SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

50.007 Machine Learning, Spring 2023
Homework 4

Due Monday 17 April 2023, 11.59pm

In this homework, we would like to look at Gaussian Mixture Model and Bayesian Network.

1 Gaussian Mixture Model [50 points]

Image segmentation is an important step towards scene understanding for autonomous systems. Figure 1 gives an example of image segmentation that decouples the image foreground from the background. The Gaussian mixture model (GMM) is one of the earliest approaches used for image segmentation.

(a) (b)

Figure 1: An example of image segmentation. (a) Original image. (b) segments of the original image.

When we formulate the image segmentation task as a Gaussian mixture model, we assume that there are K segments and each of them is modeled as a Gaussian distribution with $\mathcal{N}(\mu_k, \Sigma_k)$. These segments form a Gaussian mixture model with each segment weighted by a mixing weight π_k. We encapsulate these parameters into a parameter vector $\theta = (\pi_1, \ldots, \pi_K, \mu_1, \ldots, \mu_K, \Sigma_1, ..., \Sigma_K)$.

Since we know neither the component from which each data point (each pixel) came, nor the parameters of each model, we use the EM algorithm to alternatively solve the parameter vector θ in Expectation step (E-step) and Maximization step (M-step).

In the E-step, we compute the probability that a pixel comes from a segment based on the model parameters from M-step. In the first iteration, the model parameters are initialized randomly (Note that π_k is initialized as $1/K$). In the M-step, we compute the parameters θ of every Gaussian model based on the probabilities from E-step.

[Task Description] You are required to use the EM algorithm in GMM to segment the foreground from the background, *i.e.* the number of segments $K = 2$, for three testing images: 'fox', 'owl', and 'zebra' (in the ./data folder). We use image colors in CIE-Lab color space as the observed data $x_i \in \mathbb{R}^3$ for the image segmentation task.

- The **input** is a RGB image. You are provided with a data file in the text format (*e.g.*, 'fox.txt'), where each line gives you a coordinate and the corresponding *Lab* value.

- For each input image, you are required to generate **three outputs**: one mask image, one image with only the foreground, and one image with only background.
- You are given **an example of the output images** to check whether your result is reasonable, as shown in Figure 2. Note that your results might differ slightly from the example results due to the choice of the initial values of the parameters and the number of iterations. (This example result is also given in the data folder.)

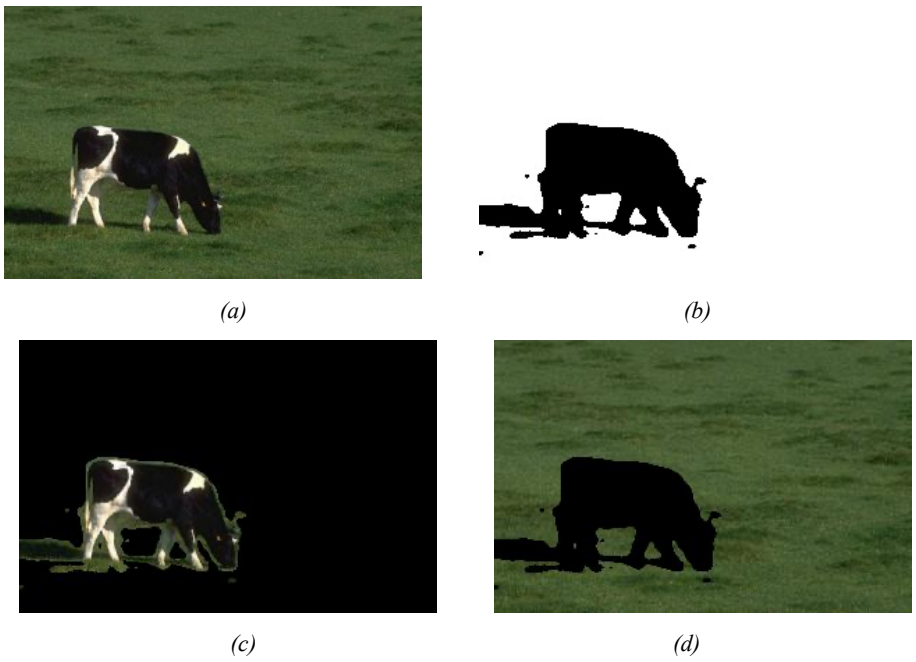


Figure 2: An example of image segmentation outputs. (a) The input image. (b) Mask image. (c) Foreground image. (d) Background image.

[Implementation] You are also given the Python source code (io_data.py) for file reading. You can use resources for mathematical operation, such as the determinant of a matrix, but you are not allowed to use any libraries or online codes for implementing the EM algorithm and Gaussian distribution. You need to write your own code to implement the EM algorithm, including parameter initialization, E-step and M-step, or convergence check (optional).

A helpful *tip* to remember is that if your method is experiencing slow convergence, you could try implementing a smart approach for parameter initialization. One potential method is to initialize the parameters μ_k and Σ_k within the range provided by the data.

[Submission] Please submit both your code and report.

1. (30 points) Submit your code together with crystal clear instructions to run the code (python version, package versions, etc.). The code must be ready to run code without requiring any changes. The TA will follow the instructions to run your code and grade accordingly.
2. (10 points) Run your program on all the three images we provide to you and submit all the output results (see the example in Figure 2) in the report.

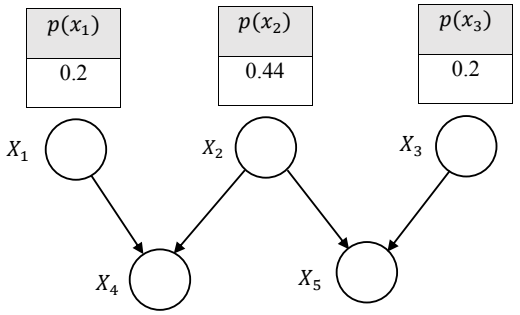
3. (10 points) In addition to the output results, you are required to briefly introduce your algorithm and equations in the report, e.g. which data feature(s) you use in EM.

2 Bayesian Network [50 points]

Given a Bayesian network shown in Figure 3, where each random variable takes a binary state, *i.e.* $x_i \in \{T, F\}$. Your task is to evaluate the following five probabilities and give the probability tables:

$$p(x_1|x5),\ p(x_2|x_4),\ p(x_3|x_2),\ p(x_4|x_3),\ p(x_5).$$

Please clearly write down all the necessary steps. (*Hint: you may find a short answer, please clearly explain it.*)



X_1	X_2	$p(x_4\|x_1, x_2)$
T	*T*	0.35
T	*F*	0.6
F	*T*	0.01
F	*F*	0.95

X_2	X_3	$p(x_5\|x_2, x_3)$
T	*T*	0.35
T	*F*	0.6
F	*T*	0.01
F	*F*	0.95

Figure 3: A Bayesian network with associated probability table for each random variable. Note that only the probability of $x_i = T$ is shown in the table.