1 Special Limits

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

2 Special Derivatives

$$\frac{\mathrm{d}[\sin x]}{\mathrm{d}x}=\cos x$$

$$\frac{\mathrm{d}[\cos x]}{\mathrm{d}x}=-\sin x$$

$$\frac{\mathrm{d}[\tan x]}{\mathrm{d}x}=\sec^2 x$$

$$\frac{\mathrm{d}[\sec x]}{\mathrm{d}x}=\sec x\tan x$$

$$\frac{\mathrm{d}[\cot x]}{\mathrm{d}x}=-\csc^2 x$$

$$\frac{\mathrm{d}[\csc x]}{\mathrm{d}x}=-\csc x\cot x$$

$$\frac{\mathrm{d}[arcsinx]}{\mathrm{d}x}=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}[arccosx]}{\mathrm{d}x}=-\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}[arctanx]}{\mathrm{d}x}=\frac{1}{1-x^2}$$

$$\frac{\mathrm{d}[arccotx]}{\mathrm{d}x}=-\frac{1}{1-x^2}$$

$$\frac{\mathrm{d}[arcsecx]}{\mathrm{d}x}=\frac{1}{|x|\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}[arccscx]}{\mathrm{d}x}=-\frac{1}{|x|\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}[\ln x]}{\mathrm{d}x}=\frac{1}{x}$$

$$\frac{\mathrm{d}[\log_a x]}{\mathrm{d}x}=\frac{1}{x\ln a}$$

$$\frac{\mathrm{d}[e^x]}{\mathrm{d}x}=e^x$$

$$\frac{\mathrm{d}[a^x]}{\mathrm{d}x}=a^x\ln a$$

3 Trig Rules

$$\int \tan x = \ln|\sec x| + c$$

$$\int \sec x = \ln|\sec x + \tan x| + c$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

4 Trig Integrals

$$\int \sin^m x \cos^n x$$

1. If n is odd $\rightarrow u = \sin x$
2. If m is odd $\rightarrow u = \cos x$
3. If n and m are even $\rightarrow$ Double Angle Formula

$$\int \tan^m x \sec^n x$$

1. If m is odd $\rightarrow u = \sec x$
2. If n is even $\rightarrow u = \tan x$

5 Trig Substitution Rules

$$\int \sqrt{a^2 - x^2} \rightarrow \sin\theta$$

$$\int \sqrt{x^2 - a^2} \rightarrow \sec\theta$$

$$\int \sqrt{x^2 + a^2} \rightarrow \tan\theta$$

6 Improper Integrals

$$\int_a^\infty f(x) = \lim_{b \to \infty} \int_a^b f(x)$$

7 Arc Length

Cartisan

$$\int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \mathrm{d}x$$

Parametric

$$\int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \mathrm{d}t$$

Polar

$$\int_a^b \sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2 + r^2}\, \mathrm{d}\theta$$

8 Area of Surface of Revolution

$$\int_a^b 2\pi r \mathrm{ds}$$

r $\rightarrow$ Radius (Could be an equation or a single variable)
ds $\rightarrow$ The Arc Length function (changes depending on type of equation given, eg. Parametric, Polar(see Section 7))

9 Parametric Equations

Tangent horizontal when $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$
Tangent vertical when $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \text{undef or} \infty$
if $y = f(x)$ then $\begin{cases} x = t \\ y = f(x) \end{cases}$
$\dfrac{y'(t)}{x'(t)} = \dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$
$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x} = \dfrac{\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}t} / \frac{\mathrm{d}x}{\mathrm{d}t}\right)}{\mathrm{d}x}$
Area under parametric curve $\displaystyle\int_a^b g(t) f'(t) \mathrm{d}t$

10 Polar Equations

Polar to Cartisan

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

Cartisan to Polar

$$\begin{cases} r^2 = x^2 + y^2 \\ \theta = tan^{-1}\left(\frac{y}{x}\right) \end{cases}$$

Area of a polar curve

$$\int_a^b \frac{1}{2}[f(\theta)]^2 \, \mathrm{d}\theta$$

11 Series

11.1 P-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} p > 1 \text{ Converges} \\ p \leq 1 \text{ Diverges} \end{cases}$$

11.2 Geometric Series

$$a, ar, ar^2, ... \sum_{n=1}^{\infty} ar^{n-1} \begin{cases} -1 < r < 1 \text{ converges at } \frac{a}{1-r} \\ \text{otherwise diverges} \end{cases}$$

Partial sum for geometric series

$$s_n = \frac{a(1-r^n)}{1-r}$$

11.3 Limit Test

if $\lim_{n\to\infty} \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges

11.4 Integral Test

$a_n = f(n)$ where f is
1. Continuous
2. Positive
3. Decreasing

Then $\sum_{n=1}^{\infty} a_n$ converges iff $\int_1^{\infty} f(x)\mathrm{d}x$ converges

11.5 Comparison Test

Suppose $a_n > b_n > 0$ for all $n \geq 1$

if $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges
if $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges
if $\lim_{n\to\infty} \frac{a_n}{b_n} =$ (a positive number) then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ have the same convergence/divergence

11.6 Alternating Series Test

$\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ where $a_n > 0$
1. if $a_n$is decreasing
2. $\lim_{n\to\infty} a_n = 0$

then $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ is convergent

11.7 Ratio Test/Root Test

Let $L = \lim_{n\to\infty} \left|\frac{b_n + 1}{b_n}\right|$
Let $L = \lim_{n\to\infty} \sqrt[n]{|b_n|}$
if $L < 1$ Series is absoluty convergent
if $L > 1$(or ∞) Series is divergent
if $L = 1$ Test is inconclusive

11.8 Taylor and Maclaurin Series

Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n(\leftarrow \text{derivative})}(a)}{n!}(x-a)^n \text{ for } |a-x| < R$$

Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n(\leftarrow \text{derivative})}(0)}{n!}(x)^n$$

12 Misc Rules

$\int \frac{1}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$

$\tan^{-1} \infty = \frac{\pi}{2}$

$\tan^{-1} -\infty = -\frac{\pi}{2}$

To find the intersection points between two curves, set them equal to each other

Area between two curves is top curve minus bottom curve