MFLua

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Abstract

We present a new implementation of METAFONT which embeds a Lua interpreter. It is fully compatible with canonical METAFONT but it has some internal "sensors" (read-only callbacks) to collect data for use in a possible post-processing. An example of post-processing that extracts the outlines of some glyphs is discussed.

1 Introduction

MFLua is a new implementation of METAFONT that embeds a Lua[3] interpreter. It doesn't introduce any new primitives so that a METAFONT file can be used with MFLua without any modification to produce exactly the same result. The Lua interpreter inside MFLua doesn't change in any way the internal state of METAFONT and it's not reachable from a METAFONT instance. This is a strict requirement: MFLua must be fully compatible at least with the current release of METAFONT (which is currently 2.718281).

The Lua interpreter is used to register the data coming from the "Lua sensors" which are, practically, read-only callbacks, i.e. functions inserted into the PascalWEB code that call external Lua scripts, which eventually do nothing. Some sensors can store the same information available with the various `tracing` instructions, but some others are placed where there are not tracing instructions — and also not all the procedures with tracing instructions have a sensor. The goal is to collect as much data as possible to produce the outlines of a METAFONT picture — most of the time a glyph.

Important note: although MFLua is able to process a full set of characters, it's still an alpha quality code — a bit more than a proof-of-concept .

2 The Lua sensors

It's well known that luaTEX embeds a Lua interpreter, and it's relatively simple to read its source code to find where and how the interpreter is initialised; this is, moreover, a particular case of a call of a C function by a PascalWEB function which is possible thanks to the automatic translation from PascalWEB to C (the WEB2C translator) and it's widely used in pdfTEX and in METAFONT too (luaTEX is currently implemented in CWEB, hence the PascalWEB to C translation became useless).

2.1 Initialization

The first step is to initialise the Lua interpreter. This is done by inserting in `mf.web` the procedure `mflua_begin_program` (without parameters) just after the `begin` of the main program; WEB2C translates it in `mfluabeginprogram` (without "`_`") and then the compiler looks for the symbol among the available sources. By convention all sensors start with `mflua` prefix and they are declared in the header `mflua.h` and implemented in the file `mflua.c`; both the files are inside the `mfluadir` folder which also contains the source of a canonical Lua distribution. In `mflua.h` we have hence:

```
extern int mfluabeginprogram();
```

and in `mflua.c` there is its implementation:

```
lua_State *Luas[];
int mfluabeginprogram()
{
  lua_State *L = luaL_newstate();
  luaL_openlibs(L);
  Luas[0] = L;
/* execute Lua external "begin_program.lua" */
  const char* file = "begin_program.lua";
  int res = luaL_loadfile(L, file);
  if ( res==0 ) {
      res = lua_pcall(L, 0, 0, 0);
    }
  priv_lua_reporterrors(L, res);
  return 0;
}
```

As we can see, the C function creates a new Lua state `L`, saves it in a global variable, loads the standard library (i.e. `math`, `string` etc.) and evaluates the external file `begin_program.lua`. This is a common pattern: the `mflua*` sensor calls an external script and evaluates it or its function; the return value is never used because it can potentially modify the state of the METAFONT process. In this way we can manage the sensor data without recompiling the program.

The script `begin_program.lua` is quite simple, just the "greetings" message:

```
print("······ mflua_begin_program says:
      'Hello world!' ······")
```

but others scripts are more complexe; for example the sensor `mfluaPRE_fill_envelope_rhs(rhs)` has one input `rhs` (of type `halfword`) and its implementation calls the script `do_add_to.lua` that contains the function `PRE_fill_envelope_rhs(rhs)`:

```
int mfluaPREfillenveloperhs P1C (halfword, rhs)
{
 lua_State *L = Luas[0];
 const char* file = "do_add_to.lua";
 int res = luaL_loadfile(L, file);
 if ( res==0 ){
   res = lua_pcall(L, 0, 0, 0);
   if (res==0){
    /* function to be called */
    lua_getglobal(L,"PRE_fill_envelope_rhs");
    /* push 1st argument */
    lua_pushnumber(L, rhs);
    /*do the call (1 arguments, 1 result)*/
    res = lua_pcall(L, 1, 1, 0) ;
    if (res==0) {
      /* retrieve result */
      int z = 0;
      if (!lua_isnumber(L, -1)){
        fprintf(stderr,
  "\n! Error:function ‘PRE_fill_envelope_rhs’
  must return a number\n",lua_tostring(L, -1));
        lua_pop(L, 1);/*pop returned value*/
        return z;
       }else{
        z = lua_tonumber(L, -1);
        lua_pop(L, 1);/*pop returned value*/
        return z;
       }
    }
   }
 }
 priv_lua_reporterrors(L, res);
 return 0;
}
```

This is the related Lua function `PRE_fill_envelope_rhs(rhs)` (it's not important to understand the details now — suffice it to say that it store the knots of an envelope):

```
function PRE_fill_envelope_rhs(rhs)
   print("PRE_fill_envelope_rhs")
   local knots ,knots_list
   local index,char
   local chartable = mflua.chartable
   knots = _print_spec(rhs)
   index =
    (0+print_int(LUAGLOBALGET_char_code()))
    +(0+print_int(LUAGLOBALGET_char_ext()))*256
   char = chartable[index] or {}
   knots_list = char['knots'] or {}
   knots_list[#knots_list+1] = knots
   char['knots'] = knots_list
   chartable[index] = char
   return 0
end
```

As general rule, every sensor has exactly one Lua function; the script is loaded and the function is evaluated each time the sensor is excited (therefore the script doesn't maintain the state between two calls). Furthermore, a sensor that has at least one input must be registered in `texmf.defines`, so we have for example

`@define procedure mfluaPREfillenveloperhs();`

but not

`@define procedure mfluabeginprogram();` .

2.2 Exporting a PascalWEB procedure via WEB2C

The files `mflua.h` and `mflua.c` fully define the implementation of the sensors and also the functions that are needed to read some of the the global data of METAFONT. For example, the number of a char is stored in the global METAFONT variables `char_code` and `char_ext` and WEB2C translates them in C as components of the global array `internal` with index `char_code` and `char_ext`, so that it's easy to read them in `mflua.c`:

```
static int
priv_mfweb_LUAGLOBALGET_char_code(lua_State *L)
{ integer char_code=18;
  integer p=
   roundunscaled(internal[char_code])%256;
  lua_pushnumber(L,p);
  return 1;
}
static int
priv_mfweb_LUAGLOBALGET_char_ext(lua_State *L)
{ integer char_ext=19;
  integer p=
   roundunscaled(internal [char_ext]);
  lua_pushnumber(L,p);
  return 1;
}
```

Both functions are then registered in the file `mfluaini.lua` as `LUAGLOBALGET_char_code` and `LUAGLOBALGET_char_ext` to be available to the Lua interpreter, so that every Lua function can use them:

```
int mfluainitialize()
{
  lua_State *L = Luas[0];
  /* register lua functions */
  :
  lua_pushcfunction(L,
   priv_mfweb_LUAGLOBALGET_char_code);
  lua_setglobal(L, "LUAGLOBALGET_char_code");
  lua_pushcfunction(L,
   priv_mfweb_LUAGLOBALGET_char_ext);
  lua_setglobal(L, "LUAGLOBALGET_char_ext");
  :
  lua_pushcfunction(L, priv_mfweb_x_coord);
  lua_setglobal(L, "x_coord");
  :
  return 0;
}
```

In this way we can make available any PascalWEB macro, procedure, function, variable, etc., as for example the info field of a memory word

```
/* @d info(#) == mem[#].hh.lh */
/*{the |info| field of a memory word} */
```

```
static int priv_mfweb_info(lua_State *L)
{
  halfword p,q;
  p = (halfword) lua_tonumber(L,1);
  q = mem [p ].hhfield.v.LH ;
  lua_pushnumber(L,q);
  return 1;
}
```

which becomes available for Lua as `info`:

```
int mfluainitialize()
{
  lua_State *L = Luas[0];
  /* register lua functions */
  :
  lua_pushcfunction(L, priv_mfweb_info);
  lua_setglobal(L, "info");
  :
  return 0;
}
```

but of course it's better to use the minimum set of sensors.

2.3 Direct translation of a PascalWEB procedure

PascalWEB and Lua are not so different and we can easily translate from one to another: for example the PascalWEB procedure `print_scaled`

```
@<Basic printing...@>=
procedure print_scaled(@!s:scaled);
{prints scaled real, rounded to five   digits}
var @!delta:scaled;
{amount of allowable inaccuracy}
begin if s<0 then
  begin print_char("-"); negate(s);
  {print the sign, if negative}
  end;
print_int(s div unity);
{print the integer part}
s:=10*(s mod unity)+5;
if s<>5 then
  begin delta:=10; print_char(".");
  repeat if delta>unity then
    s:=s+@'100000-(delta div 2);
    {round the final digit}
  print_char("0"+(s div unity));
  s:=10*(s mod unity);
  delta:=delta*10;
  until s<=delta;
  end;
end;
```

can be translated in Lua as

```
function  print_scaled(s)
 local delta
 local res = ''
 local done
 if s== nil then
  print("\nWarning from print_scale
  in mfluaini:s is nil");
  return res
 end
 if s<0 then
  res = '-'
  s=-s
 end
 -- {print the integer part}
 res = res .. print_int(math.floor(s/unity))
 s=10*(math.mod(s,unity))+5
 if s ~= 5   then
  delta=10; res = res .. '.'
  done = false
  while not done do
   if delta>unity then
     -- {round the final digit}
     s=s+half_unit-(math.floor(delta/2))
   end
   res = res .. math.floor(s/unity);
   s=10*math.mod(s,unity);
   delta=delta*10;
   if  s<=delta then done = true end
  end;
 end
 return res
end
```

3 Collecting data

To properly draw the outline of a glyph we need the following information:

1. the edge structures, i.e. the pixels of the picture;
2. the paths from the filling of a contour;
3. the paths from the drawing of an envelope with a pen;
4. the pen used in drawing an envelope.

In the fig. 1 we can see these components for the lower case 'e' of the Concrete Roman at 5 point.

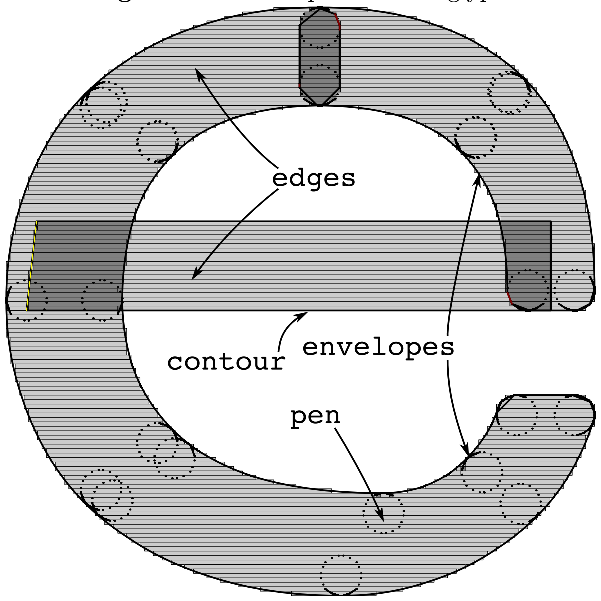
To store the edges structures we put one sensor into the procedure `ship_out(c:eight_bits)` that outputs a character into the `gf_file`:

```
procedure ship_out(@!c:eight_bits);
:
mflua_printedges(" (just shipped out)",
 true,x_off,y_off);
if internal[tracing_output]>0 then
  print_edges(" (just shipped out)",
   true,x_off,y_off);
end;
```

The Lua implementation is the function `print_edges (s,nuline,x_off,y_off)` in the file `print_edges .lua` and it is the direct translation of the PascalWEB `print_edges`:

```
function print_edges(s,nuline,x_off,y_off)
 print("\n.....Hello world
  from print_edges!.....")
 local p,q,r  --  for list traversal
```

Figure 1: The components of a glyph.



```
local n=0      --  row number
local cur_edges = LUAGLOBALGET_cur_edges()
local y =  {}
local xr = {}
local xq = {}
local f, start_row,
 end_row ,start_row_1, end_row_1
local edge
local w,w_integer,row_weight,xoff
local chartable = mflua.chartable
local index
local char
p = knil(cur_edges)
n = n_max(cur_edges)-zero_field
while p ~=  cur_edges do
 xq = {}; xr = {}
 q=unsorted(p); r=sorted(p)
 if(q>void)or(r~=sentinel) then
   while (q>void)  do
    w, w_integer,xoff = print_weight(q,x_off)
    xq[#xq+1] = {xoff,w_integer}
   end
   while r~=sentinel do
    w,w_integer,xoff = print_weight(r,x_off)
    xr[#xr+1]= {xoff,w_integer}
   end
   y[#y+1] = {print_int(n+y_off),xq,xr}
 end
 p=knil(p);n=decr(n);
end
--
-- local management of y, xq, xr
--
--f = mflua.print_specification.outfile1
index=(0+print_int(LUAGLOBALGET_char_code()))
  +(0+print_int(LUAGLOBALGET_char_ext()))*256
 char = chartable[index] or {}
 print("#xq=".. #xq)
 for i,v in ipairs(y) do
    xq,xr = v[2],v[3]
    -- for j=1, #xq, 2 do end ??
    row_weight=0
    for j=1, #xr, 1 do
     local xb = xr[j][1]
     local xwb = xr[j][2]
     row_weight=row_weight+xwb
     xr[j][3]=row_weight
    end
 end
 char['edges'] = char['edges'] or {}
 char['edges'][#char['edges']+1]=
   {y,x_off,y_off}
 :
 return 0
end
```

As we already said, a Lua script is stateless during its lifetime, but this doesn't mean that we can't store global variables: it's suffice to setup the global data by means of a sensor that is placed in the main program just before the sensors that need the global data. By convention, the global data are placed in the file `mfluaini.lua`: they have the namespace `mflua` (as in `mflua.chartable` which collects the pixels) or the prefix `LUAGLOBAL` (as in `LUAGLOBALGET_char_code()` that we have seen previously). Also `mfluaini.lua` hosts also some function like `print_int(n)` (print an integer in decimal form, directly translates from PascalWEB to Lua) or aliases like `knil=info`.

The sensors for the contours and the envelope are more complicated. It's not easy to find the optimal point where to insert a sensor, and it's compulsory to have the book *METAFONT: The program*[2] at hands (and of course also [1]). In this case the starting point is the procedure `do_add_to` where METAFONT decides, based on the current pen, to fill a contour (`fill_spec`) or an envelope (`fill_envelope`); we can hence insert a couple of sensors before and after these two points:

```
procedure do_add_to:
if max_offset(cur_pen)=0 then
 begin
  mfluaPRE_fill_spec_rhs(rhs);
  fill_spec(rhs);
  mfluaPOST_fill_spec_rhs(rhs);
  end
else
 begin
  mfluaPRE_fill_envelope_rhs(rhs);
  fill_envelope(rhs);
  mfluaPOST_fill_envelope_rhs(rhs);
 end ;
if lhs<>null then
 begin
  rev_turns:=true;
  lhs:=make_spec(lhs,max_offset(cur_pen),
       internal[tracing_specs]);
  rev_turns:=false;
  if max_offset(cur_pen)=0 then
   begin
    mfluaPRE_fill_spec_lhs(lhs);
    fill_spec(lhs);
    mfluaPOST_fill_spec_lhs(lhs);
   end
  else
   begin
    mfluaPRE_fill_envelope_lhs(lhs);
    fill_envelope(lhs);
    mfluaPOST_fill_envelope_lhs(lhs);
   end;
 end;
:
end;
```

Both `fill_spec` and `fill_envelope` have in turn another couple of sensors:

```
procedure fill_spec(h:pointer);
:
  mflua_PRE_move_to_edges(p);
  move_to_edges(m0,n0,m1,n1);
  mflua_POST_move_to_edges(p);
:
end

procedure fill_envelope(spec_head:pointer);
:
mfluaPRE_offset_prep(p,h);
{this may clobber node |q|, if it
  becomes ‘‘dead’’}
offset_prep(p,h);
mfluaPOST_offset_prep(p,h);
:
end
```

We will not show the Lua code here; suffice to say that we have followed the same strategy of the edge structures and stored the data in the global table `mflua.chartable`; the data are Bézier curves as {`p`,`c`$_1$,`c`$_2$,`q`,`offset`} which corresponds to the METAFONT path `p..controls c1 and c2..q` shifted by `offset`.
For each character `char` = `mflua.chartable`[j] we have available `char['edges']`, `char['contour']` and `char['envelope']` (the latter with its pen) for the post-processing.

4 The outlines of the glyphs

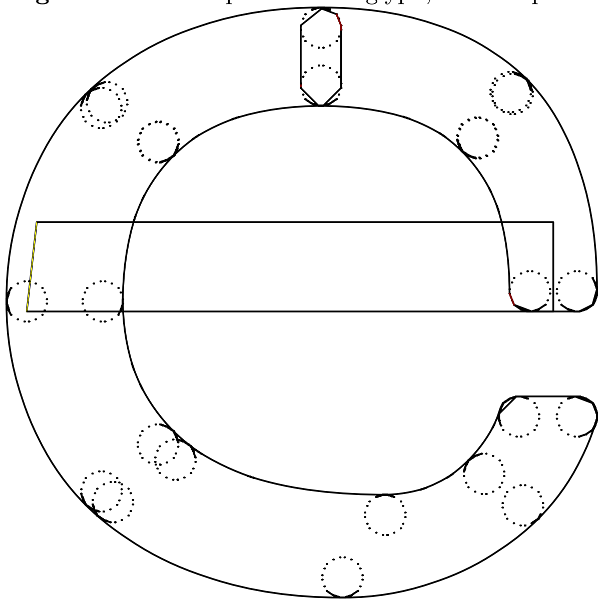
Up this point, things were relatively easy because, after all, it was to follow the PascalWEB code completely commented. The post-processing phase is easy to explain but more euristic. Briefly, for each curve we check (using the table `char['edges']`) if it is on the frontier of the picture and cut the pieces that are inside or outside. The problems stem from the fact the, by cutting a path, we are left with pending (pendent, drooping) paths that possibly should be removed; also we must have a robust algorithm to compute the intersection between two Bézier curves.

If we put the sensor `mflua_end_program` just before the end of the program, we can hence process the data collected so far. The script `end_program.lua` executes the function `end_program()` that aims to extract the contour and append as a METAPOST path to the file `envelope.tex`. We can describe the strategy as a sequence of 3 phases: preparation, compute the intersections, remove unwanted paths.

4.1 Preparation

If we remove the pixels in fig. 1 we can see the contours, the envelopes and the pens (see fig. 2: currently for a pen we will consider the polygonal closed path that joins the points).

Figure 2: The components of a glyph, without pixels.



The goal of this phase is to decide when a point of a path is inside the picture and then split the path to remove its subpaths that are inside the picture. The main tool is the de Casteljau algorithm (see, for example [4]): given a Bézier curve $\mathcal{C} = \{(\mathbf{p},\mathbf{c}_1,\mathbf{c}_2,\mathbf{q}),\ t\in[0,1]\}$, place $\mathbf{b}_0=\mathbf{p},\mathbf{b}_1=\mathbf{c}_1,\mathbf{b}_2=\mathbf{c}_2,\mathbf{b}_3=\mathbf{q}$, the de Casteljau algorithm is expressed by the recursive formula

$$\begin{cases}\mathbf{b}_i^0=\mathbf{b}_i\\ \mathbf{b}_i^j=(1-t)\mathbf{b}_i^{j-i}+t\mathbf{b}_{i+1}^{j-1},\end{cases}$$

for $j = 1, 2, 3$ and $i = 0, \ldots, 3-j$. For a fixed $t = t_1$ we have

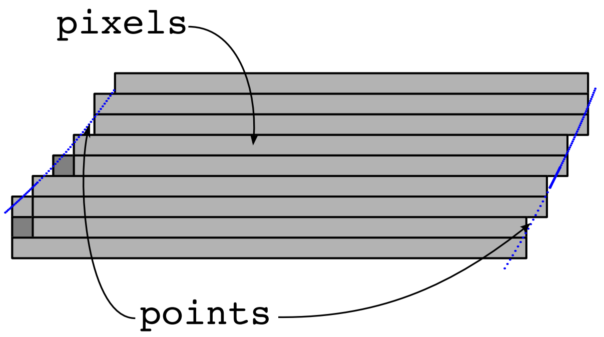
$$\begin{array}{cccc} \mathbf{b}_0^0 & \mathbf{b}_1^0 & \mathbf{b}_2^0 & \mathbf{b}_3^0 \\ \mathbf{b}_0^1 & \mathbf{b}_1^1 & \mathbf{b}_2^1 & \\ \mathbf{b}_0^2 & \mathbf{b}_1^2 & & \\ \mathbf{b}_0^3 & & & \end{array}$$

where $\mathbf{b}_0^3$ is the point on $\mathcal{C}$ at the time t_1,
$\mathcal{C}_{\text{left}} = \{(\mathbf{b}_0^0, \mathbf{b}_0^1, \mathbf{b}_0^2, \mathbf{b}_0^3),\ t \in [0, t_1]\}$,
and
$\mathcal{C}_{\text{right}} = \{(\mathbf{b}_0^3, \mathbf{b}_1^2, \mathbf{b}_2^1, \mathbf{b}_3^0),\ t \in [t_1, 1]\}$.

The Lua function `bez(p,c1,c2,q,t)` in `end_program.lua` is the immediate translation of the de Casteljau algorithm and returns `b30[1],b30[2],b00,b10,b20,b30,b21,b12,b03` where $x =$ `b30`[1] and $y =$ `b30`[2] are the coordinates of the point at time `t`.

The critical point is to decide when a point is black and it's not on the frontier; as we can see in fig. 3 some points on the frontier are white and some points are black, so we need to compute its weight and the weight if its closest neighbors and, if all of them are black, then the point is black and inside the picture (otherwise is on the frontier or outside). Another problem is that we want that a given path has "good" intersections with other paths: if we are too strict we can erroneously mark a point as not internal — and hence we can lose an intersection — and if we are too much tolerant we can have useless intersections (i.e. intersections that are internals) and the next phase is unnecessarily loaded.

Figure 3: Points (very tiny) on the frontier and pixels.

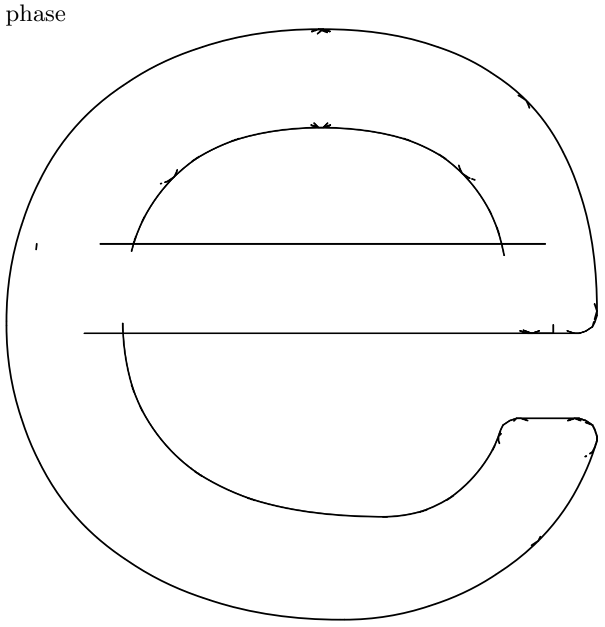


These are the steps followed in this phase:

1. associate to each path a set of time intervals that describes when the subpath is not internal;
2. adjust each interval to ensure proper intersections;
3. split each path in $\mathcal{C}_{\text{left}}$ and $\mathcal{C}_{\text{right}}$ that are not completely internal.

In fig. 4 we can see the result: there are some small isolated paths that are internal, but we can easly remove them in the next phases. Also note the effect of the non-linearity of a Bézier curve: we adjust the intervals with the same algorithm for both straight lines an semicircular lines — but the result can not be the same.

Figure 4: The components of a glyph, after the first phase



4.2 Compute the intersections

Given that METAFONT is able to calculate the intersections between two paths, it's natural to use its algorithm, but its translation in Lua or by WEB2C it's not cheap. It's better to write, for each $\texttt{path}_i$ and $\texttt{path}_j$, a simple METAFONT program like this one for $i = 2$ and $j = 1$:

```
batchmode;
message "BEGIN i=2,j=1";
path p[];
p1:=(133.22758,62) ..
   controls (133.22758,62.6250003125)
    and (133.22758,63.250000800781) ..
     (133.22758,63.875001431885);
p2:=(28.40971260273,62) ..
  controls (63.349007932129,62)
   and (98.28829,62) ..
    (133.22758,62);
numeric t,u; (t,u) = p1 intersectiontimes p2;
show t,u;
message "" ;
```

After running MFLua on it, the log

```
This is METAFONT, Version 2.718281
(Web2C 7.5.7) (base=mf 2011.1.16)
10 APR 2011 08:35
```

```
**intersec.mf
(intersec.mf
BEGIN i=2,j=1
>> 0
>> 1
```

can be easly parsed with Lua.

The number of intersections can be quite large even if $\mathtt{path}_i \cap \mathtt{path}_j = \mathtt{path}_j \cap \mathtt{path}_i$ and, if we have n paths, we compute only $\frac{n(n-1)}{2}$ intesections: for example the lower case letter 's' of the Concrete Roman at 5 point has 207 paths and on an Intel© Core$^{\mathrm{TM}}$2 Duo CPU T7250 2.00GHz with 2GByte to compute all the 21321 intersections it took around 2 seconds — that is low enough to avoid to re-implement an intersection algorithm. There is an important point to understand here: we run MFLua inside another instance of MFLua by means of the Lua function `os.execute(command)`, hence we must carefully manage shared resources (i.e. intermediary files for output as `envelope.tex`) by means of synchronization on filesystem.

4.3 Remove unwanted path

The last phase is the more euristic one. The strategy is to gradually clean up the outlines by identifying a rule for the paths to be removed and implementing it with a Lua function. The common data structures are the set of paths `valid_curves`, the set of intersections for each path `valid_curves` and the set of pen paths `valid_curves_p_set`: every time a curve is deleted these sets must be updated.

Here is a small example of rules:

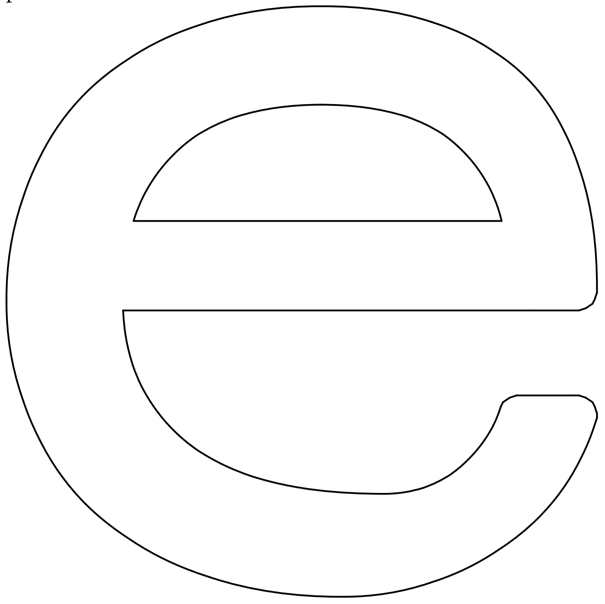
```
--
-- remove isolate paths
--
valid_curves, matrix_inters =
  _remove_isolate_path(valid_curves,
                       matrix_inters)
--
-- remove duplicate paths
--
valid_curves, matrix_inters =
  _remove_duplicate_path_I(valid_curves,
                           matrix_inters)
--
-- try to remove pen paths outside
-- the edge structure
--
valid_curves,matrix_inters =
 _open_pen_loop_0(valid_curves,
                  matrix_inters,
                  valid_curves_p_set,char)
--
-- try to remove duplicate pen paths
--
valid_curves,matrix_inters =
 _remove_duplicate_pen_path(valid_curves,
                            matrix_inters,
                            valid_curves_p_set)
```

Some rules are very specific, as the following one, one that takes care of a missing intersection for the 'y' letter (probably due to an erroneous set of time intervals)

```
--
-- a fix for an error found on ccr5 y
--
valid_curves,matrix_inters =
  _fix_intersection_bug(valid_curves,
                        matrix_inters)
```

and hence they are potentially useless for other glyphs. There are about twenty rules; after their adoption the results are the outlines of fig. 5 .

Figure 5: The components of a glyph, after the last phase



Figures 6, 7, 8 and 9 are a little gallery of results with these set of rules.

5 Conclusions

MFLua shows that it's possible to get the original outlines of a METAFONT glyph without particularly mathematical techniques and tracing algorithms. However, in view of an automatic conversion of a METAFONT source into an OpenType font there are so many details to fix that it's not opportune to focus on this for a next release. We can list the most important goals of the next release:

1. the sensors must go on a change file `mflua.ch` and not in `mf.web`;

Figure 6: The 'g' of Concrete Roman at 5 point

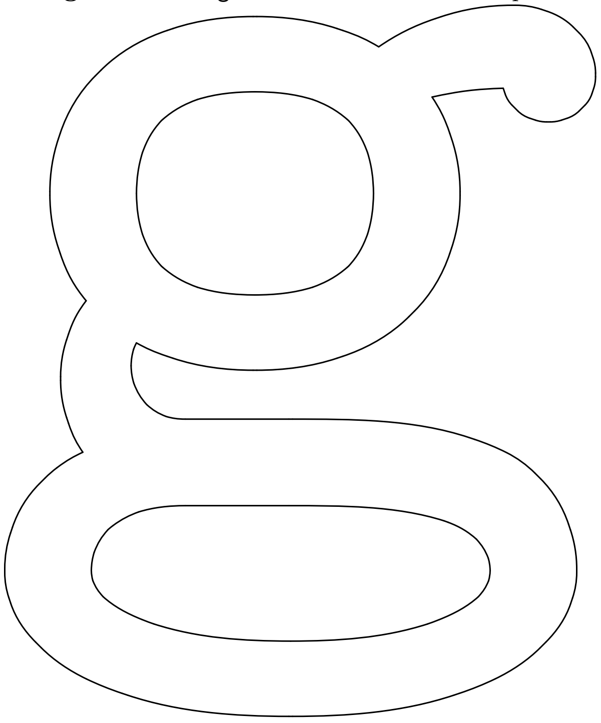


Figure 8: The 's' of Concrete Roman at 5 point. Note the approximations of the polygonal pen of upper and lower barb

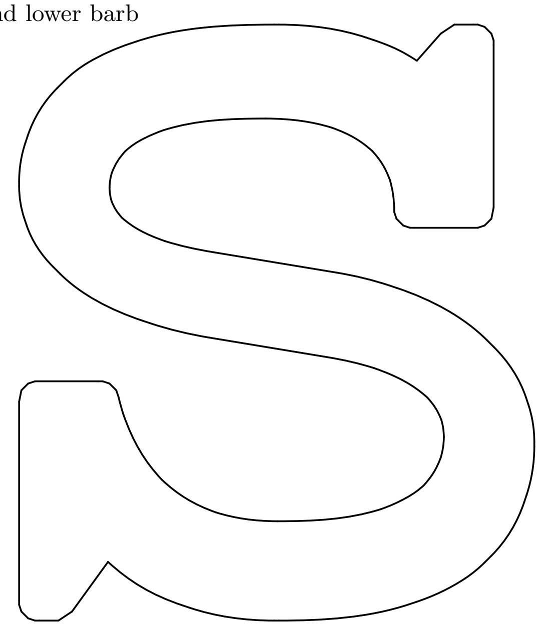


Figure 7: The 'i' of Concrete Roman at 5 point

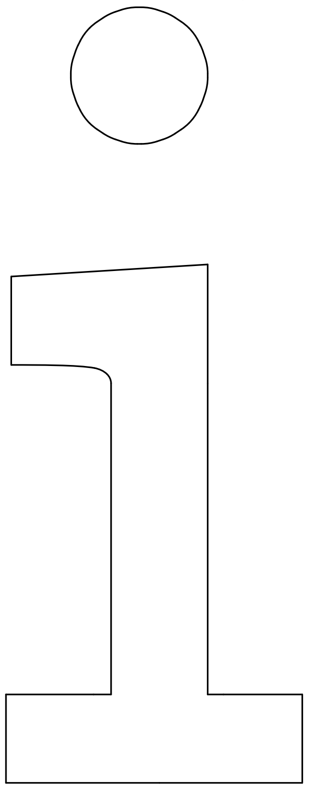
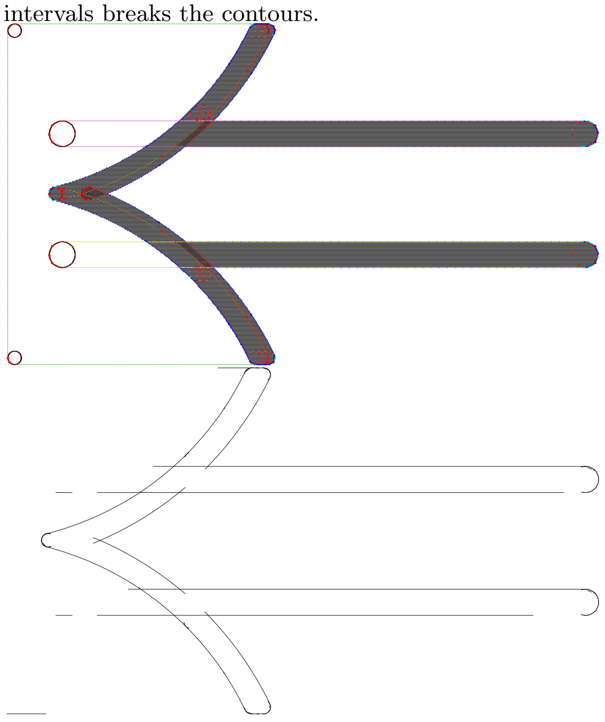


Figure 9: The '`Double leftward arrow`' of Computer Modern Math Symbols 10 point. An error of the time intervals breaks the contours.



2. MFLua should be built at least for the MicroSoft Windows© operating system, as METAFONT;
3. the function `end_program()` must be simplified: we need to test several other METAFONT sources. There are also some features still to implement, as for example a better approximation for an elliptical pen (see fig 8) or errors to fix as in fig. 9;
4. perhaps the Lua scripts should use `kpathsea`.

The Lua code needs to be made more consistent for both the name of variables and the use of tables as arrays or hashes (some bugs depended on misunderstanding of indexes as integers rather than strings).

The source code will be available for the next XIX$^{\mathtt{th}}$ EuroBachoTEX meeting in Bachotek, Poland.

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