

Analysis

$$\Phi \in \mathbb{C}^{T \times B}, B \geq T$$

Synthesis

$$\tilde{S} = S \Phi^*$$

$$S = \underbrace{\tilde{S}}_{T \times B} \underbrace{\Phi^*}_{B \times T}$$

→ wavelet

→ STFT (Short-time Fourier Transform)

→ DCT (Discrete Cosine Transform)

→ etc

→

$$X = AS + n$$

$$\rightarrow (A^*, S^*) = \arg \min_{A, S} \underbrace{\frac{1}{2} \|X - AS\|_F^2}_{\text{data loss}} + \underbrace{\lambda \|S\|_1}_{\text{sparsity regularization}}$$

$$\rightarrow (A^*, S^*) = \begin{cases} \underset{A, S}{\operatorname{argmin}} \frac{1}{2} \|X - AS\|_F^2 + \| \underline{\alpha} \|_1 \\ \text{s.t. } S = \underline{\alpha} \Phi^* \end{cases} + g(A)$$

$$\rightarrow \underset{A, S}{\operatorname{argmin}} \frac{1}{2} \|X - A \alpha \Phi^*\|_F^2 + \| \underline{\alpha} \|_1 + g(A).$$

with $g(A) = \begin{cases} 0, & \text{if } \forall \|a_i\|_2 = 1 \\ +\infty, & \text{if not} \end{cases}$

$$= \frac{a_{11}S_1 + a_{12}S_2}{a_{21}S_1 + a_{22}S_2}$$

$$A = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

$$\min \|S\|_1$$

$$\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} \cdot \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

$$\begin{pmatrix} 1000 \\ 500 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.1 \\ 0.5 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1000 & 500 \\ 500 & 100 \end{pmatrix}$$

$$\begin{cases} x_1 = a_{11}S_1 + a_{12}S_2 \\ x_2 = a_{21}S_1 + a_{22}S_2 \end{cases}$$

$$x_1 = \underline{a_{11}} \cdot P \left[\frac{1}{P} S_1 \right] + a_{12} \underline{S_2}$$

$$x_2 = \underline{\quad} = \underline{\quad}$$

$$\min_{\boxed{A, S}} \frac{1}{2} \|X - AS\|_F^2 + \underbrace{\lambda \|S\|_1}_{\uparrow} + g(A).$$

→ BCD. Block-coordinate-Descend. ~~↕~~ →

for iteration i

→ Fix A, $\min_S \frac{1}{2} \|X - AS\|_F^2 + \underbrace{\lambda \|S\|_1}_{S^{(i)}} + \underbrace{g(A)}_{\cancel{g(A)}}$

Sub-problem 1

→ Fix S = S⁽ⁱ⁾, $\min_A \frac{1}{2} \|X - AS\|_F^2 + g(A)$.

Sub-problem 2.