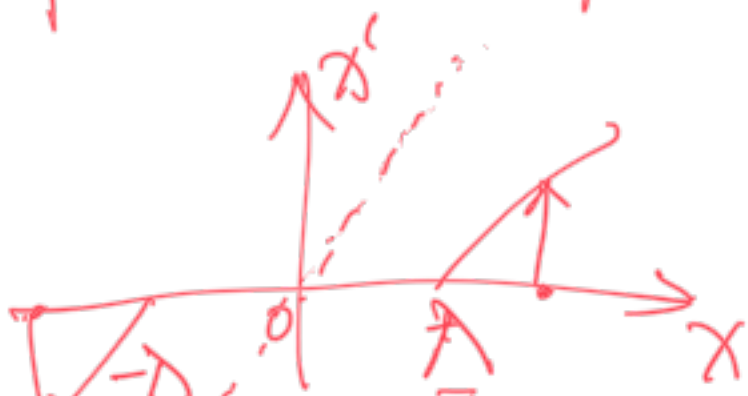
$$\underset{S}{\operatorname{argmin}} \frac{1}{2}\|X - AS\|_F^2 + \lambda\|S\|_1$$

Gradient Descend + Proximal operator.

Generalisation of Projection.

Prox of $\|\cdot\|_1$ is soft-thresholding. (Soft_λ)



$$X' = \text{Soft}_\lambda(X)$$

$\lambda > 0$.

$$X' = \begin{cases} X - \lambda, & \text{if } X \geq \lambda \\ 0, & \text{if } -\lambda \leq X < \lambda \\ X + \lambda, & \text{if } X < -\lambda \end{cases}$$

$$f(S) = \frac{1}{2}\|X - AS\|_F^2$$

$$= \langle X - AS, X - AS \rangle$$

$$= \langle X, X \rangle - \langle X, AS \rangle - \langle AS, X \rangle + \langle AS, AS \rangle$$

$$= \langle X, X \rangle - \langle A^T X, S \rangle - \langle S, A^T X \rangle$$

$$\nabla_S f = -A^T(X - AS)$$

L: Lipschitz constant of $\nabla_A f$

$$L \overset{?}{=} \|A^T A\|_F \overset{?}{=} \|A\|_2^2$$

↑ spectral norm

→ sub-problem 2.

$$\min_A \frac{1}{2}\|X - AS\|_F^2 + g(A).$$

Gradiant-Dscend + Projection

$$\nabla_A f = -(X - AS)S^T$$

$$A^{(i+1)} = \text{Proj}_{\tilde{}}\left(A^{(i)} - \frac{\nabla_A f(A^{(i)})}{L}\right)$$