frac{25}{4}} = \pm\frac{1}{2}$$

2 已知物体上A B 两点的应力状态，问A B两点的应力状态是否相同。

$$\sigma_{ij}^{A}=\begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad \sigma_{ij}^{B}=\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

A: $J_1=3$ B: $J_1=3$

$J_2=4$ $J_2=4$

$J_3=-12$ $J_3=0$

$\because J_3^A \neq J_3^B$

$\therefore$ 两点应力状态不同

3 已知物体上某点的应力状态，求该点的主应力 主方向。

$$\sigma_{ij}=\begin{bmatrix}0&2&0\\2&0&0\\0&0&3\end{bmatrix}$$

解: $J_1=3$　　$J_2=4$　　$J_3=-12$

$$\sigma^3-J_1\sigma^2-J_2\sigma-J_3=0$$

$$\sigma^3-3\sigma^2-4\sigma+12=0$$

解得 $\sigma_1=3$　$\sigma_2=2$　$\sigma_3=2$

① $\sigma_1=3$ 时.

$$\begin{cases}(\sigma_x-\sigma)l+\tau_{yx}m+\tau_{zx}n=0\\ \tau_{xy}l+(\sigma_y-\sigma)m+\tau_{zy}n=0\\ \tau_{xz}l+\tau_{yz}m+(\sigma_z-\sigma)n=0\end{cases}\Rightarrow\begin{cases}-3l+2m=0\\ 2l-3m=0\\ l^2+m^2+n^2=1\end{cases}$$

可得 $\begin{cases}l=0\\ m=0\\ n=1\end{cases}$ 或 $\begin{cases}l=0\\ m=0\\ n=-1\end{cases}$

② $\sigma_2 = 2$ 时

$$\begin{cases} (\sigma_x - \sigma)l + \tau_{yx} m + \tau_{zx} n = 0 \\ \tau_{xy} l + (\sigma_y - \sigma) m + \tau_{zy} n = 0 \\ \tau_{xz} l + \tau_{yz} m + (\sigma_z - \sigma) n = 0 \\ l^2 + m^2 + n^2 = 1 \end{cases} \Rightarrow \begin{cases} -2l + 2m = 0 \\ 2l - 2m = 0 \\ n = 0 \\ l^2 + m^2 + n^2 = 1 \end{cases}$$

可得 $\begin{cases} l = \frac{\sqrt{2}}{2} \\ m = \frac{\sqrt{2}}{2} \\ n = 0 \end{cases}$ 或 $\begin{cases} l = -\frac{\sqrt{2}}{2} \\ m = -\frac{\sqrt{2}}{2} \\ n = 0 \end{cases}$

③ $\sigma_3 = -2$ 时

$$\begin{cases} 2l + 2m = 0 \\ 5n = 0 \\ l^2 + m^2 + n^2 = 1 \end{cases} \Rightarrow \begin{cases} l = -m \\ n = 0 \\ l^2 + m^2 + n^2 = 1 \end{cases}$$

可得 $\begin{cases} l = \frac{\sqrt{2}}{2} \\ m = -\frac{\sqrt{2}}{2} \\ n = 0 \end{cases}$ 或 $\begin{cases} l = -\frac{\sqrt{2}}{2} \\ m = \frac{\sqrt{2}}{2} \\ n = 0 \end{cases}$

5 已知点的应力状态，求应力偏张量，应力球张量，等效应力。

$$\sigma_{ij} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\sigma_m = 1$$

$$\delta_{ij}\sigma_m = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma'_{ij} = \begin{bmatrix} 0 & 3 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2 + 6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$$= \sqrt{30}$$

7 判断应变状态能否存在。

(1) $\varepsilon_x = k(x^2+y^2)$ $\varepsilon_y = ky^2$ $\varepsilon_z = 0$

$\gamma_{xy} = 2kxy$ $\gamma_{yz} = 0$ $\gamma_{zx} = 0$

解：

应变协调方程：

$$\begin{cases} \dfrac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \dfrac{1}{2}\left(\dfrac{\partial^2 \varepsilon_x}{\partial y^2} + \dfrac{\partial^2 \varepsilon_y}{\partial x^2}\right) \\ \dfrac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \dfrac{1}{2}\left(\dfrac{\partial^2 \varepsilon_y}{\partial z^2} + \dfrac{\partial^2 \varepsilon_z}{\partial y^2}\right) \\ \dfrac{\partial^2 \gamma_{zx}}{\partial z \partial x} = \dfrac{1}{2}\left(\dfrac{\partial^2 \varepsilon_z}{\partial x^2} + \dfrac{\partial^2 \varepsilon_x}{\partial z^2}\right) \end{cases} \Rightarrow \begin{cases} \underline{2k \neq k} \\ \\ \\ \end{cases}$$

$\therefore$ 不存在