HW 6

Exercise 3. Use the result of Remark 3 to determine the rates of convergence of e in L_2 and H^1 when $m = 1$ and $k = 2$ (i.e., quadratic elements).

$$\|e\|_s \leq c\, h^{\beta} \|u\|_{k+1}$$

$$\beta = \min\left(k+1-s,\ 2(k+1-m)\right)$$

$k=2 \quad m=1$

for e in L_2, $s=0$

$\therefore \beta = \min(3, 4) = 3$

$\therefore \|e\|_0 \leq c\, h^3 \|u\|_3 \rightarrow$ rate of 3

for e in H^1, $s=1$

$\beta = \min(2, 4) = 2$

$\therefore \|e\|_1 \leq c\, h^2 \|u\|_3 \rightarrow$ rate of 2

Exercise 4. Determine the convergence rates of e in L_2, H^1, and H^2 for Bernoulli-Euler beam theory ($m = 2$) and Hermite cubic shape functions ($k = 3$).

$m=2 \quad k=3$

$$\|e\|_s \leq c\, h^{\beta} \|u\|_{k+1} \qquad \beta = \min\left(k+1-s,\ 2(k+1-m)\right)$$

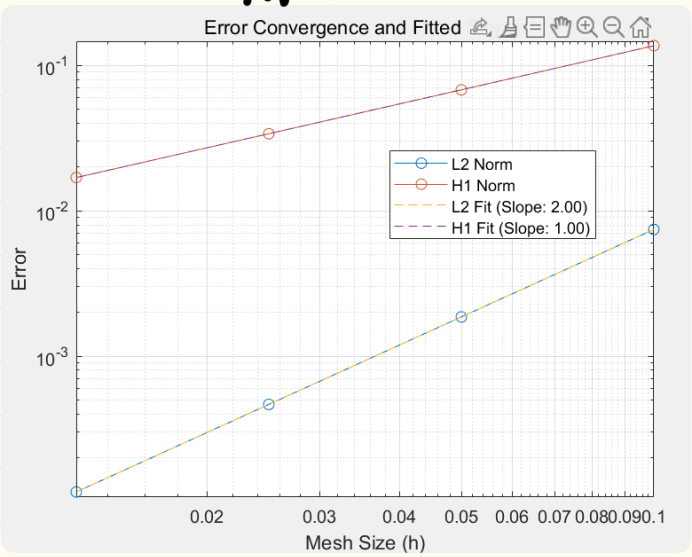
for L_2, $s=0$ $\quad \beta = \min(4, 4) = 4$

for H^1, $s=1$ $\quad \beta = \min(3, 4) = 3$

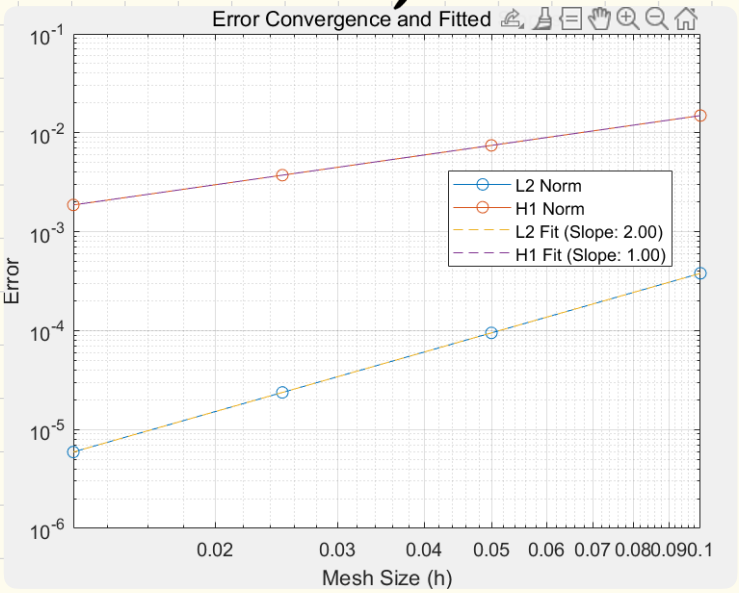
for H^2, $s=2$ $\quad \beta = \min(2, 4) = 2$

二、error：

三角形



四边形



$s=1 \rightarrow p=1$

$s=0 \rightarrow p=2$

合理