Quantifying Blur in Color Images using Higher Order Singular Values

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This letter introduces a novel framework for blind blur assessment in color images using higher order singular values. The RGB color image is seen as a 3rd-order tensor to exploit the spatial and inter-channel correlations, so that blurring effects are captured more robustly. The tensor is decomposed into different two-dimensional matrices, also called unfoldings. The conventional singular value decomposition is carried out for these unfoldings instead of computing it for the luminance component alone. The experiments were performed on several publicly available databases and the results validate the superiority of the proposed metric among different state-of-the-art blind blur assessment metrics. The proposed framework for image quality assessment (IQA) from color images fits well with the current trends and research efforts put in enhancing the quality of experience for different multimedia applications and in benchmarking new imaging and sensing technologies including camera and other vision systems with IQA capabilities.

Introduction: Digital images and videos form an important source of information for our daily living, our quality of experience, and the different social and economic aspects of society. Unfortunately, most images/videos data undergo different types of distortions due to transmission errors, compression, and post-processing operations. These distortions severely degrade image quality which results in inaccurate perceptual judgement. Among different types of distortions, blur is the most commonly observed one, due to the various limitations of acquisition and transmission systems. The role of image quality assessment (IQA) methods is not only to quantify image degradations but also in benchmarking different sensing and acquisition technologies as well as in optimizing different image processing systems and algorithms.

Different blind blur assessment techniques have been proposed in the literature based on the cumulative probability of blur detection [1], spectral radial energy [2], signal energy analysis using singular value decomposition (SVD) [3], blur similarity using SVD [4] etc.. The singular values of an image, in particular, when plotted against their indices were shown to follow an exponentially decreasing curve with the degree of the exponent varying with the amount of blur [3].

Existing IQA techniques are mostly based on either the luminance component of a color image or use separate color channels followed by pooling of the results to get the final quality score. However, for a color image, there exists a strong correlation among the Red, Green, and Blue components. Different distortions may influence different color components and disturb the correlations among these as well. The loss of color due to the different types of degradations substantially affects human perception. Moreover, the perception of blur is also different across color components. Therefore, it is inappropriate to completely ignore the correlations among the color components in IQA. The idea is to search for representations where the inter-channel correlation can be exploited to capture the effect of blur on the different color channels. Wang et al. [5] proposed the use of three color channels for full-reference (FR) quality assessment using SVD of the quaternion matrix. The quaternion matrix was generated by taking the local variance of Red, Green, and Blue channels as imaginary components and the luminance as the real component. The overall quality score was derived by computing the distance in singular values of image blocks in the reference and the distorted images (averaged over all blocks).

Motivated by the work in [3,4] on using the luminance-based SVD for blur assessment, we introduce here a new framework for IQA of color images using the so-called higher order singular values. We propose to use tensor analysis to fully represent the correlations among different color components. Tensors are used to extract useful information from high-dimensional data rather than from the two-dimensional (2D) matrices [6]. The higher order singular value decomposition (HOSVD) is an efficient tensor decomposition technique used in different image processing applications [6]. Cheng et al. [7] used tensors for IQA of color images under the simple case of FR. Here, we introduce for the first time, blind blur assessment in color images using HOSVD. We consider a given color image as a tensor and compute the higher order singular values from its unfoldings. We provide, some mathematical background of SVD, and tensors in the next section, followed by our proposed algorithm. We are convinced that the idea of using tensors for no-reference (NR) IQA of color images is new and has not been discussed in the literature.

Preliminaries on Tensors and HOSVD: A 2D image, $\mathbf{A} \in \mathbb{R}^{M \times N}$, satisfying some regularity conditions, can be decomposed using SVD as:

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{T}} \tag{1}$$

where $\mathbf{U} \in \mathbb{R}^{M \times M}$ is the matrix of left singular vectors, $\mathbf{V} \in \mathbb{R}^{N \times N}$ is the right singular matrix, and $\mathbf{\Sigma} \in \mathbb{R}^{M \times N}$ is the rectangular diagonal matrix of singular values arranged in descending order. The singular values vector can be extracted as $\mathbf{d} = \mathrm{diag}(\mathbf{\Sigma}) = [\sigma_1, \sigma_2, \cdots, \sigma_r]$ for $i = 1, 2, \cdots, r$, r being the rank of $\mathbf{A}$. The $\mathbf{U}$ and $\mathbf{V}$ give structural information along the rows and columns of $\mathbf{A}$ while the $\mathbf{d}$ represents the luminance or energy information of $\mathbf{A}$. In [3], Sang et al. showed that the singular values computed from the luminance component of natural images, when plotted against their indices, follow an exponential decay. The exponent coefficient of the decay, α, varies with the amount or the degree of blur. The final expression used as the quality score is:

$$\alpha = \sum_{k=1}^{r} \ln(d_k)\ln(k) \Big/ \sum_{k=1}^{r} \ln(k)\ln(k) \tag{2}$$

where d_k represents the singular value at index k.

While SVD is well suited to analyze 2D matrices, it cannot be used directly with higher-dimensional arrays. For such arrays, the concept of SVD has been extended using tensor theory. To formally define a tensor, let $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$ be an N^{th} order tensor with N indices where $I_1, I_2, \cdots, I_N$ are the upper limits of each dimension. Here, we propose to represent a given RGB color image as a 3rd-order tensor $\mathcal{A}$ with $I_1, I_2,$ and I_3 representing height, width, and number of color channels. The rearrangement of tensor elements into a 2D matrix is known as unfolding. For a 3rd-order tensor (an RGB image) $\mathcal{A}$, a 2D matrix or slice is obtained by fixing one of the three indices. In this way, the mode$_{(n)}$ unfolding of an RGB image is a matrix $\mathbf{A}_{(n)} \in \mathbb{R}^{I_n \times \prod_{k,k \neq n} I_k}$. For illustration, mode$_{(2)}$ unfolding or slice of a 3rd-order tensor is shown in Fig. 1. The HOSVD is the SVD of each of the tensor unfoldings [6].

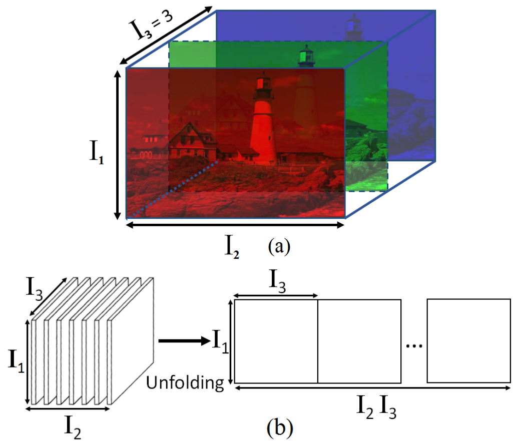

Fig. 1 *An example of* 3rd*-order tensor (a), mode*$_{(2)}$ *unfolding (b)*

The Proposed Technique: We first decompose the color image into three unfoldings and compute the higher order singular values for each unfolding. Confirming previous results, we observed the exponential decay of the singular values. Fig. 2 shows two images with different degrees of blur and the plots of higher order singular values for the original image and its five blurred versions, from the TID2013 database. From the plots, we clearly notice that the degree of the exponent varies with the degree of blur. The correlation between the degree of blur and the decay parameter is even stronger than the case of the luminance alone as considered in previous work and will be shown in our experiments. Throughout all the experiments, we observed a higher correlation between the subjective score and the blur metric computed using HOSVD of mode$_{(2)}$ unfolding. The reason is that mode$_{(2)}$ unfolding contains more spatial and inter-channel correlations than other modes. Therefore, we perform HOSVD only on mode$_{(2)}$ unfolding. The proposed technique is summarized here:

Step 1: For an RGB image, perform matrix unfoldings, $\mathbf{A}_{(2)} \in \mathbb{R}^{I_1 \times I_2 I_3}$
Step 2: Take HOSVD of unfolding $\mathbf{A}_{(2)}$. $\mathbf{A}_{(2)} = \mathbf{U}^{(2)}\mathbf{\Sigma}^{(2)}\mathbf{V}^{(2)T}$, where $\mathbf{\Sigma}^{(2)}$ is a diagonal matrix with higher order singular values corresponding to mode$_{(2)}$ unfolding of the color image.
Step 3: Compute the blur metric using (2).

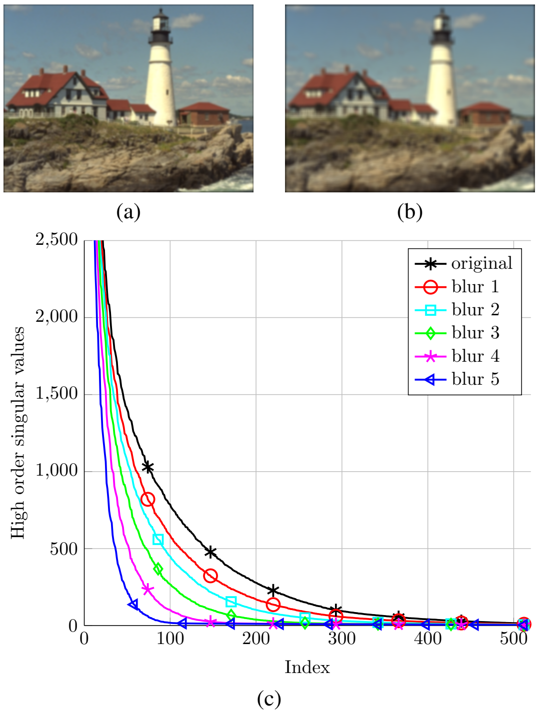

Fig. 2 *Sample images with different degree of blur from TID2013 database [8]: blur 4, MOS = 3.23, $\alpha = 1.48$ (a), blur 5, MOS = 2.16, $\alpha = 1.70$ (b), singular value curves for blurred images using HOSVD (c)*

Experimental Results: For our experiments, we used four publicly available databases [8] and [9]. Since the paper deals with blur assessment in color images, we only used the blur distorted images from these databases. The performance was evaluated using the spearman rank order correlation coefficient (SROCC), the pearson linear correlation coefficient (PLCC), and the root mean-squared error (RMSE). High values of SROCC and PLCC and low values of RMSE correspond to close relationship of the objective scores to the subjective ratings.

A 5-parameter logistic fitting function [10] is used for the calculation of PLCC and RMSE to account for the non-linearity in the subjective scores due to human opinions. The fitting function we used here is:

$$Q(q) = \beta_1 \left[\frac{1}{2} - \frac{1}{1 + \exp(\beta_2(q - \beta_3))}\right] + \beta_4(q) + \beta_5 \qquad (3)$$

where Q represents the fitted objective score after non-linear mapping, q is the calculated objective quality score, and β_k for $k = 1, 2, 3, 4, 5$ are fitting parameters. These parameters are calculated by minimizing the mean-squared error between the subjective scores and the fitted values.

We have compared our results to different state-of-the-art NR-IQA techniques, but in this letter to save space, we provide the comparisons for only those metrics that are related to blind blur assessment. The results of SROCC, PLCC, and RMSE for different databases are shown in Table 1. The proposed metric gives better performance for three of the four databases. For the LIVE2 database, the proposed metric has also consistent performance except for [4], where the results are very much comparable. From the weighted average, it is also evident that the proposed metric achieves the best overall performance in terms of prediction accuracy and monotonicity at the cost of slight increase in computational complexity. To further assess the performance of our metric using tensors, we also used two other color spaces shown to rely on weakly correlated components, namely the CIELab and YCbCr color spaces. The results we obtained were consistent across different color spaces whether these rely on strongly or weakly correlated components. To save space, we report in Table 2, our results for the CSIQ database comparing RGB, CIELab, and YCbCr color spaces. We note again that the results are consistent across different color spaces.

Table 1: Results of different evaluation metrics

Database	Metric	[1]	[2]	[3]	[4]	Proposed
CSIQ	SROCC	0.8904	0.8452	0.9167	0.9196	**0.9334**
[8]	PLCC	0.9188	0.8826	0.9433	0.9417	**0.9543**
	RMSE	0.1181	0.1407	0.0994	0.1007	**0.0894**
LIVE2	SROCC	0.9425	0.9030	0.9454	**0.9502**	0.9497
[8]	PLCC	0.9154	0.8912	0.9370	**0.9484**	0.9375
	RMSE	9.6520	10.8732	8.6241	8.0151	**7.5703**
TID2013	SROCC	0.8929	0.8152	0.8970	0.8833	**0.9086**
[8]	PLCC	0.8774	0.8023	0.8839	0.8833	**0.8928**
	RMSE	0.8826	1.0979	0.8611	0.8623	**0.8287**
CID:IQ	SROCC	0.7829	0.7120	0.8386	0.8188	**0.8730**
[9]	PLCC	0.7846	0.7542	0.8672	0.8499	**0.9008**
	RMSE	1.0842	1.1484	**0.8708**	0.9226	0.8868
Weighted	SROCC	0.8820	0.8252	0.9031	0.8977	**0.9190**
Average	PLCC	0.8793	0.8385	0.9113	0.9101	**0.9239**
	RMSE	3.0883	3.4897	2.7536	2.6003	**2.4611**

Table 2: Results for different color spaces on CSIQ database

Color Space	SROCC	PLCC	RMSE
RGB	0.9334	0.9543	0.0894
CIELab	0.9321	0.9537	0.0899
YCbCr	0.9321	0.9554	0.0884

Conclusion: We introduced, in this letter, a novel no-reference blur assessment technique for color images, using higher order singular values. The spatial and inter-channel correlations, in the color image, are exploited using tensors to quantify the amount of blur more consistently compared to luminance-only-based techniques. Our experimental results, performed on different public IQA databases, validated the power and consistency of the proposed metric across different color spaces compared to state-of-the-art no-reference blur assessment metrics.

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