

# A Faithful Mechanism for Privacy-Sensitive Distributed Constraint Satisfaction Problems\*

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**Abstract.** We consider a constraint satisfaction problem (CSP) in which constraints are distributed among multiple privacy-sensitive agents. Agents are self-interested (they may reveal misleading information/constraints if that increases their benefits) and privacy-sensitive (they prefer to reveal as little information as possible). For this setting, we design a multi-round negotiation-based incentive mechanism that guarantees truthful behavior of the agents, while protecting them against unreasonable leakage of information. This mechanism possesses several desirable properties, including Bayesian incentive compatibility and individual rationality. Specifically, we prove that our mechanism is faithful, meaning that no agent can benefit by deviating from his required actions in the mechanism. Therefore, the mechanism can be implemented by selfish agents themselves, with no need for a trusted party to gather the information and make the decisions centrally.

**Keywords:** Constraint satisfaction problems · Incentive mechanism design · Privacy

## 1 Introduction

Distributed constraint satisfaction problems (DisCSP) in which decision variables and constraints are distributed among multiple agents are common in many multi-agent systems. They are popular because they are a good representation of many real world applications including resource allocation [1], scheduling [16], electronic commerce [22] and logistics [18].

To solve distributed CSPs, agents need to exchange messages until a solution is found or until one agent finds out that there is no solution to the problem. In many cases, there is also a natural desire for the agents to minimize the amount of information revealed during the problem solving process. This is particularly true in cases where the agents are self-interested. Such privacy-sensitive encounters [10] involve the design of mechanisms that strike a balance between the amount of information revealed and the desire to reach an acceptable solution. For example exchanging no information minimizes the amount of information revealed but is

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\*This work was supported and funded by Samsung Electronics R&D Institute UK (SRUK).

unlikely to lead to a solution, whereas all agents revealing all their constraints maximizes the chance of finding a socially-optimal solution but at the cost of all privacy.

When the agents are self-interested, the mechanism needs to be robust to the possibility of receiving misleading information from the agents. However, such agents will only provide truthful information if they are motivated by relevant incentives to do so. In an incentive mechanism, the agents give each other rewards (or penalties) based on the information they share with each other. These rewards must be designed so as to align the agents' individual objectives and eventually to motivate them to not reveal fake information.

The literature on incentive mechanism design mostly focuses on centralized mechanisms where a trusted entity performs as a manager and processes the mechanism procedures centrally [3, 25]. However, in many cases, a trusted entity does not always exist. To tackle this drawback, we design a faithful incentive mechanism that can be run by selfish agents. A mechanism is faithful if an agent cannot benefit by deviating from any of his required actions, including information-revelation, computation and message passing [17].

In more detail, our faithful incentive mechanism strikes a balance between privacy and social efficiency. This mechanism is based on the *score-voting* idea which is used in the literature for designing centralized incentive mechanisms [12]. Specially, we design a multi-round negotiation-based mechanism in which at each round, the agents first rate a set of candidate solutions and then decide if any of them is acceptable. To make this voting mechanism Bayesian truthful, we present a reward function that is based on the agents' beliefs about the likely effectiveness of their votes on the final outcome. We guarantee faithfulness of the mechanism by setting non-manipulable rules and show that the minimum number of solutions being discussed at each round is a control parameter that balances the tradeoff between privacy leakage and social efficiency. We illustrate this mechanism via the domain of distributed meeting scheduling, which is a canonical example of a DisCSP with self-interested and privacy-sensitive agents.

This work presents the first faithful mechanism for a DisCSP with selfish and privacy-sensitive agents. Moreover, our mechanism has the flexibility to adjust the relative importance of privacy leakage and social efficiency. DisCSPs show quite different behaviors based on the relative importance of privacy and efficiency. Therefore, designing a unified mechanism than can plausibly handle a diverse range of DisCSPs is a key advance.

## 2 Related Literature

DisCSP was first introduced in [26]. Most existing mechanisms in this area require that all decision variables and constraints are known by the corresponding agents in advance; i.e. they are offline mechanisms. Two strands of works are prevalent in the category of offline mechanisms: complete mechanisms [7, 13] and incomplete mechanisms [23, 28]. The former are guaranteed to find the social-welfare maximizing solution, but require exponential time in the worst case. The

latter find suboptimal solutions but run quickly enough to be applied to real-world applications. However, offline mechanisms do not fit dynamic applications such as meeting scheduling, where new decision variables and constraints are introduced over time. Thus, to solve practical dynamic DisCSPs, we need to design online mechanisms that make decisions based on partial available information.

Distributed online mechanisms often use negotiation between agents to find a solution [9,21]. During such encounters, agents usually need to adjust their negotiating strategy based on their information about others' preferences to change the outcome to their favorite one. Some cooperative negotiation mechanisms assume that agents' preferences are public information [19]. In a competitive environment (non-cooperate negotiation), however, self interested agents keep their preferences private to avoid being exploited by their opponents [11,20]. Without the knowledge of opponents' preferences, agents may have difficulty in adjusting their negotiation strategies properly. This difficulty has been addressed in the literature of incentive mechanism design [25].

There is a long tradition of using centralized incentive mechanisms within distributed systems [14]. However, there are very few known methods for distributed problem solving in the presence of self-interested agents. The first steps in providing a distributed incentive mechanism were the works presented in [5,6]. However, the rules of these mechanisms are not robust to manipulation, and hence are not suitable for distributed implementation. Starting from [15], researchers have attempted to design faithful mechanisms that incentivize agents to follow all the rules [17]. These mechanisms do not consider privacy leakage and so are not directly applicable for our purposes.

There are a number of papers that are starting to address privacy issues in DisCSP [2, 10, 27]. These papers describe techniques, such as encryption [27], obfuscation [2], and codenames [2], that can be used with DisCSP algorithms such as DPOP, ADOPT, and NCBB, to provide privacy guarantees. However, these works do not take agents' selfish behavior into account.

### 3 Multi-Agent Meeting Scheduling

We view meeting scheduling as a distributed and dynamic CSP where the decisions are about when to hold each meeting and the constraints are the attendees' calendar availabilities. The problem is distributed as the agents are only aware of their own calendars and is dynamic as the needs for different meetings arise over time. In this setting, the agents need to decide about the time of different meetings one-by-one, and without knowing what will happen next. Attendees of the meetings are self interested and privacy-sensitive; they wish to maximize their own utility and reveal as little information about their availabilities and preferences as possible. Therefore, we need to design an incentive mechanism that guarantees truthful behavior of the agents, while protecting them against unreasonable leakage of information.

Formally, we model each meeting  $m$  by a tuple  $m = (A, I, l)$  where  $A$  is the set of mandatory attendees,  $I \in A$  is the initiator who is responsible for setting

the meeting time, and  $l$  is the meeting's required length in terms of time slots. We denote the set of all available time slots in a meeting scheduling problem by  $S = \{s^1, \dots, s^T\}$ , where  $s^j$  represents the  $j$ -th available time slot.

Attendees of the meetings are selfish, meaning they have some preferences over the outcomes and attend to their desires without any regard to the preferences of others. Agent  $i$ 's preferences are captured by a utility function  $U_i(\cdot)$ , which is a function of five variables:

1. Meeting start time ( $s \in S$ ): We denote agent  $i$ 's valuation for having a meeting at time  $t$  by the valuation function  $V_i(t) \in \{-\infty\} \cup [V_{min}, V_{max}]$ . The valuation is  $-\infty$  when the meeting scheduling fails or when the meeting is set at a time the agent cannot attend. Agent  $i$ 's valuation for a meeting with length  $l$  which starts at time  $s$  is the minimum value he assigns to attending a meeting at times  $s, s+1, \dots, s+l-1$ .
2. The messages sent in the mechanism ( $M_i$ ): The agents are privacy-sensitive and prefer not to share their calendar's information with others. We denote by  $L(M_i)$  the amount of agent  $i$ 's privacy which is leaked by sending messages  $M_i$ . This privacy leakage adversely affects the agent's utility. We will discuss thoroughly how to design the leakage function in Section 4.
3. Number of rounds of mechanism ( $n$ ): Each agent's utility is a decreasing function of the number of rounds. This is because, the longer the mechanism takes, more communication resources agents need to use in the process.
4. Reward received at the mechanism ( $R_i$ ): In an incentive mechanism, the agents may give some rewards to others to incentivize them to behave as they want. These rewards can come in the form of points, badges and leveling that can help the agents advance in the future [24]. In this paper, we consider rewards to be convenience points that can be used by the agents to influence the future meeting scheduling processes.
5. Convenience points spent at the mechanism ( $C_i$ ): This is the number of convenience points that agent  $i$  used to influence the outcome of the meeting scheduling process. In general, more points are required to express higher interests in a start time for a meeting.

Based on the discussion above, we model agent  $i$ 's utility function in a quasi-linear way as follows:

$$U_i(s, l, M_i, n, R_i) = \delta_i^{n-1} \min_{s \leq t \leq s+l-1} V_i(t) - \theta_i L(M_i) - C_i + R_i, \quad (1)$$

where  $\delta_i \in (0, 1)$  is agent  $i$ 's discount factor by which agent  $i$ 's future profits is multiplied to find its present value, and  $\theta_i \in (0, 1)$  is agent  $i$ 's sensitivity to his privacy. Agent  $i$ 's discount factor displays his patience in the mechanism, while his privacy sensitivity represents his attitude toward revealing his private information to others.

Agents' valuation functions and hence their calendar availabilities are assumed to be their own private information. Therefore, selfish agents have to be motivated by a suitably designed incentive mechanism to reveal their calendar availabilities truthfully. This mechanism needs to limit the leakage of agents'

privacy, as agents do not participate in a mechanism if it is overly detrimental to their privacy. In the next section, we introduce a privacy leakage function  $L(\cdot)$ . Then, in Section 5, we detail our incentive mechanism that induces honest behavior by all selfish and privacy-sensitive agents.

## 4 Privacy Leakage

In a meeting scheduling process, the agents care about the privacy of their information. They want to protect the privacy of their respective availability schedules, as well as the lists of meetings they are involved in. Moreover, the initiator who is responsible for scheduling a meeting may not want to share the details, such as the number or identities of the participants, with them before the meeting starts\*.

To satisfy these requirements, we restrict our attention to the following class of mechanisms.

**Definition 1.** Define by  $\Gamma^{1 \rightarrow 1}$  the class of incentive mechanisms that satisfy the following two properties:

1. Message passing occurs only between the initiator and the responders, and not between responders themselves. The initiator does not pass the information he receives from a responder to the others.
2. The initiator never asks the reason why an agent is free or busy at a time slot. He also never describes the meeting's details for the responders.

We call this class non-curious one-to-one (NC 1-1) mechanisms.

Restricting attention to this class of mechanisms guarantees that the details of the current meeting, as well as the other appointments or meetings the agents might have, are not leaked. However, in order to find a feasible time for the meeting, revealing some information about the free/busy (F/B) status of the agents is inevitable. In the following, we propose a function that measures the leakage of the agents' F/B information in an NC 1-1 mechanism.

In an NC 1-1 mechanism, no F/B information of a responder is leaked to the other responders. Therefore, the only possible leakage is from the initiator to the responders, and vice versa. Before revealing any information, the initiator and the responders have a prior belief about the F/B information of each other. This belief is based on the previous knowledge they may have about each other. When no such information is available, the belief assigns probability 0.5 to both free and busy status of the others for each time slot. When a meeting scheduling mechanism runs, the initiator and the responders learn some new information about each other's calendars. This new information constitutes a posterior belief about the F/B information of the other party.

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\*This leak of information may enable responders to collude with each other to alter the outcome in their favor.

We define the privacy of a responder  $i$  at each instant of time as the distance between the initiator's belief about his F/B status and his true F/B information. The privacy leakage of agent  $i$  is the difference between his privacy at the start and end of the mechanism.

To formalize this idea, we denote the true probability distribution of agent  $i$ 's availability at time slot  $s^j$  by  $t_i^j : \{F, B\} \rightarrow \{0, 1\}$ , where  $t_i^j$  assigns a probability 0 or 1 to the free ( $F$ ) and busy ( $B$ ) status of agent  $i$  at time  $s^j$ . We have  $t_i^j(F) = 1$  and  $t_i^j(B) = 0$ , if agent  $i$  is free at  $s^j$ , and  $t_i^j(F) = 0$  and  $t_i^j(B) = 1$ , if he is busy at that time.

At each instant of time, the initiator assigns a probability distribution to the F/B status of agent  $i$  for time slot  $s^j$ . We denote this probability distribution at the beginning and end of the mechanism by  $b_{I,i}^j : \{F, B\} \rightarrow [0, 1]$  and  $e_{I,i}^j : \{F, B\} \rightarrow [0, 1]$ , where  $b_{I,i}^j(F)$  and  $b_{I,i}^j(B)$  ( $e_{I,i}^j(F)$  and  $e_{I,i}^j(B)$ ) are the prior (posterior) beliefs the initiator has on the free and busy state of agent  $i$ , respectively, at time slot  $s^j$ .

Now, to define agent  $i$ 's privacy before and after running the mechanism, we compare the prior and posterior beliefs with the true distribution. We do this comparison based on the total variation distance metric [8]. For two probability distributions  $p$  and  $q$  on a random variable  $x \in X$ , the total variation distance is defined as

$$\delta(p, q) = \frac{1}{2} \|p - q\|_1 = \frac{1}{2} \sum_{x \in X} |p(x) - q(x)|, \quad (2)$$

where  $\|\cdot\|_1$  represents the  $L_1$  norm. Using this distance, we measure the privacy of responder  $i$  at the beginning and end of a mechanism as

$$Pr_i^b = \sum_{j=1}^T \delta(t_i^j, b_{I,i}^j) = \sum_{j=1}^T |t_i^j(F) - b_{I,i}^j(F)|, \quad (3)$$

and

$$Pr_i^e = \sum_{j=1}^T \delta(t_i^j, e_{I,i}^j) = \sum_{j=1}^T |t_i^j(F) - e_{I,i}^j(F)|, \quad (4)$$

respectively. The privacy leakage of responder  $i$  is the difference between his privacy at the start and end of the mechanism. That is,

$$L_i = Pr_i^b - Pr_i^e. \quad (5)$$

In a similar way, we define the initiator's privacy at the start and end of a mechanism, as:

$$Pr_I^b = \frac{1}{|A| - 1} \sum_{i \in A, i \neq I} \sum_{j=1}^T \delta(t_I^j, b_{i,I}^j) = \frac{1}{|A| - 1} \sum_{i \in A, i \neq I} \sum_{j=1}^T |t_I^j(F) - b_{i,I}^j(F)|, \quad (6)$$

and

$$Pr_I^e = \frac{1}{|A| - 1} \sum_{i \in A, i \neq I} \sum_{j=1}^T \delta(t_I^j, e_{i,I}^j) = \frac{1}{|A| - 1} \sum_{i \in A, i \neq I} \sum_{j=1}^T |t_I^j(F) - e_{i,I}^j(F)|. \quad (7)$$

The only fundamental difference between (6)-(7) and (3)-(4) is that irrespective of the responders who only communicate with the initiator, the initiator communicates with all of the responders. Therefore, his messages could affect all responders' beliefs. We define the initiator's privacy as the average of the privacy he gets in his communications with the responders. The privacy leakage of the initiator is defined as  $L_I = Pr_I^b - Pr_I^e$ .

The privacy leakage function proposed above has two main features.

1. This privacy metric takes the possible correlation among an agent's availabilities at different time slots into account. In some cases, an agent has some side information about the pattern of an agent's calendar. This side information could be the length or repeat frequency of his meetings, or the length of breaks he normally has between them. In these cases, the F/B information of one time slot may reveal parts of the F/B information of other time slots. This indirect leakage of information reflects in functions (4) and (7) through the posterior beliefs  $e_{I,i}^j$  and  $e_{i,I}^j$ . This capability is missing in most of the available privacy metrics, such as entropy and information content.
2. The privacy value of each time slot is finite and normalized to one. One of the drawbacks of the logarithmic-based privacy metrics, such as Information content and KL divergence, is that they do not provide any upper bound for the privacy leakage; by using these metrics, the privacy leakage could go to infinity even if the information of just one time slot is leaked.

Measuring privacy leakage with the function proposed above, in the next section we present our negotiation-based mechanism that guarantees truthfulness.

## 5 A Negotiation-Based Incentive Mechanism

The initiator has some candidate start times for a meeting that needs to be scheduled. The responders have different valuations and availabilities for these time intervals, but this information is not available to the initiator. To extract this information with low privacy leakage, at each round, the initiator offers at least  $L_{min}$  start times to the responders and asks them to rate the offers on a scale of 0 to  $D - 1$ , where 0 means "busy/unavailable", 1 means "Available but completely dissatisfied" and  $D - 1$  means "Available and completely satisfied".

Increasing the lower bound  $L_{min}$  increases the chance of finding a socially acceptable solution in a shorter length of time, but at the cost of a higher privacy leakage. Therefore,  $L_{min}$  is a control parameter that can be used to balance the tradeoff between speed and social efficiency on one side, and privacy leakage on the other.

The agents who rate time slot  $s$  at  $d \in \{1, 2, \dots, D - 2\}$  attend the meeting at  $s$  only if the initiator compensates them for the hardship they endure by giving them some convenience points. Two examples of hardship could be attending a meeting after work hours and rescheduling an existing meeting so as to open room for this one. The agents use these convenience points to rate future time slots. The number of points awarded by the initiator to a responder

is a decreasing function of his reported satisfaction  $d$  for that time slot, but it is also a function of the satisfaction levels he reported for the other offered time slots. Thus, if a responder announces to be generally more satisfied with the offered time slots, he will get more points if one of his undesirable time slots is selected. This rule is used so as to prevent the responders from falsely reporting low satisfaction levels in order to get more points.

In more detail, the mechanism is a multi-round negotiation, where at each round the initiator offers at most  $L_{max}$  meeting start times to the responders. If the number of offers made at round  $n$ , denoted by  $L_n$ , is greater than or equal to a lower threshold  $L_{min}$ , the initiator is permitted to go to the next round and offer some new start times, if he couldn't find a suitable time for the meeting at the current round  $n$ . However, if  $L_n < L_{min}$ , the negotiation ends at the end of round  $n$ , independent of whether or not the meeting scheduling was successful. This rule is designed to encourage the initiator to make at least  $L_{min}$  offers at each round, if he is able to do so.

We denote the time slots offered by the initiator at round  $n$  for starting the meeting by  $\{s_n^1, \dots, s_n^{L_n}\}$ . Receiving this offer, each responder should rate each of the offered times  $s_n^1, \dots, s_n^{L_n}$  on a scale of 0 to  $D - 1$ . We denote responder  $i$ 's ratings at round  $n$  by  $\mathbf{r}_{i,n} = (r_{i,n}^1, \dots, r_{i,n}^{L_n})$ , where  $r_{i,n}^j \in \{0, 1, \dots, D - 1\}$  indicates how satisfied responder  $i$  is with starting the meeting at the  $j$ -th time slot offered to him at round  $n$ .

At each round  $n$  of the mechanism, each agent  $i$  has  $b_{i,n}$  convenience points that can be used to rate the offered time slots. Giving rates 0 and 1 does not require spending points, however to give a rate  $d \geq 2$  to an offer, the agent needs to assign  $d - 1$  points to that offer. We define  $N_{i,n}^d$ ,  $d = 0, 1, \dots, D - 1$ , as the number of time slots to which responder  $i$  gives rate  $d$  at round  $n$ . Using this notation, the number of points agent  $i$  spends at round  $n$  to give rating  $\mathbf{r}_{i,n}$  can be derived as

$$C_{i,n} = \sum_{d=2}^{D-1} (d-1) N_{i,n}^d. \quad (8)$$

At each round  $n$  of negotiation, we must have  $C_{i,n} \leq b_{i,n}$ .

Let us define  $A_{i,n}$  as the number of time slots responder  $i$  announces availability at round  $n$ . We can derive this parameter as  $A_{i,n} = \sum_{d=1}^{D-1} N_{i,n}^d$ . We define the total flexibility responder  $i$  shows at round  $n$  of negotiation as follows:

$$F_{i,n} = \sum_{d=1}^{D-1} (A_{i,n} + 1)^{d-1} N_{i,n}^d. \quad (9)$$

This function gives the decimal value of number  $(N_{i,n}^{D-1}, \dots, N_{i,n}^1)$  in base  $A_{i,n} + 1$ . Therefore, a greater value of  $F_{i,n}$  means responder  $i$  is more satisfied with the time slots offered at round  $n$ . Function  $F$  is invertible; meaning that for each  $i, n$ , given  $A_{i,n} + 1$ , the vector  $(N_{i,n}^{D-1}, \dots, N_{i,n}^1)$  can be reconstructed from the flexibility  $F_{i,n}$ . We use this property and represent hereafter, the cost  $C_{i,n}$  of agent  $i$ 's rating at round  $n$  by  $C(A_{i,n}, F_{i,n})$ .



After receiving the responders' ratings, the initiator checks to see if any of  $\{s_n^1, \dots, s_n^{L_n}\}$  is a good time for the meeting. If he finds none of these time slots appealing, he can go to the next round and make some new offers, provided that  $L_n \geq L_{min}$ . But if he does so, the mechanism doesn't let him go back to time slots  $\{s_n^1, \dots, s_n^{L_n}\}$  in the future. This rule is designed to encourage the initiator to decide about the meeting time as soon as possible. If the initiator neglects a time, in which all attendees are available, and goes to the next round, there is a risk that no other feasible time slots can be found in the future, and hence the meeting scheduling fails. To avoid this risk, the initiator prefers to set up the meeting time as soon as he can.

The presence of all responders at the meeting is necessary. Therefore, the initiator does not schedule the meeting at a time at which at least one responder gave a zero rating. If the initiator chooses time slot  $s_n^j$ ,  $j = 1, \dots, L_n$ , as the meeting start time, he should award some convenience points to the following two groups of responders:

1. Responders who announce they are not completely satisfied with time slot  $s_n^j$ . These responders who rate time slot  $s_n^j$  at  $d \in \{1, 2, \dots, D-2\}$ , must receive a compensation for the hardship they will endure if they attend the meeting at interval  $[s_n^j, s_n^j + l - 1]$ ;
2. Responders who announce complete satisfaction with all offered time slots at round  $n$  at which they are available. Although these responders are completely satisfied with the possible choices and will unconditionally attend the meeting if any of them is selected, the initiator gives them a reward to appreciate their high flexibility.

The number of points that must be awarded to a responder  $i$  if time slot  $s_n^j$  is selected as the meeting start time is denoted by  $t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n)$ . This function is decreasing in  $r_{i,n}^j \in \{1, 2, \dots, D-2\}$  and increasing in  $F_{i,n}$ , when the other parameters are fixed.

To incentivize agents to rate the offered time slots truthfully, we design reward function  $t(\cdot)$  such that it satisfies the following conditions:

(a)

$$\sum_{d=1}^{D-1} P(d, A, F) t(d, A, F, L) - C(A, F) = P(1, A, A) t(1, A, A, L), \quad (10)$$

$$\forall L \leq L_{max}, \forall A \leq L, \forall F \text{ st. } F \pmod{A+1} > 0,$$

where  $P(d, A, F)$  is the probability a responder with flexibility  $F$  who announced availability at  $A$  time slots at round  $n$ , assigns to the fact that one of the time slots he rates at  $d$  will be selected by the initiator.

- (b)  $t(D-1, A, F, L) = 0$ , if  $F \neq A(A+1)^{D-2}$ .
- (c)  $t(d, A, F, L)$  is a decreasing function of  $d$  for  $d \in \{1, \dots, D-2\}$ .
- (d)  $t(\cdot)$  is invariant to shifting of the ratings. That is,  $\mathbf{r}'_{i,n} = (\mathbf{r}_{i,n} + c) \text{sign}(\mathbf{r}_{i,n})$ , where  $c \in \{1, \dots, D-2\}$ , implies that  $t(d', A', F', L) = t(d, A, F, L)^\dagger$ .

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<sup>†</sup>It is clear that by this transformation, we have  $A' = A$ .

The intuitions behind the above conditions are as follows. Condition (a) guarantees that provided the agent gives rate 1 to at least one offer, the expected number of points he gets minus the points he used depends only on the number of time slots he reports to be available, and not on the specific ratings he gives to the offers. This expectation is computed based on the agent's belief about the likely effectiveness of his ratings on the final outcome. Condition (b) ensures that a responder who is announced to be completely satisfied with the chosen meeting time receives no reward, unless he rated all the offers at  $D-1$ . Condition (c) means that the agents who are less satisfied with the selected time slot receive higher rewards. Condition (d) determines the reward for ratings with  $F \pmod{A+1} = 0$  and guarantees that the reward function is only sensitive to the relative ratings the agent give to the offers and not on the absolute values. Based on the definition provided in condition (d), we call ratings  $r$  and  $\mathbf{r}'$  shifted versions of each other, if 1) they mark the same time slots as unavailable, and 2) they differ only by a constant factor in the available time slots.

**Theorem 1.** For any fixed belief profile  $\{P(d, A, F)\}_{d,A,F}$  which is invariant to shifting of the ratings, the system of equations defined in (10) has a solution that satisfies (b)-(d). We say a belief profile is invariant to shifting if  $\mathbf{r}'_{i,n} = (\mathbf{r}_{i,n} + c) \text{sign}(\mathbf{r}_{i,n})$ , where  $c \in \{1, \dots, D-2\}$ , implies that  $P(d', A', F') = P(d, A, F)$ .

We present the proofs of all the theorems and lemmas in [4].

The probabilities  $\{P(d, A, F)\}_{d,A,F}$  depend on 1) the responders' belief about the number of other people who should attend the meeting, and 2) the initiator's strategy for selecting the meeting start time. At each round  $n$ , when the initiator receives the responders' reports  $\mathbf{r}_{i,n}$ ,  $i = 1, \dots, N$ , he evaluates all offers  $\{s_n^1, \dots, s_n^{L_n}\}$  and decides which, if any, of them are suitable to be selected as the meeting start time. Since the initiator is selfish, he does this evaluation based on his own utility. According to (1), the initiator's utility for any start time  $s_n^j$  is the difference between the discounted value interval  $[s_n^j, s_n^j + l - 1]$  has for him and the sum of his privacy leakage and the points he should spend to incentivize responders to attend the meeting at that time. This utility would be  $-\infty$ , if at least one responder cannot attend the meeting at that time. That is,

$$U_I(s_n^j) = \begin{cases} \delta_I^{n-1} \min_{s_n^j \leq t \leq s_n^j + l - 1} V_I(t) - \theta_I L(M_I) - \\ \sum_{i \in A} t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n), & \text{If } r_{i,n}^j > 0 \text{ for all } i, \\ -\infty, & \text{Otherwise.} \end{cases} \quad (11)$$

It is clear from (11) that the initiator's strategy for selecting the meeting's time and hence the probabilities  $\{P(d, A, F)\}_{d,A,F}$  depend on the reward function  $t(\cdot)$ . Therefore, for each  $L \leq L_{max}$ , to derive a reward function that satisfies the set of constraints (10) we have to run Algorithm 1. This algorithm works by first considering an arbitrary reward function  $t(\cdot)$  that satisfies conditions (c)-(d). These conditions are weak and easily satisfied. Then it calculates probabilities  $\{P(d, A, F)\}_{d,A,F}$  that matches with the selected reward function and

**Algorithm 1:** Reward Design

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1 Initialize reward function  $t(\cdot)$  such that it satisfies condition (c)-(d);
2  $err \leftarrow \infty$ ;
3 while  $err > th$  do
4   Calculate probabilities  $\{P(d, A, F)\}_{d,A,F}$  based on  $t(\cdot)$ ;
5    $t_{new} \leftarrow$  Solution of the set of equations (10) that satisfies condition (b)-(d);
6    $err \leftarrow Norm(t - t_{new})$ ;
7    $t \leftarrow t_{new}$ ;
8 end

```

---

updates function  $t(\cdot)$  based on equation (10) and conditions (b)-(d). This procedure repeats until convergence is reached. Theorem 1 ensures that the algorithm will never stick in Line 5 because of not finding a solution to the set of equations (10).

We represent the Negotiation-based Meeting Scheduling (NMS) mechanism designed in this section by  $\Gamma = (L_{min}, L_{max}, D, t(\cdot))$ . The corresponding pseudocode of this mechanism is shown by Algorithm 2. Briefly, the NMS mechanism starts with designing a reward function  $t(\cdot)$  that satisfies conditions (a)-(d) and announcing it to the agents. Then, when the need for a meeting arises, the meeting's initiator starts a negotiation process by offering some of his desirable time slots. The number of offers at each round is one of the initiator's decision variables. Receiving the offers, the responders use their convenience points to express their preferences over them. Then, the initiator evaluates each offer based on the utility it provides to him, considering the cost  $\sum_{i \in A} t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n)$  he should pay to incentivize the responders to participate in the meeting (11). If the initiator finds any of the offers acceptable, he will set up the meeting at that time and terminates the negotiation. Otherwise, he will go to the next round if  $L_n \geq L_{min}$ .

## 6 Properties of the Mechanism

In this section, we show that the NMS mechanism  $\Gamma = (L_{min}, L_{max}, D, t(\cdot))$  is faithful. To this end, we need to prove that both the responders and the initiator have no incentive to deviate from their required actions. We prove the faithfulness of the responders and the initiator in Sections 6.1 and 6.2, respectively.

### 6.1 Responders' Faithfulness

The responders must have an incentive to 1) participate in the mechanism and 2) rate the offers truthfully. The first property is called individual rationality and the second is incentive compatibility. In the following, we investigate and prove these two properties for the privacy-sensitive responders (the proofs are given in [4]).

**Algorithm 2:** NMS mechanism  $\Gamma = (L_{min}, L_{max}, D, t(.))$ 


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1 The system announces reward function  $t(.)$  to all agents.;
2 for each meeting  $m = 1, 2, \dots$  do
3   System chooses parameters  $L_{min}$  and  $L_{max}$  based on the relative
   importance of privacy and efficiency.;
4    $n \leftarrow 1$ ;
5   meeting start time  $s_m \leftarrow 0$ ;
6   while  $s_m = 0$  do
7     Initiator offers  $L_n \leq L_{max}$  time slots to the responders.;
8     Each responder  $i$  rates the offered time slots on a scale of 0 to  $D - 1$  as
        $\mathbf{r}_{i,n} = (r_{i,n}^1, \dots, r_{i,n}^{L_n})$ .;
9     if Initiator finds any of the offered time slots appealing then
10      A suitable time slot  $s_n^j$  is selected as the meeting start time.;
11       $s_m \leftarrow s_n^j$ ;
12      Initiator awards  $t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n)$  points to each responder  $i$ ;
13    else
14      if  $L_n \geq L_{min}$  then
15         $n \leftarrow n + 1$ .;
16      else
17        meeting scheduling fails.;
18         $s_m \leftarrow \infty$ .;
19      end
20    end
21  end
22 end

```

---

**Property 1 (Individual Rationality):** Individual rationality, also referred to as voluntary participation, is a desirable feature of a mechanism as it guarantees that the agents voluntarily participate in the mechanism. This property is important as agents are not forced to participate in a mechanism but can decide whether or not to participate.

**Theorem 2.** The NMS mechanism is individually rational for privacy-sensitive responders. That is, each responder prefers the outcome of the mechanism to the utility he gets when he does not participate.

**Property 2 (Incentive Compatibility):** The NMS mechanism is Bayesian incentive compatible from the responders' view point if each privacy-sensitive responder can achieve his maximum expected utility by adopting a truthful strategy. We focus on Bayesian incentive compatibility, as the agents have incomplete information and hence try to maximize their expected utility. To prove this property we first need to define what exactly a truthful strategy is.

**Definition 2.** We say that responder  $i$  is truthful in the mechanism  $\Gamma = (L_{min}, L_{max}, D, t(.))$ , if his report  $\mathbf{r}_{i,n}$  at each round  $n$  satisfies the following conditions:

- (I) For each  $j = 1, \dots, L_n$ ,  $r_{i,n}^j = 0$  if and only if the responder is busy at time  $s_n^j$ .
- (II) The ratings are non-decreasing in the value of the time slots, i.e.  $V_i(s_n^j) > V_i(s_n^k)$  implies that  $r_{i,n}^j \geq r_{i,n}^k$ .
- (III) The ratings are as discriminant as possible. That is, time slots with different values get different ratings, as long as both the number of satisfaction levels  $D$  and the budget  $b_{i,n}$  allow.

Definition 2 provides a formal description of a responder's truthful behavior. In the following, we show that the mechanism  $\Gamma$  is powerful enough to incentivize privacy-sensitive responders to adopt a truthful strategy.

**Lemma 1.** The privacy-sensitive responders do not have any incentive to lie about their availability, i.e. giving rate 0 to a time-slot is efficient for a responder if and only if he is busy at that time slot.

Lemma 1 proves that condition (I) of Definition 2 is satisfied. In the next lemma, we prove that condition (II) is also satisfied.

**Lemma 2.** It is never optimal for a responder to give a higher rating to a time slot he likes less.

To prove satisfaction of condition (III), we need the following lemma. This lemma states an important property of the proposed mechanism that is key to proving incentive compatibility.

**Lemma 3.** It is optimal for each responder  $i$  to give rate 1 to at least one offer. In this case, the expected number of points he gets at each round of the mechanism minus the number of points he spends is independent of how he rates his available time slots.

As a result of Lemma 3, when a responder wants to decide on the ratings for his available time slots, he does not need to consider the points; he only needs to consider the effect of his ratings on the selected time slot. This property helps us to prove the next lemma.

**Lemma 4.** The ratings are as discriminant as possible. That is, as long as the number  $D$  of satisfaction levels and the responder's budget allow, it is optimal for him to give unequal ratings to time slots with unequal values.

Based on Lemmas 1-4, we can state the following main theorem.

**Theorem 3.** For any  $L_{min}$ ,  $L_{max}$ , and  $D$ , the mechanism  $\Gamma = (L_{min}, L_{max}, D, t(.))$  where reward function  $t(.)$  is derived by Algorithm 1 is Bayesian incentive-compatible from the view point of privacy-sensitive responders.

## 6.2 Initiator's Faithfulness

In the NMS mechanism, the initiator is supposed to 1) participate in the mechanism voluntarily, 2) make at least  $L_{min}$  offers at each round, if he is able to do so, 3) choose a feasible start time, and 4) award convenience points to the responders according to reward function  $t(\cdot)$ . In the following, we discuss briefly why the initiator has no incentive to deviate from any of the above-mentioned actions.

Voluntarily participation of the initiator can be proved following similar steps to Theorem 2. The initiator offers at least  $L_{min}$  time slots at each round to preserve the chance of continuing the negotiation. Since otherwise, he may end up failing the scheduling, while a feasible time slot exists. In this case, the utility of the initiator is  $-\infty$  and hence, he does his best to avoid it.

Setting the meeting at a time when some agents are busy is equivalent to failing the meeting scheduling, which values  $-\infty$  to the initiator. Therefore, the initiator never chooses a time slot to which at least one agent give 0 rating. The attendance of responders at the meeting is conditioned by receiving the corresponding rewards. Therefore, if the initiator does not award the promised points to the responders, they do not participate in the meeting. This fact prevents the initiator from deviating from giving responders the promised rewards.

## 7 Conclusions

Using a score-voting approach, we described an incentive mechanism for DisCSPs with selfish and privacy-sensitive agents. Our mechanism is online and can be implemented in dynamic situations where the decision variables and constraints are evolving over time. Moreover, we showed that the mechanism is faithful and can be run by selfish agents, with no need to a central trusted entity. Devising a control parameter, we made the mechanism adjustable to different scenarios in which agents assign different weights to privacy and efficiency. We presented the mechanism via the domain of meeting scheduling, however, this mechanism can be easily applied to a wide range of multi-agent systems.

## 8 APPENDICES

### 8.1 Proof of Theorem 1

For a fixed  $L$  and  $A \leq L$ , the set of equations (10) consists of one constraint corresponding to each flexibility  $F$  such that  $F \pmod{A+1} > 0$ . When  $F = A$ , we have

$$N_{i,n}^d = \begin{cases} A, & \text{If } d = 1, \\ 0, & \text{Otherwise.} \end{cases} \quad (12)$$

Therefore,  $P(d, A, F) = 0$ , for  $d > 1$ . Moreover, based on the mechanism description and Eq. (8), giving rate 1 to offers does not require spending points.

Thus, we have  $C(A, A) = 0$ . Therefore, the constraint corresponding to  $F = A$  reduces to

$$\begin{aligned} P(1, A, A) t(1, A, A, L) = \\ P(1, A, A) t(1, A, A, L). \end{aligned} \quad (13)$$

This constraint is trivially satisfied with any value of  $t(1, A, A, L)$ . Therefore, we choose an arbitrary positive value for  $t(1, A, A, L)$ .

Now, we consider equations corresponding to  $F \neq A$ . In these cases, in order to satisfy condition (b), we define  $t(D - 1, A, F, L) = 0$ . Then, the constraint corresponding to  $F$  becomes

$$\begin{aligned} \sum_{d=1}^{D-2} P(d, A, F) t(d, A, F, L) - C(A, F) = \\ P(1, A, A) t(1, A, A, L). \end{aligned} \quad (14)$$

Substituting the chosen value of  $t(1, A, A, L)$  in (14), the constraints corresponding to different flexibilities  $F$  will have no common variable. Therefore, For each  $F \neq A$  such that  $F \pmod{A+1} > 0$ , we have a separate multi-variable linear equation with positive coefficients and positive sum, which always has a non-negative solution that satisfies condition (c). Each rating  $r$  with  $F > 0$  and  $F \pmod{A+1} = 0$ , has a shifted version  $r'$  with  $F' \pmod{A+1} > 0$ . To satisfy constraint (d), we define  $t(d', A', F', L) = t(d, A, F, L)$ .

## 8.2 Proof of Theorem 2

If an agent does not participate in the meeting scheduling process, the initiator translates this into agent's unwillingness to attend the meeting. Therefore, since the attendance of all agents are necessary for the meeting to be held, the meeting scheduling fails. In this case, the utility of all agents would be  $-\infty$ . Therefore, the agents prefer to participate in the mechanism to preserve the chance of finding a suitable time for the meeting, and eventually getting a utility greater than  $-\infty$ .

## 8.3 Proof of Lemma 1

In this problem, the responders are expected utility-maximizers. The utility of some of the outcomes are  $-\infty$ . Therefore, each responder's first priority is to reduce the chance of occurring these intolerable events. There are two such events in the meeting scheduling problem: E1) when the meeting is set at a time the responder is actually busy and hence cannot attend; and E2) when the meeting scheduling is failed.

If a responder who is busy at a time slot  $s$  reports to be free, he gives a chance to time slot  $s$  to be selected as the time of the meeting. This increases the probability of event E1 and hence is not in favor of the agent. Therefore, a responder who is busy at a time slot always rates it at 0.

On the other side, reporting to be busy when the agent is actually free at a time slot  $s$  removes the chance of that slot being selected as the time of the meeting. This time slot could be the only time slot in which all of the attendees are available. Therefore, this false report increases the probability of event E2 and hence is not aligned with the responder's preferences. As a result, a responder who is free at a time slot never rates it at 0. This completes the proof of Lemma 1.

#### 8.4 Proof of Lemma 2

We prove this lemma by contradiction. Consider two time slots  $s_n^1$  and  $s_n^2$  where  $V_i(s_n^1) > V_i(s_n^2)$ . Suppose that in round  $n$  of the mechanism, agent  $i$  is asked to rate time slots  $\{s_n^1, s_n^2, \dots, s_n^{L_n}\}$ , and he gives a higher rating to  $s_n^2$  than  $s_n^1$ , i.e.  $r_{i,n}^2 > r_{i,n}^1$ . Now, we construct another rating  $\mathbf{r}'_{i,n}$  from  $\mathbf{r}_{i,n}$  by exchanging its first and second elements (i.e.  $\mathbf{r}'_{i,n} = (r_{i,n}^2, r_{i,n}^1, r_{i,n}^3, \dots, r_{i,n}^{L_n})$ ) and show that the agent could achieve a higher expected utility if he rated the time slots with  $\mathbf{r}'_{i,n}$ . Showing this fact contradicts the rational behavior of the agent and hence proves Lemma 2.

The expected instantaneous utility agent  $i$  gets at round  $n$  of the mechanism, when he rates the offered time slots with  $\mathbf{r}_{i,n}$  is

$$\mathbf{E}[U_i] = \sum_{j=1}^{L_n} (N_{i,n}^{r_{i,n}^j})^{-1} P(r_{i,n}^j, A_{i,n}, F_{i,n}) [V_i(s_n^j) + t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n)] - \theta_i L(M_i) - C_{i,n}. \quad (15)$$

The costs of ratings  $\mathbf{r}$  and  $\mathbf{r}'$  are the same. Therefore, if agent  $i$  changes his ratings to  $\mathbf{r}'_{i,n}$ , his expected utility will be

$$\begin{aligned} \mathbf{E}[U'_i] = & (N_{i,n}^{r_{i,n}^2})^{-1} P(r_{i,n}^2, A_{i,n}, F_{i,n}) V_i(s_n^1) + (N_{i,n}^{r_{i,n}^1})^{-1} P(r_{i,n}^1, A_{i,n}, F_{i,n}) V_i(s_n^2) + \\ & \sum_{j=3}^{L_n} (N_{i,n}^{r_{i,n}^j})^{-1} P(r_{i,n}^j, A_{i,n}, F_{i,n}) V_i(s_n^j) + \\ & \sum_{j=1}^{L_n} (N_{i,n}^{r_{i,n}^j})^{-1} P(r_{i,n}^j, A_{i,n}, F_{i,n}) t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n) - \theta_i L(M_i) - C_{i,n}. \end{aligned} \quad (16)$$



Comparing these two utilities we have:

$$\begin{aligned}
\mathbf{E}[U'_i] - \mathbf{E}[U_i] &= \\
& (N_{i,n}^{r_{i,n}^2})^{-1} P(r_{i,n}^2, A_{i,n}, F_{i,n}) V_i(s_n^1) + (N_{i,n}^{r_{i,n}^1})^{-1} P(r_{i,n}^1, A_{i,n}, F_{i,n}) V_i(s_n^2) - \\
& (N_{i,n}^{r_{i,n}^1})^{-1} P(r_{i,n}^1, A_{i,n}, F_{i,n}) V_i(s_n^1) - (N_{i,n}^{r_{i,n}^2})^{-1} P(r_{i,n}^2, A_{i,n}, F_{i,n}) V_i(s_n^2) = \\
& ((N_{i,n}^{r_{i,n}^2})^{-1} P(r_{i,n}^2, A_{i,n}, F_{i,n}) - (N_{i,n}^{r_{i,n}^1})^{-1} P(r_{i,n}^1, A_{i,n}, F_{i,n})) (V_i(s_n^1) - V_i(s_n^2)) \\
& (K^2 - K^1) (V_i(s_n^1) - V_i(s_n^2)),
\end{aligned} \tag{17}$$

where  $K^d := (N_{i,n}^d)^{-1} P(d, A_{i,n}, F_{i,n})$ ,  $d = 1, 2$ , is the probability that a specific time slot that gets rate  $d$  from agent  $i$  is selected at round  $n$ . Since the subsidies are decreasing in terms of  $d$ , the initiator has more incentive to select higher-rated time slots. Therefore, we have  $K^2 > K^1$ . Using this result, we can conclude from (17) that  $\mathbf{E}[U'_i] > \mathbf{E}[U_i]$  which contradicts the rational behavior of the agent and shows that agent  $i$  achieves more utility if he rates the offered time slots in an ordering consistent with his valuation function.

### 8.5 Proof of Lemma 3

We proved in Lemma 1 that the agents announce their F/B status truthfully. Therefore, all the rational ratings a responder  $i$  would give to the offered time slots at round  $n$  have the same  $A_{i,n}$ . For each rational rating  $\mathbf{r}_{i,n}$ , the expected instantaneous reward of agent  $i$  minus the points he spends in round  $n$  is

$$\begin{aligned}
\mathbf{E}[R_i | \mathbf{r}_{i,n}] - C_{i,n} &= \\
& \sum_{j=1}^{L_n} (N_{i,n}^{r_{i,n}^j})^{-1} P(r_{i,n}^j, A_{i,n}, F_{i,n}) t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n) - C(A_{i,n}, F_{i,n}) = \\
& \sum_{d=1}^{D-1} P(d, A_{i,n}, F_{i,n}) t(d, A_{i,n}, F_{i,n}, L_n) - C(A_{i,n}, F_{i,n}),
\end{aligned} \tag{18}$$

where the last equality holds since the reputation function  $F$  is invertible.

Now, suppose that responder  $i$  rates at least one time slot with 1. In this case, we have  $N_{i,n}^1 > 0$  and hence  $F \pmod{A+1} > 0$ . According to condition (a) (Eq. (10)), we have

$$\begin{aligned}
\mathbf{E}[R_i | \mathbf{r}_{i,n}] - C_{i,n} &= \sum_{d=1}^{D-1} P(d, A_{i,n}, F_{i,n}) t(d, A_{i,n}, F_{i,n}, L_n) - C(A_{i,n}, F_{i,n}) = \\
& P(1, A_{i,n}, A_{i,n}) t(1, A_{i,n}, A_{i,n}, L_n),
\end{aligned} \tag{19}$$

for all ratings  $\mathbf{r}_{i,n}$  with the true F/B reports and  $N_{i,n}^1 > 0$ . Equation (19) shows that the expected instantaneous reward agent  $i$  gets minus the points he spends

in each round  $n$  is the same for all possible ratings over the available time slots, provided that  $N_{i,n}^1 > 0$ .

Now, to complete the proof of Lemma 3, we need to prove that it is always optimal for each responder  $i$  to rate at least one available time slot with 1. We prove this by contradiction. Suppose the statement is false. That is, suppose there exists an optimal rating  $\mathbf{r}'_{i,n}$  for responder  $i$  with  $N_{i,n}^1 = 0$ . We define  $\mathbf{r}_{i,n}$  such that  $\mathbf{r}'_{i,n} = (\mathbf{r}_{i,n} + 1) \text{sign}(\mathbf{r}_{i,n})$ . Now, according to condition (d), we have  $t(d, A, F, L) = t(d', A', F', L)$ . Since the payments are the same, ratings  $\mathbf{r}_{i,n}$  and  $\mathbf{r}'_{i,n}$  have the same impact on the initiator's decision making. Therefore,  $P(d, A, F) = P(d', A', F')$ . So, the expected number of points responder  $i$  gets at round  $n$  is the same for both ratings. However, responder  $i$  needs to spend less points to rate the offers with  $\mathbf{r}_{i,n}$ . Therefore, changing the rates to  $\mathbf{r}_{i,n}$  reduces the agent's cost while having the same impact on both the meeting scheduling process and the expected reward the agent gets. Therefore, it increases the agent's utility and contradicts the assumption that  $\mathbf{r}'_{i,n}$  is optimal. This completes the proof of Lemma 3.

### 8.6 Proof of Lemma 4

Suppose that at round  $n$ , the initiator offers time slots  $\{s_n^1, \dots, s_n^{L_n}\}$  to the responders. We assume that the numbering of these time slots are according to agent  $i$ 's preferences. That is,

$$V_i(s_n^1) \geq V_i(s_n^2) \geq \dots \geq V_i(s_n^{L_n}). \quad (20)$$

We focus on time slots  $\{s_n^1, \dots, s_n^{A_{i,n}}\}$  in which agent  $i$  is available. Responder  $i$  rates each of these time slots with an integer number between 1 and  $D - 1$ .

Suppose that agent  $i$  strictly prefers time slot  $s_n^k$  to  $s_n^{k+1}$ , i.e.  $V_i(s_n^k) > V_i(s_n^{k+1})$ , but he gives the same rating to both of these time slots, i.e.  $r_{i,n}^k = r_{i,n}^{k+1}$ . We also assume that  $N_{i,n}^1 > 0$ . We also assume  $N_{i,n}^{D-1} = 0$ , meaning that there is room for separation between  $s_n^k$  and  $s_n^k$ . In this case, we will prove that agent  $i$  could achieve a higher utility if he changes his rating to  $\mathbf{r}'_{i,n}$ , where

$$r'_{i,n} = \begin{cases} r_{i,n}^j + 1, & \text{If } j \leq k, \\ r_{i,n}^j, & \text{If } j > k. \end{cases} \quad (21)$$

We define the selection vector  $K_{i,n} = (K_{i,n}^1, \dots, K_{i,n}^{A_{i,n}})$ , where  $K_{i,n}^j = (N_{i,n}^{r_{i,n}^j})^{-1} P(r_{i,n}^j, A_{i,n}, F_{i,n})$  is the probability that a specific time slot that gets rate  $r_{i,n}^j$  at round  $n$  from agent  $i$  is selected as the meeting time. We denote the selection vector corresponding to rating  $\mathbf{r}'_{i,n}$  by  $K'_{i,n}$ . Using these new notation, we can write the expected instantaneous utility of agent  $i$  at round  $n$  when he gives ratings  $r_{i,n}^j$  and  $\mathbf{r}'_{i,n}$ , by

$$\mathbf{E}[U_i] = \sum_{j=1}^{A_{i,n}} K_{i,n}^j [V_i(s_n^j) + t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n)] - C(A_{i,n}, F_{i,n}) - \theta_i L(M_i), \quad (22)$$

and

$$\mathbf{E}[U'_i] = \sum_{j=1}^{A_{i,n}} K_{i,n}'^j [V_i(s_n^j) + t(r_{i,n}'^j, A_{i,n}, F_{i,n}', L_n)] - C(A_{i,n}, F_{i,n}') - \theta_i L(M_i), \quad (23)$$

respectively.

In Lemma 3, we proved that the expected reward the agent gets minus the cost of rating does not depend on his exact ratings. That is,

$$\begin{aligned} \sum_{j=1}^{A_{i,n}} K_{i,n}^j t(r_{i,n}^j, A_{i,n}, F_{i,n}, L_n) - C(A_{i,n}, F_{i,n}) = \\ \sum_{j=1}^{A_{i,n}} K_{i,n}'^j t(r_{i,n}'^j, A_{i,n}, F_{i,n}', L_n) - C(A_{i,n}, F_{i,n}'). \end{aligned} \quad (24)$$

Therefore, we have

$$\mathbf{E}[U'_i] - \mathbf{E}[U_i] = \sum_{j=1}^{A_{i,n}} K_{i,n}'^j V_i(s_n^j) - \sum_{j=1}^{A_{i,n}} K_{i,n}^j V_i(s_n^j) = \mathbf{E}_{K'}[V_i] - \mathbf{E}_K[V_i]. \quad (25)$$

Using the properties of the payment function, it can be shown that selection vectors satisfy the following condition:

$$\sum_{j=1}^l K_{i,n}'^j \geq \sum_{j=1}^l K_{i,n}^j, \quad (26)$$

for all  $l \leq A_{i,n}$ . This means that the selection vector  $K_{i,n}'$  has first-order stochastic dominance over  $K_{i,n}$ . Now, we can use the first-order stochastic ranking theorem stated below. This theorem is proved in the literature.

**Theorem 4.** If  $u$  is strictly increasing, and cumulative  $F$  first-order stochastically dominates cumulative  $G$ , then  $\mathbf{E}_F[u(x)] \geq \mathbf{E}_G[u(x)]$ .

Using this theorem, we can conclude that  $\mathbf{E}_{K'}[V_i] > \mathbf{E}_K[V_i]$ . Substituting this in (25) we have,  $\mathbf{E}[U'_i] > \mathbf{E}[U_i]$ . Therefore, more discriminative ratings give agents higher utilities, and hence are better for them.

### 8.7 Proof of Theorem 3

Proof of Theorem 3 is directly derived from Lemmas 1, 2, and 4.

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