#### Data-Structure

#### 1.1 fast-set

```
1 // Can correctly work with numbers in range
2 // Supports all std::set operations in O(1)
      on random queries / dense arrays, O(
      log_64(N)) in worst case (sparce array). 51
3 // Count operation works in O(1) always.
4 template < uint MAXN >
5 class fast set {
 private:
   static const uint PREF = (MAXN <= 64 ? 0 :</pre>
                  MAXN <= 4096 ? 1 :
                  MAXN <= 262144 ? 1 + 64 :
                  MAXN <= 16777216 ? 1 + 64 +
                      4096 :
                  MAXN <= 1073741824 ? 1 + 64
                      + 4096 + 262144 : 227) +
                       1;
   static constexpr ull lb(int x) {
     if(x == 64) return ULLONG MAX;
     return (1ULL << x) - 1;</pre>
   static const uint SZ = PREF + (MAXN + 63)
        / 64 + 1;
   ull m[SZ] = \{0\};
   inline uint left(uint v) const { return (v
         - 62) * 64; }
   inline uint parent(uint v) const { return
        v / 64 + 62; }
   inline void setbit(uint v) { m[v >> 6] |=
        1ULL << (v & 63); }
   inline void resetbit(uint v) { m[v >> 6]
        &= ~(1ULL << (v & 63)); }
   inline uint getbit(uint v) const { return
        m[v >> 6] >> (v \& 63) \& 1; }
   inline ull childs_value(uint v) const {
        return m[left(v) >> 6]; }
   inline int left_go(uint x, const uint c)
        const {
     const ull rem = x \& 63:
     uint bt = PREF * 64 + x;
     ull num = m[bt >> 6] & lb(rem + c):
     if(num) return (x ^ rem) | __lg(num);
     for(bt = parent(bt); bt > 62; bt =
          parent(bt)) {
        const ull rem = bt & 63;
       num = m[bt >> 6] & lb(rem);
       if(num) {
         bt = (bt ^ rem) | __lg(num);
         break;
     if(bt == 62) return -1;
     while(bt < PREF * 64) bt = left(bt) |</pre>
           __lg(m[bt - 62]);
     return bt - PREF * 64;
   inline int right go(uint x, const uint c)
        const {
     const ull rem = x \& 63;
```

```
uint bt = PREF * 64 + x;
      ull num = m[bt >> 6] \& \sim lb(rem + c);
      if(num) return (x ^ rem) |
           __builtin_ctzll(num);
      for(bt = parent(bt); bt > 62; bt =
           parent(bt)) {
        const ull rem = bt & 63;
        num = m[bt >> 6] \& \sim lb(rem + 1);
          bt = (bt ^ rem) | __builtin_ctzll(
               num);
          break:
      if(bt == 62) return -1;
      while(bt < PREF * 64) bt = left(bt) |</pre>
           builtin ctzll(m[bt - 62]);
      return bt - PREF * 64;
  public:
    fast set() { assert(PREF != 228); setbit
    bool empty() const {return getbit(63);}
    void clear() { fill(m, m + SZ, 0); setbit
         (62); }
    bool count(uint x) const { return m[PREF +
          (x >> 6)] >> (x & 63) & 1; }
    void insert(uint x) { for(uint v = PREF *
         64 + x; !getbit(v); v = parent(v))
         setbit(v); }
    void erase(uint x) {
      if(!getbit(PREF * 64 + x)) return;
      resetbit(PREF * 64 + x);
      for(uint v = parent(PREF * 64 + x); v >
           62 && !childs value(v); v = parent(v 48
           )) resetbit(v);
    int find_next(uint x) const { return
         right_go(x, \theta); } // >=
    int find prev(uint x) const { return
         left_go(x, 1); } // <=
72 };
```

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#### 1.2 lazysegtree

```
template < class S,
         S (*e)().
         S (*op)(S, S),
         class F.
         F (*id)(),
         S (*mapping)(F, S),
         F (*composition)(F, F)>
struct lazy_segtree {
 int n, size, log;
  vector<S> d; vector<F> lz;
  void update(int k) { d[k] = op(d[k << 1],</pre>
       d[k << 1 | 1]); }
  void all apply(int k, F f) {
    d[k] = mapping(f, d[k]);
    if(k < size) lz[k] = composition(f, lz[k</pre>
         1);
```

```
void push(int k) {
  all apply(k \ll 1, lz[k]);
  all_apply(k << 1 | 1, lz[k]);
  lz[k] = id();
                                              81
lazy_segtree(int _n) : lazy_segtree(vector 82
     <S>( n, e())) {}
lazy segtree(const vector<S>& v) : n(sz(v) 83
  log = __lg(2 * n - 1), size = 1 << log;
                                             84
  d.resize(size * 2, e());
                                              85
  lz.resize(size, id());
  REP(i, n) d[size + i] = v[i];
  for(int i = size - 1; i; i--) update(i); 87
void set(int p, S x) {
  p += size:
  for(int i = log; i; --i) push(p >> i);
  d[p] = x:
  for(int i = 1; i <= log; ++i) update(p</pre>
       >> i);
                                              95
S get(int p) {
  p += size;
  for(int i = log; i; i--) push(p >> i);
  return d[p];
S prod(int 1, int r) {
                                              100
  if(1 == r) return e();
                                              101
  1 += size; r += size;
  for(int i = log; i; i--) {
                                              103
    if(((1 >> i) << i) != 1) push(1 >> i); 104
    if(((r >> i) << i) != r) push(r >> i); 105
  S sml = e(), smr = e();
  while(1 < r) {</pre>
                                              107
    if(1 & 1) sml = op(sml, d[1++]);
                                              108
    if(r \& 1) smr = op(d[--r], smr);
                                              109
    1 >>= 1;
                                              110
    r >>= 1;
                                              111
  return op(sml, smr);
                                              112
                                              113
S all prod() const { return d[1]; }
                                              114
void apply(int p, F f) {
                                              115
  p += size;
                                              116
  for(int i = log; i; i--) push(p >> i);
                                             117
  d[p] = mapping(f, d[p]);
                                              118
  for(int i = 1; i <= log; i++) update(p</pre>
                                              119
       >> i);
                                              120
void apply(int 1, int r, F f) {
                                              121
  if(1 == r) return;
                                              122
  1 += size; r += size;
                                              123
  for(int i = log; i; i--) {
                                              124
    if(((1 >> i) << i) != 1) push(1 >> i); 125
    if(((r >> i) << i) != r) push((r - 1)
    int 12 = 1, r2 = r;
    while(1 < r)  {
```

if(1 & 1) all apply(1++, f);

if(r & 1) all\_apply(--r, f);

1 >>= 1:

r >>= 1;

```
1 = 12;
   r = r2:
  for(int i = 1; i <= log; i++) {</pre>
   if(((1 >> i) << i) != 1) update(1 >> i
    if(((r >> i) << i) != r) update((r -
        1) >> i);
template < class G> int max right(int 1, G g
  assert(0 <= 1 && 1 <= n && g(e());
  if(1 = n) return n;
 1 += size;
  for(int i = log; i; i--) push(1 >> i);
 S sm = e();
    while(!(1 & 1)) 1 >>= 1;
   if(!g(op(sm, d[1]))) {
      while(1 < size) {</pre>
        push(1);
        1 <<= 1:
        if(g(op(sm, d[1]))) sm = op(sm, d[
             1++1);
      return 1 - size;
    sm = op(sm, d[1++]);
  } while((1 & -1) != 1);
 return n;
template < class G> int min_left(int r, G g)
  assert(0 <= r && r <= n && g(e()));
 if(r == 0) return 0;
  r += size;
  for(int i = log; i >= 1; i--) push((r -
      1) >> i);
 S sm = e();
  do {
    while(r > 1 && (r & 1)) r >>= 1;
    if(!g(op(d[r], sm))) {
      while(r < size) {</pre>
        push(r);
        r = r << 1 | 1;
        if(g(op(d[r], sm))) sm = op(d[r])
             --], sm);
      return r + 1 - size;
    sm = op(d[r], sm);
  } while((r & -r) != r);
 return 0;
```

#### 1.3 segtree

```
i template < class S, S (*e)(), S (*op)(S, S)>
2 struct segtree {
```

```
int n, size, log;
vector<S> st;
void update(int v) { st[v] = op(st[v <<</pre>
     1], st[v << 1 | 1]); }
segtree(int _n) : segtree(vector<S>(_n, e
     ())) {}
segtree(const vector<S>& a): n(sz(a)) {
  log = lg(2 * n - 1), size = 1 << log;
  st.resize(size << 1, e());
  REP(i, n) st[size + i] = a[i];
  for(int i = size - 1; i; i--) update(i);
void set(int p, S val) {
  st[p += size] = val;
  for(int i = 1; i <= log; ++i) update(p</pre>
       >> i);
S get(int p) const {
  return st[p + size];
S prod(int 1, int r) const {
  assert(0 <= 1 && 1 <= r && r <= n);
  S sml = e(), smr = e();
  1 += size, r += size;
  while(1 < r) {</pre>
    if(1 \& 1) sml = op(sml, st[1++]);
    if(r \& 1) smr = op(st[--r], smr);
    1 >>= 1:
    r >>= 1;
  return op(sml, smr);
S all_prod() const { return st[1]; }
template < class F> int max_right(int 1, F f
    ) const {
  assert(0 <= 1 && 1 <= n && f(e()));
  if(1 == n) return n;
  1 += size:
  S sm = e();
    while(~1 & 1) 1 >>= 1;
    if(!f(op(sm, st[1]))) {
      while(1 < size) {</pre>
        1 <<= 1:
        if(f(op(sm, st[1]))) sm = op(sm,
             st[1++]);
      return 1 - size;
    sm = op(sm, st[1++]);
  } while((1 & -1) != 1);
  return n;
template < class F > int min left(int r, F f)
  assert(0 <= r \& r <= n \& f(e()));
  if(r == 0) return 0;
  r += size;
  S sm = e();
  do {
    while(r > 1 && (r & 1)) r >>= 1;
    if(!f(op(st[r], sm))) {
      while(r < size) {</pre>
        r = r \ll 1 \mid 1;
```

```
if(f(op(st[r], sm))) sm = op(st[r
                                          24
           --1, sm);
    return r + 1 - size;
  sm = op(st[r], sm);
} while((r & -r) != r);
return 0:
```

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v->r = p.first;

while(v->p != NULL) {

k++:

 $v = v \rightarrow p$ ;

1.6 動態凸包

template < bool MAX >

} else {

mutable 11 k, m, p;

const ll INF = 1e18L;

if(x->k == y->k) {

return x->p >= y->p;

void add\_line(ll k, ll m) {

if(!MAX) k = -k, m = -m;

x, y = erase(y));

return k < o.k; }</pre>

return k;

1 struct line t {

if(v == v->p->r) {

pull(v); return {v, p.second};

45 int get\_position(Node\* v) { // 0-indexed

int k = (v->1 != NULL ? v->1->sz : 0);

bool operator<(const line\_t& o) const {</pre>

struct CHT : multiset<line t, less<>>> {

bool isect(iterator x, iterator y) {

->k); // see Math

while(isect(y, z)) z = erase(z);

if(y == end()) return x->p = INF, 0;

x->p = (x->m > v->m ? INF : -INF);

 $x \rightarrow p = floor_div(y \rightarrow m - x \rightarrow m, x \rightarrow k - y)$ 

auto z = insert({k, m, 0}), y = z++, x =

if(x != begin() && isect(--x, y)) isect(

**if**(v->p->l != NULL) k += v->p->l->sz;

#### 1.4 sparse-table

```
i template < class T, T (*op)(T, T)>
 struct sparse table {
   int n;
   vector<vector<T>> b;
   sparse_table(const vector<T>& a) : n(sz(a)
      int lg = __lg(n) + 1;
      b.resize(\overline{lg}); b[0] = a;
      for(int j = 1; j < lg; ++j) {
       b[j].resize(n - (1 << j) + 1);
        REP(i, n - (1 << j) + 1) b[j][i] = op( 49
            b[j - 1][i], b[j - 1][i + (1 << (j 50)]
             - 1))]);
   T prod(int from, int to) {
      int lg = __lg(to - from + 1);
      return op(b[lg][from], b[lg][to - (1 <<
          lg) + 1]);
```

#### treap

```
1 struct Node {
     bool rev = false;
     int sz = 1, pri = rng();
     Node *1 = NULL, *r = NULL, *p = NULL;
     // TODO
   void pull(Node*& v) {
     v \rightarrow sz = 1 + size(v \rightarrow 1) + size(v \rightarrow r);
     // TODO
void push(Node*& v) {
    if(v->rev) {
       swap(v->1, v->r);
       if(v->1) v->1->rev ^= 1;
       if(v->r) v->r->rev ^= 1;
       v->rev = false:
Node* merge(Node* a, Node* b) {
    if(!a || !b) return (a ? a : b);
     push(a), push(b);
     if(a->pri > b->pri) {
       a->r = merge(a->r, b);
```

```
pull(a); return a;
                                                            while((y = x) != begin() && (--x)->p >=
    } else {
                                                                 v->p) isect(x, erase(v));
      b \rightarrow 1 = merge(a, b \rightarrow 1);
                                                     24
      pull(b); return b;
                                                          11 get(11 x) {
                                                     25
                                                            assert(!empty());
                                                     26
                                                            auto 1 = *lower bound(x);
30 pair<Node*, Node*> split(Node* v, int k) {
                                                            return (1.k * x + 1.m) * (MAX ? +1 : -1)
    if(!v) return {NULL, NULL};
    push(v):
                                                     29
    if(size(v\rightarrow 1) \rightarrow = k) {
                                                     30 };
      auto p = split(v->1, k);
      if(p.first) p.first->p = NULL;
      v \rightarrow 1 = p.second;
                                                       1.7 回滾 DSU
      pull(v); return {p.first, v};
    } else {
      auto p = split(v->r, k - size(v->l) - 1)
```

#### 1 struct RollbackDSU { if(p.second) p.second->p = NULL; int n; vi sz, tag;

```
n = n;
                                                    sz.assign(n, -1);
                                                    tag.clear();
                                                  int leader(int x) {
                                                    while(sz[x] >= 0) x = sz[x];
                                                    return x:
                                             12
                                                  bool merge(int x, int y) {
                                                    x = leader(x), y = leader(y);
                                                    if(x == y) return false;
                                                    if(-sz[x] < -sz[y]) swap(x, y);
                                                    op.eb(x, sz[x], y, sz[y]);
                                                    sz[x] += sz[y]; sz[y] = x;
                                                    return true;
                                             20
                                                  int size(int x) { return -sz[leader(x);] }
                                             21
                                                  void add_tag() { tag.pb(sz(op)); }
                                                  void rollback() {
                                                    int z = tag.back(); tag.ppb();
                                             25
                                                    while(sz(op) > z) {
                                                      auto [x, sx, y, sy] = op.back(); op.
                                                           ppb();
bool operator<(ll x) const { return p < x;</pre>
                                                      sz[x] = sx;
                                                      sz[y] = sy;
                                             28
                                             29
                                             30
```

vector<tuple<int, int, int, int>> op;

void init(int \_n) {

# 2 Flow-Matching

#### 2.1 Dinic

```
1 template < class T>
2 class Dinic {
 public:
   struct Edge {
     int from, to;
     T cap;
     Edge(int x, int y, T z) : from(x), to(y)
          , cap(z) {}
```

```
};
constexpr T INF = 1e9;
int n:
vector<Edge> edges;
vector<vi> g;
vi cur, h; // h : Level graph
Dinic(int _n) : n(_n), g(_n) {}
void add edge(int u, int v, T c) {
  g[u].pb(sz(edges));
  edges.eb(u, v, c);
 g[v].pb(sz(edges));
  edges.eb(v, u, 0);
bool bfs(int s, int t) {
 h.assign(n, -1);
  queue<int> q;
 h[s] = 0;
  q.push(s);
  while(!q.empty()) {
    int u = q.front(); q.pop();
    for(int i : g[u]) {
      const auto& e = edges[i];
      int v = e.to;
      if(e.cap > 0 && h[v] == -1) {
        h[v] = h[u] + 1;
        if(v == t) return true;
        q.push(v);
  return false;
T dfs(int u, int t, T f) {
 if(u == t) return f;
 Tr = f:
  for(int& i = cur[u]; i < sz(g[u]); ++i)</pre>
    int j = g[u][i];
    const auto& e = edges[j];
    int v = e.to;
    T c = e.cap;
    if(c > 0 & h[v] == h[u] + 1) {
     T = dfs(v, t, min(r, c));
      edges[j].cap -= a;
      edges[j ^ 1].cap += a;
      if((r -= a) == 0) return f;
  return f - r;
T flow(int s, int t, T f = INF) {
  while(f > 0 && bfs(s, t)) {
    cur.assign(n, 0);
    T cur = dfs(s, t, f);
    ans += cur;
    f -= cur;
  return ans;
```

#### 2.2 Flow 建模

- · Maximum/Minimum flow with lower bound / Circulation problem
  - 1. Construct super source S and sink T.
  - 2. For each edge (x, y, l, u), connect  $x \to y$  with capacity u-l.
  - 3. For each vertex v, denote by in(v) the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - 4. If in(v) > 0, connect  $S \rightarrow v$  with capacity in(v), otherwise, connect  $v \to T$  with capacity -in(v).
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let f be the maximum flow from Sto T. If  $f \neq \sum_{v \in V, in(v)>0} in(v)$ , there's no solution. Otherwise, the maximum flow from s to t is the answer.
    - To minimize, let f be the maximum flow from S to T. Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from S to T be f'. If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, f' is the
  - 5. The solution of each edge e is  $l_e+f_e$ , where  $f_e$ corresponds to the flow of edge e on the graph.
- · Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  - 1. Redirect every edge:  $y \to x$  if  $(x, y) \in M$ .  $x \to y$  otherwise.
  - 2. DFS from unmatched vertices in X.
  - 3.  $x \in X$  is chosen iff x is unvisited.
  - 4.  $y \in Y$  is chosen iff y is visited.
- · Minimum cost cyclic flow
  - 1. Consruct super source S and sink T
  - 2. For each edge (x, y, c), connect  $x \to y$  with (cost, cap) = (c, 1) if c > 0, otherwise con- $\operatorname{nect} y \to x \text{ with } (\cos t, cap) = (-c, 1)$
  - 3. For each edge with c < 0, sum these cost as K, then increase d(y) by 1, decrease d(x) by 1
  - 4. For each vertex v with d(v) > 0, connect  $S \rightarrow v$  with (cost, cap) = (0, d(v))
  - 5. For each vertex v with d(v) < 0, connect  $v \to T$  with (cost, cap) = (0, -d(v))
  - 6. Flow from S to T, the answer is the cost of the flow C + K
- · Maximum density induced subgraph
  - 1. Binary search on answer, suppose we're checking answer T
  - 2. Construct a max flow model, let K be the sum  $^{17}$
  - 3. Connect source  $s \to v, v \in G$  with capacity  $K^{-19}$
  - 4. For each edge (u, v, w) in G, connect  $u \to v^{-20}$ and  $v \to u$  with capacity w
  - 5. For  $v \in G$ , connect it with sink  $v \to t$  with capacity  $K+2T-(\sum_{e\in E(v)}w(e))-2w(v)$  23
  - 6. T is a valid answer if the maximum flow f < 24K|V|
- · Minimum weight edge cover
  - 1. For each  $v \in V$  create a copy v', and connect  $u' \to v'$  with weight w(u, v).

- 2. Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where 28  $\mu(v)$  is the cost of the cheapest edge incident to 29
- 3. Find the minimum weight perfect matching on 31
- Project selection problem
  - 1. If  $p_v > 0$ , create edge (s, v) with capacity  $p_v$ ; otherwise, create edge (v, t) with capacity  $-p_v$ . 35
  - 2. Create edge (u, v) with capacity w with w being  $_{36}$ the cost of choosing u without choosing v.
  - 3. The mincut is equivalent to the maximum profit 38 of a subset of projects.
- 0/1 quadratic programming

can be minimized by the mincut of the following graph:

- 1. Create edge (x, t) with capacity  $c_x$  and create edge (s, y) with capacity  $c_y$ .
- 2. Create edge (x, y) with capacity  $c_{xy}$ .

static constexpr T INF = numeric limits<T</pre>

3. Create edge (x, y) and edge (x', y') with capacity  $c_{xyx'y'}$ .

```
else if(!check(x)) return;
REP(x, n) if(!vl[x] \&\& d > slk[x]) d =
      slk[x]:
```

if(d) slk[x] = d;

```
REP(x, n) {
 i\hat{f}(v1[x]) h1[x] += d;
 else slk[x] -= d;
 if(vr[x]) hr[x] -= d;
```

REP(x, n) if(!v1[x] && !s1k[x] && !check(x)) return;

T solve() { fill(all(fl), -1); fill(all(fr), -1);

fill(all(hr), 0); REP(i, n) hl[i] = \*max element(all(w[i]) REP(i, n) bfs(i);

T ans = 0; REP(i, n) ans += w[i][fl[i]]; // i 跟 fl [i] 配對

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53 };

#### 2.3 $\mathbf{K}\mathbf{M}$

| template<class T>

2 struct KM {

```
>::max():
int n, ql, qr;
vector<vector<T>> w;
vector<T> hl, hr, slk;
vi fl, fr, pre, qu;
vector<bool> v1, vr;
KM(int n) : n(n), w(n, vector<T>(n, -INF))
     , hl(n), hr(n), slk(n), fl(n), fr(n),
     pre(n), qu(n), vl(n), vr(n) {}
void add edge(int u, int v, int x) { w[u][
     v] = x; } // 最小值要加負號
bool check(int x) {
  vl[x] = 1;
  if(fl[x] != -1) return vr[qu[qr++] = fl[
  while(x != -1) swap(x, fr[fl[x] = pre[x
       11);
  return 0;
                                             16
void bfs(int s) {
  fill(all(slk), INF);
  fill(all(vl), 0);
  fill(all(vr), 0);
  ql = qr = 0, qu[qr++] = s, vr[s] = 1;
  while(true) {
                                             20
   T d;
                                            21
    while(ql < qr) {</pre>
      for(int x = 0, y = qu[ql++]; x < n;
                                            23
        if(!vl[x] \&\& slk[x] >= (d = hl[x]
                                            25
            + hr[y] - w[x][y])) {
```

pre[x] = y;

#### **2.4** MCMF

return ans;

```
i template < class S, class T>
2 class MCMF {
3 public:
   struct Edge {
     int from, to;
      S cap;
      Edge(int u, int v, S x, T y) : from(u),
          to(v), cap(x), cost(y) {}
    const 11 INF = 1e18L;
    int n:
   vector<Edge> edges;
   vector<vi> g;
    vector<T> d;
   vector<bool> ing;
   vi pedge:
    MCMF(int _n) : n(_n), g(_n), d(_n), inq(_n
        ), pedge(_n) {}
    void add edge(int u, int v, S cap, T cost)
      g[u].pb(sz(edges));
      edges.eb(u, v, cap, cost);
     g[v].pb(sz(edges));
     edges.eb(v, u, 0, -cost);
    bool spfa(int s, int t) {
      bool found = false:
      fill(all(d), INF);
      d[s] = 0;
```

```
inq[s] = true;
      queue<int> q;
      a.push(s):
      while(!q.empty()) {
        int u = q.front(); q.pop();
        if(u == t) found = true;
        inq[u] = false;
        for(auto& id : g[u]) {
          const auto& e = edges[id];
          if(e.cap > 0 && d[u] + e.cost < d[e.</pre>
               to]) {
            d[e.to] = d[u] + e.cost;
            pedge[e.to] = id;
            if(!inq[e.to]) {
              q.push(e.to);
              inq[e.to] = true;
      return found;
    pair<S, T> flow(int s, int t, S f = INF) {
     S cap = 0;
     T cost = 0;
      while(f > 0 && spfa(s, t)) {
        S send = f;
        int u = t;
        while(u != s) {
          const Edge& e = edges[pedge[u]];
          send = min(send, e.cap);
          u = e.from;
        while(u != s) {
          Edge& e = edges[pedge[u]];
          e.cap -= send;
          Edge& b = edges[pedge[u] ^ 1];
          b.cap += send;
          u = e.from;
        cap += send;
        f -= send;
        cost += send * d[t];
      return {cap, cost};
73
74 };
        一般圖最大匹配
```

```
i struct GeneralMaxMatch {
   vector<pii> es;
   vi g, vis, mate; // i 與 mate[i] 配對 (
        mate[i] == -1 代表沒有匹配)
   GeneralMaxMatch(int n) : n(n), g(n, -1),
        mate(n, -1) {}
   bool dfs(int u) {
     if(vis[u]) return false;
     vis[u] = true;
     for(int ei = g[u]; ei != -1; ) {
```

```
auto [x, y] = es[ei]; ei = y;
    if(mate[x] == -1) {
      mate[mate[u] = x] = u;
      return true;
  for(int ei = g[u]; ei != -1; ) {
    auto [x, y] = es[ei]; ei = y;
    int nu = mate[x];
    mate[mate[u] = x] = u;
    mate[nu] = -1;
    if(dfs(nu)) return true;
    mate[mate[nu] = x] = nu;
    mate[u] = -1;
  return false;
void add_edge(int a, int b) {
  auto f = [&](int a, int b) {
    es.eb(b, g[a]);
   g[a] = sz(es) - 1;
  f(a, b); f(b, a);
int solve() {
  vi o(n);
  iota(all(o), 0);
  int ans = 0:
  REP(it, 100) {
    shuffle(all(o), rng);
   vis.assign(n, false);
    for(auto i : o) if(mate[i] == -1) ans
        += dfs(i);
  return ans;
```

#### 一般圖最小權完美匹配

```
1 struct Graph {
   // Minimum General Weighted Matching (
         Perfect Match) 0-base
   static const int MXN = 105;
   int n. edge[MXN][MXN];
   int match[MXN], dis[MXN], onstk[MXN];
   vector<int> stk;
   void init(int n) {
      for(int i=0; i<n; i++)</pre>
        for(int j=0; j<n; j++)</pre>
          edge[i][j] = 0;
   void add_edge(int u, int v, int w) { edge[
         u][v] = edge[v][u] = w; }
   bool SPFA(int u){
     if(onstk[u]) return true;
      stk.push_back(u);
      onstk[u] = 1;
      for(int v=0; v<n; v++){</pre>
       if(u != v && match[u] != v && !onstk[v
             1){
          int m = match[v];
```

```
if(dis[m] > dis[u] - edge[v][m] +
21
                edge[u][v]){
             dis[m] = dis[u] - edge[v][m] +
                  edge[u][v];
             onstk[v] = 1;
23
             stk.push back(v);
24
25
             if(SPFA(m)) return true;
             stk.pop back();
             onstk[v] = 0;
        }
      onstk[u] = 0;
      stk.pop back();
33
      return false;
34
    int solve() {
      for(int i = 0; i < n; i += 2) match[i] =</pre>
            i + 1, match[i+1] = i:
       while(true) {
        int found = 0;
        for(int i=0; i<n; i++) dis[i] = onstk[ 30</pre>
        for(int i=0; i<n; i++){</pre>
           stk.clear();
           if(!onstk[i] && SPFA(i)){
             found = 1;
             while(stk.size()>=2){
               int u = stk.back(); stk.pop_back
               int v = stk.back(); stk.pop_back
               match[u] = v;
               match[v] = u;
        if(!found) break;
      int ans = 0;
      for(int i=0; i<n; i++) ans += edge[i][</pre>
           match[i]];
      return ans / 2;
58 }graph;
```

# 2.7 二分圖最大匹配

```
struct bipartite_matching {
   int n, m; // 二分圖左右人數 (0 ~ n-1), (0
        \sim m-1)
   vector<vi> g:
   vi lhs, rhs, dist; // i 與 Lhs[i] 配對 (
        Lhs[i] == -1 代表沒有配對)
   bipartite_matching(int _n, int _m) : n(_n)
        , m(_m), g(_n), lhs(_n, -1), rhs(_m,
        -1), dist(_n) {}
   void add_edge(int u, int v) { g[u].pb(v);
   void bfs() {
     queue<int> q;
     REP(i, n) {
```

# Geometry

**if**(lhs[i] == -1) {

q.push(i);

} else {

bool dfs(int u) {

for(auto v : g[u]) { **if**(rhs[v] == -1) {

return true;

for(auto v : g[u]) {

return true;

return false;

int ans = 0;

while(true) {

int aug = 0;

ans += aug;

return ans;

if(!aug) break;

int solve() {

bfs();

rhs[v])) {

rhs[lhs[u] = v] = u;

dist[i] = 0;

dist[i] = -1;

while(!q.empty()) {

for(auto v : g[u]) {

q.push(rhs[v]);

rhs[lhs[u] = v] = u;

int u = q.front(); q.pop();

**if**(rhs[v] != -1 && dist[rhs[v]] ==

**if**(dist[rhs[v]] == dist[u] + 1 && dfs(

REP(i, n) if(lhs[i] == -1) aug += dfs(

dist[rhs[v]] = dist[u] + 1;

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#### 3.1 convex-hull

```
void convex_hull(vector<P>& dots) {
   sort(all(dots));
   vector<P> ans(1, dots[0]);
   for(int it = 0; it < 2; it++, reverse(all(</pre>
      for(int i = 1, t = sz(ans); i < sz(dots)</pre>
            ans.pb(dots[i++])) {
        while(sz(ans) > t && ori(ans[sz(ans) -
              2], ans.back(), dots[i]) < 0) {
```

```
ans.ppb();
ans.ppb();
swap(ans, dots);
```

#### point-in-convex-hull

```
i int point in convex hull(const vector<P>& a,
       P p) {
   // -1 ON, 0 OUT, +1 IN
   // 要先逆時針排序
   int n = sz(a);
   if(btw(a[0], a[1], p) || btw(a[0], a[n -
        1], p)) return -1;
   int 1 = 0, r = n - 1;
   while(1 <= r) {
     int m = (1 + r) / 2;
     auto a1 = cross(a[m] - a[0], p - a[0]);
     auto a2 = cross(a[(m + 1) % n] - a[0], p
           - a[0]);
     if(a1 >= 0 && a2 <= 0) {
       auto res = cross(a[(m + 1) % n] - a[m
           ], p - a[m]);
       return res > 0 ? 1 : (res >= 0 ? -1 :
     if(a1 < 0) r = m - 1;
     else l = m + 1;
   return 0;
```

### point

```
i using P = pair<11, 11>;
3 P operator+(P a, P b) { return P{a.X + b.X,
      a.Y + b.Y; }
4 P operator-(P a, P b) { return P{a.X - b.X,
      a.Y - b.Y}; }
5 P operator*(P a, 11 b) { return P{a.X * b, a
       .Y * b}; }
6 P operator/(P a, 11 b) { return P{a.X / b, a
       .Y / b}; }
7 | 11 dot(P a, P b) { return a.X * b.X + a.Y *
      b.Y; }
8 11 cross(P a, P b) { return a.X * b.Y - a.Y
       * b.X; }
9 11 abs2(P a) { return dot(a, a); }
double abs(P a) { return sqrt(abs2(a)); }
int sign(ll x) { return x < 0 ? -1 : (x == 0)</pre>
        ? 0 : 1); }
12 int ori(P a, P b, P c) { return sign(cross(b
        - a, c - a)); }
13 bool collinear(Pa, Pb, Pc) { return sign( 17
      cross(a - c, b - c)) == 0; }
```

```
14 bool btw(P a, P b, P c) {
    if(!collinear(a, b, c)) return 0;
    return sign(dot(a - c, b - c)) <= 0;</pre>
bool seg_intersect(P a, P b, P c, P d) {
    int a123 = ori(a, b, c);
    int a124 = ori(a, b, d);
    int a341 = ori(c, d, a);
    int a342 = ori(c, d, b);
    if(a123 == 0 && a124 == 0) {
      return btw(a, b, c) || btw(a, b, d) ||
           btw(c, d, a) || btw(c, d, b);
    return a123 * a124 <= 0 && a341 * a342 <=
27 }
  P intersect(P a, P b, P c, P d) {
    int a123 = cross(b - a, c - a);
    int a124 = cross(b - a, d - a);
    return (d * a123 - c * a124) / (a123 -
         a124):
33 }
```

#### 3.4 polar-angle-sort

```
bool cmp(P a, P b) {
  #define ng(k) (sign(k.Y) < 0 || (sign(k.Y)
        == 0 \&\& sign(k.X) < 0)
  int A = ng(a), B = ng(b);
  if(A != B) return A < B;</pre>
  if(sign(cross(a, b)) == 0) return abs2(a)
       < abs2(b);
  return sign(cross(a, b)) > 0;
```

#### 3.5 最近點對

```
1 const 11 INF = 9e18L;
 vector<P> a:
 sort(all(a), [](P a, P b) \{ return a.x < b.x \}
      ; });
 11 solve(int 1, int r) {
   if(1 + 1 == r) return INF;
   int m = (1 + r) / 2;
   11 d = min(solve(1, m), solve(m, r));
   inplace_merge(a.begin() + 1, a.begin() + m
        , a.begin() + r, [](P a, P b) {
      return a.y < b.y;</pre>
   #define SQ(x) ((x) * (x))
   vector<P> p;
   for(int i = 1; i < r; ++i) if(SQ(a[i].x -
        a[m].x) <= d) p.pb(a[i]);
   REP(i, sz(p)) {
      for(int j = i + 1; j < sz(p); ++j) {</pre>
       if((p[i].y - p[j].y) * (p[i].y - p[j].
            y) > d) break;
        d = min(d, SQ(p[i].x - p[j].x) + SQ(p[i].x)
            i].y - p[j].y));
```

```
19
   return d; // 距離平方
21 }
```

# Graph

#### 4.1 2-SAT

```
1 struct two sat {
   int n; SCC g;
    vector<bool> ans;
    two_sat(int _n) : n(_n), g(_n * 2) {}
    void add_or(int u, bool x, int v, bool y)
      g.add_edge(2 * u + !x, 2 * v + y);
      g.add_edge(2 * v + !y, 2 * u + x);
    bool solve() {
      ans.resize(n);
      auto id = g.solve();
      REP(i, n) {
        if(id[2 * i] == id[2 * i + 1]) return
        ans[i] = (id[2 * i] < id[2 * i + 1]);
      return true;
18 };
```

#### centroid-tree

```
pair<int, vector<vi>>> centroid tree(const
       vector<vi>& g) {
    int n = sz(g);
    vi siz(n);
    vector<bool> vis(n);
    auto dfs sz = [&](auto f, int u, int p) ->
                                                 25
          void {
      siz[u] = 1:
      for(auto v : g[u]) {
        if(v == p || vis[v]) continue;
        f(f, v, u);
        siz[u] += siz[v];
12
    auto find_cd = [&](auto f, int u, int p,
         int all) -> int {
      for(auto v : g[u]) {
        if(v == p || vis[v]) continue;
        if(siz[v] * 2 > all) return f(f, v, u,
              all);
      return u;
19
    };
    vector<vi> h(n);
    auto build = [&](auto f, int u) -> int {
      dfs_sz(dfs_sz, u, -1);
```

```
int cd = find_cd(find_cd, u, -1, siz[u])
      vis[cd] = true;
      for(auto v : g[cd]) {
        if(vis[v]) continue;
        int child = f(f, v);
        h[cd].pb(child);
      return cd;
31
    int root = build(build, 0);
    return {root, h};
```

vi siz, par, depth, top, tour, fi, id;

#### 4.3 HLD

1 struct HLD {

int n;

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vector<vi> g;

```
sparse table<pii, min> st;
HLD(int _n) : n(_n), g(_n), siz(_n), par(
     _n), depth(_n), top(_n), fi(_n), id(_n
  tour.reserve(n);
void add_edge(int u, int v) {
  g[u].push back(v);
  g[v].push_back(u);
void build(int root = 0) {
  par[root] = -1;
  top[root] = root;
  vector<pii> euler tour;
  euler_tour.reserve(2 * n - 1);
  dfs sz(root);
  dfs link(euler tour, root);
  st = sparse table<pii, min>(euler tour);
int get lca(int u, int v) {
  int L = fi[u], R = fi[v];
  if(L > R) swap(L, R);
  return st.prod(L, R).second;
bool is anc(int u, int v) {
  return id[u] <= id[v] && id[v] < id[u] +</pre>
        siz[u];
bool on path(int a, int b, int x) {
  return (is ancestor(x, a) || is ancestor
       (x, b)) && is ancestor(get lca(a, b)
       , x);
int get_dist(int u, int v) {
  return depth[u] + depth[v] - 2 * depth[(
       get_lca(u, v))];
int kth_anc(int u, int k) {
  if(depth[u] < k) return -1;</pre>
  int d = depth[u] - k;
  while(depth[top[u]] > d) u = par[top[u
  return tour[id[u] + d - depth[u]];
```

```
int kth node on path(int a, int b, int k)
     int z = get_lca(a, b);
     int fi = depth[a] - depth[z];
     int se = depth[b] - depth[z];
     if(k < 0 \mid | k > fi + se) return -1;
     if(k < fi) return kth anc(a, k);</pre>
     return kth anc(b, fi + se - k);
   vector<pii> get path(int u, int v, bool
        include lca = true) {
     if(u == v && !include_lca) return {};
     vector<pii> seg;
     while(top[u] != top[v]) {
       if(depth[top[u]] > depth[top[v]]) swap
       seg.eb(id[top[v]], id[v]);
       v = par[top[v]];
     if(depth[u] > depth[v]) swap(u, v); // u
           is Lca
     if(u != v || include_lca) seg.eb(id[u] +
           !include lca, id[v]);
     return seg;
   void dfs sz(int u) {
     if(par[u] != -1) g[u].erase(find(all(g[u
          ]), par[u]));
     siz[u] = 1;
     for(auto& v : g[u]) {
       par[v] = u;
       depth[v] = depth[u] + 1;
       dfs_sz(v);
       siz[u] += siz[v];
       if(siz[v] > siz[g[u][0]]) swap(v, g[u
            ][0]);
   void dfs_link(vector<pii>& euler_tour, int
     fi[u] = sz(euler_tour);
     id[u] = sz(tour);
     euler tour.eb(depth[u], u);
     tour.pb(u);
     for(auto v : g[u]) {
       top[v] = (v == g[u][0] ? top[u] : v);
       dfs link(euler tour, v);
       euler tour.eb(depth[u], u);
 4.4 lowlink
| struct lowlink {
   int n, cnt = 0, tecc_cnt = 0, tvcc_cnt =
   vector<vector<pii>>> g;
   vector<pii> edges;
```

```
vi roots, id, low, tecc id, tvcc id;
vector<bool> is bridge, is cut,
    is_tree_edge;
```

```
lowlink(int _n) : n(_n), g(_n), is_cut(_n, 68
     false), id(_n, -1), low(_n, -1) {}
void add edge(int u, int v) {
  g[u].eb(v, sz(edges));
  g[v].eb(u, sz(edges));
  edges.eb(u, v);
  is bridge.pb(false);
  is tree edge.pb(false);
  tvcc id.pb(-1);
void dfs(int u, int peid = -1) {
  static vi stk;
  static int rid;
  if(peid < 0) rid = cnt;</pre>
  if(peid == -1) roots.pb(u);
  id[u] = low[u] = cnt++;
  for(auto [v, eid] : g[u]) {
    if(eid == peid) continue;
    if(id[v] < id[u]) stk.pb(eid);</pre>
    if(id[v] >= 0) {
     low[u] = min(low[u], id[v]);
    } else {
      is_tree_edge[eid] = true;
      dfs(v, eid);
      low[u] = min(low[u], low[v]);
      if((id[u] == rid && id[v] != rid +
           1) | | (id[u] != rid && low[v] >= 94 |
           id[u])) {
        is_cut[u] = true;
      if(low[v] >= id[u]) {
        while(true) {
          int e = stk.back();
          stk.pop_back();
          tvcc id[e] = tvcc cnt;
          if(e == eid) break;
        tvcc_cnt++;
void build() {
  REP(i, n) if(id[i] < 0) dfs(i);</pre>
  REP(i, sz(edges)) {
    auto [u, v] = edges[i];
    if(id[u] > id[v]) swap(u, v);
    is bridge[i] = (id[u] < low[v]);</pre>
vector<vi> two ecc() { // 邊雙
  tecc_cnt = 0;
  tecc id.assign(n, -1);
  vi stk;
  REP(i, n) {
   if(tecc id[i] != -1) continue;
    tecc_id[i] = tecc_cnt;
    stk.pb(i);
    while(sz(stk)) {
     int u = stk.back(); stk.pop_back();
      for(auto [v, eid] : g[u]) {
        if(tecc id[v] >= 0 || is bridge[
                                             25
             eid]) {
                                             26
          continue;
                                             27
```

```
tecc id[v] = tecc cnt;
       stk.pb(v);
   tecc_cnt++;
 vector<vi> comp(tecc cnt);
 REP(i, n) comp[tecc_id[i]].pb(i);
 return comp;
vector<vi> bcc_vertices() { // 點雙
 vector<vi> comp(tvcc cnt);
 REP(i, sz(edges)) {
   comp[tvcc_id[i]].pb(edges[i].first);
   comp[tvcc id[i]].pb(edges[i].second);
 for(auto& v : comp) {
   sort(all(v));
   v.erase(unique(all(v)), v.end());
 REP(i, n) if(g[i].empty()) comp.pb({i});
 return comp;
vector<vi> bcc edges() {
 vector<vi> ret(tvcc cnt);
 REP(i, sz(edges)) ret[tvcc id[i]].pb(i);
 return ret;
```

#### 4.5 SCC

```
1 struct SCC {
   int n:
   vector<vi> g, h;
   SCC(int _n) : n(_n), g(_n), h(_n) {}
   void add edge(int u, int v) {
     g[u].pb(v);
     h[v].pb(u);
   vi solve() { // 回傳縮點的編號
     vi id(n), top;
     top.reserve(n);
     #define GO if(id[v] == 0) dfs1(v);
     function<void(int)> dfs1 = [&](int u) {
       id[u] = 1;
       for(auto v : g[u]) GO;
       top.pb(u);
     };
     REP(v, n) GO;
     fill(all(id), -1);
     function<void(int, int)> dfs2 = [&](int
          u, int x) {
       id[u] = x;
       for(auto v : h[u]) {
         if(id[v] == -1) {
           dfs2(v, x);
     };
     for(int i = n - 1, cnt = 0; i >= 0; --i)
```

```
int u = top[i];
        if(id[u] == -1) {
          dfs2(u, cnt):
          cnt += 1;
33
      return id;
37 };
```

#### Math

#### **5.1** Chinese-Remainder

```
1 // (rem, mod) {0, 0} for no solution
2 pair<11, 11> crt(11 r0, 11 m0, 11 r1, 11 m1)
    r0 = (r0 \% m0 + m0) \% m0;
    r1 = (r1 \% m1 + m1) \% m1:
    if(m0 < m1) swap(r0, r1), swap(m0, m1);</pre>
    if(m0 \% m1 == 0) {
      if(r0 % m1 != r1) return {0, 0};
    11 g, im, qq;
    g = ext_gcd(m0, m1, im, qq);
    11 u1 = (m1 / g);
    if((r1 - r0) % g) return {0, 0};
    11 x = (r1 - r0) / g % u1 * im % u1;
    r0 += x * m0:
    m0 *= u1:
    if(r0 < 0) r0 += m0:
16
    return {r0, m0};
17
```

#### 5.2 Discrete-Log

```
i int discrete log(int a, int b, int m) {
    if(b == 1 | m == 1) return 0;
    int n = sqrt(m) + 2, e = 1, f = 1, j = 1;
    unordered_map<int, int> A; // becareful
    while(j <= n && (e = f = 1LL * e * a % m)</pre>
         != b) A[1LL * e * b % m] = j++;
    if(e == b) return j;
    if(__gcd(m, e) == __gcd(m, b)) {
      for(int i = 2; i < n + 2; ++i) {
        e = 1LL * e * f % m;
        if(A.find(e) != A.end()) return n * i
             - A[e];
11
12
13
    return -1;
```

#### 5.3 extgcd

```
1 // ax + by = acd(a, b)
2 | 11 ext_gcd(11 a, 11 b, 11& x, 11& y) {
   if(b == 0) {
     x = 1, y = 0;
     return a;
   ll x1, y1;
   11 g = ext_gcd(b, a % b, x1, y1);
  x = y1, y = x1 - (a / b) * y1;
   return g;
```

#### 5.4 Floor-Sum

```
| | // sum_0^{n-1} floor((a * i + b) / m) in log
      (n + m + a + b)
2 ll floor_sum(ll n, ll m, ll a, ll b) {
   11 \text{ ans} = 0;
   if(a >= m) ans += (n - 1) * n * (a / m) /
       2, a %= m;
   if(b >= m) ans += n * (b / m), b %= m;
   11 y_max = (a * n + b) / m, x_max = (y_max)
         * m - b);
   if(y max == 0) return ans;
   ans += (n - (x max + a - 1) / a) * y max;
   return ans + floor_sum(y_max, a, m, (a -
        x max % a) % a);
```

#### Miller-Rabin

```
| bool is prime(ll n, vector<ll> x) {
   ll d = n - 1;
    d >>= builtin ctzll(d);
    for(auto a : x) {
      if(n <= a) break;</pre>
      11 t = d, y = 1, b = t;
      while(b) {
        if(b \& 1) y = i128(y) * a % n;
        a = i128(a) * a % n;
        b >>= 1;
      while(t != n - 1 && y != 1 && y != n -
        y = i128(y) * y % n;
      if(y != n - 1 && t % 2 == 0) return
           false:
    return true;
20 bool is prime(ll n) {
    if(n <= 1) return false;</pre>
    if(n % 2 == 0) return n == 2;
    if(n < (1LL << 30)) return is prime(n, {2,
          7, 61});
    return is_prime(n, {2, 325, 9375, 28178,
         450775, 9780504, 1795265022});
```

#### 5.6 Pollard-Rho

if(n == 1) return;

```
if(is prime(n)) return mp[n]++, void();
 if(n % 2 == 0) {
   mp[2] += 1;
   PollardRho(mp, n / 2);
   return;
 11 x = 2, y = 2, d = 1, p = 1;
 #define f(x, n, p) ((i128(x) * x % n + p)
 while(true) {
   if(d != 1 && d != n) {
     PollardRho(mp, d);
     PollardRho(mp, n / d);
     return;
   p += (d == n);
   x = f(x, n, p), y = f(f(y, n, p), n, p);
   d = \_gcd(abs(x - y), n);
 #undef f
vector<ll> get_divisors(ll n) {
 if(n == 0) return {};
 map<11, int> mp;
 PollardRho(mp, n);
 vector<pair<11, int>> v(all(mp));
 vector<11> res;
 auto f = [&](auto f, int i, ll x) -> void
   if(i == sz(v)) {
     res.pb(x);
     return;
   for(int j = v[i].second; ; j--) {
     f(f, i + 1, x);
     if(j == 0) break;
     x *= v[i].first;
 f(f, 0, 1);
 sort(all(res));
 return res;
```

void PollardRho(map<11, int>& mp, 11 n) {

#### 5.7 Primes

1 /\* 12721 13331 14341 75577 123457 222557 556679 999983 1097774749 1076767633 100102021 999997771 1001010013 1000512343 987654361 999991231 999888733 98789101 987777733 999991921 1010101333 1010102101 1000000000039 1000000000000037 2305843009213693951 4611686018427387847 9223372036854775783 18446744073709551557 \*/

#### 5.8 估計值

#### Estimation

- The number of divisors of n is at most around 100 for n < 5e4,500 for n < 1e7,2000 for n < 1e10, 200000 for n < 1e19.
- The number of ways of writing n as a sum of positive integers, disregarding the order of the summands. 1, 1, 2, 3, 5, 7, 11, 15, 22, 30 for  $n = 0 \sim 9,627 \text{ for } n = 20, \sim 2e5 \text{ for }$  $n = 50, \sim 2e8 \text{ for } n = 100.$
- Total number of partitions of n distinct elements: B(n)

#### 5.9 定理

· Cramer's rule

$$ax + by = e \qquad x = \frac{ed - bf}{ad - bc}$$

$$cx + dy = f \Rightarrow y = \frac{af - ec}{ad - bc}$$

· Vandermonde's Identity

$$C(n+m,k) = \sum_{i=0}^{k} C(n,i)C(m,k-i)$$

· Kirchhoff's Theorem

Denote L be a  $n \times n$  matrix as the Laplacian matrix of graph G, where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where c is the number of edge (i, j) in G.

- The number of undirected spanning in G is  $|\det(L_{11})|$ .
- The number of directed spanning tree rooted at r in G is  $|\det(\tilde{L}_{rr})|$ .
- · Tutte's Matrix

Let D be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if i < j and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{rank(D)}{2}$  is the maximum matching on G.

- · Cayley's Formula
  - Given a degree sequence  $d_1, d_2, \ldots, d_n$  1 1 floor\_div(11 a, 11 b) {
  - $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!} \text{ spanning trees.} \\ \text{ Let } T_{n,k} \text{ be the number of labeled forests on}$ n vertices with k components, such that vertex  $1, 2, \ldots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

. Erd □s-Gallai theorem

be represented as the degree sequence of a finite simple graph on n vertices if and only if  $d_1 + \cdots + d_n$  is even and  $\sum d_i \leq k(k-1) + \sum \min(d_i, k)$  holds for  $\overline{i-1}$ 

A sequence of nonnegative integers  $d_1 \geq \cdots \geq d_n$  can

#### · Gale-Ryser theorem

every  $1 \le k \le n$ .

A pair of sequences of nonnegative integers  $a_1 >$  $\cdots \geq a_n$  and  $b_1, \ldots, b_n$  is bigraphic if and only if  $\begin{array}{c} n & \text{distinct} & \text{elements.} & D(h) & -\frac{1}{2} \\ 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570 \\ 27644437, 190899322, \dots & \sum_{i=1}^{k} a_i & \sum_{i=1}^{n} b_i \text{ and } \sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k) \text{ holds} \end{array}$ 

· Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \ldots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \geq \cdots \geq a_n$  is digraphic if and only  $\text{if } \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i \text{ and } \sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k$ 

 $\sum \min(b_i, k)$  holds for every  $1 \le k \le n$ .

M□bius inversion formula

$$\begin{array}{lll} -\ f(n) &=& \sum_{d\mid n} g(d) &\Leftrightarrow & g(n) &=\\ &\sum_{d\mid n} \mu(d) f(\frac{n}{d}) \\ -\ f(n) &=& \sum_{n\mid d} g(d) &\Leftrightarrow & g(n) &=\\ &\sum_{n\mid d} \mu(\frac{d}{n}) f(d) \end{array}$$

- · Spherical cap
  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap,  $\theta$ : arcsin(a/r).
  - Volume =  $\pi h^2(3r h)/3 = \pi h(3a^2 + h)$
  - $h^2)/6 = \pi r^3 (2 + \cos \theta)(1 \cos \theta)^2/3.$  Area =  $2\pi rh = \pi(a^2 + h^2) = 2\pi r^2(1 \sin \theta)$  $\cos \theta$ ).

#### 5.10 整數除法

```
for each labeled vertices, there are 2 return a/b - ((a^b) < 0 && a%b != 0);
                                   4 ll ceil div(ll a, ll b) {
                                     return a/b + ((a^b) > 0 && a%b != 0);
```

else if(1 < r) seg[v] = seg[v \* 2] + seg[v

8 void update(int ql, int qr, int x, int v =

1, int l = 0, int r = N - 1) {

update(q1, qr, x, v \* 2, 1, m);

vector<array<int, 4>> ev(2 \* n);

cin >> x >> y >> x2 >> y2;

 $ev[2 * i] = \{x, y, y2, +1\};$ 

prev = ev[i][0], i = j - 1;

 $ev[2 * i + 1] = \{x2, y, y2, -1\};$ 

ans += (ev[i][0] - prev) \* seg[1];

while(j < 2 \* n && ev[i][0] == ev[j][0])

update(ev[j][1], ev[j][2] - 1, ev[j

x += N / 2; y += N / 2;x2 += N / 2; y2 += N / 2;

update(q1, qr, x, v \* 2 + 1, m + 1, r);

 $if(ql > r \mid | qr < 1) return;$ **if**(q1 <= 1 && r <= qr) { tag[v] += x; } else {

int m = (1 + r) / 2;

pull(v, 1, r);

int n; cin >> n;

sort(all(ev));

**int** j = i;

j++;

REP(i, 2 \* n) {

int x, y, x2, y2;

11 ans = 0, prev = 0;

cout << ans << "\n";</pre>

REP(i, n) {

19 int main() {

17

29

18 }

#### 5.11 數字

· Bernoulli numbers

$$B_0 - 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$B_0 - 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^{m} {m+1 \choose j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + \left\lfloor \frac{b}{c} \right\rfloor \cdot (n+1) \\ +f(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \\ 0, & n < 0 \lor^2 \\ 0, & n < 0 \lor^2 \\ nm - f(c, c - b - 1, a, m - 1), \end{cases}$$
#define all(v) (v).begin(), (v).end()
$$\begin{pmatrix} n & 0 \lor^2 \\ mm - f(c, c - b - 1, a, m - 1), \\ mm - f(c, c - b - 1, a, m - 1), \end{cases}$$
#define pb push\_back
$$\begin{pmatrix} n & 0 \lor^2 \\ mm - f(c, c - b - 1, a, m - 1), \\ mm - f(c, c - b - 1, a, m - 1), \end{cases}$$
#define pb pop back

$$\begin{split} S_m(n) &= \sum_{k=1}^n k^m &= g(a,b,c,n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor \\ \frac{1}{m+1} \sum_{k=0}^m {m+1 \choose k} B_k^+ n^{m+1-k} &= \int_{l=0}^n \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)}{l} \\ +g(a \bmod c, n) &= \sum_{k=0}^n i \lfloor \frac{ai+b}{c} \rfloor \end{split}$$

· Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$\begin{split} S(n,k) &= S(n-1,k-1) + kS(n-1,k), S(n,1) = \\ S(n,n) &= 1 \\ S(n,k) &= \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} {k \choose i} i^n \\ x^n &= \sum_{i=0}^n S(n,i)(x)_i \end{split}$$

Pentagonal number theorem

$$\begin{array}{lll} \displaystyle \prod_{n=1}^{\infty} (1 & - & x^n) & = & 1 & + \\ \displaystyle \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right) & & \end{array}$$

Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly k elements are greater than the previous element. k j:s s.t.  $\pi(j) > \pi(j+1), k+1$  j:s s.t.  $\pi(j) \ge j, k$  j:s s.t.  $\pi(i) > i$ .

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{i=0}^{k} (-1)^{i} {n+1 \choose i} (k+1-j)^{n}$$

#### 歐幾里得類算法

- $m = |\frac{an+b}{a}|$
- Time complexity:  $O(\log n)$

### Misc

#### 6.1 default

```
5 #define ppb pop_back
                                                                                                                  6 #define REP(i, n) for(int i = 0; i < int(n);</pre>
                                                                                                                  8 using vi = vector<int>;
= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} & \text{s} \mid \text{using vi = vect} \\ +g(a \bmod c, b \bmod c, c, n), & a \ge c \lor b \ge c \\ 0, & n < 0 \lor a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c - b - 1, a, m - 1) \\ -h(c, c - b - 1, a, m - 1)), & \text{6.2} \text{PBDS} \end{cases}
                                                                                                                 9 using pii = pair<int, int>;
```

```
| #include <ext/pb_ds/assoc_container.hpp>
                                                                        using namespace __gnu_pbds;
                                                                         3 tree<11, null_type, less<11>, rb_tree_tag,
 = \begin{cases} \lfloor \frac{a}{c} \rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \lfloor \frac{b}{c} \rfloor^2 \cdot (n+1) \\ + \lfloor \frac{a}{c} \rfloor \cdot \lfloor \frac{b}{c} \rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{a}{c} \rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \lfloor \frac{b}{c} \rfloor \cdot f(a \bmod c, b \bmod c, c, n), \\ 0, \end{cases} 
                                                                                tree_order_statistics_node_update> st;
                                                                         4 // find_by_order order_of_key
                                                                         5 // __float128_t
                                                                         6 for(int i = bs._Find_first(); i < bs.size();</pre>
                                                                          i = bs._Find_next(i));
                                                                             a \ge c \lor b \ge c
       (-2f(c,c-b-1,a,m-1)-f(a,b,c,n), 6.3 rwispython
```

```
| from decimal import Decimal, getcontext
getcontext().prec = 1000000000
a = pow(Decimal(2), 82589933) - 1
```

#### 6.4 readchar

```
\begin{array}{lll} -A(rx)\Rightarrow r^na_n & & & & \\ -A(x)+B(x)\Rightarrow a_n+b_n & & \\ -A(x)B(x)\Rightarrow \sum_{i=0}^n a_ib_{n-i} & & \\ -A(x)^k\Rightarrow \sum_{i_1+i_2+\dots+i_k=n}a_{i_1}a_{i_2}\dots a_{i_k} & \\ -xA(x)'\Rightarrow na_n & & \\ -\frac{A(x)}{1-x}\Rightarrow \sum_{i=0}^n a_i & & \\ \end{array}
                                                                                                         static char *p = buf, *end = buf;
if(p == end) end = buf + fread(buf, 1, sz,
• Exponential Generating Function A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x_i
                                                                                                                    stdin), p = buf;
                                                                                                                  return *p++;
```

# - $A(x) + B(x) \Rightarrow a_n + b_n$ - $A^{(k)}(x) \Rightarrow a_{n+k}$ - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$ - $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n}^{n} \binom{n}{i_1,i_2,\dots,i_k} a_{i_1} a_{i_2}$ **5.5** $a_{i_k}$ 矩形覆蓋面積 - $xA(x) \Rightarrow na_n$

· Special Generating Function

 $-A(rx) \Rightarrow r^n a_n$ 

 $f(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor$ 

 $h(a,b,c,n) = \sum_{i=0}^{n} \lfloor \frac{ai+b}{c} \rfloor^{2}$ 

5.13 生成函數

```
- (1+x)^{n} = \sum_{i\geq 0} {n \choose i} x^{i} 

- \frac{1}{(1-x)^{n}} = \sum_{i\geq 0} {n \choose i-1} x^{i}
```

• Ordinary Generating Function  $A(x) = \sum_{i>0} a_i x^i$ 

```
1 const int N = 2e6 + 5; // [-1e6, 1e6]
int tag[N * 4], seg[N * 4];
3 void pull(int v, int l, int r) {
4 seg[v] = 0;
if(tag[v] > 0) seg[v] = r - 1 + 1;
```

# **String**

#### 7.1 AC

#### 7.2 KMP

```
i vi KMP(const vi& a) {
   int n = sz(a);
   vi k(n);
   for(int i = 1; i < n; ++i) {</pre>
     int j = k[i - 1];
     while(j > 0 && a[i] != a[j]) j = k[j -
          1];
     j += (a[i] == a[j]);
     k[i] = j;
```

```
return k;
```

#### LCP

```
vector<int> lcp(const vector<int>& s, const
      vector<int>& sa) {
   int n = sz(s), h = 0;
   vi rnk(n), lcp(n - 1);
   REP(i, n) rnk[sa[i]] = i;
   REP(i, n) {
     h -= (h > 0);
     if(rnk[i] == 0) continue;
     int j = sa[rnk[i] - 1];
     for(; j + h < n && i + h < n; h++) if(s[</pre>
          j + h] != s[i + h]) break;
     lcp[rnk[i] - 1] = h;
   return lcp;
```

#### manacher

```
ı|// length: (z[i] - (i & 1)) / 2 * 2 + (i &
vi manacher(string t) {
   string s = "&";
   for(char c : t) s.pb(c), s.pb('%');
   int 1 = 0, r = 0;
    vi z(sz(s));
    REP(i, sz(s)) {
     z[i] = r > i ? min(z[2 * 1 - i], r - i)
      while(s[i + z[i]] == s[i - z[i]]) z[i
     if(z[i] + i > r) r = z[i] + 1, l = i;
12
   return z;
```

# rolling-hash

```
| const | 1 M = 911382323, mod = 972663749;
2 11 Get(vector<11>& v, int 1, int r) {
  if(!1) return h[r]; // p[i] = M^i % mod
   ll ans = (h[r] - h[l - 1] * p[r - l + 1])
        % mod;
   return (ans + mod) % mod;
 vector<ll> Hash(string s) {
   vector<ll> ans(sz(s));
   ans[0] = s[0];
   for(int i = 1; i < sz(s); i++) ans[i] = (</pre>
        ans[i - 1] * M + s[i]) % mod;
   return ans;
```

#### **7.6 SAIS**

```
// mississippi
// 10 7 4 1 0 9 8 6 3 5 2
vi SAIS(string a) {
  #define QQ(i, n) for(int i = (n); i >= 0;
      i--)
  int n = sz(a), m = *max_element(all(a)) +
      1;
  vi pos(m + 1), x(m), sa(n), val(n), lms;
  for(auto c : a) pos[c + 1]++;
  REP(i, m) pos[i + 1] += pos[i];
  vector<bool> s(n);
  00(i, n - 2) s[i] = a[i] != a[i + 1] ? a[i]
      ] < a[i + 1] : s[i + 1];
  auto ind = [&](const vi& ls){
    fill(all(sa), -1);
    auto L = [&](int i) { if(i >= 0 && !s[i
         ]) sa[x[a[i]]++] = i; };
    auto S = [\&](int i) \{ if(i >= 0 \&\& s[i]) \}
         sa[--x[a[i]]] = i; };
    REP(i, m) x[i] = pos[i + 1];
    QQ(i, sz(ls) - 1) S(ls[i]);
    REP(i, m) x[i] = pos[i];
    L(n - 1);
    REP(i, n) L(sa[i] - 1);
    REP(i, m) x[i] = pos[i + 1];
    QQ(i, n - 1) S(sa[i] - 1);
  auto ok = [&](int i) { return i == n || (! 32|}
       s[i - 1] && s[i]); };
  auto same = [&](int i,int j) {
     if(a[i++] != a[j++]) return false;
   } while(!ok(i) && !ok(j));
    return ok(i) && ok(j);
  for(int i = 1; i < n; i++) if(ok(i)) lms.
  ind(lms);
  if(sz(lms)) {
    int p = -1, w = 0;
    for(auto v : sa) if(v && ok(v)) {
     if(p != -1 && same(p, v)) w--;
     val[p = v] = w++;
    auto b = lms;
    for(auto& v : b) v = val[v];
    b = SAIS(b);
    for(auto& v : b) v = lms[v];
    ind(b);
  return sa;
```

#### smallest-rotation

SA[p].go[c] = x;

p = SA[p].link;

```
string small_rot(string s) {
   int n = sz(s), i = 0, j = 1;
   s += s;
   while(i < n && j < n) {</pre>
     int k = 0;
     while(k < n \&\& s[i + k] == s[j + k]) k
     if(s[i + k] <= s[j + k]) j += k + 1;
     else i += k + 1;
     if(i == j) j++;
   int ans = i < n ? i : j;</pre>
   return s.substr(ans, n);
```

4 void sa\_init() { SA[0].link = -1, SA[0].len

while(p != -1 && SA[p].go[c] == 0) {

 $if(SA[p].len + 1 == SA[q].len) {$ 

SA[u].first\_pos = SA[u].len = SA[p].len +

return z;

= 0, sz = 1;

SA[p].go[c] = u;

p = SA[p].link;

SA[u].link = 0;

int q = SA[p].go[c];

SA[u].link = q;

SA[x].len = SA[p].len + 1;

SA[q].link = SA[u].link = x;

while(p != -1 && SA[p].go[c] == q) {

int u = sz++;

SA[u].cnt = 1;

**if**(p == -1) {

return u;

return u:

int x = sz++;

SA[x] = SA[q];

SA[x].cnt = 0;

return u;

12

int sa extend(int p, int c) {

#### 7.9 7

```
vi z_algorithm(const vi& a) {
  int n = sz(a);
   vi z(n);
   for(int i = 1, j = 0; i < n; ++i) {</pre>
     if(i \leftarrow j + z[j]) z[i] = min(z[i - j], j
           + z[j] - i);
      while(i + z[i] < n && a[i + z[i]] == a[z
           [i]]) z[i]++;
      if(i + z[i] > j + z[j]) j = i;
```

#### SAM

```
1// cnt 要先用 bfs 往回推, 第一次出現的位置是
      state.first_pos - |S| + 1
2 struct Node { int go[26], len, link, cnt,
      first pos; };
3 Node SA[N]; int sz;
```

ACM ICPC			1.5 treap	2 2	4	<b>Graph</b> 4.1 2-SAT	<b>5</b> 5		歐幾里得類算法 生成函數	8
Team Reference Angry Crow	-		Flow-Matching 2.1 Dinic	<b>2</b> 2		4.2 centroid-tree          4.3 HLD          4.4 lowlink          4.5 SCC	5 5 6 6	6.2 1 6.3 1	default	8 8 8 8
Takes Flight! Contents			2.3       KM	3 4 4 4	5	Math 5.1 Chinese-Remainder 5.2 Discrete-Log 5.3 extgcd 5.4 Floor-Sum 5.5 Miller-Rabin	6 7	6.5 5 7 <b>String</b> 7.1 7.2	矩形覆蓋面積	8 8 8 8
		3	ometry	4		5.6 Pollard-Rho	7		LCP	9
1 Data-Structure1.1 fast-set1.2 lazysegtree1.3 segtree1.4 sparse-table	1 1 1 1 2		3.1 convex-hull	4 5 5 5 5		5.7 Primes          5.8 估計值          5.9 定理          5.10 整數除法          5.11 數字	7 7 7 7 8	7.5 1 7.6 5 7.7 5 7.8 5	rolling-hash	9 9 9 9

# ACM ICPC Judge Test Angry Crow Takes Flight!

#### C++ Resource Test

```
#include <bits/stdc++.h>
using namespace std;

namespace system_test {

const size_t KB = 1024;
const size_t MB = KB * 1024;
const size_t GB = MB * 1024;
```

```
chrono::duration<double> diff = end -
10 size t block size, bound;
                                                          begin;
  void stack size dfs(size t depth = 1) {
                                                     return diff.count():
   if (depth >= bound)
                                                   void runtime error 1() {
    int8_t ptr[block_size]; // 若無法編譯將
                                                     // Segmentation fault
         block size 改成常數
                                                     int *ptr = nullptr;
    memset(ptr, 'a', block_size);
                                                     *(ptr + 7122) = 7122;
    cout << depth << endl;</pre>
                                                 42 }
    stack_size_dfs(depth + 1);
                                                   void runtime_error_2() {
                                                     // Segmentation fault
  void stack_size_and_runtime_error(size_t
                                                     int *ptr = (int *)memset;
       block size, size t bound = 1024) {
                                                     *ptr = 7122;
    system test::block size = block size;
                                                 48 }
    system_test::bound = bound;
    stack size dfs();
                                                   void runtime_error_3() {
                                                     // munmap_chunk(): invalid pointer
                                                     int *ptr = (int *)memset;
  double speed(int iter num) {
                                                     delete ptr;
    const int block_size = 1024;
    volatile int A[block size];
    auto begin = chrono::high resolution clock
                                                   void runtime_error_4() {
         ::now();
                                                     // free(): invalid pointer
    while (iter_num--)
                                                     int *ptr = new int[7122];
      for (int j = 0; j < block_size; ++j)</pre>
                                                     ptr += 1;
                                                     delete[] ptr;
    auto end = chrono::high resolution clock::
```

```
63 void runtime error 5() {
    // maybe illegal instruction
    int a = 7122, b = 0;
    cout << (a / b) << endl;</pre>
  void runtime error 6() {
    // floating point exception
    volatile int a = 7122, b = 0;
    cout << (a / b) << endl;
73 }
  void runtime error 7() {
    // call to abort.
    assert(false);
78 }
  } // namespace system test
82 #include <sys/resource.h>
void print_stack_limit() { // only work in
       Linux
    struct rlimit 1;
    getrlimit(RLIMIT STACK, &1);
    cout << "stack_size = " << l.rlim_cur << "</pre>
          byte" << endl;</pre>
87 }
```