Appendix for Power-law in Sparsified Deep Neural Networks

Anonymous Author(s)
Affiliation
Address
email

A Procedure for TPL Fitting

We follow the method in [1] to estimate $x_{\min}$, $x_{\max}$ and α in (4). Specifically, $x_{\min}$ and $x_{\max}$ are chosen by minimizing the difference between the probability distribution of the observed data and the best-fit power-law model as measured by the Kolmogorov-Smirnov (KS) statistic [2]:

$$D = \max_{x \in \{x_{\min}, x_{\min}+1, \dots, x_{\max}-1, x_{\max}\}} |S(x) - P(x)|,$$

where $S(x)$ and $P(x)$ are the CCDF's of the observed data and fitted power-law model for $x \in \{x_{\min}, x_{\min}+1, \dots, x_{\max}-1, x_{\max}\}$. To reduce the search space, we search $x_{\min}$ in the $\lfloor (n \times k\%) \rfloor$ smallest degree values, where n is the number of nodes in that layer, and $x_{\max}$ in the $\lfloor (n \times k\%) \rfloor$ largest degree values, respectively. In the experiments, $k = 30$ is used. For each $(x_{\min}, x_{\max})$ pair, we estimate α using the method of maximum likelihood as in [1].

B Degree Calculation in Convolutional Neural Networks

As an example, consider a feature map (node) in the first convolutional layer (conv1) for CNN on MNIST in Section 3. As the input image is of size 28×28, each such feature map is of size 24×24. To illustrate the counting more easily, we consider the unpruned network. We first count its connections to the input layer. Recall that in conv1, (i) the filter size is 5×5; (ii) each filter weight is used $24 \times 24 = 576$ times; and (iii) there is only one channel in the grayscale MNIST image. Thus, each node has $25 \times 576 \times 1 = 14,400$ connections to the input layer. Similarly, for connections to the conv2 layer, (i) the conv2 filter size is 5×5; (ii) each filter weight is used $8 \times 8 = 64$ times (size of each conv2 feature map); and (iii) there are 32 feature maps in conv2. Thus, each conv1 node has $25 \times 64 \times 32 = 51,200$ connections to the conv2 layer. Hence, the degree of each conv1 node (in an unpruned network) is $14,400 + 51,200 = 65,600$. Note that the pooling layers do not have learnable connections. They are never sparsified, and we do not need to study their degree distributions.

C Proofs

C.1 Proposition 4.1

Proof 1 *For node i at layer l, let $d_i(t)$ be its degree at time t. Out of these $d_i(t)$ connections, let $d_i^{\uparrow}(t)$ be connected to the upper layer, and $d_i^{\downarrow}(t)$ to the lower layer*[1]*. Using (6), the increase of its degree due to new connections to the upper layer is:*

$$\frac{\mathrm{d}d_i^{\uparrow}(t)}{\mathrm{d}t} = \sum_m \Delta_t^l(d_i(t), d_m(t)) = N^l a^l d_i(t) \frac{\sum_m d_m(t)}{(\sum_s d_s(t))(\sum_m d_m(t))} = \frac{N^l a^l d_i(t)}{\sum_s d_s(t)},$$

[1]For simplicity, we only consider hidden layers here. Analysis for the other layers can be easily modified and are not detailed here.

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where m and s are indices to all the nodes in layer $(l+1)$ and layer l, respectively. Similarly, the increase of degree due to new connections to the lower layer is:

$$\frac{\mathrm{d}d_i^{\downarrow}(t)}{\mathrm{d}t} = \sum_r \Delta_t^{l-1}(d_r(t), d_i(t)) = \frac{N^{l-1}a^{l-1}d_i(t)}{\sum_s d_s(t)},$$

where r and s are indices to all the nodes in layer $(l-1)$ and layer l, respectively. The total number of new connections for nodes in layer l can be obtained as:

$$\sum_s d_s(t) = \sum_s d_s(0) + \int_0^t (N^l a^l + N^{l-1}a^{l-1})dt = \sum_s d_s(0) + (N^l a^l + N^{l-1}a^{l-1})t.$$

Combining all these, we have

$$\frac{\mathrm{d}d_i(t)}{\mathrm{d}t} = \frac{\mathrm{d}d_i^{\uparrow}(t)}{\mathrm{d}t} + \frac{\mathrm{d}d_i^{\downarrow}(t)}{\mathrm{d}t} = \frac{(N^l a^l + N^{l-1}a^{l-1})d_i(t)}{\sum_s d_s(0) + (N^l a^l + N^{l-1}a^{l-1})t}.$$

After integration and simplification, we obtain

$$d_i(t) = d_i(0)c^l(t). \tag{1}$$

C.2 Corollary 1

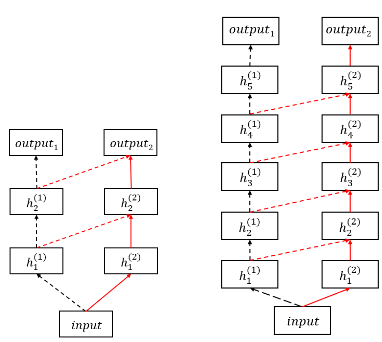
Proof 2 *If the degree distribution of layer l at $t=0$ (denoted p_0^l) follows the power law (standard or TPL), i.e., $p_0^l(d) = Ad^{-\alpha}$ for some $A>0$ and $\alpha>1$. Then, from (1), its degree distribution at time t is*

$$p_t^l(d) = p_0^l\left(\frac{d}{c^l(t)}\right) = A\left(\frac{d}{c^l(t)}\right)^{-\alpha} = (c^l(t))^{\alpha}Ad^{-\alpha}, \tag{2}$$

which also follows the same power law as p_0^l, but scaled by the factor $(c^l(t))^{\alpha}$.

D Progressive Neural Network

In a progressive neural network, we first train a column for task A, using the method in [3]. connections of this column are then fixed. a new column is instantiated for task B. let $h_l^{(1)}$ and $h_l^{(2)}$ be the lth hidden layer of the first and second columns, respectively. layer $h_l^{(2)}$ receives input from both $h_{l-1}^{(2)}$ and $h_{l-1}^{(1)}$ via lateral connections. The progressive neural networks for MNIST and CIFAR are shown in Figure 1.



(a) MLP on MNIST. (b) CNN on CIFAR.

Figure 1: Progressive network. In each model, the left column (with black dashed arrows for its connections) is for task A, while the right column is for task B (with red solid lines for connections within the task B column, and red dashed lines for lateral connections connecting to the task A column).

References

[1] A. Clauset, C. R. Shalizi, and M. E. J. Newman. Power-law distributions in empirical data. *SIAM Review*, 51(4):661–703, 2009.

[2] Anna Deluca and Álvaro Corral. Fitting and goodness-of-fit test of non-truncated and truncated power-law distributions. *Acta Geophysica*, 61(6):1351–1394, 2013.

[3] S. Han, J. Pool, J. Tran, and W. Dally. Learning both weights and connections for efficient neural network. In *Advances in Neural Information Processing Systems*, pages 1135–1143, 2015.