Institute of Theoretical Physics
São Paulo State University

What is the Universe Made of?

IV Journeys Into Theoretical Physics
Prof. Cliff Burgess
July 6-12, 2019
Níckolas de Aguiar Alves

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Level: Undergraduate
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What is the Universe made of?
Cliff Burgess - Perimeter Institute

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- Introduction to ΛCDM cosmology
- Evidence for DM and DE
- Primordial Fluctuations → close distances: things could move around and transmit heat; far distances: not enough time to move, primordial

Field formulations (GR, EM, etc): needed due to SR because information has a limited speed

If you know the distribution of mass in the Universe, you can find the overall geometry

↳ firstly we assumed isotropic + homogeneous for simplicity → confirmed by CMB (cosmic microwave background)

↳ FLRW Universe → Friedmann-Lemaître-Robertson-Walker

↳ general line element

$-+++$

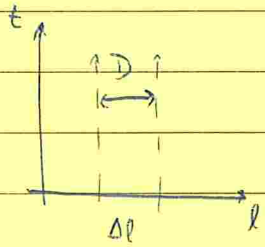
$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-k\frac{r^2}{R_0^2}} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$= -dt^2 + a^2(t)\left[dl^2 + r^2(l)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$dl = \frac{dr}{\sqrt{1-k\frac{r^2}{R_0^2}}}$$

↳ "comoving radius"

$k = 0, 1, -1$
plane, sphere, ↳ hyperboloid



$D = \Delta l\, a(t)$
↳ physical

sometimes we do $r \to \hat{r} R_0$ for a dimensionless $\hat{r}$ and put dimensions into $a(t)$

"scale factor"

What is $a(t)$? besides, $a(t_{now}) = 1$

Galaxy $l(t)$

$$\frac{d}{dt}D(t) = \frac{d}{dt}\left[a(t)\,l(t)\right] = \frac{\dot{a}}{a}D + a\frac{dl}{dt}$$

$H(t)$; $a\frac{dl}{dt}$: peculiar velocity → negligible for cosmological distances; not on a geodesic

Hubble Law
$V = HD$

Hubble flow

$H(t_0) = 70\ km\, s^{-1} Mpc^{-1}$
$= h \cdot 100\ km\, s^{-1} Mpc^{-1}$

$r(l) = R_0 \sin(l/R_0)$ $k=1$
$r(l) = l$ $k=0$
$r(l) = R_0 \sinh(l/R_0)$ $k=-1$

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↳ not useful (we want to measure light emitted → now)

use redshift ↳ H is abundant and has very known lines

$$\frac{\lambda_{obs}}{\lambda_{emitted}} = \frac{a(t_{now})}{a(t_{em})} = \frac{a_0}{a(t)} \; ; \quad z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{a_0}{a(t)} - 1 \Rightarrow \frac{a_0}{a(t)} = 1 + z$$

Einstein's equations $\qquad \hbar = c = k_B = 1$

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{Curvature}} = \underbrace{8\pi G}_{1/M_p^2} T_{\mu\nu}$$

$1/M_p^2$ "reduced Planck mass" $\qquad M_p \simeq 10^{18}\,\mathrm{GeV}$

Missing Λ? It "hides" in $T_{\mu\nu}$, for we are allowing vacuum to have energy (which is what Λ did before, in a different way)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \; , \quad J^\mu = \begin{pmatrix} \rho \\ \vec{J} \end{pmatrix} \Rightarrow \partial_\mu J^\mu = 0$$

$E, \vec{p}$ are conserved, but different from charge, not everyone agrees in how much for each one (not Lorentz invariant)

$$\begin{pmatrix} E \\ \vec{p} \end{pmatrix} = p^\mu \xrightarrow[\text{currents}]{} T^{\mu\nu} \Rightarrow \begin{matrix} \partial_\mu T^{\mu\nu} = 0 & \text{flat} \\ \nabla_\mu T^{\mu\nu} = 0 & \text{curved} \end{matrix}$$

$$T^\mu{}_\nu = g_{\nu\lambda} T^{\mu\lambda}$$

For isotropic and homogeneous

$$T^\mu{}_\nu = \begin{pmatrix} \rho(t) & 0 & 0 & 0 \\ 0 & p(t) & 0 & 0 \\ 0 & 0 & p(t) & 0 \\ 0 & 0 & 0 & p(t) \end{pmatrix}$$

homogeneity, energy density ($\rho(t)$); hom. + isotropic (zeros); pressure ($p(t)$)

↳ needed form for hom. + iso.

In principle, Einstein would give 10 eqs, but we get 2 due to energy conservation (stress-energy, in fact)

↳ just like Maxwell's and charge cons.

Friedmann equations

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \; ; \quad \frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p)$$

00 + trace

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3M_p^2}$$

00 component of Einstein Equations

02

$\nabla_\mu T^{\mu\nu} = 0 \xrightarrow{FLRW} \dot{\rho} + 3H(\rho + p) = 0$ → with first Friedmann eq, implies second Friedmann eq

$$\underbrace{\frac{d}{dt}(\rho a^3)}_{\text{total energy}} = \underbrace{- p \frac{d}{dt}(a^3)}_{\text{"work done"}}$$

We have two eqs, but three variables → we still need the equation of state that relates ρ and p

$$p = p(\rho)$$

Options for eqs. of state

photons ("radiation", "relativistic particles")

$$\rho = a_B T^4$$
$$p = \frac{1}{3} a_B T^4$$
$$w \equiv \frac{p}{\rho} = \frac{1}{3}$$

a_B → Stefan-Boltzmann

nonrelativistic matter

(protons, neutrons, electrons, dark matter)

$$p = nT$$ → n numerical density

$$\rho = nm + \frac{nT}{\gamma - 1} \qquad \gamma = \frac{c_p}{c_v}$$

We need $m \gg T$, or the atoms would be relativistic. Thus,

$$w = \frac{p}{\rho} = \frac{T}{m} \ll 1 \Rightarrow w \approx 0$$

vacuum → Lorentz invariant $\Rightarrow T_{\mu\nu} \propto g_{\mu\nu}$

$$T_{\mu\nu} = -\rho_{vac}\, g_{\mu\nu} \Rightarrow \begin{array}{l} \rho_{vac} = cte \\ p_{vac} = -\rho_{vac} \end{array} \Rightarrow w = -1$$

For $p = w\rho$ with w independent of time, we get

Eq of energy conservation

$$\dot{\rho} + 3H(\rho + p) = 0 \Rightarrow \rho = \rho_0 \left(\frac{a}{a_0}\right)^{-s}, \quad s = 3(1+w)$$

$$\rho_{vac} = cte, \ \rho_m \propto \frac{1}{a^3}, \ \rho_{rad} \propto \frac{1}{a^4}$$

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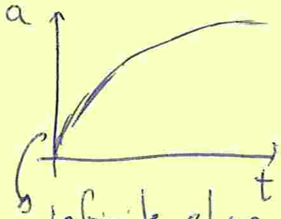
For the Friedmann eq (with ~~k~~ $k=0$)

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3M_p^2} \Rightarrow \left.\begin{array}{ll} a \sim t^{2/3} & \text{matt} \\ a \sim t^{1/2} & \text{rad} \\ a \sim e^{Ht} & \text{vac} \end{array}\right\}$$ if Universe respects only one eq. state in the list provided

$$a = a_0 \left(\frac{t}{t_0}\right)^q \qquad q = \frac{2}{3(1+w)}$$



infinite slope

↳ universe expands faster than light ↳ ok, no information transfer

For matt & rad, $\ddot{a} < 0$. For vacuum, $\ddot{a} > 0$.

The energy densities are going to add, though.

$$\rho(a) = \rho_{v_0} + \rho_{m_0}\left(\frac{a_0}{a}\right)^3 + \rho_{r_0}\left(\frac{a_0}{a}\right)^4$$

→ if they don't interact (conserved each of them alone)

↳ pretty true

$$p(a) = -p_{v_0} + 0 + \frac{1}{3}\rho_{r_0}\left(\frac{a_0}{a}\right)^4$$

$$H^2 = \frac{8\pi G}{3}\rho = H_0^2\left[\Omega_R + \Omega_M + \Omega_{vac} - \cancel{\Omega_{curv}}\right]$$

→ universe is pretty flat

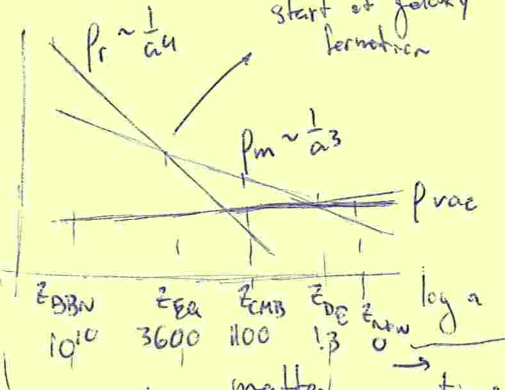
$$= H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_V\right]$$

$$\rho_c = \frac{3H_0^2}{8\pi G} = 3H_0 M_p^2$$

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

↳ $\Omega_{R_0} + \Omega_{M_0} + \Omega_{V_0} = 1$ ↳ now

log ρ



$\rho_{\gamma_0} = 0.261\ \mathrm{MeV/m^3}$

$\rho_{\nu_0} = 0.18\ \mathrm{MeV/m^3}$

$\rho_{B_0} = 210\ \mathrm{MeV/m^3}$

$\rho_{D_0} = 1350\ \mathrm{MeV/m^3}$

$\rho_{V_0} = 3600\ \mathrm{MeV/m^3}$

↓

$\Omega_{\gamma_0} = 5\cdot 10^{-5}$

$\Omega_{\nu_0} = 3.1\cdot 10^{-5}$ $\quad \Omega_{D_0} = 0.26$

$\Omega_{B_0} = 0.04$ $\quad \Omega_{V_0} = 0.7$

$n_{\gamma_0} = 4.1\cdot 10^8/\mathrm{m^3}$

$n_{B_0} = 0.22/\mathrm{m^3}$

↳ last digit uncertainty

Crossovers:

$$H^2 = H_{DE}^2\left[1 + \left(\frac{a_0}{a}\right)^3\right]$$ → after radiation

$$\Rightarrow a(t) = a_0 \sinh^{2/3}\left(\frac{3H_{DE}t}{2}\right)$$

$\simeq t^{2/3}$ for $H_{DE}t \ll 1$

$\simeq e^{H_0 t}$ for $H_{DE}t \gg 1$

→ redshift and it tells us when things happened → and temperatures

→ power law

↳ if something gets smaller than sth else, it gets <u>really</u> smaller

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Two kinds of radiation → relativistic stuff

- photons
  - thermo distribution around 2,7 K → can calculate ρ_r
- neutrinos
  - not quite same temperature (weak interactions, decouple from many things)
  - around 2,9 K (we knew they were in equilibrium long time ago, can get ρ_ν from them)

Non-Relativistic Matter

- baryons
  - can measure how many there are independent of seeing or gravity
  - there are much more photons than baryons, but the latter are much more massive and thus carry more energy

Dark Energy

$\Omega_{r_0} + \Omega_{\nu_0} + \Omega_{B_0} + \Omega_{DM_0} \approx 0.3$ → something else is there, for $\sum_i \Omega_i = 1$

We can count r and ν, but how do we know if (and how much) baryons, dark matter and/or dark energy is out there?

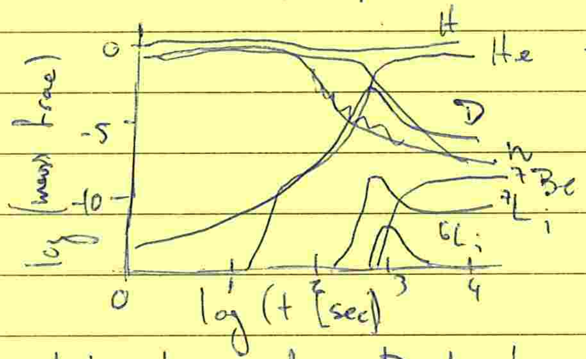
Counting Baryons

Hot Big Bang: p^+ & n^0 create nuclei, but they separate very fast. Eventually things are cool enough for nuclei to stand around

D → fragile $\quad n^0 \to p^+ + e^- + \bar{\nu}_e \qquad p^+ + n^0 \to {}^2_1D + r \qquad {}^2_1D + p^+ \to {}^3_2He + r$

He → very stable $\quad {}^2_1D + {}^2_1D \to {}^3_2He + n^0 \qquad {}^2_1D + {}^2_1D \to {}^3_1T + p^+ \qquad {}^3_1T + {}^2_1D \to {}^4_2He + n^0$

as soon as you have enough deuterium, you get a lot of helium



There is also ${}^3He$ & ${}^3H$ around (omitted from sketch)

As you accumulate D, you can do other things. Eventually lines are constant due to inflation

↳ everyone is far, no reactions

It takes time for D to be stable enough for other things to form

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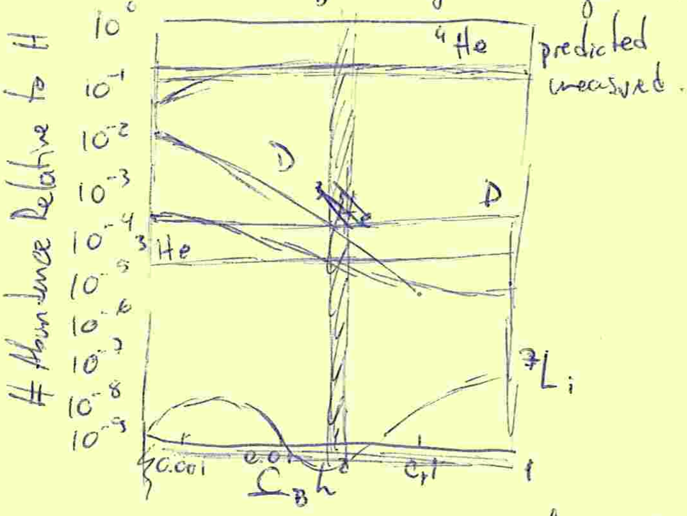
Hot Big Bang → # each element in terms of $\Omega_B h^2$

Data for various elements; get a fit

More baryons than we see

↳ makes sense, we can't see everyone, the other way around (we see more than prediction) would be weird

$\Omega_B \approx 0.4$ gets all right



↳ varying ~~percentage~~ percentage of Baryons in Universe

From CMB

Speed of sound in H (CMB depends on)

↳ depends on H density

Fit H properties from CMB

What about Dark Matter?

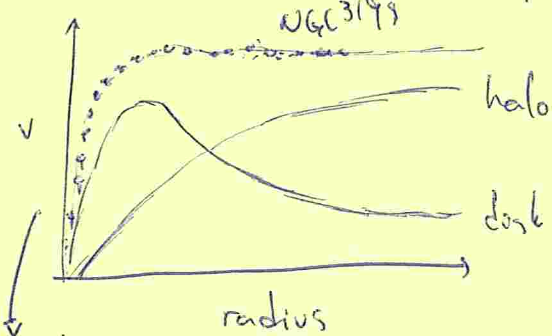
Galaxies: hydrogen halo (goes even further than stars) → we can measure

Newtonian Gravity: if one knows G, you can measure $M_\odot$ by measuring the periods of planets

↳ same reasoning to get M_{galaxy}

↳ measure blue/redshift

↳ rotation curves



as you go away

$\frac{av^2}{r} = \frac{GM}{r^2} \Rightarrow v^2 = \frac{GM}{r}$

what?!

Visible matter behaves as expected, but something doesn't → dark matter

Proposal: If accel on stars reaches a minimum, dynamics change → Modified Newtonian Dynamics → MoND ↳ really successful for galaxies

$F = ma^2$

Groups of galaxies: also lacks mass for measured velocities

↳ MoND doesn't solve

↳ velocities measured by redshift

↳ mass necessary to trap H seen by X-ray → same result

↳ gravitational lensing allows to analyse where is matter

↳ same result

↳ shock of galaxy clusters: gas left behind → still, great mystical mass is with galaxies

↳ doesn't interact with itself strongly

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Baryonic Acoustic Oscillations (BAO)

- correlation between hot/cold @ CMB and where galaxies are
  - ↳ gravity amplify fluctuations
    - ↳ doesn't happen until matter dominated universe

After matter starts dominating, DM starts amplifying, but not baryons, bc they ~~are~~ still shackled to photons. However, after CMB, they decouple and amplify → baryons start falling at DM potentials

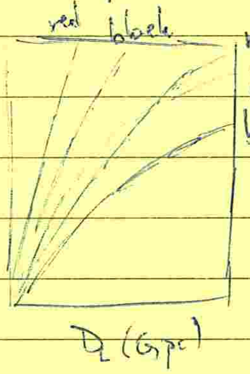
Baryonic wave with photons is amplified by DM → travels at almost speed of light. After CMB, photons and baryons decouple and wave "stops" (much slower and non-relativistic) ↳ photon wave carrying baryons, actually

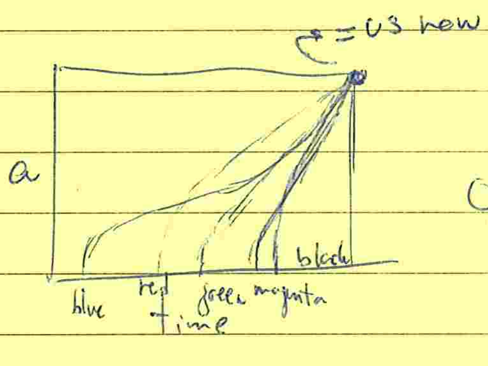
What about Dark Energy?

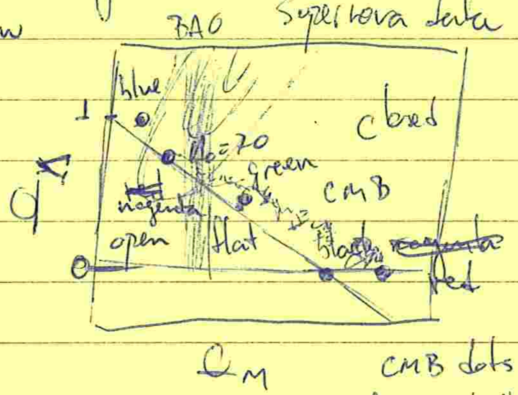
- ↳ Found by testing Hubble Law
  - ↳ wants something very far, but visible, to see if further ⇒ more redshift
    - ↳ supernovae → a single star gets as bright as a galaxy

One expects expansion to slow down due to gravity

Each color represents a diff. amount of DM & DE







CMB dots for each H_0 (Hubble cte)

Setting $H_0 = 70$ (measured) for CMB measurement, we get the same answer as flat universe which gives same answer as the supernova data

Particle Dark Matter

- → it is simple and useful to have thermal equilibrium at BB
  - ↳ favorite models keep this → "WIMP"
    - ↳ Weakly Interacting Massive Particles
      - ↳ mass as W boson or top quark, e.g.
      - ↳ interact only weakly

well motivated, but not observed yet

can we really ask thermal equilibrium or is it too strong

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Thermal history of particles on expanding universe

Detailed balance $\longrightarrow$ equilibrium distribution

$$A + B \leftrightarrow C + D$$

design as to be equal for both ways, no matter cross-section

# of parts per volume per d^3p ↳ volume in momentum space

$$f(\vec{p}) = \frac{1}{e^{\beta(\epsilon-\mu)} \pm 1}$$

$+$ → fermions, $-$ → bosons

$\beta = \frac{1}{T}$, μ = chemical potential

Using f, we can get again

$\rho_{rad} = a_B T^4$

$n_{rad} \propto T^3$

How does T change with time?

energy conservation: $\rho \propto \frac{1}{a^4} \Rightarrow T \propto \frac{1}{a}$

entropy density: $s \propto n_{rad} \propto T^3$

total entropy: $s a^3 \propto T^3 a^3$

unless some irreversible process is happening within universe, total entropy remains constant for radiation dominated universe

Why would photons be in equilibrium if they are not scattering anymore?

for photons, $\mu = 0$

$$f(\vec{p}) = \frac{1}{e^{|\vec{p}|/T} - 1}$$

$$\left.\begin{aligned} p &\propto \frac{1}{\lambda} \propto \frac{1}{a} \\ T &\propto \frac{1}{a} \end{aligned}\right\}$$ distribution doesn't change on time

For non-relativistic particles

$$f(p) = \frac{1}{e^{p^2/2mT} \pm 1}, \quad n = \int \frac{d^3p}{e^{p^2/2mT} \pm 1} \Rightarrow n_m = (mT)^{3/2} e^{-m/T}$$

$$\rho_m \propto (mT)^{8} e^{-m/T}$$

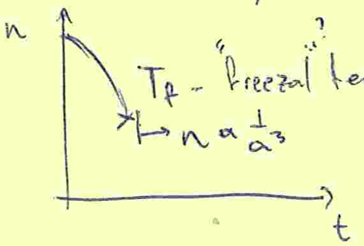
low temperature: particles and antiparticles find each other and annihilate

Eventually, particles are too far and not in equilibrium anymore

energy cons: $\rho_m \sim \frac{1}{a^3}$

for high temp, creation and annihilation happen

T decreases in time



$$n(t) = \left(\frac{a_f}{a}\right)^3 n(t = t_f) = \left(\frac{a_f}{a}\right)^3 (m T_f)^{3/2} e^{-m/T_f}$$

what is T_f? → expansion rate > annihilation rate

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A reaction remains in equilibrium if $\Gamma > H$ at any time (Γ → reaction rate; H → Hubble rat)

$$\Gamma = \langle n \sigma v \rangle \approx (mT)^{3/2} e^{-m/T}$$

(n → # density, σ → cross-section, v → velocity)

e.g. weak interaction annihilation

$$\sigma = G_F^2 (mT)$$

(G_F → Fermi constant)

non relativistic

$v \approx 1$

$$\Gamma = (mT)^{3/2} e^{-m/T} G_F^2 (mT)$$

Friedmann eq

$$H^2 = \frac{8\pi G}{3}\rho \xrightarrow{\text{rad dom}} H \sim \frac{T^2}{M_P}$$

Weak interactions

$$\eta_X = \frac{n_x}{n_\gamma} \approx 0.1\,\eta_B \quad \text{if } m \gtrsim m_B$$

(B → baryon)

Z ~~and~~ top can't be DM because are unstable

$$\Omega_X = 0.1\left(\frac{m}{m_B}\right)\Omega_B \nearrow \text{good value for } m \sim 100 m_B$$

↳ tip: introduce new charge and give only to DM. Since has to conserve, can't decay

WIMP Models get all observations right

Structure Formation

↳ perturbations on homogeneity → ~~homogeneous~~ also depends on position, but here time is more important

$$g_{\mu\nu}(\vec{x},t) = g_{\mu\nu}^{FRW}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$T_{\mu\nu}(\vec{x},t) = t_{\mu\nu}(t) + \delta T_{\mu\nu}(\vec{x},t)$$

($t_{\mu\nu}$ ↳ $p(t)$, $\rho(t)$)

linearize Einstein Equations to get sols. with FRW background

seperate perturbations

$$\delta g_{\mu\nu} = \underbrace{\delta_S g_{\mu\nu}}_{\text{scalar}} + \underbrace{\delta_V g_{\mu\nu}}_{\text{vector}} + \underbrace{\delta_T g_{\mu\nu}}_{\text{tensor}}$$

$$\delta_T g_{\mu\nu} = a^2 \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}$$

↳ traceless

↳ divergenceless

$$\partial^i h_{ij} = h_i^{\ i} = 0$$

$$\delta_V g_{\mu\nu} = a^2 \begin{pmatrix} 0 & v_i \\ v_j & \partial_i w_j + \partial_j w_i \end{pmatrix}$$

$$\delta_S g_{\mu\nu} = a^2 \begin{pmatrix} 2\Phi & \partial_i B \\ \partial_j B & 2\Psi\delta_{ij} + \partial_i\partial_j E \end{pmatrix}$$

h_{ij} → gravitational waves

$$\begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \overleftrightarrow{0} \end{pmatrix}$$

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By linearizing Einsteins, h_{ij} gets an equation which, in Fourier space, reads

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \underbrace{\frac{k^2}{a^2}}_{\text{momenta redshift}} h_{ij} = 0 \qquad h_{ij}(x) = \int \frac{d^3k}{(2\pi)^3} e^{ikx} h_{ij}(k)$$

Hubble friction $\rightarrow$ expanding universe extracts energy from wave

Flat universe allows to use momentum space

Slow motions gets amplified by gravity

↳ too fast $\Rightarrow$ passes directly through potential

Scaler perturbations $\rightarrow$ non-relativistic limit

↳ only Φ is relevant (newtonian potential), other scalars die out

Equations of a fluid:

$$\text{energy cons:} \quad \frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\text{momentum cons:} \quad \rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] + \nabla p + \rho\nabla\Phi = 0$$

$$\text{Newton's gravity:} \quad \nabla^2\Phi = 4\pi G\rho \qquad D_t = \frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla$$

$\vec{v} = \vec{v}_0 + \delta\vec{v}, \quad \rho = \rho_0 + \delta\rho \;\cdots$ } linearize

Hubble flow $\vec{a}$

$$D_t\delta\rho + 3H\delta\rho + \rho_0\nabla\cdot\delta\vec{v} = 0 \quad \rightarrow \text{en. cons}$$

$$\rho_0(D_t\delta\vec{v} + H\delta\vec{v}) + \nabla\delta p + \rho_0\,\delta\Phi = 0$$

$$\nabla^2\delta\Phi = 4\pi G\,\delta\rho$$

Eliminate $\delta\vec{v}$ and $\delta\Phi$ (diff en. cons) and a Fourier Transform

$$C_s^2 = \left(\frac{\partial p}{\partial\rho}\right)_s \sim \frac{T}{m} \quad \text{speed of sound} \qquad \delta_k = \int \frac{\delta\rho}{\rho_0} e^{-ikx} d^3x$$

$$\ddot{\delta}_k + \underbrace{2H\dot{\delta}_k}_{\text{damping}} + \left(\frac{c_s^2k^2}{a^2} - 4\pi G\rho_{m0}\right)\delta_k = 0$$

redshift (k^2/a^2); ↳ non-relativistic matter

Solutions with $c_s^2(k/a)^2 \gg H^2\, 4\pi G\rho_{m0}$: $\delta_k \simeq a^{-1/2}\exp\left[\pm ikc_s\int^t \frac{dt'}{a(t')}\right] \rightarrow$ gravitational waves

For small k (large λ): grows instead of oscillating ↳ damped due to expansion of universe

↳ Jeans instability $c_s^2(k/a)^2 \ll H^2\,8\pi G\rho_{m0}$

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$$\ddot{\delta}_k + 2H\dot{\delta}_k \simeq 0$$

$$\frac{d}{dt}\left(a^2\dot{\delta}_k\right) = 0$$

$$\dot{\delta}_k = \frac{c_1}{a^2} \Rightarrow \delta_k = c_1 + c_2\int\frac{dt}{a^2(t)}$$

Jeans Instability

for instability with $\delta_k \propto a$

$\delta_k \sim t^{2/3}$ ($\sim a$ for matt dom) $\to$ grows

$\delta_k \sim t^{-1}$ ($\sim a^{-3/2}$)

if matter dominated

$\frac{c_s k}{a} \ll H \to$ drop $\frac{c_s^2 k^2}{a^2}$

~~$k/a \gg H \to$ non relativistic~~ $\quad c_s \ll 1$

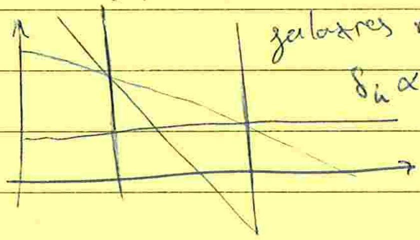
non relativistic matter

for relativistic, $c_s \approx 1$ and $k/a \gg H$ incompatible with $c_s \frac{k}{a} \ll H$

Otherwise, no instability

start $\delta_k \propto 10^{-5}$ - CMB

galaxies require $\delta_k \propto 1$



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